Theory of recombination radiation in 2D systems

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A theory is derived for recombination radiation from a quasi-two-dimensional ($2D$) system in a strong magnetic field for the case of a three-electron cluster in a short-range model (the first two quasipotentials are taken into account). In the recombination process, one of the electrons is captured by a local center. The latter accordingly acquires an electric charge, and the two other electrons move in the field of this charge. Since the competing energies are all on the same order of magnitude, the intensities of the transitions to the ground state of the system and to its excited states are comparable. The intensity distribution in the spectrum depends on the initial angular momentum and spin of the cluster; it also depends on the interaction of the electrons with the center in the final state. For numerical reasons, the transition to the ground state of the system is usually the strongest, but the intensities of the other transitions may be comparable in magnitude. Under these conditions the transmission band must be asymmetric, with a more gently sloping low-frequency side. The event in which light is emitted is accompanied by a strong interaction of the internal angular momentum and the angular momentum of the center of mass of the cluster. Trial functions with an arbitrary spin are proposed for a hard-core potential. There is the possibility of pronounced fluctuations in the spin ordering of the electrons, as a result of fluctuations in the Coulomb potential with a length scale on the order of several times the magnetic length (magnetic polarons).

Recent years have seen additions to the list of the systems and phenomena in which the electron-electron interaction in a $2D$ electron phase dominates. Coming under study in addition to Laughlin's spin-polarized liquid[1] have been spin-unpolarized or partially polarized phases,[2–6] whose formation in n-GaAs is promoted by the small value of the g-factor.[7] There are pieces of experimental evidence based on transport measurements which point to the appearance of such phases.[8,9] Optical methods which have recently been developed for studying the effects of electron–electron interactions in $2D$ systems[10–12] are opening up some new opportunities for observing restructuring of spectra near critical values of the filling factor ν, for measuring gaps, for observing the competition between the Zeeman and Coulomb interactions (which is manifested in a field dependence of the g-factor enhancement), and for determining the energies of elementary excitations from the spectra of recombination luminescence.

A circumstance of much importance for spectroscopic experiments of this type is that in a quantizing magnetic field H all the electron–electron interaction energies are of the same order of magnitude: $e^2/\varkappa l(H)$, where $\varkappa$ is the dielectric constant, and $l(H)$ the magnetic length. In a situation of this sort, the shake-up processes which accompany an optical transition in the electron gas are strong and cannot be dealt with by perturbation theory (in contrast with the case $H=0$, when one can carry out an expansion in[13,14] ε_F^{-1}, where ε_F is the Fermi energy). An important question in this connection is the relation among the intensities of the optical transition to the ground state of the system and of the transitions which are accompanied by Auger processes. Also of importance are the specific differences in the emission from spin-polarized and spin-unpolarized (or partially polarized) state. Finally, there is particular interest in the structural features which are caused in the spectrum in the region of fractional quantization by the presence of a condensate and gaps in the spectrum.

So far, there has been no comprehensive theoretical study of these questions, which pose some major difficulties. The spectral pattern should obviously depend very strongly on ν. In this paper we examine the emission spectrum on the basis of a model of a three-electron cluster. The optical transition involves the capture of one of the electrons by a center, which is neutral in the initial state and has an electric charge in the final state. An advantage of this extremely simplified model is that it can be solved analytically, so explicit expressions can be found for the transition intensities and frequencies. From this model we can draw several qualitative conclusions regarding the first two of the questions formulated above; the third requires further study. Some of these conclusions appear to be of general applicability. This is true, for example, of the construction of trial functions with various degrees of spin ordering and also of the conclusion that the spin ordering in the $2D$ phase is highly sensitive to variations in the electric potential.

1. THREE-PARTICLE SYSTEM IN A PERFECT CRYSTAL

The coordinate part of the wave function of the three-particle system in a strong field H can be written in the form

$$\psi(z_1, z_2, z_3)=F_M(z)\exp\{-{}^1/_4(|z_1|^2+|z_2|^2+|z_3|^2)\}, \quad (1)$$

$$F_M(z)=\sum_{\{m\}} c_{\{m\}} z_1^{m_1} z_2^{m_2} z_3^{m_3}, \quad m_1+m_2+m_3=M, \quad m_i\geqslant 0, \quad (2)$$

where z_i are the complex coordinates of the particles. A symmetric gauge has been adopted for the vector potential; the charge of the particles is $e>0$; the particles are in the lower Landau level; and $l(H)=1$. The Zeeman energy has been omitted, in accordance with a small g-factor (the situation in

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the case of n-GaAs). As in Ref. 15, in the construction of an equation for the coefficients $c_{\{m\}}$, the matrix elements of the binary-interaction potential,

$$\langle m_1 m_2 | V(|z_1 - z_2|) | m_1' m_2' \rangle,$$

written in the basis

$$\psi_m(z) = (2\pi 2^m m!)^{-1/2} z^m \exp\{-\tfrac{1}{4}|z|^2\}, \quad (3)$$

are conveniently rewritten through the introduction of a relative coordinate $(z_1 - z_2)$. Quasipotentials[15,16] corresponding to various angular momenta of the relative motion of the two particles then arise in a natural way:

$$V_m = \langle \varphi_m | V(|z|) | \varphi_m \rangle, \quad \varphi_m = (\pi m!)^{-1/2} 2^{-(m+1)} z^m \exp(-|z|^2/8). \quad (4)$$

For a Coulomb potential we would have $V_m = \pi^{1/2}(2m-1)!!/m! 2^{m+1}$; in particular, $V_0 = \pi^{1/2}/2 \approx 0.89$ and $V_1 = \pi^{1/2}/4 \approx 0.44$. The basic results can be derived through the use of only the two lowest angular momenta, i.e., through the retention of only V_0 and V_1. We then write

$$\lambda c_{\{m\}} \equiv \lambda c_{m_1 m_2 m_3} = \frac{(m_1+m_2)!}{m_1! m_2!} 2^{-(m_1+m_2)} \times \sum_{m_1' m_2'} \left\{ V_0 + \frac{(m_1-m_2)(m_1'-m_2')}{m_1+m_2} V_1 \right\} c_{m_1' m_2' m_3} + \text{c.p.}, \quad (5)$$

where $m_1' + m_2' = m_1 + m_2$, and the two similar terms, differing through a cyclic permutation (c.p.), have not been written out explicitly.

A spin-polarized system ($S = 3/2$) has been studied for short-range[15] and Coulomb[17] potentials. The quasipotential V_0 drops out by virtue of the Pauli principle, and from (5) we find

$$F_M \propto f_J(z_1, z_2, z_3) Z^{M_c}, \quad M = J + M_c,$$
$$f_J = (z_1 - z_2) \tilde{z}_3^{J-1} + \text{c.p.} \quad (6)$$

Here J and M_c are the angular momenta of the internal motion and of the center-of-mass motion, $Z = (z_1 + z_2 + z_3)/3$, $\tilde{z}_i = z_i - Z$, and there is degeneracy with respect to M_c. The function F_M is found from the Fok determinant for a spin-polarized gas, and it is proportional to the Vandermond determinant $W(z_1, z_2, z_3)$ by virtue of the well-known theorem.[18] The energy spectrum is described by

$$\lambda_J^{(1/2)} = V_1\{1 + (3J-4)/(-2)^{J-1}\}. \quad (7)$$

The levels oscillate as functions of J; there is degeneracy with respect to M_c.

In the state $S = 1/2$, $S_z = 1/2$ the coordinate wave function multiplying the spin factor $\alpha(1)$, $\alpha(2)\beta(3)$ is

$$f_J = a_0(\tilde{z}_1^J - \tilde{z}_2^J) + \tfrac{1}{3}a_1(Jv)^{1/2}\{\tilde{z}_1^{J-1}(z_2 - z_3) + \tilde{z}_2^{J-1}(z_3 - z_1) - 2\tilde{z}_3^{J-1}(z_1 - z_2)\}. \quad (8)$$

The function F_M is related to f_J as in (6), and $v = V_1/V_0$. A dispersion relation follows from this system:

$$\{\lambda - V_0[1 - (-2)^{-J}]\}a_0 - 3(-2)^{-J}(Jv)^{1/2} V_0 a_1 = 0,$$
$$-3(-2)^{-J}(Jv)^{1/2} V_0 a_0 + \{\lambda - V_1[1 + (3J-4)(-2)^{-J}]\}a_1 = 0. \quad (9)$$

For $J \leqslant 3$ there is a single polynomial which satisfies the symmetry conditions of the permutation group. Accordingly, the two terms in (8) are the same, and there is a single energy level:

$$\lambda_J^{(1/2)} = V_0[1 - (-2)^{-J}] + V_1[1 + (-2)^{-J}(3J-4)]. \quad (10)$$

For $J > 3$ the number of levels increases to two in accordance with the general properties of a system with $N = 3$, $S = 1/2$ (N is the number of particles; §63 in Ref. 19). The existence of an incompressible fluid results from the behavior of the hard-core potential,[16] which reduces, roughly speaking, to the dominance of the V_m with the smallest m. For a Coulomb potential $V(|z|)$, this condition actually holds. We accordingly assume $V_0 \gg V_1$. We see that for $J \leqslant 3$ we have $\lambda_J^{(1/2)} \sim V_0$ for all J. At $J \geqslant 4$, however, the smaller of the roots is

$$\lambda_J^{(1/2)} \approx V_1\left\{1 + \frac{3J-5}{(-2)^J} - \frac{3J-1}{2^{2(J-1)}}\right\} \Big/ \{1 - (-2)^{-J}\}, \quad (11)$$

and the corresponding wave function is

$$f_J \propto W(z_1, z_2, z_3)(z_1 + z_2 - 2z_3). \quad (12)$$

To leading order in $v \ll 1$, the function f_J thus has a first-order zero in terms of all differences $|z_i - z_j|$, regardless of the spin state of the corresponding electrons. In contrast with the $S = 3/2$ case, the appearance of W here is not dictated by the permutation-symmetry conditions. The scale value $\lambda_J^{(1/2)} \sim V_1$ arises because of the zeros in (12) with respect to all differences $|z_i - z_j|$. The upper root of Eq. (9) is described by $\lambda_j^{(1/2)} \sim V_0$.

It is interesting to note that according to (11) the quantity $\lambda_J^{1/2}$ oscillates as a function of J; the oscillations in (7) and (11) are out of phase. At a given value of J, the energies are very different; for example, $\lambda_5^{(3/2)} \approx 3\lambda_5^{(1/2)}$. If, however, we compare the minimum values of $\lambda^{(3/2)}$ and $\lambda^{(1/2)}$, namely $\lambda_6^{(3/2)} = 0.5625$ and $\lambda_5^{(1/2)} = 0.6136$, we find that the difference between them is considerably smaller. We also need to allow for the circumstance that a difference in J indicates a difference in the average densities of these states, as follows from an estimate based on the customary formula $\nu = N(N-1)/2M$. If we equate the values of M and ν, considering a state $S = 1/2$ with $J = 5$, $M_c = 1$, and $M = 6$, we find that the difference remains at a level $\sim 10\%$. If, however, we assume $M_c = 0$ and adopt a dependence $\lambda \propto \nu^{1/2}$, we find that, by virtue of the value $(6/5)^{1/2}\lambda_6^{(3/2)} = 0.6162$, the difference from $\lambda_5^{(1/2)}$ decreases to $\sim 0.5\%$. Remarkably, for homogeneous systems the typical difference in the energies between states with different spin orderings is $\sim 1\%$ (Refs. 5 and 6). In all of these comparisons one must of course bear in mind that for a cluster, which is a highly inhomogeneous system, the reduction to an average ν is extremely arbitrary. Furthermore, one should not underestimate the pronounced difference between $\lambda_J^{(1/2)}$ and $\lambda_J^{(3/2)}$ at a given value of J. It seems natural to suggest that this difference indicates a pronounced change in the magnetic order in regions with a significant short-range change in the Coulomb potential, which leads to the formation of clusters. We are essentially talking about the formation of magnetic polarons in such regions.

The observation above that it is possible to represent the wave functions of the low-lying states as in (12) can be gen-

eralized to systems with an arbitrary number of particles N and an arbitrary total spin S. Such states of course exist under the condition $\nu<1$; for $N=3$, this condition corresponds to $J\geqslant 4$. We construct the functions in accordance with the customary Young tableau.[19] We assume that the determinant $W(z)$ of all arguments appears in f as a factor. We denote the coordinates z_j which correspond to the first column of the Young coordinate tableau by x_i, while those corresponding to the second column are y_i. The function of N arguments (for definiteness, we let N be even)

$$f(z)\propto W(x,y)\prod_{i=1}^{N/2-S}(x_i-y_i) \tag{13}$$

is symmetric with respect to all rows. This symmetry follows from the antisymmetry of W with respect to all arguments. A subsequent antisymmetrization in both columns leads to the expression

$$f(z)\propto W(z)\sum_k(-1)^k(2S+k)!\left(\frac{N}{2}-S-k\right)!\sigma_{N/2-S-k}(x)\sigma_k(y), \tag{14}$$

where σ_k are symmetric polynomials of degree k; e.g.,

$$\sigma_2(y)=y_1y_2+y_1y_3+y_2y_3+\ldots. \tag{15}$$

With $S=0$, expression (14) reduces to

$$f(z)\propto W(z)\,\mathrm{per}\|x_i-y_j\|. \tag{16}$$

The permanent (per) in (16) differs from a determinant in that a symmetrization instead of an antisymmetrization is carried out in it. Function (14) describes a state with a spin S, an energy $\lambda\sim V_i$ and $\nu<1$. Since the degree of $W(z)$ is $N(N-1)/2$, while the degree of the second factor in (14) is $(N/2)-S$, in the macroscopic limit $N\to\infty$ the filling factor approaches unity, $\nu\to 1$. The function given by (16) (multiplied by W^{2p}, where p is an integer) was recently proposed[4] as a trial function for the state $S=0$, $\nu=1/(2p+1)$. Its accuracy for a many-electron system can be judged to some extent on the basis of the results for $N=3$. $V_0=2V_1$, then the exact value is $\lambda_4^{(1/2)}=1.125V_1$. The use of (12) as a trial function yields $\lambda_4^{(1/2)}=1.25$; i.e., the error is $\approx 10\%$. The approximate expression (11) leads to $\lambda_4^{(1/2)}\approx 1.35$. A considerably poorer result, $\lambda_4^{(1/2}\approx 1.5$, is found if we omit the first term in (8). That term corresponds to configurations which are unfavorable from the energy standpoint, in which two particles are simultaneously in an $m=0$ orbital. The mechanism for the lowering of the energy by $\approx 30\%$ associated with the incorporation of this term might be interpreted in terms of resonant mixing.[5] The incorporation of unfavorable configurations makes possible a resonance $z_1^4z_2^0\rightleftarrows z_1^3z_2$, which results in a substantial lowering of the total energy.

2. TWO-PARTICLE SYSTEM IN THE FIELD OF A DEFECT

If there is a three-particle system in the initial state, and if one of the particles recombines with a neutral center, than the final state will have a two-particle system which is moving in the field of a charge center (a situation analogous to that of Refs. 10 and 11). The system of equations corresponding to (5) is

$$\lambda c_{m_1m_2}=\frac{(m_1+m_2)!}{m_1!m_2!}2^{-(m_1+m_2)}\times\sum_{m_1'm_2'}\left\{V_0+\frac{(m_1-m_2)(m_1'-m_2')}{m_1+m_2}V_1\right\}c_{m_1'm_2'}+(U_{m_1}+U_{m_2})c_{m_1m_2}. \tag{17}$$

Here

$$U_m=\langle\psi_m|U(|z|)|\psi_m\rangle, \tag{18}$$

$U(|z|)$ is the potential of the charge center, and the functions ψ_m are determined by (3). If we assume that $U(|z|)$ is the potential of a Coulomb center which is lying in the plane of a $2D$ layer, then we have $U_m=2^{1/2}V_m$; i.e., the potential of the defect is not small. This potential does weaken if the defect lies outside the $2D$ layer. We will restrict the discussion below to the first two pseudopotentials, U_0 and U_1. When there is a symmetric defect, the only quantum number is the total angular momentum M. In the triplet state (a spin-polarized system) we would have

$$\lambda_1^{(1)}=V_1+(U_0+U_1),\ F_1\propto(z_1-z_2).$$

The dispersion relation for the higher-order angular momenta is

$$\frac{V_1}{\lambda}\left\{1+\frac{1}{2^{M-1}}\left[M\frac{U_0}{\lambda-U_0}+(M-2)^2\frac{U_1}{\lambda-U_1}\right\}=1,\ M\geqslant 2. \tag{19}$$

At $M\leqslant 4$ there are only two independent functions which satisfy the antisymmetry condition, so the number of roots of Eq. (19) is two (the trivial root $\lambda=0$ has no physical meaning). A third root appears at $M=5$. The wave functions are

$$F_M^{(1)}\propto(z_1-z_2)(z_1+z_2)^{M-1}+\frac{U_0}{\lambda-U_0}(z_1^M-z_2^M)+(M-2)\frac{U_1}{\lambda-U_1}z_1z_2(z_1^{M-2}-z_2^{M-2}). \tag{20}$$

In the $S=0$ singlet state at $M\geqslant 3$ the dispersion relation takes the form

$$\frac{\lambda}{V_0}=1+\frac{1}{2^{M-1}}\left\{\frac{U_0}{\lambda-U_0}+M\frac{U_1}{\lambda-U_1}\right\}, \tag{21}$$

and the wave functions become

$$F_M^{(0)}\propto(z_1+z_2)^M+\frac{U_0}{\lambda-U_0}(z_1^M+z_2^M)+M\frac{U_1}{\lambda-U_1}z_1z_2(z_1^{M-2}+z_2^{M-2}). \tag{22}$$

The fact that the dispersion relation for a spin-unpolarized state contains V_0 alone (not V_1) for all values of M is a specific feature of the two-particle problem. If the number of particles were large, the lower part of the spectrum with $\lambda\sim V_1$ would have developed, in total analogy with Sec. 1. This is a significant limitation of this model, which may leave its imprints on an analysis of the emission spectrum (Sec. 3).

It is convenient to write the eigenfunctions in terms of the coordinates of the particles [(8), (20), and (22)] because the expressions will then hold for arbitrary M. Practical calculations are often more conveniently carried out in terms of relative coordinates.

3. OPTICAL TRANSITIONS

Here are the basic assumptions which we will make below in the calculation of transition probabilities: Transitions are allowed; the spatial size of the wave function of the impurity center at the point $z = 0$ is small in comparison with $l(H)$; the Zeeman splitting at the impurity center can be ignored; and M is conserved by virtue of the presence of a symmetry axis. The probability w for a transition from the state M, J, S_i of the three-particle system to the state M, S of the two-particle system satisfies the proportionality

$$w \propto (\Phi_{MJ}^{(S_i)}, \Psi_M^{(S)})^2, \tag{23}$$

where $\Psi_M^{(S)}(1,2)$ is the complete coordinate-spin wave function of the final state. In order to find $\Phi_{MJ}^{(S_i)}$ from the complete wave function of the initial state, $\Psi_{MJ}^{(S_i)}(1,2,3)$, we need to set $Z_3 = 0$ in the latter and discard the spin wave function of the corresponding electron. The wave functions $\Psi_M^{(S)}$ and $\Psi_{MJ}^{(S_i)}$ are constructed in the standard way from the functions F_M and f_J given in Secs. 1 and 2. We adopt states which are low on the energy scale, with a fixed value of J, as initial states.

The initial state $S_i = 3/2$

We set $U_1 = 0$, and for the moment we restrict the discussion to the case $M_c = 0$. Transitions are possible only to a triplet state. The behavior of w as a function of J at $J \geqslant 3$ (average density $\nu \gtrsim 1$) is given by

$$w \propto \frac{1}{J}\left(\frac{2}{3}\right)^{J-1}\left[1+\frac{3J-4}{(-2)^{J-1}}\right]\left(\eta^2+\frac{2^{J-1}}{J}-1\right)^{-1}, \tag{24}$$

where $\eta_\pm = \lambda_\pm/(\lambda_\pm - U_0)$, and $\lambda_\pm$ are the roots of Eq. (19) ($\lambda_+ > \lambda_-$). The difference between the values of w for transitions to the levels $\lambda_\pm$ is determined by the last factor. A calculation of the electron density at the point $z = 0$ on the basis of the function (20) yields

$$\rho(z=0) \propto \eta^2\left[\eta^2+\frac{1}{J}2^{J-1}-1\right]^{-1}. \tag{25}$$

Since ρ increases monotonically with increasing η, w reaches a maximum for that root for which $\rho(z=0)$ is at a minimum. Since the capture by the impurity occurs at the point $z = 0$, $\rho(z)$ is lowered at this point, and the result found here shows that the transition probability is at a maximum for a transition to that state in which this dip cannot be "filled in" by the electron–electron interaction. Less definite is the answer to the question of which of the roots λ_+, λ_- corresponds to the larger value of w. As a rule, w is larger for the transition to the lower level, λ_-. The opposite situation is possible only if $(1 - M/2^{M-2})V_1 > U_0$. A necessary condition for the satisfaction of this inequality is $J \geqslant 5$. In the most favorable case, $J \to \infty$, the condition $V_1 > U_0$ is sufficient. The comparative estimates of V_i and U_i given above show that in general we would naturally expect the opposite inequality to hold.

Figure 1a shows the behavior of the energy levels and of the intensities of the transitions to these levels from the $J = 3$, $M_c = 0$ state as a function of the parameter $u = U_0/V_1$. The intensity of the transition to level λ_+ falls off rapidly with increasing u. Parts b and c of Fig. 1 show the corresponding behavior for the cases in which the initial states are the $M_c = 1$ and $M_c = 2$ states. With increasing M_c, the relative intensity of the transitions to the excited

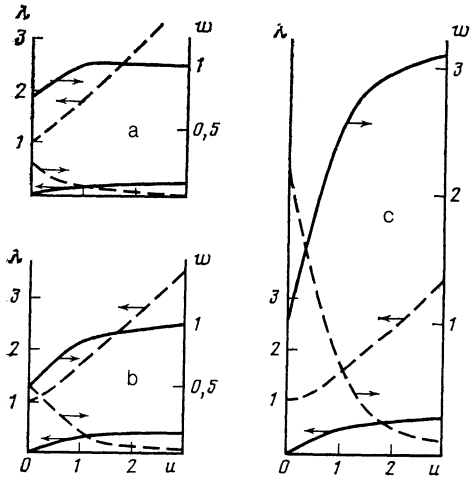

FIG. 1. The energy levels $\lambda_\pm$ of a two-particle system in the field of a quasipotential U_0 in the $S = 1$ state and probabilities for transitions to these levels from the $S = 3/2$, $J = 3$ three-particle state versus the dimensionless potential of the defect, $u = U_0/V_1$. Solid lines—The lower level, λ_-; dashed lines—λ_+. The λ scale is in units of V_1; the intensity w is in arbitrary units, the same for all three parts of the figure. a) $M_c = 0$; b) $M_c = 1$; c) $M_c = 2$.

state of the system increases; at $M_c = 2$, these transitions dominate if $u \ll 1$. Their relative importance falls off rapidly with increasing u, however, so that the relation between the intensities becomes the opposite at $u \gtrsim 1$. This behavior is also seen at larger values of M_c. If, for example, we adopt as the initial states the states with $J = 3$ and arbitrary M_c then we have

$$w \propto \frac{1}{2}\left(\frac{4}{3}\right)^{M_c}\frac{(M-1)(M-2)/M}{\eta^2+(2^{M-1}/M)-1}, \qquad M=3+M_c. \tag{26}$$

Determining $\eta_\pm$ from (19) with $U_1 = 0$, we see that the increase in w with M_c at small values of u which is evident from Fig. 1 gives way to a decrease in the limit $M_c \to \infty$. In this region the exponential decrease in w with increasing M_c occurs over the entire range of values of u. For the stronger band the behavior is $(2/3)^{M_c}$, and that for the weaker band is $(1/3)^{M_c}$. In the region $u < 1$, the low-frequency band, which corresponds to a transition to the excited state of the center, is strong, while for $u > 1$ the high-frequency band, corresponding to a transition to the ground state of the center, is strong.

Interestingly, in the course of the recombination at the defect there is an intense interaction of the angular momenta J and M_c, which correspond to the internal motion and to the center-of-mass motion. The two motions are coupled through the recombining particle, which is involved in both motions. This effect is general in nature. For example, in the macroscopic limit, in regions of fractional quantization, it corresponds to an exchange of angular momentum between quasiparticles and the condensate.

The initial state $S_i = 1/2$, $J = 4$, $M_c = 0$

We chose the $J = 4$ state as the initial state, since at this value of J the three-particle system first acquires levels with $\lambda \sim V_1$. A distinctive feature of this situation is that the emission spectrum consists of two parts, which correspond to

transitions to singlet ($S=0$) and triplet ($S=1$) final states. All the expressions for $\lambda_{\pm}$ and $w_{\pm}$ can be derived on the basis of Secs. 1 and 2. At $M=4$, transitions to the $S=1$ state are allowed even in the very simple model with $U_1=0$. Transitions to the levels $\lambda_{\pm}$, with $S=0$, on the contrary, are allowed only in first order in U_1/V_1 and V_1/V_0. For $S=0$, $U_1=0$, however, there is in addition a strong allowed transition to the $\lambda=0$ state, which acquires,for small U_1, a finite energy $\lambda\approx 3U_1/7$. Figure 2 shows the level scheme of the two-particle system and the distribution of the amplitudes A corresponding to the intensities of transitions to the corresponding levels ($w\propto A^2$) for two sets of parameter values. We note the following aspects of these spectra. First, transitions to the singlet (spin-unpolarized) state are on the whole stronger than the transitions to the triplet state. This is true despite the fact that two of the three transitions to the singlet state are forbidden in the lowest-order theory. The parameter values which allow these transitions, however, are actually 1. Second, although the transitions to the states which lie low on the energy scale are the most intense in all of the spectra in Fig. 2 (at any rate, within the "singlet" or "triplet" series), transitions to excited states are fully competitive in terms of intensity. This tendency is particularly clear in the singlet spectrum in Fig. 2a, which corresponds to a realistic relation among the values of the competing parameters. The distance between the frequencies of the individual bands is on the order of V_1—the characteristic energy of the Coulomb interaction.

We have one final comment, of a general nature. We did not introduce a Zeeman energy above, so we did not calculate the g-factor enhancement. It is nevertheless clear that since the frequencies of all the transitions include the difference between the Coulomb energies in the initial and final states, it is this difference which will determine the g-factor enhancement which is optically measurable. The relative magnitudes of the various contributions depend on the specific optical experiment. On the whole, however, we would not expect the results found in optical measurements to agree with those found in transport[20] measurement, since the latter are determined by the exchange energy[21] only in the state which is the initial state in a luminescence experiment.

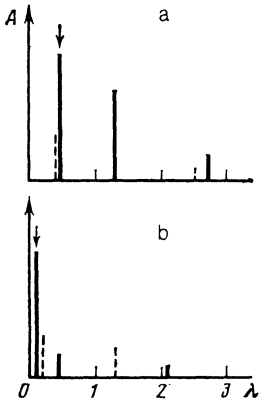

FIG. 2 Positions of the energy levels λ of the two-particle system in the field of quasipotentials U_0 and U_1 and probability amplitudes A for transitions to these states from the $S=1/2$, $J=4$, $M_c=0$ level of the three-particle system. Solid lines—Amplitudes of transitions to the $S=0$ states; dashed lines—to the $S=1$ states (the λ scale is in units of V_1; A is given in arbitrary units, but the same for both parts of the figure). The arrows mark transitions to the low-lying singlet level, $\lambda\approx 3U_1/7$. a) $V_0\approx 2V_1$, $U_0\approx 2V_1$, $U_1\approx V_1$; b) $V_0\approx 2V_1$, $U_0\approx 0.5V_1$, $U_1\approx 0.25V_1$.

4. CONCLUSION

In the problem of the spectroscopy of $2D$ liquids, it is an extremely complicated matter to describe optical transitions which involve a strongly interacting electronic system. For example, one cannot use the approximation of a self-consistent field here, in contrast with the spectroscopy of multiexciton-impurity complexes in the Dean-Kirzhenov model. In contrast with the Mahan-Nozières problem in the theory of x-ray spectra, we cannot ignore the Coulomb interaction of the electrons in this case. Our recourse in this situation consists of palliative approaches which start from an examination of very simple models. Analysis of the model of a three-particle cluster leads to the following basic conclusions.

In agreement with the fact that in a strong magnetic field all the competing energies in the electronic system are comparable in magnitude to the energy of the Coulomb (exchange) interaction, the intensities of the recombination transitions to the ground state and to electronically excited states are comparable in magnitude. For numerical reasons, the energy of the transition to the ground state of the system is usually larger, but transitions to the excited states may be competitive in terms of intensity. One would thus expect an asymmetry of the emission band, with an extended long-wavelength side. The half-width of the band would be on the order of the characteristic energy of the electron–electron interaction. If the initial state of a cluster is spin-unpolarized ($S=1/2$), then the spectrum of transitions to the singlet final state will be the most intense. In the course of the emission of light, there is a strong interaction between the internal angular momentum of the cluster and the angular momentum of the center of mass; the shape of the spectrum depends strongly on the magnitude of the latter angular momentum in the initial state of the system.

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