Structure of lateral waves formed on reflection of light beams by absorbing and amplifying media

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The nature of lateral waves formed on reflection of light beams from amplifying media is found using the Sommerfeld radiation principle. It is shown that discrepancies with the results of other authors are due to differences in the selection of the solutions when describing the reflection of plane waves from amplifying media. The prerequisites for critical experiments are discussed.

INTRODUCTION

Identification of the origin of lateral waves requires an analysis not only of the reflected field, but also of fields excited by the incident radiation penetrating beyond the interface into an optically less dense medium.[1] When a light beam is reflected from an amplifying medium, we can expect qualitative differences in the structure of lateral waves compared with the case of passive media. Moreover, in principle, experimental observations of lateral waves can be used to identify the correct one from three mutually exclusive selection rules for waves refracted under total internal reflection conditions, suggested in Refs. 2–4.

Alternative predictions of the structure of lateral waves were obtained in Ref. 5 for two, proposed by that time,[2,3] selection rules governing the nature of refracted waves. The nature of lateral waves has been identified relatively recently[6] on the basis of the causality relationships governing the reflected field, but ignoring the waves in an amplifying medium. The results obtained in Ref. 6 are close to one of the variants of the theory discussed in Ref. 5 representing the point of view put forward in Ref. 2. We shall determine the structure of lateral waves in accordance with the Sommerfeld radiation principle in the case of light beams reflected from absorbing and amplifying media. This structure agrees with the general rule, which was proposed in Ref. 4 and justified in Refs. 7 and 8 on the basis of the causality considerations, which was called (in Ref. 8) the Mandel'shtam radiation principle.[9] In most cases this principle is equivalent to the Sommerfeld radiation principle and is identical with the latter in the case of the usual models of amplifying media.

It is generally known that in the problem of reflection of a plane wave the conditions at the interface between two media determine only the square of the normal component of the refracted wave (k_{2z}):

$$k_{2z}^2=(\omega/c)^2\varepsilon_2(\omega)-k_x^2=a+ib, \tag{1}$$

where $\varepsilon_2(\omega)$ is the permittivity of the reflecting medium; ω is the frequency; k_x is the tangential component of the wave vector of the incident wave. In general, application of the Sommerfeld radiation principle in order to remove the ambiguity in the definition of k_{2z} in Eq. (1) reduces to two operations: 1) taking a cut in the complex k_{2z}^2 plane along the negative part of the real axis ($a<0$), which corresponds to the following rule for extracting the root:

$$k_{2z}=\pm(k_s'+ik_s''), \tag{2a}$$

$$k_s'=2^{-1/2}[(a^2+b^2)^{1/2}+a]^{1/2},\qquad k_s''=2^{-1/2}[(a^2+b^2)^{1/2}-a]^{1/2}\operatorname{sign} b, \tag{2b}$$

2) such a selection of the sign on the right-hand side of Eq. (2a) that in the adopted coordinates the value of k_{2z}' corresponds to the propagation of the refracted wave from the interface. The rule proposed in Ref. 2 can be reduced to making a cut along the negative part of the imaginary axis ($b<0$), where in Ref. 3 this involves a cut along the positive part of the real axis in the complex plane of k_{2z}^2 ($a>0$). An analysis of the validity of the rules proposed in Refs. 2–4 can be found in Refs. 5–8, 10, and 11, where the relevant references are cited. It should be pointed out that the known criteria for distinction between decaying and growing waves[12,13] can be adopted formally only after decision about the nature of the $k_{2z}(\omega)$ branches, i.e., after adoption of one of the rules from Refs. 2–4.

Essentially, the cut in the k_{2z}^2 plane, reflected in Eq. (2b), and fixing of the sign in Eq. (2a) completes the basic physical formulation of the problem. The next stage, which is essentially mathematical, we have to consider k_{2z} as a function of parameters. Then, in the relevant complex planes we have to make cuts which are mapped in the k_{2z}^2 plane on the negative part of the real axis ($a<0$).

NATURE OF LATERAL WAVES

Before considering the case of reflection from amplifying media, we shall deal with details such as that the application of the Sommerfeld radiation principle to absorbing media gives the same result as the traditional approach of Ref. 1, where the direction of the refracted plane waves is selected on condition that they decay away from the interface (i.e., using the principle of maximum absorption). For the sake of simplicity, we shall use s to denote a polarized two-dimensional beam incident on a slit separated by a distance z_0 from the interface between two media (see Fig. 1), which creates a field

$$E_i(x)=Q(x)\exp(ik_1x\sin\varphi-i\omega t), \tag{3}$$

where φ is the angle of incidence, k_1 is the wave number, and the function $Q(x)$ governs the transverse structure of the beam. Following Ref. 1, we shall expand the incident beam in terms of plane waves characterized by complex angles of incidence ν ($k_x=k_1\sin\nu$) and, dropping the time factor, we find that the reflected wave is described by

$$E_r(x,z)=(2\pi)^{-1}\int_{\Gamma_1}d\nu\cos\nu\tilde{E}_i(\nu)V(\nu)\exp[ik_1R\cos(\nu-\nu_0)], \tag{4a}$$

where $\tilde{E}_i(\nu)$ is the angular spectrum of the incident beam corresponding to Eq. (3), R is the distance from the point of

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observation P to the mirror image of the center of the slit (Fig. 1), ν_0 is the angle of observation,

$$x=R\sin\nu_0,\quad z+z_0=R\cos\nu_0, \tag{4b}$$

$V(\nu)$ is the reflection coefficient of plane waves:

$$V(\nu)=(p_1-p_2)/(p_1+p_2),\quad k_{iz}=-k_1p_i\quad (i=1,2), \tag{4c}$$

$$p_1=\cos\nu,\quad p_2=(n^2-\sin^2\nu)^{1/2},\quad n^2=\varepsilon_2/\varepsilon_1, \tag{4d}$$

ε_1 is the permittivity of the medium from which the beam is incident and it is assumed to be a real quantity; the integration contour Γ_1 is represented in Fig. 2 by a broken line going from $-\pi/2+i\infty$ to $\pi/2+i\infty$. The cuts in the complex plane ν, which ensure that the Sommerfeld radiation principle $p'_{2z}>0$ is satisfied on the "upper" physical sheet, are represented by continuous curves beginning from the branching points of the function $p_2(\nu)$, denoted by A_1 and A_2. The dashed lines show the generally accepted method used to make cuts.[1]

Integration of Eq. (4a) for a reflected field is carried out, as usual,[1] by the steepest descent method on the assumption that $k_1R\gg 1$. In the case of observation points characterized by angles ν_0 exceeding the critical angle $\delta=\arcsin n'$ the steepest-descent contour intersects a cut originating from the point A_1. Continuous deformation of the initial contour Γ_1 until it assumes the steepest-descent form Γ creates an additional loop Γ_2 (Fig. 2), which bypasses the cut and horizontal segments from $-\pi/2+\nu_0$ to $\pi/2$ going in opposite directions on the upper and lower sheets. Integration along the steepest-descent contour Γ limited to the first term of the asymptotic expansion yields a wave corresponding to reflection in accordance with the laws of geometrical optics

$$E_{r0}=(2\pi)^{-1}(k_1R)^{-1}V(\nu_0)\tilde{E}_i(\nu_0)\cos\nu_0\exp(ik_1R+i\pi/4). \tag{5}$$

Integration along the loop Γ_2 reduces to integration along a path Γ'_2 of the steepest descent, coinciding with that employed in the traditional approach[1]:

$$E_p=(2\pi)^{-1}\int_{\Gamma'_2} d\nu\cos\nu\tilde{E}_i(\nu)[V_u(\nu) - V_l(\nu)]\exp[ik_1R\cos(\nu-\nu_0)], \tag{6}$$

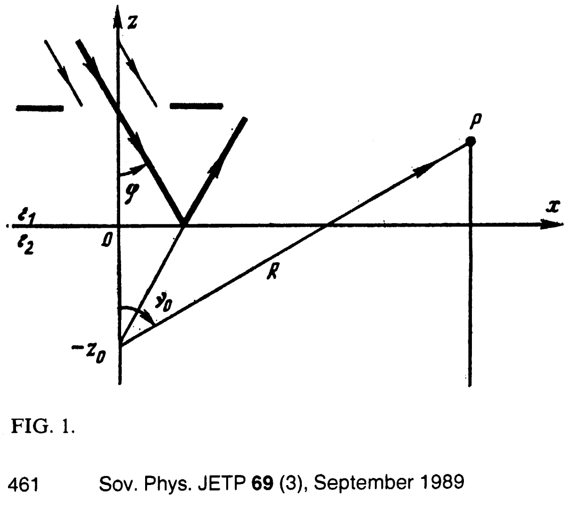

FIG. 1.

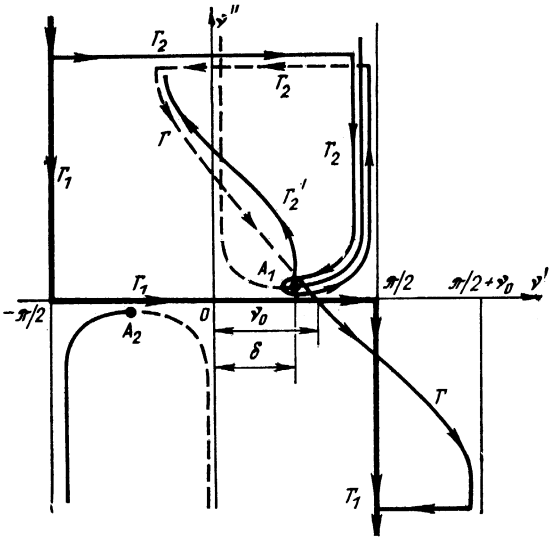

FIG. 2.

where $V_u(\nu)$ is the value of the function (4c) on the upper sheet, corresponding to the positive branch of the square root in Eq. (4d) and $V_l(\nu)$ is the value of $V(\nu)$ on the lower sheet found from V_u by reversing the sign of p_2. Formally, when this treatment of V_u and V_l is adopted where Eq. (6) differs in respect of the sign from the corresponding expression obtained by traditional cuts.[1] If the quantity in Eq. (6) is calculated, as in Ref. 1, assuming that the absorption is weak ($|n''|\ll n'$) and the changes in $\tilde{E}_i(\nu)$ are slow, we find that Eq. (6) reduces to an integral over a small part of the path in the vicinity of the point A_1, where the contour Γ'_2 is almost parallel to the imaginary axis, so that we obtain

$$E_p=-2\pi^{-1}i\tilde{E}_i(\delta)\exp[ik_1R\cos(\delta-\nu_0)]\int_0^\infty dy\,(-2in'y\cos\delta]^{1/2} \times\exp[-k_1Ry\sin(\nu_0-\delta)], \tag{7}$$

where the root of $-i$ gives, in accordance with the rule (2), $\exp(-i\pi/4)$ in contrast to the rule used in Ref. 1 which gives a quantity opposite in sign. The final expression is then identical with that obtained for the case in question by adopting the traditional approach[1]:

$$E_p=0,\quad \nu_0<\delta,$$

$$E_p=-[2n(1-n^2)^{1/2}/\pi]^{1/2}[k_1R\sin(\nu_0-\delta)]^{-3/2}\tilde{E}_i(\delta)\times\exp[ik_1R\cos(\nu_0-\delta)+i\pi/4],\quad \nu_0>\delta. \tag{8}$$

We shall now calculate lateral waves in the controversial case of the reflection of a beam from an amplifying medium [Eq. (3)] using the just tested Sommerfeld radiation principle, which is formulated exactly in the same way as in the case of absorbing media. The reflected field is still given by the integral of Eq. (4a) along the same initial contour Γ_1. The only difference is in the positions of the branching points A_1 and A_2 and of the cuts originating from them (Fig. 3). The dashed curves in Fig. 3 represent cuts corresponding to the "principle of maximum amplification," which is ob-

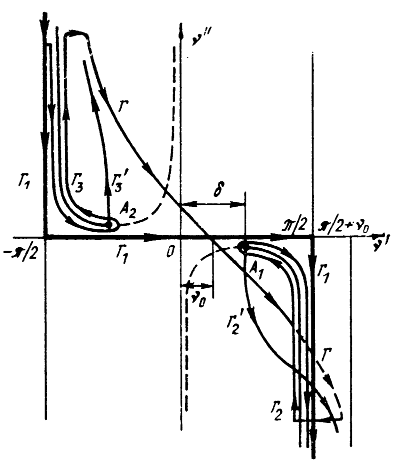

FIG. 3.

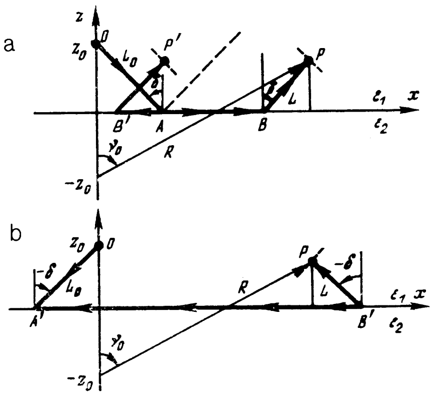

FIG. 4.

tained from the traditional principle of maximum absorption when we go over to linear amplifying media. The situation differs qualitatively from the case of passive media discussed above. In fact, for any angle of observation we have, in addition to the integral along the steepest-descent contour Γ, also integrals along the loop Γ_3 which goes round the cut originating from A_2 lying in the upper half-plane. When the angle of observation ν_0 exceeds the critical value ($\nu_0 > \delta$), the steepest-descent contour Γ intersects twice the cut from A_1 which is now in the lower half-plane and can be joined without difficulty to the ends of the original profile Γ_1. The need to go around the loop Γ_2 arises now at angles of observation lower than the critical value ($\nu_0 < \delta$) when the steepest-descent contour Γ intersects only once the cut in question. The corresponding integral along Γ_2 gives the second lateral wave.

The integrals along the loops Γ_2 and Γ_3 can be calculated using the steepest-descent paths Γ_2' and Γ_3' (see Fig. 3) using the same approximations as in the case of passive media ($|n''| \ll n'$):

$$E_3=-[2\pi^{-1}n(1-n^2)^{1/2}]^{1/2}[k_1R\sin(\delta+\nu_0)]^{-3/2}\tilde{E}_i(-\delta)\times\exp[ik_1R\cos(\delta-\nu_0)-i\pi/4], \qquad \nu_0<\delta, \tag{9a}$$

$$E_2=0, \qquad \nu_0>\delta, \tag{9b}$$

$$E_3=-[2\pi^{-1}n(1-n^2)^{1/2}]^{1/2}[k_1R\sin(\delta+\nu_0)]^{-3/2}\tilde{E}_i(-\delta)\times\exp[ik_1R\cos(\delta+\nu_0)-i\pi/4]. \tag{10}$$

Comparison of Eqs. (8) and (9) shows that the corresponding lateral wave E_p and E_2, associated with going around the cuts in the region of positive angles ν', different from zero along two mutually complementary parts of half-space. We can assume that on going over from an absorbing to an amplifying medium there is a change in the range of existence of the lateral wave E_p of Eq. (8) and this wave is converted into E_2 of Eq. (9). This is accompanied by the appearance of a new lateral wave E_3 which exists throughout the half-space.

The structure of lateral waves can be described conveniently by the ray representation. It is known[1] that in the case of reflection from passive media the phase of a lateral wave of Eq. (8) can be represented by an optical path along a ray *OABP* (Fig. 4a):

$$k_1R\cos(\nu_0-\delta)=k_1[(L_0+L)+n'L_1], \tag{11a}$$

where

$$L_0+L=(z_0+z)/\cos\delta, \tag{11b}$$

$$L_1=\mathrm{abs}[R\sin\nu_0-(z+z_0)\,\mathrm{tg}\,\delta]. \tag{11c}$$

In the case of the lateral wave described by Eq. (9) and formed as a result of reflection from an amplifying medium the phase shift (advance) occurs along a loop-like ray *OAB'P'*, for which the optical path is given by the same expressions as Eq. (11). The difference between these two cases lies in the sign of the argument of the absolute value on the right-hand side of Eq. (11c). In the section *AB'* there is amplification by a factor $\exp(-k_1n''L_1)$.

The ray representation of a "negative–angular" lateral mode E_3 of Eq. (10) is given in Fig. 4b. The phase shift (advance) for this wave is

$$k_1R\cos(\delta+\nu_0)=k_1[(L_0+L)-n'L_2], \tag{12a}$$

where

$$L_2=R\sin\nu_0+(z_0+z)\,\mathrm{tg}\,\delta, \tag{12b}$$

can be interpreted as the optical path along a "ray" *OA'B'P* with the section *A'B'* passing through the amplifying medium where the phase decreases and the sections *OA'* and *B'P* where it increases. A ray arrives at the point of observation *P* from the side opposite to the source (slit) and decreases in the region *A'B'* by a factor $\exp(k_1n''L_2)$. It readily follows from Eqs. (3) and (10) and from Figs. 1 and 4b that in order to observe a negative angular lateral wave E_3 of Eq. (10) the incident beam (3) is best directed at the total internal reflection angle $\varphi = -\delta$ in the direction away from the point of observation.

DISCUSSION OF RESULTS

We shall compare our results with the prediction of the structure of lateral waves made in Refs. 5 and 6. The fact that the predictions of Ref. 5 should generally differ from ours follows from the difference between the initial assumptions. However, the main source of discrepancies from Ref. 6, where as in our case the causality relationships are employed, must be identified more specifically. This is because in ensuring the causality for a reflected field in Ref. 6, no attempt was made to do the same for the refracted wave. In particular, the integration contour over the spatial spectrum, where an account is taken of the changes in the positions of the branching points in the complex plane k_x as a result of a shift of the contour of integration with respect to the complex frequencies ω, is justified in Ref. 6 regarding the directions of the cuts in the k_x plane as constant and the same as those characteristic of absorbing media.[2] However, application of the Sommerfeld radiation principle to the field in an amplifying medium, when the principle itself can be regarded as a consequence of causality,[7,8] requires a change of the direction of the cuts originating from the branching point k_{2z} in the k_x plane when they intersect the real axis, giving rise to results qualitatively different from those reported in Ref. 6.

In a comparison of the predictions in the present paper and those given in Refs. 5 and 6 the main interest is in the question how large are the differences which enable experimental confirmation of one specific theory. From this point of view a negative angular lateral wave of the E_3 type described by Eq. 10 is hardly a suitable object for observation, because its existence was predicted also in Refs. 5 and 6 and its amplitude is small in all the theories. Distinguishing characteristics more suitable for an experimental check are exhibited by a positive angular lateral wave (of Eq. 9) which—according to our results—disappears in the range of observation angles ν_0 greater than the total internal reflection angle δ, whereas the results of Refs. 5 and 6 predict a lateral wave of higher intensity in the same range.

In view of the difficulty of setting up experiments which would fit well the theoretical model, used here and in Refs. 5 and 6, of a semiinfinite homogeneous amplifying medium,[4] we checked whether the difficulties of obtaining such a medium can be avoided and a situation typical of reflection from an amplifying medium can be modeled by ensuring the same relative complex refractive index for reflection of a light beam incident from a strongly absorbing onto a weakly absorbing medium. An analysis of the more general case shows that the structure of lateral waves is close to E_p of Eq. (8), i.e., of an ordinary wave typical of reflection from passive media, which shows that the proposed way of overcoming the difficulty will not work. Clearly, a discussion of the reflection from amplifying media is most likely to become theoretical and aimed to resolve more specifically the problems of describing reflection from strongly resonant absorbing media mentioned in Ref. 7.

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