Turbulence in Stokesian flows

G. I. Barenblatt, S. I. Voropaev, I. A. Filippov, and S. I. Chuvil'chikov

P. P. Shirshov Oceanology Institute of the USSR Academy of Sciences
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The possibility that turbulence arises in Stokesian flows is demonstrated both theoretically and experimentally. The nonlinearity necessary for the appearance of turbulence is provided by the mobility of the boundaries. Scaling parameters and criteria for transition to turbulence are discussed.

1. According to classical conceptions the nature of hydrodynamic turbulence is related to two factors: nonlinearity and viscosity. Therefore at a first glance it might seem that turbulence is impossible in Stokesian flows. Indeed, the main source of nonlinearity—inertial forces and the associated quadratic terms in the hydrodynamic equations are, by definition, negligibly small in Stokesian flows.

Such reasoning assumes tacitly an important, but not specified, assumption: that the boundaries of the flow are immobile, or are subject to motion according to a prescribed law. If the boundaries of the flow themselves are moving under the action of the hydrodynamic forces, and their positions are not *a priori* prescribed, a strong nonlinearity appears. Therefore the possibility of a transition to turbulence, i.e., a stochastization of vortex flows, may occur even for the very small Reynolds numbers which are characteristic for Stokesian flows.

2. A typical phenomenon of this type is turbulence in a mixture of a viscous fluid with solid particles, assumed for simplicity to have the same density, for small Reynolds numbers.

Let us imagine (Fig. 1) a shear flow of a fluid with large solid particles suspended in it, defined by the average shear velocity

$$\gamma=\partial_y U. \tag{1}$$

Here $U(y)$ is the mean flow velocity in the longitudinal direction x, and y is the transverse coordinate. As is well known, the shear velocity determines the angular velocity of the rotation of a small particle encompassing the given point. The motion of each individual rigid particle is not prescribed and is determined by the hydrodynamic forces **F** and torques **M**, which are in turn determined by the motion of the fluid and of the surrounding particles. This gives rise to the nonlinearity and we emphasize—for arbitrarily small Reynolds numbers of the flow. It is therefore natural to pose the problem of transition to turbulence (stochastization of the vortex flow) in such a flow, and about criteria for this transition to happen.

The flow regime is determined by the following parameters: the average shear velocity γ of the underlying flow, the average size d of the particles and their concentration n (the relative volume they occupy), the dynamic viscosity of the fluid η, and the latter's density ρ. The density enters among the determining parameters, since the micromotion is essentially nonstationary, and the term $\rho\partial_t \mathbf{u}$ in the fundamental equation of motion is essential. The general size of the region of motion is considered sufficiently large compared to d, but at the same time is sufficiently small so that the global Reynolds number is small.

From the enumerated determining parameters one can form two dimensionless scaling parameters:

$$\Pi_1=\gamma d^2/\nu=\mathrm{Br},\ \Pi_2=n \tag{2}$$

($\nu=\eta/\rho$ is the kinematic viscosity of the fluid). The physical meaning of these parameters is transparent. The quantity d^2/ν characterizes the viscous relaxation time of a particle with respect to the ambient motion. Therefore the parameter Br represents the rotation angle of the particle during the viscous relaxation time. If the rotation angle is small the particles behave with respect to streamlining almost as if they were motionless, and it is normal to expect a laminar regime for the flow. If Br is large, then during the viscous relaxation time the particle undergoes a large angular displacement, entraining the adjacent fluid and deflecting it from the underlying flow. For a sufficiently large concentration of particles, the deflected portion of fluid will rapidly reach the influence region of the next particle and will in turn be deflected by it: a possible stochastization mechanism arises. According to Eq. (2) the criterion for onset of turbulence has the form

$$\mathrm{Br}=\gamma d^2/\nu=\Phi(n), \tag{3}$$

where the function $\Phi(n)$ must exhibit a certain universality. It should be noted that the importance of a parameter which was close in meaning to Br was introduced outside its relation to turbulence by G. K. Batchelor (Ref. 1) when he considered the motion of a sphere in a deformational viscous flow—hence the designation Br.

3. An experimental test of the considerations made above is of interest—first of all a verification whether such turbulence indeed arises in Stokesian flows. For this purpose the following experiment was carried out. An annular basin with exterior diameter $D_1 = 33.8$ cm and interior diameter $D_2 = 23.8$ cm was filled with a two-layered fluid. The fluid in the lower layer of 2 cm thickness was heavy (density

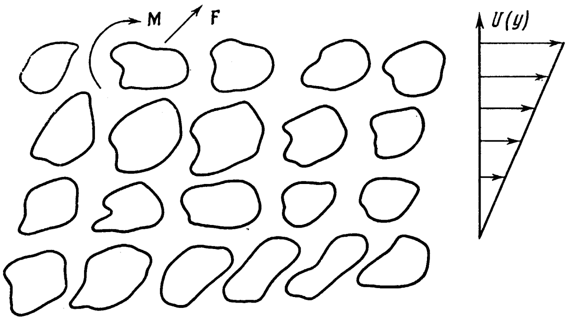

FIG. 1. Shear flow with particles—schematic.

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$\rho = 1.58$ g/cm^3) low-viscosity (kinematic viscosity $\nu = 0.56 \times 10^{-2}$ cm^2/s) carbon tetrachloride. The fluid in the thin (thickness 0.6 cm) upper layer was viscous ($\nu = 410 \times 10^{-2}$ cm^2/s) light ($\rho = 1.25$ g/cm^3) glycerine. The exterior wall of the basin rotates with a prescribed linear velocity U_0 together with the bottom. The interior wall remains at rest. As shown by the experiment, after a sufficiently short time from the start of the rotation, a region with approximately constant shear velocity $\gamma = 2U_0(D_1 - D_2)^{-1}$ is formed. For the speeds U_0 used in the experiment the centrifugal force is small and the level of the fluid remains close to horizontal.

Round rubber disks of diameter d were placed on the surface of the glycerine layer. The density of the disks is insignificantly lower than that of glycerine and they were practically completely submerged in the fluid. The experiment studied quantitatively the evolution of an initially round spot of colored glycerine placed in the flow.

A typical picture observed in the absence of discs ($n = 0$) is shown in Fig. 2. First the round spot (indicated by the arrow in Fig. 2a) is extended into a line by the rotation ($\gamma = 0.56$ s^{-1}), Fig. 2b. As the rotation is reversed the colored liquid gathers again into a round spot (Fig. 2c) just as in the classical experiment of G. B. Taylor. Moreover it returns to the same position where the spot was initially placed. A similar picture of laminar motion was observed when discs of small diameter $d = 0.54$ cm were placed into the flow with low concentration $n = 0.014$ (Br = 0.04), see Fig. 3. However an increase of the size of the disks to $d = 1.26$ cm, of the concentration of discs to $n = 0.44$ and of the shear velocity to $\gamma = 0.76$ s^{-1} (Br = 0.29) sharply changed the observed picture: as shown by the distribution of color, the flow becomes manifestly stochastic. The criterion for the onset of turbulence was considered to be, in accord with the considerations presented above, a splitting of the initially round spot of colored fluid (Fig. 4a,b). When the motion with a split colored region was reversed, the color no longer gathered back into a spot (Fig. 4c), but distributed itself through the cross section of the flow.

The data show, in our opinion that the existence of Stokesian turbulence can be considered as established. It is understood that a detailed quantitative investigation of the function $\Phi(n)$ is necessary.

The phenomenon under consideration may have sever-

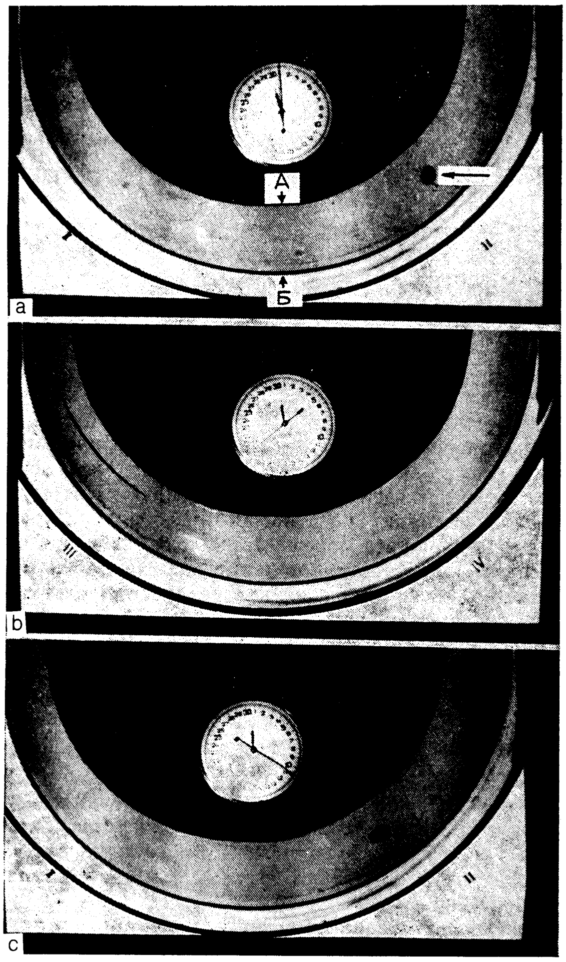

FIG. 2. The evolution of a colored glycerine spot (marked by the arrow) in a shear flow ($\gamma = 0.56$ s^{-1}) without particles ($n = 0$). View from above: a—start of the experiment, b—after a half-rotation at the instant of start of the reversion; c—return of the spot to its previous position. The inner cylinder (A) is at rest, the outer cylinder (cyrillic B) is rotating, carrying out a half-rotation clockwise, followed by a half-rotation counterclockwise. The Roman numerals in the corners label the sectors of the platform which is comoving with the cylinder.

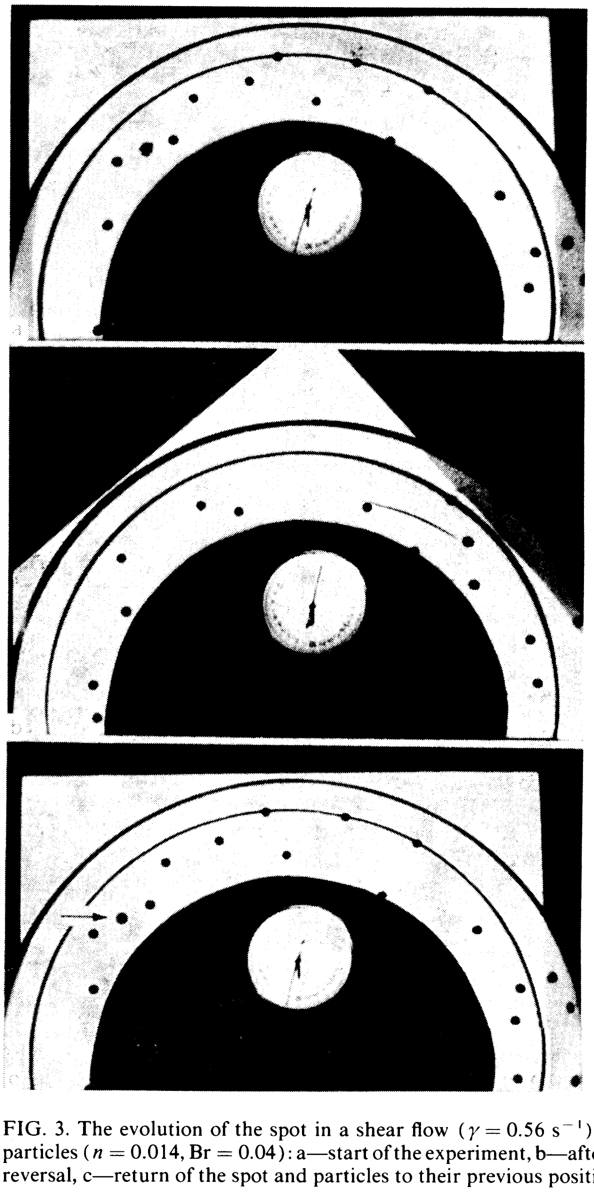

FIG. 3. The evolution of the spot in a shear flow ($\gamma = 0.56$ s^{-1}) with particles ($n = 0.014$, Br $= 0.04$): a—start of the experiment, b—after the reversal, c—return of the spot and particles to their previous positions.

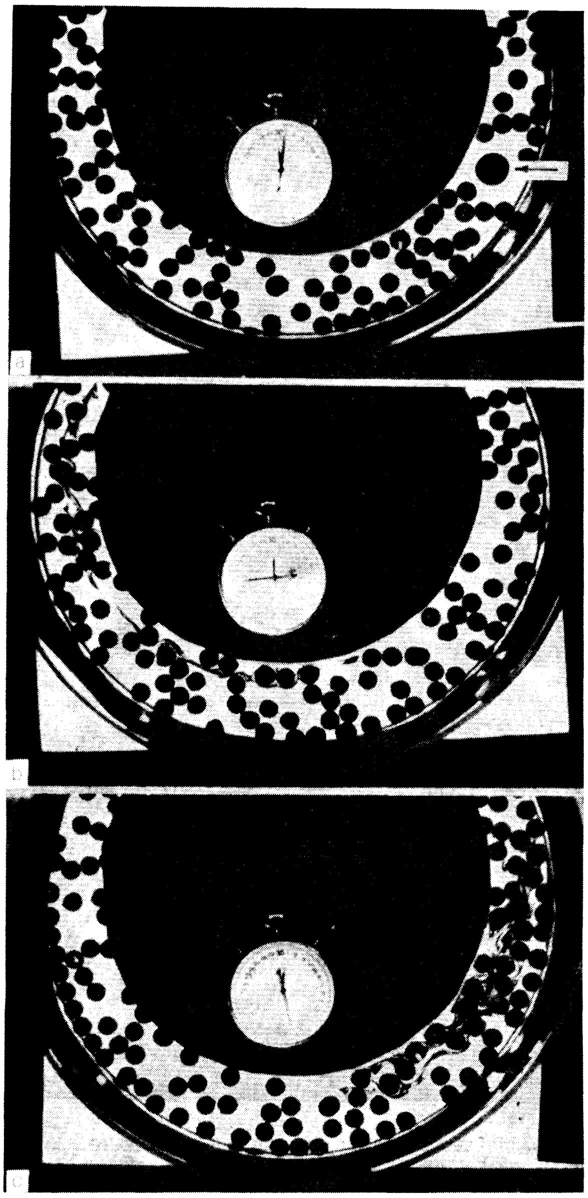

FIG. 4. The evolution of the spot in a shear flow ($\gamma = 0.76\ s^{-1}$) with particles ($n = 0.44$, Br $= 0.29$): a—start of the experiment, b—after 3/4 turns at the start of the reverse motion, c—stochastization of the colored liquid when the external cylinder returns to its initial position.

al applications. We note two possible geophysical ones.

1. Turbulent motions in the lithosphere. The motion of lithospheric plates and their fragments in a medium which is modeled by geologists as a liquid of large viscosity, takes place at very small Reynolds numbers. However, the concentration is large in this case, and the sizes of the plates and fragments is also significant, so that the conditions for the onset of turbulence may be realized. In the opinion of V. I. Keĭlis-Borok it seems possible that seismicity may be related to the transition to Stokesian turbulence.

2. The turbulentization of the motion of an ensemble of coherent structures in the layer of the ocean adjacent to the surface. The discovery of mushroom-like structures in the surface layer of the ocean (Fedorov structures), namely, long-lived formations which do not get destroyed by collisions, was an important event in physical oceanology. The mechanism of formation and evolution of individual mushroom-like structures under the influence of localized disturbances has been investigated in detail under natural conditions by means of space photographs (Ref. 3), and in laboratory experiments (Refs. 4 and 5). It was shown in Ref. 6 that the mushroom-like structures in natural and laboratory conditions can be adequately described in the Stokesian approximation. The next natural step is the creation of models of an ensemble of mushroom-like structures. It seems that transition to turbulence of the type described above in such ensembles is a natural phenomenon: the mushroom-like structures here play the same role that the discs played in the experiment, and the capture of fluid by the mushroom-like structures must additionally facilitate stochastization.

In conclusion we note that the flow considered here differs substantially from so-called chaotic advection in the flow of an ideal fluid (Ref. 7) and from Lagrangian turbulence in Stokesian flow (Ref. 8). Indeed, in our case there is a direct nonlinearity for arbitrarily small Reynolds numbers, related to moving boundaries of the flow region. At the same time the hydrodynamic systems described in Refs. 7 and 8, are linear, and the stochastization occurs on account of nonstationarity.

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