The structure of short-range order near a defect in systems with spin-density waves

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We investigate the problem of how short-range order arises near a defect in an itinerant antiferromagnet with a spin-density wave. We analyze the specific properties of the localized states for the case of weak pinning of the phase of the complex order parameter, and discuss the possible role of the short-range order effect in chromium-based alloys.

1. INTRODUCTION

The self-consistent description of electronic and magnetic structures near defects in crystals with charge- or spindensity waves (CDW or SDW) is a very difficult problem. This is due to the fact that not only the magnitude but also the shape of the order parameter $\Delta(\mathbf{r})$ in these systems depends on the defect potential and varies in a complicated fashion in its vicinity.

At the same time, a number of questions can be resolved qualitatively within the framework of various non-self-consistent approaches. The most widely used approximation, i.e., a rigid shape for $\Delta(\mathbf{r})$ and a non-self-consistent defect potential U, is correct in the limit of small values of the potential $|U| \ll |\Delta|$ far from the transition point in the region of the ordered phase (see, e.g., one of the early papers, Ref. 1). For a Peierls system with a CDW the problem of reconstruction of the electron density near a nonmagnetic impurity was investigated in Ref. 2, using the approximation of a rigid shape for $\Delta(\mathbf{r})$ (although with a rather artificial form for the matrix element of the impurity potential), and for impurities with frozen-in magnetic moments in Ref. 3. The problem of redistribution of the spin density in the neighborhood of a magnetic impurity in an antiferromagnet with a SDW was formulated (but not completely solved) in Ref. 4. An investigation analogous to Ref. 2 was undertaken in Ref. 5 for a quasi-one-dimensional system with a SDW.

The general conclusion to be drawn from these papers is that it is impossible in practice to describe the charge and spin redistribution in the neighborhood of a defect analytically without using an approximate rigid form of the order parameter $\Delta(\mathbf{r})$. The applicability of this approximation is not obvious in the general case; however, it is surely incorrect near the phase transition point where $\Delta(\mathbf{r})$ begins to vary rapidly on the scale of the correlation length $\zeta_T \gg a$ in the neighborhood of the defect (a is the radius of influence of the defect potential). If $\Delta(\mathbf{r})$ is small and slowly-varying in the function space, we can use the Ginzburg-Landau expansion for the thermodynamic potential $\Omega(\Delta)$.

In what follows, for concreteness we will investigate a model of a quasi-one-dimensional SDW in a metal with planar "nested" portions of the Fermi surface which coincide when one portion is translated by a vector \mathbf{Q} into the other. We will assume that the width W_{\parallel} of the conduction band is large in the direction \mathbf{Q} compared to its width W_{\perp} in the transverse directions. In this case the motion of electrons near the planar portions of the Fermi surface takes on a quasi-one-dimensional character and to within terms $\sim W_{\perp}/W_{\parallel}$ the order parameter can be represented in the form

 $\Delta(\mathbf{r}) = \varphi(\rho)\Delta(x)$ (where ρ is the transverse coordinate). The applicability of the mean-field approximation, which we will use in what follows, is contingent on the suppression of one-dimensional thermodynamic fluctuations; these are expected to be small for $T_N/W_{\perp} \leq 1$ (T_N is the antiferromagnetic transition temperature). This situation obtains, e.g., in the AF_1 -1Q state for the SDW in chromium (i.e., the octahedral model).

A scheme for calculating the distorted function $\Delta(x)$ near a defect (i.e., the long-wavelength envelope of the SDW) in an itinerant antiferromagnet was considered for this system in Ref. 7, based on a general phenomenological approach.⁶ In this scheme, under certain conditions there can exist a region of nearly antiferromagnetic order near the defect on the scale $\sim \zeta_T$ (i.e., a localized SDW state) even above the volume transition point (in this case the Néel point).

In this paper we discuss some specifics of the formation of a localized state which are connected with the peculiarities of a system with a DSW. The topics of discussion in Secs. 2 and 3 are situations (above and below the volume transition point, respectively) where the phase of the order parameter $\Delta(x) = \Delta_1(x) + i\Delta_2(x)$ of a linearly polarized SDW is weakly pinned, and where the structure of $\Delta(x)$ which forms in the vicinity of an isolated point defect is distorted not only in amplitude distortion, as was discussed in Ref. 7, but also in phase.

The physical meaning of a state with a complex value of $\Delta(x)$ depends on how the spin density associated with $\Delta_1(x)$ and $\Delta_2(x)$ gets redistributed. Thus, in the single-band model with simple halving of the period of the antiferromagnetic structure, $\Delta_1(x)$ corresponds to a SDW with maxima at the lattice points, while $\Delta_2(x)$ represents a SDW with maxima at points midway between the latter. In the two-band model without period-halving $\Delta_2(x)$ describes an ordered distribution of fluxes of spinlike quasiparticles between the different bands, i.e., a state with a spin-current density wave,^{8,9} while $\Delta_1(x)$ describes the interband spin density distribution.

In the concluding Sec. 4 we discuss a number of experiments with ternary alloys of chromium both above and below the Néel point, with a view to confirming our theoretical predictions with regard to the role of short-range order effects.

2. AMPLITUDE AND PHASE DISTORTIONS OF A LOCALIZED SDW ABOVE THE NÉEL POINT IN THE VICINITY OF A NONMAGNETIC DEFECT

A description of localized states above the phase transition point was given on Ref. 7 in terms of a single-parameter functional $\Omega(\Delta_1)$ (the case of a rigid phase function, i.e., $\Delta_1 \neq 0, \Delta_2 \equiv 0$). When we can neglect higher powers of the order parameter Δ and its derivatives in the expansion of the thermodynamic potential Ω (see Ref. 7 for details), the problem coincides to within a change of notation with that of localized superconductivity.¹⁰ In what follows we will analyze a different kind of functional, which in the case of weak phase pinning takes into account the possibility of phase slipping of Δ . Note that the dependence of the thermodynamic potential $\Omega(\Delta)$ on the phase of the order parameter is specific for the model of electron-hole pairing and has no analog in the theory of localized superconductivity.¹⁰ From a microscopic point of view the dependence of Ω on the phase of the complex antiferromagnetic order parameter is connected with the various possible relations between the interelectron interaction potentials in the dispersive and annihilating channels.⁸ In the model of an itinerant antiferromagnet with a SDW wave vector Q equal to half a reciprocal lattice vector of the crystal, G/2, the meaning of the parameters Δ_1 and Δ_2 becomes clear once we take out the rapidly-varying components in the expression for the spin density $S(\mathbf{r})$:

$$S(\mathbf{r}) \approx \varphi(\rho) \left[\Delta_1(x) \cos Qx + \Delta_2(x) \sin(Qx) \right].$$

Thus, Δ_1 and Δ_2 are the slowly-varying envelopes of the spin density components which have antinodes at the lattice points and the midpoints between them, respectively.

In this analysis, for simplicity we will limit ourselves to the region of the itinerant-antiferromagnet phase diagram near the boundary for a transition to the uniform (commensurate) SDW phase, where the higher powers of the functions Δ_1 and Δ_2 and their derivatives do not play an essential role in the Ginzburg-Landau expansion. Then the two-parameter functional $\Omega(\Delta_1, \Delta_2)$ in the absence of the defect has the form

$$\Omega_{0} = \int \left[C_{1}^{(1)} \Delta_{1}^{2} + C_{1}^{(2)} \Delta_{2}^{2} + C_{2} (\Delta_{1}^{\prime 2} + \Delta_{2}^{\prime 2}) + C_{2} (\Delta_{1}^{2} + \Delta_{2}^{2})^{2} \right] dx.$$
(1)

Specific expressions for the coefficients of the functional (1) in terms of the microscopic parameters of the model are described, e.g., in Ref. 7. For us it is important that $C_2 > 0$ hold everywhere in the parameter region under investigation, while we have $C_1^{(1)}$, $C_1^{(2)} \sim T - T_{1,2}^0$ where $T_{1,2}^0$ is the temperature at which the paramagnetic phase is absolutely unstable relative to formation of the bulk structures with $\Delta_1 \neq 0$ and $\Delta_2 \neq 0$. Let us assume that $T_1^0 > T_2^0$ everywhere; the following exact relation holds between $C_1^{(1)}$ and $C_1^{(2)}$:

$$C_1^{(1)} - C_1^{(2)} = \bar{g}_1^{-1} - \bar{g}_2^{-1} < 0,$$

where $\bar{g}_{1,2}$ are the interaction constants corresponding to the structures with $\Delta_1 \neq 0$ and $\Delta_2 \neq 0$ in the SDW model.⁸

Let us consider the (T,μ) phase diagram, where T is the temperature and μ is the noncongruency parameter (i.e., the deviation from half-occupancy in the single-band model). The intersection of the lines $C_1^{(1)}(T,\mu)$ and $C_2(T,\mu)$ gives a Lifshits point (T_1^*, μ^*) , near which the expansion (1) is valid. Let us assume that $|T_1^0 - T_2^0| \ll T_1^0 \sim T_1^*$ holds, i.e., in the absence of the defect a transition should occur to the phase with Δ_1 , although the point where the paramagnetic phase becomes unstable relative to the formation of the Δ_2 phase is close in temperature.

The correction $\Omega_{imp}(\Delta_1, \Delta_2)$ connected with a point source of the order parameter is of "local transition temperature" type; in the microscopic SDW model it is calculated by standard Green's function methods. For a nonmagnetic defect with a short-range potential we have

$$\Omega_{imp}(\Delta_1, \Delta_2) = -\int \left[\frac{\Gamma_1}{2}\Delta_1^2(x) + \frac{\Gamma_2}{2}\Delta_2^2(x)\right] \delta(x) dx. \quad (2)$$

The potentials Γ_1 and Γ_2 are calculated in the Appendix in terms of the parameters of the microscopic SDW model. In the limit of weak electron-impurity scattering, to second order in the potential U we have the important relation:

$$\Gamma_1(U) = \Gamma_2(U) - dU^2, \quad d > 0, \tag{3}$$

i.e., $\Gamma_1 < \Gamma_2$; this relation does not depend on the sign of the potential U (the signs of the potentials Γ_1 and Γ_2 , of course, are directly related to the sign of U). On the other hand, a localized state at the defect can appear above the bulk transition point only for $\Gamma_1 > 0$ and $\Gamma_2 > 0$. It is clear from (2) and (3) that for $\Gamma_2 > 0$ the relation $\Gamma_1 < \Gamma_2$ implies that the tendency to form a localized state with the structure Δ_2 is enhanced even in the presence of a bulk transition to the SDW phase with structure Δ_1 . Consequently, near the defect the bulk order parameter can suffer not only amplitude modulation but also phase slipping at the same time on the macroscopic scale $\sim \zeta_T$. A more rigorous criterion for the formation of one or another type of localized state will be formulated below.

By minimizing the functional $\Omega = \Omega_0 + \Omega_{imp}$,

$$\frac{\delta\Omega(\Delta_1, \Delta_2)}{\delta\Delta_1} = 0, \quad \frac{\delta\Omega(\Delta_1, \Delta_2)}{\delta\Delta_2} = 0, \tag{4}$$

we obtain the following system of equations for the functions $\Delta_1(x)$ and $\Delta_2(x)$:

$$\Delta_{i}'' - (2\Delta^{2} + A_{i})\Delta_{i} = -\frac{\Gamma_{i}}{2C_{2}}\Delta_{i}(0)\delta(x), \qquad (5)$$

$$\Delta_2'' - (2\Delta^2 + A_2)\Delta_2 = -\frac{\Gamma_2}{2C_2}\Delta_2(0)\delta(x), \qquad (6)$$

$$\Delta^{2} = \Delta_{1}^{2} + \Delta_{2}^{2}, \quad A_{1,2} = C_{1}^{(1,2)} / C_{2}.$$
(7)

The region of instability of the paramagnetic phase against the formation of localized states with $\Delta_1 \neq 0$ and $\Delta_2 \neq 0$ for $T > T_1^0$ is determined by the condition that a nontrivial solution exists for Eqs. (5) and (6) linearized with respect to Δ_1 and Δ_2 :

$$4[C_{1,2}(T_{1,2})C_{2}(T_{1,2})]^{\frac{1}{2}} \leq \Gamma_{1,2}.$$
(8)

Let us investigate the temperature region above the temperature T_1^0 for the bulk transition to the uniform SDW state. By virtue of Eqs. (3), (8) a situation is possible in which there is a higher temperature $T_2 > T_1^0 > T_2^0$, determined by the condition

$$4[C_1^{(2)}(T_2)C_2(T_2)]^{\frac{1}{2}}=\Gamma_2,$$

for which short-range order with the Δ_2 structure arises at the defect (i.e., maxima in the spin density at points midway between the lattice points), although the system is unstable against the formation of long-range order of type Δ_1 (i.e., maxima in the spin density at the lattice points). Actually, for $\Gamma_1 > \Gamma_2 > 0$, if $T_1^0 > T_2^0$, Eq. (6) admits a nontrivial solu-



FIG. 1. Spin density distribution in the neighborhood of a point defect: $a - \Delta_1(x) \neq 0$, $\Delta_2(x) = 0$; b- $\Delta_1(x) = 0$, $\Delta_2(x) \neq 0$.

tion for lower temperatures T having the well-known form¹⁰ $\Delta_2(x) = A_2^{1/2} \operatorname{sh}(A_2^{1/2} | x | + \Phi), \quad \operatorname{cth} \Phi = \Gamma_2 / 4 [C_1^{(2)} C_2]^{1/2} \ge 1. (9)$

In this case we have $\Delta_1 \equiv 0$, and at the point where the shortrange order appears, i.e., when $T \rightarrow T_2$, we have

$$\operatorname{cth} \Phi \to 1$$
, $\operatorname{sh} \Phi \to \infty$, $\Delta_2(x) \to 0$.

Let us clarify the physical meaning of the localized state with $\Delta_2(x) \neq 0$. For this state the spin density around a substitutional defect

$$S(x) \sim \Delta_2(x) \sin Qx$$

is redistributed antisymmetrically and its maxima shift toward points midway between the lattice sites (Fig. 1b). Note that in the case of a localized state at the defect with $\Delta_1(x) \neq 0$ the spin density

$$S(x) \sim \Delta_1(x) \cos Qx$$

is redistributed symmetrically (Fig. 1a).

In the direction $\rho \perp \mathbf{Q}$ the scale of localization of the spin density is determined by the "nesting" parameters of the electron spectrum, and for the case of a short-range defect potential it can be verified that the magnitude of this scale is $\sim (T/V_{\perp})^{-1} \ll \zeta_T$ (where V_{\perp} is the transverse velocity at the Fermi surface).

We now discuss how a further localized state can form at the defect and whether a $\Delta_1(x)$ can form on top of a $\Delta_2(x)$ background. Solving a version of Eq. (5) linearized with respect to $\Delta_1(x)$ with the boundary conditions $\Delta_1(\pm \infty) \rightarrow 0$ far from the defect, we obtain

$$\Delta_{1}(x) = \Delta_{1}(0) \exp(-A_{1}^{\frac{1}{2}}|x|) - \frac{\alpha + \operatorname{cth}(A_{2}^{\frac{1}{2}}|x| + \Phi)}{\alpha + \operatorname{cth} \Phi}, \quad (10)$$

$$\alpha = (A_1/A_2)^{1/2}.$$
 (11)

Note that $\Delta_1(x)$ decreases more slowly than $\Delta_2(x)$. By matching (10) smoothly at the coordinate origin, we find the condition for existence of a nontrivial solution $\Delta_1(x)$ against the background $\Delta_2(x)$:

$$4[C_1^{(1)}C_2]^{\frac{1}{2}} \leq \Gamma_1 - (\Gamma_2^2 - 16C_1^{(2)}C_2) / [\Gamma_2 + 4(C_1^{(1)}C_2)^{\frac{1}{2}}].$$
(12)

It is clear from this that above the volume transition point the presence of $\Delta_2(x)$ leads to a decrease in the temperature T_1 at which $\Delta_1(x)$ appears compared to the case where $\Delta_2(x)$ is absent. Recall that our discussion is limited to the conditions $T_1 \ge T_1^0$. The limiting case to which Eq. (12) applies is realized when $T_1 = T_1^0$, i.e., $C_1^{(1)} = 0$, or

$$16C_2C_1^{(2)} \ge \Gamma_2(\Gamma_2 - \Gamma_1). \tag{13}$$

If this relation is not fulfilled, then above the bulk transition point a state with $\Delta_1(x)$ cannot arise.

Now let us suppose that for temperatures $T_1 > T_1^0 > T_2^0$ the state $\Delta_1(x)$ nevertheless does arise; however, we will assume as before $\Gamma_2 > \Gamma_1 > 0$ and $C_1^{(2)} > C_1^{(1)} > 0$. In order to find the temperature T_2 [for $\Delta_2(x)$ to appear superposed on $\Delta_1(x)$] we can use the same Eqs. (9)–(12) as before, with an obvious change of indices; as a final result we obtain

$$4(C_1^{(2)}C_2)^{\frac{1}{2}} \leq \Gamma_2 - (\Gamma_1^2 - 16C_1^{(1)}C_2) / [\Gamma_1 + 4(C_1^{(2)}C_2)^{\frac{1}{2}}].$$
(14)

The applicability of (14) is limited by the requirement $T_2 > T_1^0$, and in the limit $T_2 = T_1^0$ this implies

$$16C_{1}^{(2)}C_{2}/[\Gamma_{1}+4(C_{1}^{(2)}C_{2})^{\frac{1}{2}}] \leq \Gamma_{2}-\Gamma_{1}.$$
(15)

However, if this relation is violated, then as in (13) we once again must investigate the case $T < T_1^0$, i.e., the possibility of a localized state appearing below the bulk transition point.

3. DEVIATIONS IN THE SDW AMPLITUDE AND PHASE IN THE VICINITY OF A NONMAGNETIC DEFECT BELOW THE NÉEL POINT

Let us turn now to the question of how nonuniform structure in the two-component order parameter $\Delta_1 + i\Delta_2$ arises near a point nonmagnetic defect below the transition temperature T_1^0 . Equations (5) and (6) are valid as previously, only now it is necessary to apply different boundary conditions in solving them:

$$x \rightarrow \pm \infty$$
, $\Delta_2(x) \rightarrow 0$, $\Delta_1(x) \rightarrow (|A_1|/2)^{\frac{1}{2}}$, $A_1 < 0$.

Let us first determine how $\Delta_1(x)$ is reconstructed near the defect in the absence of a phase distortion [i.e., $\Delta_2(x) \neq 0$]. The solution to Eq. (5) with a source on the right side is now possible (in contrast to the situation above the Néel point) both for $\Gamma_1 > 0$ and $\Gamma_1 < 0$. Taking into account the newly-introduced boundary conditions, for $\Gamma_1 > 0$ this solution has the form

$$\Delta_{1}(x) = \frac{\Delta_{1}(0)}{\operatorname{cth} \psi_{+}} \operatorname{cth} \left[\left(\frac{|A_{1}|}{2} \right)^{\frac{1}{2}} |x| + \psi_{+} \right], \tag{16}$$

where

$$\operatorname{cth} \psi_{+} = \gamma_{1} + (1 + \gamma_{1}^{2})^{\frac{\gamma_{2}}{2}}, \quad \gamma_{1} = \frac{\Gamma_{1}}{8C_{2}} \left(\frac{2}{|A_{1}|}\right)^{\frac{\gamma_{2}}{2}}, \quad (17)$$
$$\Delta_{1}(0) = \left(\frac{|A_{1}|}{2}\right)^{\frac{\gamma_{2}}{2}} \operatorname{cth} \psi_{+},$$

while for $\Gamma_1 < 0$,

$$\Delta_{i}(x) = \frac{\Delta_{i}(0)}{\operatorname{th} \psi_{-}} \operatorname{th} \left[\left(\frac{|A_{i}|}{2} \right)^{\frac{1}{2}} |x| + \psi_{-} \right], \quad (18)$$



where

th
$$\psi_{-} = (1 + \gamma_{1}^{2})^{\frac{1}{2}} - |\gamma_{1}|, \quad \Delta_{1}(0) = \left(\frac{|A_{1}|}{2}\right)^{\frac{1}{2}} \text{th } \psi_{-}.$$
 (19)

The quantity $\Delta_1(x)$ is thus determined by the sign and magnitude of the dimensionless parameter γ_1 . In Fig. 2 we show the function $\Delta_1(x)$ for $\gamma_1 > 0$ and $\gamma_1 < 0$, respectively. For $\gamma_1 > 0$ an increase in the amplitude $\Delta_1(x)$ occurs in the neighborhood of the defect on a scale $\zeta_T \sim |A_1|^{-1/2}$, i.e., the short-range antiferromagnetic order is enhanced. However, for $\gamma_1 < 0$ the value of $\Delta_1(x)$ in the vicinity of the defect decreases, and for $|\gamma_1| \ge 1$ we have $\Delta_1(0) \ll |A_1|/2)^{1/2}$, i.e., a "hole" forms in the SDW structure and the antiferromagnetic short-range order is disrupted.

We will discuss the question of phase distortion below the temperature T_1^0 only for the case $\Gamma_1 > 0$ and $\Gamma_2 > 0$. Let us assume that a localized state $\Delta_2(x)$ cannot form at the temperature T_1^0 , and that the conditions (14) and (8) are not fulfilled. Then, substituting $\Delta_1(x)$ from (16) and (6) and solving an approximate version of Eq. (6) linearized in $\Delta_2(x)$, we find

$$\Delta_{2}(x) = \Delta_{2}(0) \frac{B + \operatorname{cth}[(|A_{1}|/2)^{\frac{y_{1}}{2}}|x| + \psi_{+}]}{B + \operatorname{cth}\psi_{+}}$$
$$\times \exp[-(|A_{1}| + A_{2})^{\frac{y_{1}}{2}}|x|], \qquad (20)$$

$$B=2(1+A_2/|A_1|)^{\frac{1}{2}}.$$
 (21)

Matching the solution to (20) at the point x = 0, we obtain conditions for instability against the formation of a $\Delta_2(x)$ against a $\Delta_1(x)$ background below the Néel point:

$$2\gamma_2 = B + \frac{\operatorname{cth}^2 \psi_+ - 1}{B + \operatorname{cth} \psi_+}, \quad \gamma_2 = \frac{\Gamma_2}{8C_2} \left(\frac{2}{|A_1|}\right)^{\psi_2}.$$
(22)

Analysis of condition (22) is quite awkward in the general case. However, we can rather easily verify that even in the limit $\Gamma_1 \rightarrow 0$ the instability condition (22) contradicts the original assumption that formation of $\Delta_2(x)$ above T_1^0 is impossible. Thus, for $\Gamma_1 \rightarrow 0$ we have from (22) that

$$\Gamma_2 \ge 4 [C_2(C_1^{(2)} - C_1^{(1)})]^{\frac{1}{2}}, \tag{23}$$

and since $C_1^{(1)} < 0$ holds below T_1^0 the condition (23) necessarily implies that (14) is fulfilled for this Γ_2 . Thus, if a $\Delta_2(x)$ does not appear above T_1^0 , then it cannot appear below this temperature either. The presence of a source for $\Delta_1(x)$ with $\Gamma_1 > 0$ only worsens the situation, as we can verify in analogy with (14).

Finally, let us discuss the case where $\Delta_2(x)$ nevertheless does appear above T_1^0 , condition (8) for Γ_2 is fulfilled, and, what is more, for $T > T_1^0$ a localized state in $\Delta_1(x)$ does FIG. 2. SDW amplitude distribution below the Néel point in the neighborhood of a point defect: $a - \gamma_1(x) > 0$; $b - \gamma_1(x) < 0$.

not arise, i.e., condition (12) is violated. Assuming that below T_1^0 the amplitude $\Delta_2(x)$ is rather large $(|\Delta_2(x)| \ge |\Delta_1(x) - (|A_1|/2)^{1/2}|)$ compared to the local correction to $\Delta_1(x)$ at the defect, we obtain from (6) a solution of type (9) with the replacement $A_1 \rightarrow |A_1| + A_2$, $\Phi \rightarrow \eta$, for which

 $\operatorname{cth} \eta = 2\gamma_2/B. \tag{24}$

Taking into account the microscopic relations for the coefficients $C_1^{(1)}$ and $C_1^{(2)}$ (and correspondingly for A_1 and A_2), it is not difficult to verify that below T_1^0 an abrupt slowing occurs in the variation in the shape and amplitude of $\Delta_2(x)$ (i.e., the sum $|A_1| + A_2$ depends very weakly on temperature, and only through the function $C_2(T)$.

4. CONCLUSION

The effect of short-range order on the thermodynamic properties of itinerant antiferromagnets is apparently detectable in several dilute chromium alloys. In particular, for $\operatorname{Cr}_{1-x} V_x$ alloys with x < 4%, which possess long-range antiferromagnetic order below the Néel temperature $T_N(x)$, the following phenomena have been observed: an unusual growth in the Sommerfeld coefficient $\gamma(x)$ as x decreases in the composition range 4% < x < 10% (Ref. 11), along with an anomalous "tail" in the temperature dependence of the nuclear spin relaxation rate $(T_1T)^{-1}$ for $T > T_N$ and x < 4% (Ref. 12). In addition, the magnetic susceptibility curve $\chi(x,T)$ of $\operatorname{Cr}_{1-x} V_x$ alloys for $T > T_N(x)$ follows the Curie-Weiss law.¹³

All these results can in principle be interpreted in a unified fashion using our model (see, e.g., the discussion of $\chi(x,T)$ in Ref. 14). In fact our picture of short-range magnetic order at fluctuations in composition and the formation of an inhomogeneous spin density distribution is quite close to the picture of thermodynamic spin fluctuations in the statistical approximation to the self-consistent renormalization scheme of Moriya and Kawabata for antiferromagnets¹⁷ used by the authors of Refs. 15 and 16. Of course, in our case the physical reason for the fluctuations themselves is different and is connected with the spatial inhomogeneity of the impurity potential distribution in the alloy. An indirect indication of the correctness of our interpretation of the peculiarities in the properties of $\operatorname{Cr}_{1-x} V_x$ alloys is the absence or smallness of these peculiarities in pure chromium.

Also noteworthy are the investigations of the magnetic susceptibility of $\operatorname{Cr}_{1-x} \operatorname{Rh}_x$ alloys reported in Ref. 18 over a wide range of temperature and composition. In their experiments these authors found that as the rhodium concentration increases, the susceptibility $\chi(x,T)$ in the paramagnetic phase near the Néel point changes character from Pauli-like

to Curie-Weiss-like (for $x \ge 5\%$). In their opinion, this unusual behavior of $\chi(x,T)$ indicates the formation of local moments induced by the rhodium impurities (more precisely, in our view, by composition fluctuations in the $\operatorname{Cr}_{1-x} \operatorname{Rh}_{x}$).

An investigation of the concentration and temperature dependence of $\chi(x,T)$ in the ternary system $\operatorname{Cr}_{1-x-y} V_x \operatorname{Co}_y$ could serve as an indirect indication of the possible existence of static short-range order in itinerant antiferromagnets. It is known that the magnetic moment at a cobalt atom is rather strongly coupled to the antiferromagnetic structure of chromium, ¹⁹ and therefore can serve as a unique "probe" for observing the short-range order for $T > T_N$. It might be expected that the effective magnetic moment induced in a cobalt atom, which we infer from analysis of the function $\chi(x,T)$, will decrease as x increases, because with increasing x a larger and larger fraction of the magnetic impurities will fall into the localized SDWs forming around vanadium-rich regions and be "frozen" into the SDWs. This may perhaps explain the differences previously observed in the curves of $\chi(x,T)$ at x = 0% and x = 77% in the Cr–Co– V alloy system.¹⁹

The behavior of the magnetic moment of iron in dilute alloys of the type $Cr_{1-x}Fe_x$ is quite different. In the ternary system Cr - V + 0.34% Fe, the impurity component of the inverse susceptibility shows almost no change in its linear character: $\chi_{imp}^{-1}(T) \propto T$ both above and below the Néel point. Data on the Mossbauer effect in $Cr_{1-x}Fe_x$ alloys (x = 0.5% to 1.5%) indicates a weak exchange coupling of the iron magnetic moment with the SDW,²⁰ attesting to the small value of the time-averaged effective magnetic field at the Fe⁵⁷ nucleus.

The model we have investigated here of a localized SDW distribution (and specifically its suppression below the Néel point in the neighborhood of a defect with $\Gamma_1 < 0$) allows us to interpret the "free" behavior of Fe spins observed in Ref. 20 (in contrast to Co) in the magnetizing field of the antiferromagnetic Cr matrix without resorting to the assumption that the matrix element for the exchange interaction between the impurity spins and the itinerant electrons is anomalously small; in our model, we need only take into account scattering by the composition fluctuations of the alloy.

The NMR measurements presented in Ref. 21 in $Cr_{1-x}Mo_x$ alloys show that the SDW amplitude at lattice sites occupied by molybdenum atoms, which are isoelectronic to chromium, is approximately three times smaller than at those sites where the Cr are located. This fact also is found to be in qualitative agreement with prediction of our model that SDW at defects below the Néel point are suppressed.

APPENDIX

The coefficients of the functional Ω_{imp} in (4) are expressed in terms of the parameters of the microscopic SDW model in the following way:

$$\begin{split} \frac{\Gamma_{i}}{2} &= \varphi_{i} \left[\frac{\overline{U}}{1 + \overline{U}^{2}} - \frac{\overline{U}^{3}}{(1 + \overline{U}^{2})^{2}} \right] \\ &- \varphi_{0} \left[\frac{\overline{U}^{2}}{1 + \overline{U}^{2}} + \frac{1}{2} \overline{U}^{2} \frac{1 - \overline{U}^{2}}{(1 + \overline{U}^{2})^{2}} \right], \\ \frac{\Gamma_{2}}{2} &= \varphi_{i} \frac{\overline{U}}{1 + \overline{U}^{2}} - \varphi_{0} \frac{\overline{U}^{2}}{1 + \overline{U}^{2}}. \end{split}$$

Here $\overline{U} = 2\pi UN$ is a dimensionless constant for the electron-impurity interaction, N is the density of states at the Fermi surface, and

$$\phi_0 = 2\pi T \sum_n \frac{\omega_n}{\omega_n^2 + \mu^2}, \quad \phi_1 = 2\pi T \sum_n \frac{\mu}{\omega_n^2 + \mu^2}, \\ \omega_n = \pi T(2n+1), \quad n = 0, 1, 2, \dots$$

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