Theory of resonance saturation of IR absorption in semiconductors with degenerate resonance bands in electric and magnetic fields

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The influence of a weak electric and/or magnetic field on the coefficient of nonlinear IR absorption in semiconductors with degenerate valence bands (such as *p*-Ge) is considered. The absorption is due to vertical transitions of holes between valence-band subbands. At sufficiently high intensities this absorption saturates, decreasing with increase of intensity, owing to equalization of the distribution functions of the heavy and light holes at the vertical-transition point. It is shown that in the nonlinear region even a weak electric or magnetic field applied to the semiconductor increases the absorption coefficient (in proportion to the applied field) up to its linear value. There are two modes of resonance saturation of the absorption in an external field. The first is coherent, takes place at high IR intensities, and is accompanied by inversion of the distribution function in the resonance region. The second is incoherent, occurs in weaker fields and accordingly at lower radiation intensities. It is shown that in both cases the nonlinear-absorption intensity threshold is determined by one and the same dependence on the field (it is proportional to the field strength), and the absorption in the nonlinear region at intensities above threshold is inversely proportional to the IR intensity. It is emphasized that an important role can be played in nonlinear absorption even by low densities of the charged impurities in the semiconductor, before they can influence transport phenomena substantially.

1. INTRODUCTION

It is known that, at not too low frequencies ω, IR absorption in semiconductors having degenerate valence bands (such as *p*-Ge) is due to vertical transitions of holes between subbands of the valence band (Fig. 1). With increased IR emission intensity, this absorption becomes nonlinear and decreases (saturates), owing to equalization of the distribution functions of the light and heavy holes at the vertical-transition point. This decrease of the absorption coefficient was observed at sufficiently high intensities in a large number of experiments on various semiconductors and in the range from nitrogen to room temperature.[1–14] So far, however, there is no meeting of minds concerning the nonlinearity mechanism. Two such mechanisms exist. The first can be called resonance saturation of the absorption. It is due to equalization of the distribution functions of the heavy and light holes in a relatively narrow region of momentum space near the equal-energy surface

$$\varepsilon_{1\mathbf{p}}-\varepsilon_{2\mathbf{p}}-\hbar\omega=0, \qquad (1)$$

corresponding to the resonance conditions. Here $\varepsilon_{1\mathbf{p}}$ and $\varepsilon_{2\mathbf{p}}$ are respectively the energies of a heavy and light hole with momentum $\mathbf{p}$. Each hole with momentum $\mathbf{p}$ can then be regarded as a two-level system with energy (distance between levels) $\varepsilon_{1\mathbf{p}}-\varepsilon_{2\mathbf{p}}$. An ensemble of holes in the semiconductor is then equivalent to an ensemble of independent two-level systems with a state density proportional to the square root of the energy (for a parabolic dispersion law). It can be regarded as a constant near the equal-energy surface (1). Resonance saturation of the absorption is due to equalization of the populations of two levels of such two-level systems with $\varepsilon_{1\mathbf{p}}-\varepsilon_{2\mathbf{p}}\approx\hbar\omega$.

The nonlinear absorption coefficient α calculated on the basis of such premises decreases like $\alpha\sim I^{-1/2}$ at sufficiently high IR intensities I with increase of the intensity.[2–5,10–14]

Owing to the "corrugation" of the valence band, any scattering process (even elastic) takes the hole out of resonance.[1)] A light hole is transformed with overwhelming probability into a heavy one because of the large difference between the densities of states. Optical pumping causes thus a nonequilibrium distribution of the energy of the heavy holes in a region far from resonance. The hole-hole or hole-phonon collisions cause the distribution to relax gradually to its equilibrium value. If the rate of this energy relaxation is low enough compared with that of the optical pumping, the heavy-hole band can become depleted in the low energy region, and the absorption coefficient can therefore become nonlinear. At sufficiently high intensities this coefficient decreases like $\alpha\sim I^{-1}$.[7–9]

This nonlinearity mechansim, in contrast to the first, can be called nonresonant saturation of the absorption. The difference between them is that in nonresonant absorption the distribution functions of the light and heavy holes become equalized in a rather large region of momentum space near the resonance transition (1).

A theory developed in Ref. 15 takes into account these two mechanisms jointly and shows that in pure semiconductors of the *p*-Ge type resonance saturation of the absorption sets in earlier.

The difference between resonance and nonresonance saturation is distinguished in experiment primarily by the

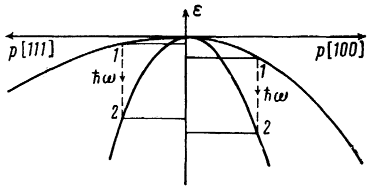

FIG. 1. Spectrum of *p*-Ge holes in the two crystallographic directions [100] and [111], and optical-transition scheme.

intensity dependence of the absorption coefficient $\alpha(I)$. This requires, however, measurements to be made in a rather large range of intensities.

There is, however, another possibility. It is based on the fact that even a weak electric or magnetic field applied to the semiconductor destroys the resonance saturation and leads to an increase of the absorption coefficient (up to its linear value). The reason is that by changing the hole momentum the external field takes it out of the resonance region. This increases in turn the difference between the distribution functions of the heavy and light holes in the region of the vertical transition.

In the case of an electric field, one of the mechanisms whereby it influences the resonance coherent saturation of the absorption was considered theoretically by Kumekov and Perel[16] for optical transitions between the valence and conduction bands of a semiconductor. In a semiconductor with a degenerate valence band there is realized, as a rule, an incoherent saturation of the resonance absorption. This regime differs from the coherent one in that no coherent Rabi oscillations of the resonant holes are produced by the IR radiation, and no mutual conversion of the heavy and light holes occurs. The influence of the electric field on incoherent resonance saturation of absorption will therefore also be different. The cause of this situation is as a rule the strong difference, at low temperatures, between the relaxation times of the light and heavy holes in the vicinity of the vertical transition. Electric fields that influence strongly the nonlinear-absorption coefficient can reach in pure semiconductors 1 V/cm (for 10.6-μm IR radiation).

The reason why a magnetic field affects the resonance saturation of the absorption, i.e., takes the holes off-resonance, is the corrugation, noted already in Ref. 15, of the valence band of the semiconductor. Since the resonance region is narrow, the magnetic fields needed for this purpose are also weak.

We develop in the present paper, by solving a system of kinetic equations for the hole density matrix in electric and magnetic fields, a consistent theory of the influence of these fields on resonance saturation of IR absorption in semiconductors with degenerate valence bands. Both the coherent and incoherent regimes of resonance saturation are considered, and the differences between them are analyzed. We precede the quantitative treatment with a brief qualitative exposition of the main result of the paper.

2. QUALITATIVE CONSIDERATION

To be specific, we carry out the qualitative analysis of the situation using as an example the influence of only an electric field on the resonance saturation. We begin, however, by recalling briefly how resonance saturation of absorption occurs in the absence of a field.[15]

At not too high IR intensities (the appropriate limit will be discussed later) the probability of an optical transformation of a heavy hole into a light one per unit time is determined by the simple quantum-mechanical equation[17]

$$W(\varepsilon)=\frac{A_p^2}{2}\frac{1/\tau_p}{\varepsilon^2+(\hbar/\tau_p)^2}. \qquad (2)$$

Here ε is the detuning from resonance:

$$\varepsilon=\varepsilon_{1p}-\varepsilon_{2p}-\hbar\omega, \qquad (3)$$

$A_p/2$ is the matrix element of the vertical transition and is proportional to the electromagnetic-wave amplitude $\varepsilon_0 \sim I^{1/2}$ (Refs. 15 and 18)

$$\frac{1}{\tau_p}=\frac{1}{2}\left(\frac{1}{\tau_{1p}}+\frac{1}{\tau_{2p}}\right), \qquad (4)$$

where τ_{1p} and τ_{2p} are the hole lifetimes in states 1 and 2, respectively (see Fig. 1). A fairly common situation is one in which these times differ quite strongly:

$$\tau_{1p}\gg\tau_{2p}. \qquad (5)$$

For p-Ge, for example, at an IR wavelength 10.6 μm and at nitrogen temperature, we have the ratio $\tau_{1p}/\tau_{2p}\approx 10$ (Ref. 15).

It follows from Eq. (2) that the probability of the optical transition per unit time is maximal in the resonance energy region $\varepsilon \lesssim \hbar/\tau_p$ and its order of magnitude is

$$A_p^2\tau_p/2\hbar^2. \qquad (6)$$

If this maximum probability is much smaller than $1/\tau_{1p}$, i.e.,

$$A_p\ll\hbar/(\tau_{1p}\tau_{2p})^{1/2} \qquad (7)$$

[we take here (4) and (5) into account], the optical transitions only disturb weakly the heavy-hole distribution function in the detuning region $\varepsilon \lesssim \hbar/\tau_p$, so that it is close there to its equilibrium value. The same holds obviously also for the light-hole distribution function. When condition (7) is met, the absorption coefficient is therefore close to its linear value α_0.

If

$$\hbar/(\tau_{1p}\tau_{2p})^{1/2}<A_p<\hbar/\tau_p, \qquad (8)$$

the probability (6) is significantly larger than $1/\tau_{1p}$, and optical transitions in a certain energy region $|\varepsilon| \lesssim \varepsilon^*$ (which we call in this case the resonance region) occur more frequently than departures from this region as a result of scattering. The so-defined width ε^* of the resonance region is in this case obviously larger than $\hbar/\tau_p$. It is determined from the condition $W(\varepsilon^*) = 1/\tau_{1p}$, whence

$$\varepsilon^*=\frac{A_p}{2}\left(\frac{\tau_{1p}}{\tau_{2p}}\right)^{1/2}>\frac{\hbar}{\tau_p}. \qquad (9)$$

Note that ε^* increases with intensity like $I^{1/2}$.

In this case the heavy-hole distribution function in the resonance region differs substantially from its equilibrium value. The absorption is therefore nonlinear and the absorption coefficient decreases with intensity in inverse proportion to $I^{1/2}$ (Ref. 15). The quantity $A_p = \hbar/(\tau_{1p}\tau_{2p})^{1/2}$ is thus indicative of the amplitude threshold of the nonlinear absorption in the absence of a field.

At even higher intensities, when

$$A_p>\hbar/\tau_p, \qquad (10)$$

Rabi oscillations are produced: the hole reverses helicity periodically, with frequency $A_p/\hbar$, and is alternately heavy and light. The width, relative to ε, of the region of such oscillations is then of order A_p. Coherent saturation of the absorption takes place in this detuning region, and Eq. (2) no longer holds for the transition probability. Since, however, we always have $A_p<\varepsilon^*$, the onset of Rabi oscillations does not influence noticeably the distribution functions and the dependences of the absorption coefficient on the intensity.[2)]

In an electric field $\mathbf{E}$ the detuning ε [Eq. (3)] is linear in time. The rate of change of the detuning (the rate of going off-resonance) is in this case

$$\dot{\varepsilon}=(\mathbf{v}_1-\mathbf{v}_2)\dot{\mathbf{p}}=(\mathbf{v}_1-\mathbf{v}_2)e\mathbf{E}. \quad (11)$$

Here $\mathbf{v}_1$ and $\mathbf{v}_2$ are the velocities of the heavy and light holes respectively at the vertical transition point.

If, during the lifetime τ_{1p} of the heavy hole, the electric field does not take the hole out of the resonance region, whose minimum width in the absence of a field is of order $\hbar/\mathbf{p}$, i.e.,

$$|\dot{\varepsilon}|<\hbar/\tau_{1p}\tau_p, \quad (12)$$

the field has practically no effect at all on the absorption coefficient, and on the distribution function of the heavy and light holes. We call such an electric field weak.

If this condition is not met, i.e., $|\dot{\varepsilon}|>\hbar/\tau_{1p}\tau_p$, the heavy hole goes off resonance before it has time to interact with the phonon. The electric field alters then substantially the dependence of the absorption coefficient on the intensity. First, the nonlinearity threshold shifts towards higher intensities. The characteristic amplitude A_p above which nonlinear effects become noticeable in absorption, is now of the order of $(2\hbar|\dot{\varepsilon}|)^{1/2}>\hbar/(\tau_{1p}\tau_p)^{1/2}$. Second, the $\alpha(I)$ dependence acquires in the linear region a new section in which the absorption coefficient decreases with increase of intensity in inverse proportion to I. All these changes are shown schematically in Fig. 2.

The physics of the influence of the electric field on the nonlinear-absorption coefficient turns out, however, to be different in different electric-field ranges. If

$$\hbar/\tau_{1p}\tau_p<|\dot{\varepsilon}|<\hbar/\tau_p^2 \quad (13)$$

(we call these fields intermediate) the electric field has no substantial effect on the probability of the quantum-mechanical transition (2). The absorption coefficient α will equal its linear value α_0 if the maximum probability is less than the reciprocal time $|\dot{\varepsilon}|\tau_p/\hbar$ in which the hole goes off resonance under the influence of the electric field, i.e., at

$$A_p<(2\hbar|\dot{\varepsilon}|)^{1/2}. \quad (14)$$

When this condition is not met, i.e., $A_p>(2\hbar|\dot{\varepsilon}|)^{1/2}$, the hole completes the optical transition before the electric field takes it off resonance. The resonance width ε^* is now determined from the condition $W(\varepsilon^*)=|\dot{\varepsilon}|/\varepsilon^*$, so that

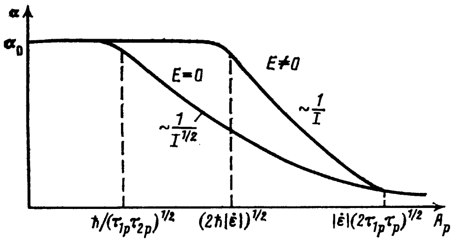

FIG. 2. Qualitative behavior of the absorption coefficient as a function of the electromagnetic-wave amplitude (the abscissas are the values of the corresponding matrix element).

$$\varepsilon^*=A_p^2/2|\dot{\varepsilon}|\tau_p. \quad (15)$$

In this case all the heavy holes "delivered" by the electric field to the resonance region, undergo with near-unity probability an optical transition, such that the absorbed power is independent of the intensity and the absorption coefficient is inversely proportional to the intensity I. This continues until, with increase of intensity, the time of passage of the particle through the resonance absorption region $\varepsilon^*/|\dot{\varepsilon}|=A_p^2/2\varepsilon^2\tau_p$ exceeds the lifetime τ_{1p} of the heavy hole, i.e., until A_p becomes larger than $|\dot{\varepsilon}|(2\tau_{1p}\tau_p)^{1/2}$.

If

$$A_p>|\dot{\varepsilon}|(2\tau_{1p}\tau_p)^{1/2} \quad (16)$$

we return to the situation in the absence of an electric field. The width of the resonance region is determined in this case by Eq. (9), and the absorption coefficient is inversely proportional to $I^{1/2}$. Note that in the case of intermediate electric fields the region of the Rabi oscillations produced when $A_p>\hbar/\tau_p$ again turns out to be narrower than the resonance region. Their appearance therefore does not change the described picture.

If the electric field is strong enough so that

$$|\dot{\varepsilon}|>\hbar/\tau_p^2 \quad (17)$$

(we call such a field strong) in an appreciable range of the IR intensities, the time $\varepsilon^*/|\dot{\varepsilon}|$ that the particle stays in the resonance region turns out to be shorter than the lifetime of either the heavy or the light hole. Therefore phonon relaxation of the holes in the resonance region does not influence at all the IR absorption in this case. The absorption-nonlinearity threshold is located as before at $A_p=(2\hbar|\dot{\varepsilon}|)^{1/2}$, but its physical nature is entirely different, since the electric field now influences strongly the probability of the quantum-mechanical transition.[3)] Namely, if the amplitude A_p is located in the interval

$$\hbar/\tau_p<A_p<(2\hbar|\dot{\varepsilon}|)^{1/2} \quad (18)$$

[this amplitude interval occurs only in the case of strong fields, see Eq. (17)], the electric field suppresses the Rabi oscillations that would be present in the absence of the field (since $A_p>\hbar/\tau_p$). The reason is that during a time equal to the Rabi-oscillation period $\hbar/A_p$ the hole is subjected in the electric field to a detuning considerably larger than the width A_p of the region of the oscillations in the absence of the field.

The width of the resonance region is determined in this case by the applied field and is equal to $\varepsilon^*\approx(\hbar|\dot{\varepsilon}|)^{1/2}$. This value is obtained for ε^* by comparing the time $\varepsilon/|\dot{\varepsilon}|$ of passage through the resonance with the value of $\hbar/\varepsilon$. If $\varepsilon/|\dot{\varepsilon}|<\hbar/\varepsilon$, a transition can take place, with a probability on the order of unity, between the adiabatic terms (Fig. 3), i.e., the heavy hole, absorbing a photon, can turn into a light one (and vice versa). When the amplitude A_p exceeds the threshold value $(2\hbar|\dot{\varepsilon}|)^{1/2}$ but is not too large, so that the following condition is met

$$(2\hbar|\dot{\varepsilon}|)^{1/2}<A_p<2|\dot{\varepsilon}|\tau_p, \quad (19)$$

the hole passes under the influence of the electric field slowly (adiabatically) through the resonance region (whose width is of order A_p), with enough time for many Rabi oscillations,

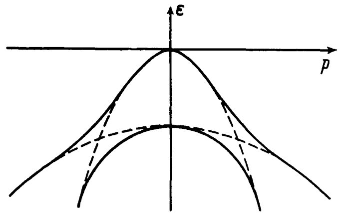

FIG. 3. Hole quasi-energy spectrum (schematic) in the field of an electromagnetic wave. The solid and dashed lines show respectively the adiabatic and diabatic levels.

but not enough time to be scattered [right-hand side of inequality (19)]. In such an adiabatic and collisionless passage through the resonance region, the heavy particle is converted with unity probability into a light one, and vice versa.[4)] This leads in turn to an interesting physical phenomenon—inversion of the distribution functions of the light and heavy holes in the immediate vicinity of the vertical transition. The number of light holes exceeds then that of the heavy ones (we refer, of course, to the magnitudes of their distribution functions). If $\dot{\varepsilon} > 0$, this takes place for values of **p** satisfying the condition $\varepsilon_{1p} - \varepsilon_{2p} > \hbar\omega$, i.e., on the right of the resonance. If $\dot{\varepsilon} < 0$, the distribution-function inversion takes place on the left of the resonance, at $\varepsilon_{1p} - \varepsilon_{2p} < \hbar\omega$. Lastly, in the region of sufficiently large amplitudes, at $A_p > 2|\dot{\varepsilon}|\tau_p$, the time $\varepsilon^*/|\dot{\varepsilon}|$ of stay of the hole in the resonance region becomes longer than τ_p. The phonon relaxation of the light hole in the resonance region can no longer be neglected, and we return to the physical picture considered in the case of intermediate fields. The width of the resonance region is determined by Eq. (15) and turns out to be larger than the width A_p of the Rabi-oscillations region so that they can be disregarded in the calculation of the absorption.

It is interesting to note that in the entire amplitude range

$$(2\hbar|\dot{\varepsilon}|)^{1/2} < A_p < |\dot{\varepsilon}|(2\tau_{1p}\tau_p)^{1/2} \tag{20}$$

the absorption coefficient is inversely proportional to the intensity and is described by one and same expression (60)—the same as for intermediate fields. This holds even though the physics of the phenomenon is different on the left and on the right of the point $2|\dot{\varepsilon}|\tau_p$ in the interval (20). As we shall show, only the form of the distribution function is sensitive in this sense to the amplitude $2|\dot{\varepsilon}|\tau_p$. In particular, the distribution-function inversion referred to above takes place in the left-hand side of the interval (20), but not in the right.

The effect of a magnetic field on the resonance saturation of the absorption is similar to that of an electric one. We shall show that the rate at which the holes go off resonance is in this case $(e/c)\ [\mathbf{v}_2\times\mathbf{H}]\mathbf{v}_1$. It differs from zero only for a corrugated equal-energy surface. In the absence of corrugation we have $\mathbf{v}_1 \| \mathbf{v}_2$ and $\dot{\varepsilon}$ vanishes.

The nonresonance absorption-saturation regime will not be considered in the present paper, i.e., we shall assume that the smooth parts of the distribution functions of the light and heavy particles (see Ref. 15) relax rapidly enough and are equal as a result to their equilibrium values.

3. EQUATIONS FOR THE DENSITY MATRIX

The absorption coefficient $\alpha(I)$ is determined by the off-diagonal component of the hole density matrix

$$\hat{F}_p = \begin{pmatrix} f_{1p} & \Psi_p e^{i\omega t} \\ \Psi_p^* e^{-i\omega t} & f_{2p} \end{pmatrix} \tag{21}$$

as follows:

$$\alpha(I) = -\frac{2\omega}{IV}\sum_p A_p \,\mathrm{Im}\,\Psi_p, \tag{22}$$

where $A_p/2$ is the vertical-transition matrix element, I is the IR intensity, ω its frequency, and V the crystal volume. The off-diagonal components of the density matrix Ψ_p and the distribution functions f_{1p} and f_{2p} of the heavy and light holes are determined from the system of kinetic equations. It is more convenient to represent the distribution functions f_{1p} and f_{2p} as sums of a smooth (gently sloping) and abrupt (steep) parts (see Ref. 15):

$$f_{1p} = f_{10} + \varphi_1, \quad f_{2p} = f_{20} + \varphi_2. \tag{23}$$

Here f_{j0} $(j = 1,2)$ is the smooth part of the distribution function. We take it to be an equilibrium Boltzmann function (see the end of Sec. 1) so that it varies over energy scales of the order of the temperature T, while φ_j are the abrupt parts of the distribution functions. They depend on the detuning ε [Eq. (3)] and vary in an energy scale $\delta\varepsilon \ll T$. The off-diagonal component of the density matrix Ψ_p likewise varies in an scale small compared with T.

It can be shown that in the stationary case the system of kinetic equations for the density-matrix components is of the form

$$\beta_p \frac{d\varphi_1}{d\varepsilon} + \frac{\varphi_1}{\tau_{1p}} = \frac{A_p}{\hbar}\,\mathrm{Im}\,\Psi_p, \tag{24a}$$

$$\beta_p \frac{d\varphi_2}{d\varepsilon} + \frac{\varphi_2}{\tau_{2p}} = -\frac{A_p}{\hbar}\,\mathrm{Im}\,\Psi_p, \tag{24b}$$

$$\beta_p \frac{d\Psi_p}{d\varepsilon} + \frac{\Psi_p}{\tau_p} - \frac{i}{\hbar}\varepsilon\Psi_p - \frac{i}{\hbar}\frac{A_p}{2}(\varphi_2 - \varphi_1) = \frac{iA_p}{2\hbar}(f_{20} - f_{10}), \tag{24c}$$

where β_p is defined as

$$\beta_p = e\mathbf{E}(\mathbf{v}_1 - \mathbf{v}_2) + \frac{e}{c}[\mathbf{v}_2\times\mathbf{H}]\mathbf{v}_1, \tag{25}$$

and $\mathbf{v}_1$ and $\mathbf{v}_2$ are respectively the velocities of the heavy and light holes at the vertical-transition point. The quantity β_p has the meaning of the rate at which the particles go off-resonance under the influence of the electric and magnetic fields [cf. Eq. (11)]. The contribution of the magnetic field to β_p differs from zero only when the corrugation of the valence band is taken into account. If the equal-energy surfaces $\varepsilon_{1p}, \varepsilon_{2p}$ = const are spheres, we have $\mathbf{v}_1 \| \mathbf{v}_2 \| \mathbf{p}$ and the contribution of the magnetic field to β_p vanishes.

The times τ_{1p}, τ_{2p} are the hole departure relaxation times from the states 1 and 2 respectively, due to the hole–phonon interaction, at the vertical-transition point (see Fig. 1). Expressions for them were obtained in Ref. 15. The relaxation time τ_p of the off-diagonal component of the density matrix is connected with them by the relation (4).

The system (24) was derived in the so-called resonance approximation, when $\delta\varepsilon \ll \hbar\omega, T$. The arrival terms in the kinetic equations need not be taken into account, since the resonance region is narrow.[5] For the same reason, we have left out of the kinetic equations the field terms with derivatives, with respect to momentum, of the smooth parts of the distribution functions f_{10} and f_{20}, and differentiated with respect to $\mathbf{p}$ only the abrupt parts φ_1 and φ_2:

$$\mathbf{F}_j \frac{\partial \varphi_j}{\partial \mathbf{p}} = \mathbf{F}_j \frac{d\varphi_j}{d\varepsilon}\frac{\partial \varepsilon}{\partial \mathbf{p}} = \frac{d\varphi_j}{d\varepsilon}(\mathbf{v}_1-\mathbf{v}_2)\mathbf{F}_j, \qquad j=1,2,$$

where

$$\mathbf{F}_j = e\mathbf{E} + \frac{e}{c}[\mathbf{v}_j \times \mathbf{H}]$$

is the Lorentz force acting on the holes in the band j.

As to the field term in the equation for the off-diagonal component of the density matrix $\Psi_{\mathbf{p}}$, it takes the form

$$\frac{1}{2}(\mathbf{F}_1+\mathbf{F}_2)\frac{\partial \Psi_{\mathbf{p}}}{\partial \mathbf{p}} = \frac{1}{2}(\mathbf{F}_1+\mathbf{F}_2)(\mathbf{v}_1-\mathbf{v}_2)\frac{d\Psi_{\mathbf{p}}}{d\varepsilon}$$

(see, e.g., the paper by D'yakonov and Khaetskiĭ[19]).

It is easy to verify that all three quantities

$$\mathbf{F}_1(\mathbf{v}_1-\mathbf{v}_2), \qquad \mathbf{F}_2(\mathbf{v}_1-\mathbf{v}_2), \qquad \frac{\mathbf{F}_1+\mathbf{F}_2}{2}(\mathbf{v}_1-\mathbf{v}_2)$$

are equal to one another and to $\beta_{\mathbf{p}}$. We arrive thus at the set (24).

We conclude this section by noting that the density-matrix components depend not only on the detuning ε but also on the angles, via the quantities $\tau_{2\mathbf{p}}, A_{\mathbf{p}}, f_{10}, f_{20}, \beta_{\mathbf{p}}$. These angular dependences, however, in contrast to the dependence on ε, are smooth. It was this which allowed us to change from a system of partial differential equations with respect to the components of the momentum $\mathbf{p}$ to a system of ordinary differential equations in which the angular dependences enter only via the parameters listed above and taken at $\varepsilon = 0$.

We begin the analysis of the system (24) with the case of low intensities. Even though the absorption coefficient is in this case practically equal to its linear value, we can assess, with this case as an example, the characteristic parameters that enter in our problem.

4. LOW INTENSITIES

The system (24) can be solved in this case by iteration with respect to $A_{\mathbf{p}}$. The first nonvanishing approximation for $\Psi_{\mathbf{p}}$ is here proportional to $A_{\mathbf{p}}$:

$$\Psi_{\mathbf{p}} = -iQ\int_0^\infty dx \exp[-x+ix(\tilde{\varepsilon}-bx/2)], \qquad (26)$$

where

$$Q = \frac{A_{\mathbf{p}}(f_{10}-f_{20})\tau_{\mathbf{p}}}{2\hbar}, \qquad \tilde{\varepsilon} = \frac{\varepsilon\tau_{\mathbf{p}}}{\hbar}, \qquad b = \frac{\beta_{\mathbf{p}}\tau_{\mathbf{p}}^2}{\hbar}.$$

The absorption coefficient (22) is determined by the imaginary part of the off-diagonal component $\Psi_2 = \text{Im}\,\Psi_{\mathbf{p}}$ of the density matrix and is obviously equal, in the approximation considered, to its linear value α_0. It is easy to verify that it is independent of the electric or magnetic field. The function $\Psi_2(\varepsilon)$ itself, however, is found to be very sensitive to E and H.

It is convenient to analyze $\Psi_2(\varepsilon)$ by writing it in the form

$$\Psi_{\mathbf{p}} = -\frac{Q}{2}(1+i)\left(\frac{\pi}{b}\right)^{1/2}(1-\text{erf}\, z)\exp z^2, \qquad (27)$$

where

$$z=(1-i\tilde{\varepsilon})/(1+i)b^{1/2},$$

and erf z is the error function of complex variable. We consider first the case $\beta_{\mathbf{p}} > 0$. In intermediate and weak fields, $\beta_{\mathbf{p}} < \hbar/\tau_{\mathbf{p}}^2 (b<1)$, the $\Psi_2(\varepsilon)$ dependence is Lorentzian just as in the absence of a field:

$$\Psi_2 = \text{Im}\,\Psi_{\mathbf{p}} = -Q/(\tilde{\varepsilon}^2+1). \qquad (28)$$

The effective width of this function, i.e., the width of the resonance region, is $\varepsilon^* \approx \hbar/\tau_{\mathbf{p}}$. In a strong field, for $\beta_{\mathbf{p}} > \hbar/\tau_{\mathbf{p}}^2 (b>1)$, there are several intervals of the energy ε, in which the function $\Psi_2(\varepsilon)$ behaves differently.

If $|\varepsilon| \ll (\hbar\beta_{\mathbf{p}})^{1/2} (|\tilde{\varepsilon}| \ll b^{1/2})$, then Ψ_2 is independent of ε:

$$\Psi_2 = -Q(\pi/4b)^{1/2}. \qquad (29a)$$

For $(\hbar\beta_{\mathbf{p}})^{1/2} \ll |\varepsilon| < \beta_{\mathbf{p}}\tau_{\mathbf{p}} (b^{1/2} \ll |\tilde{\varepsilon}| < b)$ we have

$$\Psi_2 = -Q\begin{cases} (2\pi/b)^{1/2}\cos(\tilde{\varepsilon}^2/2b)\exp(-\tilde{\varepsilon}/b), & \varepsilon>0 \\ b/|\tilde{\varepsilon}|^3, & \varepsilon<0 \end{cases}. \qquad (29b)$$

In this case $\Psi_2(\varepsilon)$ oscillates if ε is positive, and the period of the oscillations decreases as the detuning increases from a value on the order of $(2\hbar\beta_{\mathbf{p}})^{1/2}$ to the value $\hbar/\tau_{\mathbf{p}}$. For $\varepsilon < 0$, the value of $\Psi_2(\varepsilon)$ decreases in inverse proportion to $|\varepsilon|^3$.

Finally, if $|\varepsilon| \gg \beta_{\mathbf{p}}\tau_{\mathbf{p}} (|\tilde{\varepsilon}| \gg b)$, we get

$$\Psi_2 = -Q/\tilde{\varepsilon}^2, \qquad (29c)$$

i.e., it is the same as in the absence of external fields.

For $\beta_{\mathbf{p}} < 0$ we need only put $\varepsilon \to -\varepsilon$ in all the results. This property, as follows from the system (24), holds true both for the function $\Psi_2(\varepsilon)$ and for the functions $\varphi_1(\varepsilon)$ and $\varphi_2(\varepsilon)$ with arbitrary $A_{\mathbf{p}}$.[6]

The function $\Psi_2(\varepsilon)$ is thus influenced only by a strong field. It follows from (29a) that the width of the resonance region ε^* is determined in this case by the applied field:

$$\varepsilon^* \approx (\hbar|\beta_{\mathbf{p}}|)^{1/2}. \qquad (30)$$

By virtue of the inequality $|\beta_{\mathbf{p}}| > \hbar/\tau_{\mathbf{p}}^2$ the width ε^* exceeds the width $\hbar/\tau_{\mathbf{p}}$ of the resonance region in the absence of a field. Another conclusion that can be drawn from (29b) is that in the presence of a strong field the function is strongly asymmetric about the point $\varepsilon = 0$. It oscillates on the right side of the resonance region (at $\beta_{\mathbf{p}} > 0$) and decreases rapidly with increase of $|\varepsilon| (\sim |\varepsilon|^{-3})$ on the left.

The cause of the oscillations of $\Psi_2(\varepsilon)$ in the region $(\hbar\beta_{\mathbf{p}})^{1/2} \ll \varepsilon < \beta_{\mathbf{p}}\tau_{\mathbf{p}}$ (at $\varepsilon > 0$) is that, after passing through resonance, a hole of definite species goes over into a superposition of heavy- and light-hole states. It is interference of these states which leads to the indicated oscillations. They take place up to an energy $\varepsilon \approx \beta_{\mathbf{p}}\tau_{\mathbf{p}}$. This energy is acquired by the light hole in the field prior to collision with a phonon. At high energies the oscillation amplitude decreases exponentially (in the scale of $\beta_{\mathbf{p}}\tau_{\mathbf{p}}$) and Ψ_2 reaches its asymptote (29c), which is no longer field-dependent. Prior to passage through resonance there are no oscillations of $\Psi_2(\varepsilon)$, since

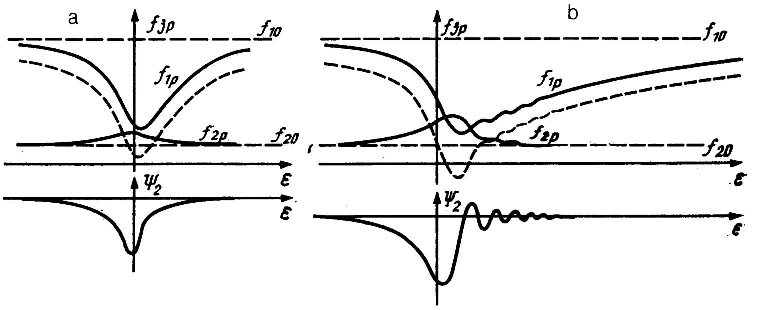

FIG. 4. Results of numerical computer solution of the kinetic equations (24) for p-Ge at $\lambda = 0.6\,\mu$m, $T = 77.4$ K [$\tau_{1p} = 5.1\times10^{-12}$ s, $\tau_{2p} = 0.52\times10^{-12}$ s (Ref. 15)]: $a - b = \beta_p\tau_p/\hbar = 0.6$, $a = A_p\tau_p/\hbar = 2.6$; $6 - b = 4$, $a = 3$; the difference $f_{1p} - f_{2p}$ is shown dashed.

the "light" and "heavy" states of the hole are not correlated. A correlation sets in only after passage through resonance, and is induced by an alternating field. Figure 4 shows the computer-calculated dependence of the imaginary part of the off-diagonal component of Ψ_2 on the detuning, obtained by numerical solution of the system (24). The case shown in this figure corresponds to the higher intensity, where perturbation theory no longer holds, but the $\Psi_2(\varepsilon)$ still remains oscillatory.

We analyze now the abrupt parts of the distribution functions of the light and heavy holes. At low intensities we obtain for them from (24)

$$\varphi_j = (-1)^j B\int_0^\infty dx\,\frac{e^{-x}}{\gamma_j^2 + b^2x^2}\,[\gamma_j\cos a(x) + bx\sin a(x)], \quad (31)$$

where $j = 1$ or 2, and

$$B = A_p^2(f_{10} - f_{20})\tau_p^2/2\hbar^2,\quad a(x) = x(\tilde{\varepsilon} - bx/2),\quad \gamma_j = \tau_p/\tau_{jp}. \quad (32)$$

As a rule, the light-hole relaxation time at the vertical-transition point is significantly shorter than that of the heavy one [see Eq. (5)]. In this case, taking Eq. (4) into account, we find that $\gamma_1 \approx 2\tau_{2p}/\tau_{1p} \ll 1$, but $\gamma_2 \approx 2$. We shall therefore analyze (31) first for the heavy-hole distribution function. To be specific we consider, as before, the case $\beta_p > 0$. In contrast to $\Psi_2(\varepsilon)$, it is necessary here to consider three field regions: weak, intermediate and strong.

In weak fields,

$$\beta_p < \hbar/\tau_{1p}\tau_p\,(b < \gamma_1), \quad (33)$$

$$\varphi_1 = -B/\gamma_1(1 + \tilde{\varepsilon}^2), \quad (34)$$

i.e. the same as in the absence of a field.

In the intermediate-field region

$$\hbar/\tau_{1p}\tau_p < \beta_p < \hbar/\tau_p^2 \quad (\gamma_1 < b < 1) \quad (35)$$

there exist several ranges of the energy ε in each of which the function $\varphi_1(\varepsilon)$ behaves differently. In the case $|\varepsilon| \ll \hbar/\tau_p$, $|\tilde{\varepsilon}| \ll 1$

$$\varphi_1 = -\pi B/2b \quad (36)$$

and is independent of ε. If $\hbar/\tau_p \ll |\varepsilon| \ll \beta_p\tau_{1p}(1 \ll |\tilde{\varepsilon}| \ll b/\gamma_1)$ the function $\varphi_1(\varepsilon)$ is asymmetric about the point $\varepsilon = 0$:

$$\varphi_1 = -B\begin{cases}\pi/b, & \varepsilon > 0\\ 1/b|\tilde{\varepsilon}|, & \varepsilon < 0\end{cases}, \quad (37)$$

i.e., φ_1 is independent of ε in the region $\varepsilon > 0$ and decreases slowly with increase of $|\varepsilon|$ at $\varepsilon < 0$. Lastly, in the case $|\varepsilon| \gg \beta_p\tau_{1p}(|\tilde{\varepsilon}| \gg b/\gamma_1)$ the distribution function is the same as in the absence of fields:

$$\varphi_1 = -B/\gamma_1\tilde{\varepsilon}^2. \quad (38)$$

In the strong-field region, $\beta_p > \hbar/\tau_p^2\,(b > 1)$, the situation is similar. When $|\varepsilon| \ll (\hbar\beta_p)^{1/2}(|\tilde{\varepsilon}| \ll b^{1/2})$ is a constant:

$$\varphi_1 = -\pi B/4b. \quad (39)$$

In the region $(\hbar\beta_p)^{1/2} \ll |\varepsilon| \ll \beta_p\tau_{1p}(b^{1/2} \ll |\tilde{\varepsilon}| \ll b/\gamma_1)$ the function $\varphi_1(\varepsilon)$ is asymmetric:

$$\varphi_1 = -B\begin{cases}\pi/b, & \varepsilon > 0\\ \dfrac{1}{b}\left(\dfrac{1}{|\tilde{\varepsilon}|} + \dfrac{b}{2\tilde{\varepsilon}^2}\right), & \varepsilon < 0\end{cases}, \quad (40)$$

with the second in the parentheses predominant in the region to the left of the resonance, and only the first term is important if $b \ll |\tilde{\varepsilon}| \ll b/\gamma_1$. Finally, for $|\varepsilon| \gg \beta_p\tau_{1p}(|\tilde{\varepsilon}| \gg b/\gamma_1)$ the picture is symmetric, just as in the absence of fields; the expression for φ_1 coincides in this case with (38).

We consider now the distribution function φ_2 of the light holes. Its dependence on the detuning ε is now easily obtained from the following considerations. Expression (31) with $j = 2$ differs from φ_1 in sign and by the substitution $\gamma_1 \to \gamma_2$. Therefore all the expressions (34), (39), (40), and (38) (in the latter expression for $b > 1$ and $|\tilde{\varepsilon}| \gg b$) turn out to be valid for φ_2 apart from reversal of the sign of φ_1 and the substitution $\gamma_1 \to \gamma_2$; the corresponding inequalities on which these equations are based are also valid. However, since $\gamma_2 \approx 2$, there is obviously no intermediate-field region in this case and the field influences noticeably the distribution function of the light holes only if it is strong.

An important conclusion of the foregoing analysis is that the distribution function of the heavy holes, in view of their long lifetime τ_{1p}, turns out to be more sensitive to the field action than the distribution function $\varphi_2(\varepsilon)$ of the light holes and the off-diagonal component $\Psi_2(\varepsilon)$ of the density matrix. The action of the field on $\varphi_1(\varepsilon)$ becomes noticeable under the condition $b_p > \gamma_1$ or $\beta_p > \beta_c$, where the critical field β_c is given by

$$\beta_c = \hbar/\tau_{1p}\tau_p. \quad (41)$$

These are substantially weaker fields, by an approximate factor τ_{1p}/τ_{2p}, than those defined by the condition $\beta_p = \hbar/\tau_p^2\,(b = 1)$, at which the field influences strongly the light-hole distribution function and the off-diagonal component of the density matrix $\Psi_2(\varepsilon)$. The condition $\beta_p > \beta_c$ means that within its lifetime τ_{1p} the heavy hole is moved by the field far out of a resonance region whose width (for $\beta_p < \hbar/\tau_p^2$) is equal to $\hbar/\tau_p$ [see (28)]. The detuning ε built up during this time in the field is of order $\beta_p \tau_{1p}$; this casts light on the physical meaning of this energy in the corresponding inequalities for intermediate and strong fields. It becomes clear also why this energy is indicative of the width of the heavy-hole distribution function to the right of the resonance at $\beta_p > \beta_c$. To the left of the resonance, the range of this function coincides, obviously, with the width of the resonance region, i.e., with $\hbar/\tau_p$ in the intermediate-field region and with $(\hbar\beta_p)^{1/2}$ in the strong-field region.

The field influences noticeably the light-hole distribution function only if it is strong, $\beta_p > \hbar/\tau_p^2$, i.e., when it influences also the function $\Psi_2(\varepsilon)$. The field makes $\varphi_2(\varepsilon)$ asymmetric: its "width" to the left of the resonance is equal to the width of the resonance region, and to the right it is equal to the energy $\beta_p \tau_{2p}$ acquired by the light hole in the field during the free-path time.

To conclude this section, we discuss the conditions under which the employed perturbation theory with respect to an alternating electromagnetic field is valid. Obviously, this requires that the corrections φ_1 and φ_2 to the difference $f_{10} - f_{20}$ be small. These corrections are maximal in the resonance region, i.e., near $\varepsilon = 0$. Using Eqs. (34), (36), and (39) we find that in the weak-field region the perturbation theory is applicable under the condition

$$A_p \ll \hbar/(\tau_{1p}\tau_p)^{1/2} \tag{42}$$

just as in the absence of a field.

For intermediate and strong fields, the perturbation theory is valid in a wider range corresponding to the requirement

$$A_p \ll (\hbar|\beta_p|)^{1/2}. \tag{43}$$

Thus, in this case the nonlinearity threshold, i.e., the intensity starting with which the absorption becomes nonlinear, increases up to values

$$A_p \approx (\hbar|\beta_p|)^{1/2} > \hbar/(\tau_{1p}\tau_{2p})^{1/2}.$$

We proceed now to consider nonlinear absorption at intensities for which perburbation theory is inapplicable. We begin with weak and intermediate fields.

5. HIGH INTENSITIES (WEAK AND INTERMEDIATE FIELDS)

For weak and intermediate fields we can neglect in (24b) and (24c) the terms containing derivatives with respect to the detuning (in view of the condition $|\beta_p| < \hbar/\tau_p^2$). These equations become then algebraic. Solving them for Ψ_2 and substituting in (24a), we obtain a simple differential equation for the distribution function φ_1. We can write its solution in the form

$$\varphi_1(\bar{\varepsilon}) = -\frac{B}{b}\int_0^\infty \frac{dx}{(\bar{\varepsilon}-x)^2+c_p^2}\Phi(\bar{\varepsilon},x), \tag{44}$$

where

$$\Phi(\bar{\varepsilon},x) = \exp\left\{-\frac{\beta_c}{\beta_p}\left[x+\frac{d_p^2}{c_p}\left(\operatorname{arctg}\frac{\bar{\varepsilon}}{c_p}-\operatorname{arctg}\frac{\bar{\varepsilon}-x}{c_p}\right)\right]\right\},$$

with coefficients

$$c_p = \left(1+\frac{A_p^2\tau_p\tau_{2p}}{2\hbar^2}\right)^{1/2}, \quad d_p = \frac{A_p}{\hbar}\left(\frac{\tau_p\tau_{1p}}{2}\right)^{1/2}. \tag{45}$$

In weak fields (33), for $|\beta_p| < \beta_c$, Eq. (44) leads to Eq. (16) of Ref. 15:

$$\varphi_1(\bar{\varepsilon}) = -\frac{\tau_{1p}}{\tau_p}\frac{B}{\bar{\varepsilon}^2+1+A_p^2\tau_{1p}\tau_{2p}/\hbar^2}, \tag{46}$$

i.e., the result is independent of the field here.

In the region of intermediate fields (35) ($\beta_c < \beta_p < \hbar/\tau_p^2$) and for $A_p \ll (2\hbar|\beta_p|)^{1/2}$ we obtain from (44) the results of the preceding section, i.e., as already mentioned, perturbation theory can be used here and the absorption coefficient is equal to its linear value α_0.

For $A_p > (2\hbar|\beta_p|)^{1/2}$ the field influences strongly the distribution function φ_1 if the amplitude A_p lies in the interval

$$(2\hbar|\beta_p|)^{1/2} < A_p < |\beta_p|(2\tau_{1p}\tau_p)^{1/2}. \tag{47}$$

In this case, at energies

$$|\varepsilon| \ll \varepsilon^* = A_p^2/2\beta_p\tau_p \tag{48}$$

$\varphi_1(\varepsilon)$ is independent of the detuning:

$$\varphi_1 = -(f_{10}-f_{20}) \tag{49}$$

and saturation sets in: the distribution functions of the light and heavy holes become equal in the region of the vertical transition.

If

$$A_p^2/2\beta_p\tau_p \ll |\varepsilon| \ll \beta_p\tau_{1p} \tag{50}$$

the heavy-hole distribution function becomes asymmetric with respect to the resonance:

$$\varphi_1(\bar{\varepsilon}) = \begin{cases} -(f_{10}-f_{20}), & \varepsilon > 0 \\ -B/b|\bar{\varepsilon}|, & \varepsilon < 0 \end{cases}. \tag{51}$$

Lastly, for

$$|\varepsilon| \gg \beta_p\tau_{1p} \tag{52}$$

the distribution function is independent of the field and is given by

$$\varphi_1(\bar{\varepsilon}) = -B/\gamma_1\bar{\varepsilon}^2, \tag{53}$$

which agrees with the corresponding asymptote of Eq. (46).

In the large-amplitude region

$$A_p > |\beta_p|(2\tau_{1p}\tau_p)^{1/2}, \tag{54}$$

the field does not influence substantially the form of the distribution function

$$\varphi_1(\bar{\varepsilon}) = \begin{cases} -(f_{10}-f_{20}), & |\varepsilon| \ll A_p\tau_{1p}^{1/2}/2\tau_{2p}^{1/2} \\ -B/\gamma_1\bar{\varepsilon}^2, & |\varepsilon| \gg A_p\tau_{1p}^{1/2}/2\tau_{2p}^{1/2} \end{cases} \tag{55}$$

(cf. the results obtained from (46), recognizing that in the intermediate-field region at $A_p > |\beta_p|(2\tau_{1p}\tau_p)^{1/2}$ the condition $A_p^2\tau_{1p}\tau_{2p}/\hbar^2 > 1$ is automatically met).

As to the light-hole distribution function, it coincides in the intermediate-field region [see (24b)], accurate to a factor $-A_p\tau_{2p}/\hbar$, with the imaginary part of the off-diagonal component of the density matrix $\Psi_2(\bar{\varepsilon})$:

$$\varphi_2 = -A_p\tau_{2p}\Psi_2/\hbar. \qquad (56)$$

The latter, as already stated, can be determined from Eq. (24c) from which we omit the term with the derivative with respect to ε. As a result, using (44) which we integrate by parts, we get

$$\Psi_2(\tilde{\varepsilon}) = \frac{-A_p(f_{10}-f_{20})}{2|\beta_p|\tau_{1p}(\tilde{\varepsilon}^2+c_p^2)}\int_0^\infty dx\,\Phi(\tilde{\varepsilon},x). \qquad (57)$$

In weak fields, for $|\beta_p| < \beta_c$, Eq. (57) leads to Eq. (18) of Ref. 15:

$$\Psi_2(\tilde{\varepsilon}) = -Q/(\tilde{\varepsilon}^2+1+A_p^2\tau_{1p}\tau_{2p}/\hbar^2). \qquad (58)$$

In the intermediate field range ($\beta_c < |\beta_p| < \hbar/\tau_p^2$)Eq. (57) with $A_p \ll (2\hbar|\beta_p|)^{1/2}$ leads to the perturbation-theory result (28). Finally in the region of the amplitudes (47) ($(2\hbar|\beta_p|)^{1/2} < A_p < |\beta_p|(2\tau_{1p}\tau_p)^{1/2}$) we have from (57) (for $\beta_p > 0$)

$$\Psi_2(\tilde{\varepsilon}) = -(f_{10}-f_{20})\begin{cases} \hbar/A_p\tau_{1p}, & |\varepsilon| \ll A_p^2/2\beta_p\tau_p \\ A_p\tau_p/2\hbar\tilde{\varepsilon}^2, & \varepsilon<0,\ |\varepsilon| \gg A_p^2/2\beta_p\tau_p \\ A_p/2\beta_p\tau_{1p}\tilde{\varepsilon}, & A_p^2/2\beta_p\tau_p \ll \varepsilon \ll \beta_p\tau_{1p} \\ A_p\tau_p/2\hbar\tilde{\varepsilon}^2, & \varepsilon \gg \beta_p\tau_{1p} \end{cases}. \qquad (59)$$

It can be seen that at these amplitudes the function $\Psi_2(\tilde{\varepsilon})$ is asymmetric and its minimum is shifted to the left of the resonance (see Fig. 4a).

In the region of large amplitudes $A_p > \beta_p(2\tau_{1p}\tau_p)^{1/2}$ the field does not affect Ψ_2 noticeably and we arrive as a result at Eq. (58) (the unity in the denominator can obviously be neglected in this case).

From Ψ_2 we can now determine with the aid of (56) the light-hole distribution function. Figure 4a shows by way of illustration the distribution functions of the light and heavy holes, calculated on a computer from the system (24), in this case for p-Ge. Owing to the difference between the relaxation times τ_{1p} and τ_{2p} the light-hole distribution function φ_2 turns out to be noticeably smaller than $|\varphi_1|$.

We proceed now to calculate the absorption coefficient. A weak field, $|\beta_p| < \beta_c$, does not affect the distribution function and the off-diagonal density-matrix component [see (58)]. It therefore does not affect the absorption coefficient, which is determined by the theory developed in Ref. 15, and which turns out at $A_p > \hbar/(\tau_{1p}\tau_{2p})^{1/2}$ to be inversely proportional to the square root of the IR intensity.[7)]

At intermediate fields (35) ($\beta_c < \beta_p < \hbar/\tau_p^2$) interest attaches to the intensity region where $A_p > (2\hbar\beta_p)^{1/2}$ and the absorption coefficient depends on the intensity. In the interval (47) the main contribution to $\alpha(I)$ of Eq. (22) is made by a region having a width of order $A_p^2/2\beta_p\tau_p$ in the vicinity of the detuning $\varepsilon \approx -A_p^2/2\beta_p\tau_p$. In this region we can neglect in the left-hand side of (24a) the relaxation term φ_1/τ_{1p}. The absorption coefficient, which is proportional to the value of the integral

$$\int_{-\infty}^{+\infty} d\varepsilon\,\Psi_2(\varepsilon),$$

is determined in this case simply by the difference between the values of φ_1 far enough to the left and right from the point $\varepsilon \approx -A_p^2/2\beta_p\tau_p$. This difference, as follows from (51), is equal to $f_{10}-f_{20}$. As a result, the absorption coefficient in this amplitude range is inversely proportional to the intensity and directly proportional to the applied field:

$$\alpha(I) = \frac{2\hbar\omega}{IV}\sum_p |\beta_p|(f_{10}-f_{20})\delta(\varepsilon_{1p}-\varepsilon_{2p}-\hbar\omega). \qquad (60)$$

At still higher intensities, where $A_p > |\beta_p|(2\tau_{1p}\tau_p)^{1/2}$, we return to the results of Ref. 15, viz., the absorption coefficient is independent of the field and is inversely proportional to $I^{1/2}$ (see footnote 7).

Thus, summarizing the analysis, we note that in contrast to the low-intensity case, the intermediate-region field (35) influences at high intensities in the amplitude range (47) not only the heavy-hole distribution function φ_1, but also the distribution function φ_2 of the light holes and the off-diagonal component Ψ_2 of the density matrix, as well as the absorption coefficient which is proportional in the nonlinear region (47) to the applied field and is inversely proportional to the intensity.

We see thus [see (49)] that the heavy-hole distribution function φ_1 is equal in the resonance region to the density of the smooth parts of $f_{20}-f_{10}$. The difference between the complete distribution functions of the heavy and light holes is then practically zero (see Fig. 4a). The characteristic width of the dip in the heavy-hole distribution function is equal to the width of the resonance region on the left of the resonance. On the right it is significantly larger and equals the detuning energy $\beta_p\tau_{1p}$ acquired in the field during the free-path time of the heavy hole.

The distribution function of the light holes has, on the contrary, a maximum in the vicinity of the point $\varepsilon = -A_p^2/2\beta_p\tau_p$. The probability of a vertical transition is large in this region and the number of heavy holes is still large. To the left of this point, the function φ_2 is decreased in proportion to ε^2 because of the decreased probability of the optical transitions [see (59)]. On the right, it decreases abruptly (exponentially) over detuning scales of the order of the width of the resonance region. The exponential decrease gives way next to a power-law decrease, first like $1/\varepsilon$ and then like $1/\varepsilon^2$.

On the other hand, the dependence of Ψ_2 on the detuning coincides with the $\varphi_2(\varepsilon)$ dependence.

It follows from (60) that at ampliltudes from the interval (47) the absorption coefficient in the intermediate-field region is inversely proportional to the intensity and directly proportional to the applied field. This fact can be illustratively interpreted as follows.

The applied field attracts the heavy holes into the resonance region, from which they go off with a probability of order unity into the light subband as a result of an optical transition following absorption of a photon $\hbar\omega$. The total absorbing power is in this case independent of the intensity, since the number of heavy holes "supplied" by the field to the resonance region is constant and is determined only by the applied field strength. The absorption coefficient is therefore inversely proportional to I and directly proportional to the field $|\beta_p|$.

6. HIGH INTENSITIES ("STRONG" FIELDS)

We consider now absorption in the "strong" field region

$$|\beta_p| > \hbar/\tau_p^2.$$

At $A_p \ll (2\hbar|\beta_p|)^{1/2}$, as already mentioned in Sec. 4, perturbation theory is applicable and the absorption coefficient is equal to its linear value α_0. For $A_p > (2\hbar|\beta_p|)^{1/2}$ there exist three ranges of the amplitude A_p, with different physical behavior of the nonlinear absorption.

In an amplitude region

$$(2\hbar|\beta_p|)^{1/2} < A_p < 2|\beta_p|\tau_p \quad (61)$$

of width of order A_p near resonance, Rabi oscillations are produced when the hole goes over periodically, with frequency $A_p/\hbar$, from one subband to the other. These oscillations are not suppressed by phonons, since the inequality $A_p > \hbar/\tau_p$ is satisfied by virtue of the conditions $|\beta_p| > \hbar/\tau_p^2$ and $A_p > (2\hbar|\beta_p|)^{1/2}$. That is to say, the period of the oscillations is shorter than the relaxation time τ_p of the off-diagonal component of the density matrix. Nor are they suppressed by the field, since many Rabi oscillations can take place during the time, of order of $A_p/|\beta_p|$, of passage through the resonance region.

By virtue of the right-hand side of inequality (61), the width of the resonance region in the considered amplitude range is smaller than the characteristic energy $|\beta_p|\tau_p$, meaning the detuning to which the hole is subjected during the relaxation time τ_p. The hole moves in this energy region without collisions and it is possible in this case to neglect in the equations (24) for the density matrix all the relaxation terms $\varphi_1/\tau_{1p}, \varphi_2/\tau_{2p}$ and Ψ_p/τ_p. The solution of the resultant system of equations, with boundary conditions (for $\beta_p > 0$):

$$\varphi_1 = \varphi_2 = \Psi_1 = \Psi_2 = 0, \quad \text{for } \varepsilon = -\infty \quad (62)$$

takes the form

$$\varphi_1 = -\varphi_2 = -{}^1/_2 (f_{10} - f_{20})(1 + \varepsilon/(\varepsilon^2 + A_p^2)^{1/2}), \quad (63)$$

$$\Psi_1 = -A_p (f_{10} - f_{20})/2(\varepsilon^2 + A_p^2)^{1/2}, \quad (64)$$

$$\Psi_2 = -\hbar|\beta_p| A_p (f_{10} - f_{20})/2(\varepsilon^2 + A_p^2)^{3/2}. \quad (65)$$

The physical meaning of the boundary condition (62) is that $d\varepsilon/dt > 0$ if $\beta_p > 0$ and the holes far to the left of the resonance (with $\varepsilon < 0$) have a density matrix close to the equilibrium value. On the contrary, the holes to the right of the resonance (with $\varepsilon > 0$) were "drawn" by the field through the resonance region. Their density matrix in the absence of relaxation can therefore differ greatly from equilibrium.

An important feature of the solutions obtained in these energy ranges is the presence of inversion of the distribution functions of the light and heavy holes (see Fig. 4b). Actually, as follows from (63), for $\varepsilon \gg A_p$ the total distribution functions $f_j = f_{j0} + \varphi_j$ ($j = 1,2$) of the heavy and light holes (23) are equal to

$$f_1 = f_{20}, \; f_2 = f_{10}, \quad (66)$$

i.e.,

$$f_1 - f_2 = -(f_{10} - f_{20}) < 0. \quad (67)$$

This means that a heavy hole passing through the resonance region is converted with near-unity probability into a light hole, and a light hole correspondingly into a heavy one. Therefore at $\beta_p < 0$ the region of inversion of the distribution functions is on the left of the resonance if $\varepsilon < 0$.

The physical cause of the inversion is that if the condition $A_p > (2\hbar|\beta_p|)^{1/2}$ is met the probability w of a transition between adiabatic terms (see Fig. 3) is, according to Zener's equations,[20] exponentially small:

$$w = \exp(-\pi A_p^2/2\hbar|\beta_p|), \quad (68)$$

since the hole passes through the resonance region slowly (adiabatically), in a time of order $A_p/|\beta_p|$, and executes many Rabi oscillations. Remaining on one and the same adiabatic term, a heavy hole is thus converted into a light one after passing through the resonance, and a light one into a heavy one (see Fig. 4b).

The above "collisionless" solutions (63)–(65) for the density-matrix components are valid, as already stated, in the energy region $|\varepsilon| < |\beta_p|\tau_p$. At energies $|\varepsilon| > |\beta_p|\tau_p$ on the contrary, relaxation terms are more important in Eqs. (24b) and (24c), and the influence of the field can be neglected. We return in this case to the system of equations whose solution was obtained in Sec. 5 [see Eqs. (44), (56), and (57)]. Analysis of these solutions as applied to the considered case (61) leads to the following results (at $\beta_p > 0$).

For the heavy-hole distribution function:

$$\varphi_1 = -(f_{10} - f_{20}) \begin{cases} 1, & \varepsilon > 0, \quad \beta_p\tau_p \ll \varepsilon \ll \beta_p\tau_{1p} \\ A_p^2/2\beta_p\tau_p|\varepsilon|, & \varepsilon < 0, \quad \beta_p\tau_p \ll |\varepsilon| \ll \beta_p\tau_{1p}. \\ A_p^2\tau_{1p}/2\tau_p\varepsilon^2, & |\varepsilon| \gg \beta_p\tau_{1p} \end{cases} \quad (69)$$

For the light-hole distribution function:

$$\varphi_2 = (f_{10} - f_{20}) \begin{cases} A_p^2/4\beta_p\tau_{1p}\varepsilon, & \varepsilon > 0, \quad \beta_p\tau_p \ll \varepsilon \ll \beta_p\tau_{1p} \\ A_p^2/4\varepsilon^2, & \varepsilon < 0, \quad |\varepsilon| \gg \beta_p\tau_p \\ & \varepsilon > 0, \quad \varepsilon \gg \beta_p\tau_{1p} \end{cases} \quad (70)$$

The off-diagonal component of the density matrix Ψ_2 at $|\varepsilon| > \beta_p\tau_p$ is connected with the light-hole distribution function by Eq. (56). Thus, in a strong field, the characteristic energy scale $\beta_p\tau_p$ which was missing in the intermediate-field region for $A_p > (2\hbar|\beta_p|)^{1/2}$, reappears in the density matrix in the amplitude interval (61). If $|\varepsilon| > \beta_p\tau_p$, the light-hole distribution function on the right of the resonance no longer coincides in absolute value with the heavy-hole distribution function which is "drawn" by the field into the region of high energies of order $\beta_p\tau_{2p}$ (Fig. 4b).

It is easy to obtain the absorption coefficient in the considered amplitude region. To this end it is necessary to substitute in (22) the function Ψ_2 from (65) and integrate with respect to ε. We arrive then at the result (60) of the preceding section. The reason is that in this case, just as in the case of intermediate fields, in the amplitude interval (47) all the heavy holes that pass through the resonance region are transformed into light ones by absorbing a photon, so that the absorbed power is independent of the intensity. It depends only on the difference between the heavy- and light-hole fluxes along the energy axis ε, a difference proportional to the applied field and determined by the values of the distribution functions far from resonance, i.e., by the quantities f_{10} and f_{20}.[8)] In this case the form of the distribution functions f_{1p} and f_{2p} in the resonance region is immaterial to the calculation of the absorption.

There is, however, a fundamental difference between these cases. In intermediate fields the probability of light-hole scattering by a phonon greatly exceeds the probability of its return to the heavy subband with emission of an IR photon, so that the light-hole distribution function φ_2 is always smaller than $|\varphi_1|$. In the considered case of a strong field, in the resonance region, the interaction of the holes with phonons can be neglected, and inversion of the distribution functions takes place: $\varphi_1 = -\varphi_2$. This difference, however, does not affect the value of the absorption coefficient.

We proceed now to consider the next amplitude region:

$$2|\beta_p|\tau_p < A_p < |\beta_p|(2\tau_p\tau_{1p})^{1/2}. \qquad (71)$$

In this case the width $A_p^2/2|\beta_p|\tau_p$ of the resonance region exceeds the widths A_p of the region in which of Rabi oscillations take place and $\beta_p\tau_p$ of the collisionless-motion region. The density-matrix elements are therefore not noticeably changed at these small energy scales, and terms with derivatives with respect to detuning can be neglected in Eqs. (24b) and (24c). But then we arrive at the result of Sec. 4, at Eqs. (44), (56), (57), and (60).

The situation is analogous for

$$A_p > |\beta_p|(2\tau_p\tau_{1p})^{1/2}, \qquad (72)$$

when the width of the resonance region is $A_p(\tau_{1p}/2\tau_p)^{1/2}$ and the field influences substantially neither the density-matrix elements nor the absorption coefficient, the latter being inversely proportional to the square root of the intensity.[15]

It is interesting to note that as the amplitude A_p goes through the value $2|\beta_p|\tau_p$ no change takes place in the intensity dependence of the absorption coefficient, whereas the dependence of the density-matrix components on the detuning ε changes radically.

7. CONCLUSION

We obtain thus a rather complicated picture of the behavior of the hole density matrix as a function of the detuning ε at various intensities of IR radiation and of electric and magnetic fields. At the same time the absorption coefficient α has a relatively simple dependence on the intensity I. In the linear absorption region, in fields exceeding the threshold value β_c [Eq. (41)], the plot of $\alpha(I)$ has only two branches (see Fig. 2). On the first (at lower intensities) the absorption coefficient is inversely proportional to the intensity and directly proportional to the applied field. On the second, $\alpha(I)$ is independent of the field and decreases approximately like $I^{-1/2}$. The boundary between these sections is proportional to the square of the applied field, while the below-threshold intensity of the nonlinear absorption is proportional to the first power of the field intensity ($A_p = (2\hbar|\beta_p|)^{1/2}$).

Let us estimate the minimum electric and magnetic fields, starting with which the nonlinear-absorption coefficient begins to depend on the field. Thus, in p-Ge at an IR wavelength $\lambda = 10.6$ μm and at $T = 77.4$ K the times τ_{1p} and τ_{2p} are respectively $5\cdot 10^{-12}$ s and $0.52\cdot 10^{-12}$ s.[15] From this and from Eq. (41) we obtain $\beta_c \approx 0.22\cdot 10^{-3}$ erg/s. In terms of the electric or magnetic field, neglecting all the angular dependences, we obtain according to (25) $E_c \approx 1.4$ V/cm and $H_c \approx 10$ Oe. If the electric and (or) magnetic field intensities in the semiconductor exceed these values, $E \gg E_c$ and (or) $H \gg H_c$, these fields will influence substantially the absorption coefficient in the intensity range $I_1 \ll I \ll (\beta_p/\beta_c)^2 I_1$. The critical intensity I_1 (corresponding to the amplitude $A_p = \hbar/(\tau_{1p}\tau_{2p})^{1/2}$ for p-Ge in this situation is of the order of 15 kW/cm^2 (Ref. 5).

We have considered so far the influence of external fields on the nonlinear-absorption coefficients in pure semiconductors and disregarded hole scattering by charged impurities and by one another. Yet their role is extremely important, for owing to the corrugation of the valence band small-angle scattering of heavy holes can take the latter out of the resonance region. By virtue of the specific features of Coulomb scattering, the holes depart diffusely with a diffusion coefficient D_p proportional to the charged-impurity density N_i. It can be shown that, in order of magnitude,

$$D_p \approx \frac{e^4 N_i m_1 (\hbar\omega)^{1/2}}{\varepsilon_0^2 m_2^{3/2}} \Lambda, \qquad (73)$$

where Λ is the Coulomb logarithm, m_1 and m_2 are the effective masses of the heavy and light holes, respectively, and ε_0 is the dielectric constant of the semiconductor.

Charged impurities can be disregarded under the condition

$$D_p < \hbar^2/\tau_{1p}\tau_p^2, \qquad (74)$$

i.e., when the diffusion length for the detuning ε, equal to $(D_p\tau_{1p})^{1/2}$, is less than the minimum width $\hbar/\tau_p$ of the resonance region. In p-Ge this corresponds, for the situation considered above, to the rather low charged-impurity density[9)] $N_i \lesssim 10^{11}$ cm^{-3}.

At higher densities, the charged impurities play an important role in resonance saturation of IR absorption in the absence of the field, by shifting the nonlinearity threshold towards higher intensities. We hope to assess their influence in detail in a separate paper. Yet it is clear that the influence of an external field on the nonlinear-absorption coefficient will be exactly the same also in that case. This influence, however, will manifest itself in stronger fields, when the departure from resonance under the influence of the field is faster than by diffusion. This calls only for satisfaction of the condition

$$\beta_p > D_p\tau_p/\hbar, \qquad (75)$$

and then the theory developed above is applicable also to "dirty" semiconductors. Here, however, the region of intermediate fields may become "closed" (if $D_p\tau_p/\hbar > \hbar/\tau_p^2$), but the dependence of the absorption coefficient on the intensity remains the same as before.

We note in conclusion that the effects predicted here may apparently be more easily revealed by the influence of a magnetic field on the absorption coefficient, for in that case there is no heating of the carriers and no change of the smooth parts of the distribution function.

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[1)] An exception is small-angle scattering by charged impurities. This mechanism will not be dealt with in the present article.

[2)] This means in turn that the equations obtained in Ref. 15 for the nonlinear-absorption coefficient are valid also for incoherent IR radiation. To obtain the correct answer if the spectrum width of this radiation is $\Delta\omega > 1/\tau_p$, it is necessary to replace τ_p by $1/\Delta\omega$ in all the equations of Ref. 15.

[3)] This mechanism of the influence of an electric field on the nonlinear-absorption coefficient was considered by Kumekov and Perel'.[16]

[4)]The deviation of this probability from unity equals $\exp(-\pi A_p^2/2\hbar|\dot{\varepsilon}|)$, i.e., is exponentially small.

[5)]They are significant only in the equation for the smooth parts of the distribution functions f_{10} and f_{20} (see Ref. 15).

[6)]$\Psi_1(\varepsilon) \equiv \mathrm{Re}\,\Psi_p$ reverses sign when $\varepsilon \to -\varepsilon$ and $\beta_p \to -\beta_p$.

[7)]If it is assumed that the energy relaxation in the heavy band is fast enough (see Ref. 15).

[8)]Such a flux was actually calculated by Kumekov and Perel' [16] to obtain the absorption coefficient in an electric field.

[9)]The value of α is also small in this case. It can be measured, however, by using for example the drag effect (see Ref. 9 for details.)

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