Gravitational wave pulses with "velocity-coded memory"

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We prove the existence in principle of gravitational wave pulses whose passage can leave two initially stationary test particles with nonvanishing final velocities. We show that one possible source of such a burst with "velocity-coded memory" might be the flight of an attracting body through some sort of gravitationally enhanced configuration—a star passing through the accretion disk surrounding a black hole, for example. We examine the physical properties of pulses with velocity-coded memory, and furnish numerical estimates of the expected experimental effect.

1. INTRODUCTION

Astrophysical sources of gravitational radiation are many and varied, and they yield signals that differ in amplitude, duration, time history, and frequency of occurrence. The methods employed to detect them therefore span a very wide range (recent reviews may be found in Refs. 1 and 2). In view of the exceptional difficulties of detecting gravitational waves experimentally, one must necessarily start out with as complete a theoretical picture as possible of the properties that the gravitational signals might possibly have. It may turn out in practice, for example, that rather than the strongest, the most likely signals to be detected are comparatively weak (but relatively more frequent) gravitational wave bursts or bursts displaying some particular time profile.

Sources of gravitational radiation associated with cosmic catastrophes are most promising from the standpoint of detectability. Far from the source, a passing pulse of radiation induces a momentary change in separation between a pair of free bodies. These bodies might, for instance, by the mirrors of a laser interferometer employed in a gravitational wave detection experiment. After a conventional pulse has departed, the test objects return to the positions they occupied prior to the pulse's arrival.

It has been shown elsewhere, however, that there exist certain unconventional pulses, so-called "pulses with memory," or more precisely, pulses with position-coded memory. After such a pulse has passed, free test particles remain slightly displaced from their original positions; in other words, their new positions record the fact of pulse passage. Detailed theoretical descriptions of pulses with position-coded memory and their experimental advantages are given in Refs. 3–7.

It has been pointed out[1,5] that in addition to gravitational wave pulses possessing position-coded memory, there should exist another even more exotic type of pulse, namely one with velocity-coded memory. The passage of such a pulse would leave a set of free test particles with nonvanishing, constant relative velocities. Pulses with velocity-coded memory have another experimental advantage, the long-term progressive change in separation of a pair of free test particles. Following the passage of such a pulse, propagating for example in the x^1 direction and linearly polarized, the distance between free particles systematically increases in another direction (say along the x^2 axis), and it systematically decreases in the third (along the x^3 axis). We would be justified in saying that all this takes place right after the pulse has passed, since the curvature tensor is then zero to first order, and spacetime is flat to the same approximation.

In the present paper, we examine in some detail the properties of pulses with velocity-coded memory, studying their spectral characteristics, comparing them with analogous pulses of electromagnetic radiation, suggesting specific astrophysical sources, and providing numerical estimates.

2. PULSES WITH VELOCITY-CODED MEMORY: GENERAL PROPERTIES

We rcall the form taken by the metric of a weak gravitational plane wave propagating in the x^1-direction:

$$ds^2=c^2dt^2-(dx^1)^2-(1-a)(dx^2)^2-(1+a)(dx^3)^2+2b\,dx^2dx^3,$$
$$a=a(u),\quad b=b(u),\quad u=x^0-x'. \tag{1}$$

Let us examine a finite-duration pulse—that is, we assume that to first order in the small quantities $a(u)$, $b(u)$, the curvature tensor is nonzero only in a region $u_1\leqslant u\leqslant u_2$. Prior to the arrival of the pulse $(u\leqslant u_2)$, we may take $a(u)=0$ and $b(u)=0$. After the pulse has passed $(u>u_2)$, spacetime is again flat, and the functions $a(u)$ and $b(u)$ can therefore only have the general form

$$a(u)=a_1u+a_2,\quad b(u)=b_1u+b_2. \tag{2}$$

As noted in Ref. 5, the combination of nonzero a_2, b_2 with $a_1=0$, $b_1=0$ specifies a pulse with position-coded memory, while nonzero a_1, b_1 yields a pulse with velocity-coded memory. The functions $a(u)$ and $b(u)$ constitute a special case of the weak wave field described by the corrections $h_{\mu\nu}$ to the Minkowski metric $\eta_{\mu\nu}$:

$$ds^2=(\eta_{\mu\nu}+h_{\mu\nu})dx^\mu dx^\nu. \tag{3}$$

The actual form of the wave quantities $h_{\mu\nu}$ is dictated by the properties of the radiation source. The quadrupole approximation far from the source[8,9] gives

$$h_{ik}=-\frac{2G}{3c^4r}\ddot{D}_{ik}\left(t-\frac{r}{c}\right), \tag{4}$$

where r is the distance from the source and D_{ik} is the reduced quadrupole moment. A pulse with position-coded memory will be produced by a source for which the asymptotic values of the $\ddot{D}_{ik}$ (as $t\to\pm\infty$) are not identical. Likewise, a pulse with velocity-coded memory is produced by a source with non-identical asymptotic values (as $t\to\pm\infty$) of $\dddot{D}_{ik}$. The

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initial values of h_{ik} and $\dot{h}_{ik}$ (as $t \to -\infty$) may be assumed to vanish.

At the observation point, the passage of a pulse with position-coded memory is signaled by the fact that a typical component of the wave field $h(t)$ changes from its initial vanishing value to some constant, finite nonzero value. Similarly, the passage of a pulse with velocity-coded memory shows up through a change in $\dot{h}(t)$ from its initial zero value to some constant, finite nonzero value. Both types of pulses may be said to have infinite memory. Admittedly, we are dealing with the linear approximation to h, and are ignoring higher-order corrections. In general, the latter render such infinite-memory pulses impossible, as the approximate expressions (1) and (2) lose their validity. Specifically, in the quadratic approximation to h, the focusing effect of the energy transport by the gravitational wave field comes into play, and the behavior of the test particles no longer corresponds to (1) and (2). In practice, however, this effect is absolutely negligible: the important point is that the values of h and $\dot{h}$ can remain sensibly constant for a very long time.

We now return to a more detailed consideration of the characteristics of pulses with velocity-coded memory. The typical behavior of $\dot{h}(t)$ has been plotted in Fig. 1.

The time T over which the value of $\dot{h}$ holds constant is much greater than the duration Δt of the pulse. Recall that the linearized curvature tensor is nonzero over the interval Δt and zero over the interval T. The actual value of T is determined by the domain of applicability of the linear approximation, or more likely (and more realistically) by the nature of motion within the source itself. We shall be dealing with just such sources below. In any event, we are interested here in systems for which T exceeds any reasonable signal-observation time τ.

It is well known that a pulse with position-coded memory is radiated in a noncentral collision (flyby) of a pair of particles,[3] a phenomenon that is easy to apprehend qualitatively. At $t = -\infty$, let the two bodies be approaching one another in the x direction with some nonzero impact parameter. The component D_{xy} of the quadrupole moment of the mass m, which moves in the $x-y$ plane, is

$$D_{xy}=mx(t)y(t),$$

whereupon

$$\ddot{D}_{xy}=m(\ddot{x}y+2\dot{x}\dot{y}+x\ddot{y}).$$

Both before and after the collision, i.e., at $t = \pm\infty$, we have $\ddot{x}=0$, $\ddot{y}=0$. But for both of the bodies, $\dot{y}=0$ at $t=-\infty$ and $\dot{y}\neq 0$ at $t=+\infty$. Thus, we have for the component D_{xy} of the whole system

$$\ddot{D}_{xy}(t=-\infty)\neq\ddot{D}_{xy}(t=+\infty).$$

The amplitude of the pulse produced in this way is determined by the jump in the value of $\dot{y}$, or in other words by the jump in the y-component of the velocity.

Similarly, it can be shown that maintaining a condition such as

$$\dddot{D}_{xx}(t=-\infty)\neq\dddot{D}_{xx}(t=+\infty),$$

requires a jump in the acceleration $\ddot{x}$ (with the reasonable assumption that $\ddot{x}=0$ at $t=\pm\infty$). In fact,

$$\dddot{D}_{xx}=2m(\dddot{x}x+3\dot{x}\ddot{x}). \qquad (5)$$

One example of a system in which a jump in acceleration is possible would be the flight of a body through a gravitational system with enhanced density. The acceleration of the body below the attracting plane would have the same magnitude as above, but it would have the opposite sign. The velocity of the body after the traversal would be the same as it was before.

Consider a disk of thickness d and diameter $B(d \ll B)$, with surface mass density σ. Let an object of mass m move at velocity v perpendicular to the disk (Fig. 2). At distances x from the disk that are not too large ($x \ll B$), the gravitational potential of the disk is $\varphi \approx 2\pi G\sigma x$, and the resulting gravitational acceleration is $\ddot{x} \approx 2\pi G\sigma$. The moving object will pass through this x-region in a time $T \lesssim B/v$. The object will either traverse the disk (thickness d) or collide with it in a much shorter time: $\Delta t \approx d/v \ll T$. For this type of motion of a mass within the source, we obtain a gravitational pulse with velocity-coded memory. The time-dependent of $\dot{h}$ resembles that shown in Fig. 1.

Let us estimate the value of $\dot{h}$; since

$$\dot{h}\approx 2G\dddot{D}/3c^4r, \text{ a } \dddot{D}\approx 2m\dot{x}\ddot{x}\approx 4\pi m\dot{x}G\sigma,$$

we have

$$\Delta\dot{h}\approx\frac{8\pi r_g}{3r}\frac{Gv\sigma}{c^2},$$

where $r_g = 2Gm/c^2$ is the gravitational radius of the object

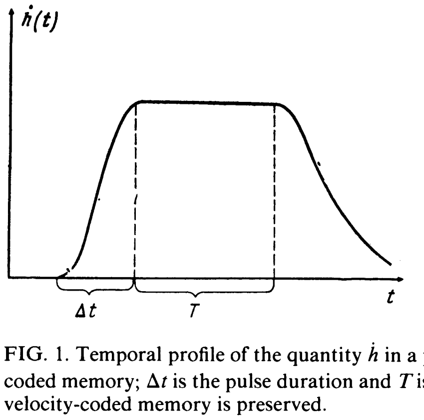

FIG. 1. Temporal profile of the quantity $\dot{h}$ in a pulse leading to velocity-coded memory; Δt is the pulse duration and T is the time over which the velocity-coded memory is preserved.

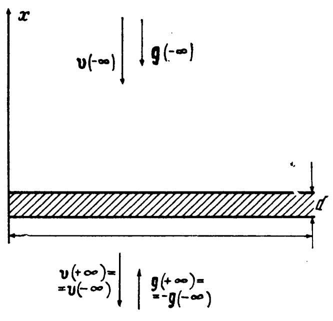

FIG. 2. Passage of a gravitating point object through a density-enhanced gravitating system, generating a pulse with velocity-coded memory: $D_{xx}(-B/v) \propto g(-\infty)v(-\infty) = -g(+\infty)v(+\infty) \propto -D_{xx} \times (+B/v)$.

of mass m. The value of $\dot{h}$ remains essentially constant over the time T. The magnitude of h for the most rapidly varying part of the pulse is typically $h \sim \dot{h}\Delta t \sim \dot{h}d/v$; i.e.,

$$h \sim \frac{r_g}{r}\frac{v_{\rm eff}}{c^2} \tag{6}$$

where $v_{\rm eff} = (\frac{8}{3}\pi G\sigma d)^{1/2}$. The quantity Δt determines the characteristic frequency of the pulses, $\nu \sim \Delta t^{-1}$.

Experimentally, the advantage of a pulse with velocity-coded memory is that the distance ratio $\Delta l/l$ for a pair of test particles is determined not just by the quantity h as given by (6), but also by the long interaction time (or observation time) τ of the pulse; since $(\Delta l)^{\dot{}} \sim \dot{h}l$, we have a time τ that $\Delta l(\tau) \sim (\Delta l)^{\dot{}}\,\tau \sim \dot{h}/\tau$, and

$$\frac{\Delta l}{l}(\tau) \sim h\frac{\tau}{\Delta t}, \qquad \Delta t \ll \tau \ll T. \tag{7}$$

The implication of (7) is that the relative change in length induced by interaction with a pulse having characteristic frequency $\nu \sim \Delta t^{-1}$ and amplitude h at that frequency, $h|_{\nu \approx \Delta t^{-1}}$, increases by a factor of N, where $N = \tau/\Delta t$ is the typical number of Δt pulse periods contributing within an overall observation time τ. Another way to interpret Eq. (7) would be to say that $\Delta l/l$ is the result of a simple interaction with a pulse having a characteristic frequency $\nu = \tau^{-1}$ and amplitude $h|_{\nu \approx \Delta t^{-1}} \approx h|_{\nu = \tau^{-1}}N$. The latter interpretation is consistent with the spectral representation of the pulse shown in Fig. 1 (see below). Substituting (6) into (7), we can rewrite (7) in the form

$$\frac{\Delta l}{l}(\tau) \sim \frac{r_g}{r}\varepsilon\frac{\tau}{T}, \tag{8}$$

where $M = \pi\sigma B^2$ is the mass of the disk, $R_g = 2GM/c^2$ is its gravitational radius, and $\varepsilon = R_g/B$ is the "relativistic" parameter of the disk. We see from (8) that even with the maximum possible signal integration time $\tau = T$, the expected value of $\delta l/l$ is a factor of ε smaller than what is attainable in principle when an object of mass m collides with a massive black hole that has a gravitational radius R_g. Under actual astrophysical conditions, however, a collision between stars with disk systems is a much more likely (and common) occurrence than a collision with a black hole.

We now consider an example—a massive disk surrounded by a dense star cluster. In principle, such a system might exist at the center of the Milky Way. We take the mass and diameter of the disk to be $M \approx 10^5\,M_\odot$ and $B \approx 10^{-3}$ pc $\approx 3\times10^{15}$ cm, so that $\varepsilon \approx 10^{-5}$. We assume that the disk is located at the center of a dense globular cluster with mass $M_{\rm cl} \approx 10^6\,M_\odot$ and cluster radius $R_{\rm cl} \approx 10^{-1}$ pc. The rms stellar velocity within the cluster is $v \approx 10^{-3}$ c. Some star will cross the plane of the disk an average of once every three years, radiating a pulse with velocity-coded memory in the process. The duration of the memory segment is $T \approx B/v \approx 3$ yr. Plugging these parameters into (8), we obtain

$$\frac{\Delta l}{l}(\tau) \approx 10^{-17}\cdot 10^{-5}\frac{\tau}{T}, \tag{9}$$

and for $\tau \approx 1$ yr, we have $\Delta l/l \approx 3\times10^{-23}$.

This is not an overly encouraging result, but it does indicate that such events might be observable with improved laser interferometers (see Refs. 1, 2). Somewhat farther afield, the presumed structure of certain active galactic nuclei implies larger expected values of $\Delta l/l$, despite the distances to those sources (see Sec. 3).

We now move on to a spectral description of pulses with velocity-coded memory. It is not hard to show that the temporal profile of the pulses under consideration (Fig. 1) corresponds to a Fourier expansion

$$(\dot{h})_\omega = \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty} dt\,\dot{h}(t)e^{i\omega t} \tag{10}$$

(here $\omega = 2\pi\nu$), with the low-frequency asymptotic behavior

$$(\dot{h})_\omega \approx \frac{\Delta\dot{h}}{\sqrt{2\pi}}\frac{e^{i\omega T}-1}{i\omega}, \qquad \omega\Delta t \ll 1, \tag{11}$$

which does not depend on the actual profile of the jump itself, but only on the "storage" time T and the height of the jump $\Delta\dot{h} = \dot{h}(\Delta t) - \dot{h}(-\infty)$ (the jump is assumed to take place at $t = 0$). Thus, at frequencies $\omega \leqslant 1/\Delta t$, we have $(\dot{h})_\omega \sim \omega^{-1}$ and $h_\omega \sim \omega^{-2}$; that is, the dimensionless Fourier amplitude $h(\omega) \equiv h_\omega\omega$ takes the form $h(\omega) \sim \omega^{-1}$ (and $h(\nu) \sim \nu^{-1}$), as previously indicated in the discussion of Eq. (7).

The asymptotic high-frequency behavior ($\omega\Delta t \gg 1$) involves an exponential falloff of amplitude with frequency, and depends heavily on the temporal profile of the jump; nevertheless, the asymptotic behavior makes only a minor contribution to the quantities of interest.

It is intriguing to compare the effects of velocity-coded memory for the gravitational and electromagnetic cases. To do so, we first examine the influence of a burst of electromagnetic radiation on a free charged particle. The particle acceleration is

$$a = eE/m, \tag{12}$$

so the velocity jump Δv is proportional to the potential difference:

$$\Delta v = \int_{-\infty}^{\infty} a(t)\,dt = \frac{e}{m}\int_{-\infty}^{\infty} E(t)\,dt = \frac{e}{m}\Delta A. \tag{13}$$

A nonzero jump in velocity is therefore obtained from an electromagnetic pulse localized in time, and having different asymptotic values of the potential A. There would seem to be an exploitable distinction between the effects of velocity-coded memory in the electromagnetic and gravitational cases, since in the latter instance it is the quantity h that takes on the role of the potential A. A difference in asymptotic values of h would induce a position-coded memory effect, while velocity-coded memory requires a difference in the values of $\dot{h}$. But the analogy is completely retrievable if we bear in mind that (12) involves an absolute acceleration—i.e., the acceleration of a charged particle relative to a stationary neutral one. In the gravitational case, on the other hand, we are dealing with the relative acceleration of two test particles, inasmuch as there is no such thing as a gravitationally neutral particle. If we consider the relative acceleration of two charged test particles, we find that they behave in exactly the same way as in the gravitational case. Explicitly, the relative acceleration is given by

$$a_{\rm rel}\approx\frac{e}{m}(\nabla_\perp E)\,l\approx\dot{E}l\approx\ddot{A}l. \qquad (14)$$

Consequently, velocity-coded memory is governed by the asymptotic values of $\dot{A}$, as in the gravitational case, where it is the asymptotic values of $\dot{h}$ that are important.

3. SOURCES OF GRAVITATIONAL WAVE PULSES WITH VELOCITY-CODED MEMORY

The passage of a star through the thin accretion disk surrounding a massive black hole at the center of a dense star cluster could serve as an actual astrophysical source of a gravitational wave pulse with velocity-coded memory. Models of this sort have been extensively discussed in the literature (see Ref. 10, for example) as they relate to the observed properties of active galactic nuclei and quasars.

In analyzing the feasibility of detecting such pulses it is important, as noted in the Introduction, to assess not only the magnitude of the effect itself, but also how frequently such events might actually take place. For a rough estimate, we may employ the following simple model. A massive black hole of mass $M_{\rm BH}=10^7 M_\odot m_{\rm BH}$ is surrounded by a disk of mass $M_D=10^5 M_\odot m_D$ (by the mass of the disk we mean the mass of that part of the disk which may be considered thin). It is well known that $M_D \ll M_{\rm BH}$.[11-13] We shall assume that under realistic conditions, $M_D\approx 0.01 M_{\rm BH}$, i.e., $m_D\approx m_{\rm BH}$. The diameter of the indicated portion of the disk is $B\approx R_g/\varepsilon$, where R_g and ε were introduced in Sec. 2. Let the density of the star cluster surrounding the black hole be $N_s=10^7\,n$ pc^{-3}. Data pertaining to both the Milky Way[14] and active galactic nuclei[15-17] suggest that $n\sim 1$. A typical stellar mass is $M_s=M_\odot m_s$, so we may provisionally take $m_s\approx 1$. We shall assume that the velocity of the star as it traverses the accretion disk, $V=v\times 300$ km/sec, is of the order of that given by the virial theorem, so $v\sim 1$.

Then to order of magnitude, the time between successive stellar traversals of the accretion disk is

$$\tau_*\approx(\pi B^2NV)^{-1}\approx 10^{12}\varepsilon^2(m_D{}^2nv)^{-1}\ \text{yr}. \qquad (15)$$

The effect itself (Eq. (8)) is equal to

$$\frac{\Delta l}{l}\approx\frac{r_g}{r}\,\varepsilon\min\left\{1,\frac{\tau}{T}\right\}\approx 10^{-17}m_s\left(\frac{r}{10\ \text{kpc}}\right)^{-1}\varepsilon. \qquad (16)$$

In Eq. (16) we have already put $\tau\gtrsim T\approx B/V$.

If we are interested in just one individual cluster, situated for example at the center of the Milky Way, then the typical time $\hat\tau$ between successive pulses of the type described as they arrive at the earth is obviously τ_*. Choosing a baseline of one astronomical unit between test objects, as will presumably be feasible in spaceborne experiments in the near future, and stipulating that $\hat\tau\lesssim 1$ yr, we find from (15) that

$$\varepsilon<10^{-6}\ m_D(nv)^{1/2} \qquad (17)$$

and

$$\Delta l\,[\text{cm}]\leqslant 10^{-10}\ m_D(nv)^{1/2}. \qquad (18)$$

In choosing parameters $m_D\sim v\sim 1$, $v\sim 10$, we have returned to the estimates of Sec. 2. The effect given by (18) is very small, and is most unlikely to be measurable using Doppler shifts in the near future.

Consider now the possibility of detecting pulses with velocity-coded memory from the nuclei of distant galaxies and quasars, generated by the same mechanism—the passage of a star through an accretion disk. Here we lose out in terms of distance, but there is some hope of recovery: there is a gain due first to the relativistic factor ε, and due second to the density of the star cluster and the mass $m_{\rm BH}$ of the black hole under the exotic conditions to be found in active nuclei. Furthermore, there is also some hope in reducing the time $\hat\tau$ as a result of the large number of objects $N_0=\frac{4}{3}\pi r^3 N_g$, where N_g is the density of objects with active nuclei. However, those objects comprise no more than 1% of all galaxies,[18] i.e., $N_g\lesssim 10^{-3}$ Mpc^{-3}. Therefore, requiring that $N_0>1$, we obtain for the size of the region to be considered

$$R>(3/4\pi N_g)^{1/3}\gtrsim 10\ \text{Mpc},$$

i.e., the reduction in the observed effect due to distance, keeping all other parameters the same, is at least three orders of magnitude as compared with the center of the Milky Way. If there were a great many systems, the mean time between successive pulses at the earth would be reduced. In that case,

$$\hat\tau=\tau_*/N_0=10^{14}\varepsilon^2(m_D{}^2nv)^{-1}\left(\frac{r}{10\ \text{Mpc}}\right)^{-3}. \qquad (19)$$

If we require that $\hat\tau\lesssim 1$ yr, just as we did for the center of the Milky Way, then (19) yields

$$\varepsilon<10^{-8}m_D(nv)^{1/2}(r/10\ \text{Mpc})^{3/2} \qquad (20)$$

and

$$\Delta l\ [\text{cm}]\leqslant 10^{-13}m_D(nv)^{1/2}(r/10\ \text{Mpc})^{1/2}. \qquad (21)$$

The conditions in active galactic nuclei and quasars are such that the parameters m_D, n, and v may be substantially greater than unity; even in that case, however, the effect is quite small. Without going into the issue of how to obtain some proposed value of the parameter ε, Eqs. (20) and (21) give, for r comparable with the Hubble radius R_H (that is, $r\approx R_H\approx 10^4$ Mpc),

$$\varepsilon\leqslant 3\cdot 10^{-2}\ m_D(nv)^{1/2} \qquad (22)$$

and

$$\Delta l\ [\text{cm}]\leqslant 3\cdot 10^{-12}\ m_D(nv)^{1/2}, \qquad (23)$$

which points up the immense difficulty of detecting such pulses.

In conclusion, we remark that although the velocity-coded memory effect due to the astrophysical sources that we have considered is quite small, the effect may still be of interest, at least in principle. Pulses with velocity-coded memory are clearly distinctive in their effects on test bodies, a circumstance that may prove useful experimentally.

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