Characteristics of resonant *LO* scattering in disordered semiconductor solid solutions

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(Submitted 14 September 1987; resubmitted 21 November 1988)
Zh. Eksp. Teor. Fiz. **96**, 330–343 (July 1989)

An analysis is made of the influence of a random potential with Gaussian white noise statistics on light scattering accompanied by the excitation of a longitudinal optical (*LO*) phonon, which is of interest in studies of mixed semiconductor crystals where a random potential appears as a result of fluctuations of crystal components. A theoretical description of exciton states in a random potential is developed allowing for their multiple scattering by this potential. The probability of the scattering of light in the range of localized exciton states is calculated by a new method suitable for multistage processes and based on the one-instanton approximation utilizing the approach of Lipatov, Brezin and Parisi. The results can be simplified greatly in the region of the ground state of an exciton, both below and above the maximum of an absorption band due to this stage. Such properties of the 1*LO* scattering process as the dependence of the cross section on the frequency of the exciting light at the temperature of a crystal can then be expressed in terms of simple physical characteristics of the exciton stages. It is shown that a combined analysis of the data on the scattering and absorption of light in the region of the ground state of an exciton provides full information on the exciton lifetime both in the range of localized states and in the range of positive energies where an exciton is scattered weakly by a random potential. A good agreement is obtained with the available experimental data.

1. INTRODUCTION

Investigations of disordered isoelectronic solid solutions of semiconductor crystals[1–7] have shown that exciton states undergo major changes under the influence of a fluctuation potential created by a random distribution of atoms of the components of a crystal. The results of such investigations require the use of modern field theory methods, which go beyond perturbation theory,[8–13] and provide interesting opportunities for experimental confirmation of the theoretical predictions in a wide range of the values of the coupling constant.

Optical experimental methods currently used in studies of solid solutions include reflection, absorption, luminescence, and resonance scattering of light. Information on the spectral density of excitons is provided by the deflection and absorption spectra, but systematic data on the absorption are not yet available. In many cases it is easier to study the processes of emission of light as a result of resonant excitation.[4–7] Such processes carry information on the nature and properties of the resonant states of an exciton, and also on the lifetimes and symmetry of its localized states.[6] Scattering characterized by the loss of one longitudinal optical vibration quantum (1*LO* scattering) is observed in a spectral range quite far from an exciting line. This and the low dispersion of the *LO* branch make it easier to identify and study it. In the case of excitation to localized exciton states the 1*LO* scattering process is indistinguishable from luminescence.

The 1*LO* scattering in the "forbidden" geometry, when the interaction of an exciton with *LO* phonons is described by the intraband Fröhlich interaction, is affected by a random potential to a greater extent than other scattering processes of the first or higher orders. This is due to a redistribution of the contributions made to the scattering amplitude by various states in the Rydberg exciton series.

In the case of perfect crystals the amplitude of the 1*LO* scattering in the forbidden geometry is a complex function of the wave vector $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ transferred in the scattering process, where $\mathbf{k}$ and $\mathbf{k}'$ are the wave vectors of the incident and scattered photons, and of the frequency ω of the exciting light.[14–16] Consequently, in a wide region $\omega \sim \varepsilon_g$ near a resonance (ε_g is the width of the band gap) the main contribution to the amplitude comes from those exciton states which satisfy the condition $aqn^2 \approx 1$, where a is the Bohr radius of an exciton, n is the principal quantum number, and deeper states with n such that $aqn^2 < 1$ are effectively excluded from the scattering process.

In the case of isoelectronic solid solutions[17,18] and crystals containing defects[19–21] there is no short-range order and, therefore, the law of conservation of momentum is not obeyed so that $\mathbf{q} \neq \mathbf{k} - \mathbf{k}'$. Consequently, the main contribution to the scattering amplitude comes from deep states of an exciton and the form factors of these states impose fewest restrictions on the phonon wave vector, particularly on the ground state of an exciton. The 1*LO* scattering process resembles a two-phonon process because of the occurrence of the elastic scattering of excitons by defects or by a random potential.

Our aim will be to derive a relationship between the characteristics of the 1*LO* scattering process in disordered solid solutions excited in the range of localized excitons and the corresponding characteristics of delocalized states of excitons, and also such characteristics of excitons as the spectral density and the probability of scattering accompanied by the emission of two *LO* phonons.

2. MAIN RELATIONSHIPS

We shall assume that the motion of an exciton in a solid solution is governed by the large-scale potential with the Gaussian white noise statistics. The problem of the motion of an exciton in such a random potential can be reduced to

0038-5646/89/070186-08$04.00

the single-particle case by assuming that the potential acts on a heavy particle, i.e., on a hole, and that the hole mass is considerably greater than the electron mass $m_v \gg m_c$. Then, in the limiting case of low localization energies of an exciton $r_l \gg a$ (r_l is the exciton localization radius) the random potential acts on the motion of the center of mass of an exciton and in the limiting case of $r_l \ll a$ an electron is bound by the Coulomb potential to a deeply localized hole. When the difference between the electron and hole masses is large, the constants of the interaction with the random potential and the Coulomb energies differ little in these two limiting cases: the difference is of the order of m_c/m_v, so that we can expect a single-particle approach to be valid in a wide range of parameters. The real situation in isoelectronic solutions with anion substitution[1,3-6] is probably close to that discussed here.

We shall consider only the first of these limiting cases and represent the Hamiltonian of an exciton in the form

$$H=H_{\mathbf{R}}+h_{\mathbf{r}}, \quad H_{\mathbf{R}}=-\frac{\hbar}{2M}\Delta_{\mathbf{R}}-V(\mathbf{R}),$$
$$h_{\mathbf{r}}=-\frac{\hbar}{2\mu}\Delta_{\mathbf{r}}-\frac{e^2}{\varepsilon_0 r},$$
$$\langle V(\mathbf{R})V(\mathbf{R}')\rangle=w^2\delta(\mathbf{R}-\mathbf{R}'), \quad \langle V(\mathbf{R})\rangle=0, \qquad (1)$$
$$M=m_c+m_v, \quad \mu=m_c m_v/(m_c+m_v).$$

Here, H_r and h_r are the Hamiltonians of the motion of the center of mass of an exciton and of its relative motion; ε_0 is the effective permittivity of the investigated crystal. The other limiting case gives the Hamiltonian described by Eq. (1), but with M replaced with m_v and μ with m_c.

The Green function of an exciton can be represented in the form

$$\mathscr{G}_{\mathbf{kk}'}(\mathbf{r},\mathbf{r}',\omega)=\sum_\lambda \varphi_\lambda(\mathbf{r})\mathscr{G}^\lambda_{\mathbf{kk}'}(\omega-\omega_\lambda)\varphi_{\lambda'}^*(\mathbf{r}'), \qquad (2)$$

where $\varphi_\lambda(r)$ represents the normalized wave functions of the relative motion of an exciton, which are not affected by the random potential when the Hamiltonian is selected in the form given by the system (1); ω_λ are the eigenvalues of the Hamiltonian h_r; $\mathscr{G}^\lambda_{kk'}$ is the Green function of the motion of the center of mass of an exciton and differs for different states λ only by a shift of the energy by ω_λ. In the absence of the fluctuation potential, we have

$$\mathscr{G}_{\mathbf{kk}'}(\omega-\omega_\lambda)=\delta(\mathbf{k}-\mathbf{k}')g_{\mathbf{k}}^\lambda(\omega), \qquad (3)$$

$$g_{\mathbf{k}}^\lambda(\omega)=\left[\omega-\frac{\hbar k^2}{2M}-\omega_\lambda+i\delta\right]^{-1}. \qquad (4)$$

We shall measure energies from the bottom of the band of the $1s$ state in the virtual-crystal approximation, i.e., we shall assume that $\omega_{1s}=0$.

The interaction of an exciton with longitudinal optical phonons can be represented in the form

$$H^F_{\lambda\lambda'}(\mathbf{q})=g_{LO}\Omega_0{}^{LO}F^{\lambda\lambda'}_{\mathbf{q}}, \quad g_{LO}=\left[\frac{e^2}{2a\varepsilon_0}\frac{1}{\hbar\Omega_0{}^{LO}}\left(\frac{\varepsilon_0}{\varepsilon_\infty}-1\right)\right]^{1/2}, \qquad (5)$$

$$F_{\mathbf{q}}(\mathbf{r})=\frac{(4\pi)^{1/2}}{qa}[\exp(i\mathbf{q}^c\mathbf{r})-\exp(-i\mathbf{q}^v\mathbf{r})], \quad q^{c,v}=q\frac{\mu}{m_{c,v}}.$$

The amplitude of the scattering accompanied by the emission of one phonon can be written as follows:

$$A_{\mathbf{k}-\mathbf{k}'}(\omega)=g_{LO}\Omega_0{}^{LO}H^{em}\mathscr{G}_{\mathbf{kp}}(0,\mathbf{r};\omega)F_{\mathbf{q}}(\mathbf{r})$$
$$\times\,\mathscr{G}_{\mathbf{p}-\mathbf{q};\,\mathbf{k}'}(\omega')H^{em}2\pi\delta(\omega-\omega'-\Omega_{\mathbf{q}}{}^{LO}). \qquad (6)$$

Here, H^{em} is the Hamiltonian of the electromagnetic interaction, and ω and ω' are the frequencies of the incident and scattered photons. Equation (6) implies integration with respect to $d^3\mathbf{r}$ and $d^3\mathbf{p}$. Using the representation of Eq. (2) and omitting factors which will be unimportant in subsequent analysis, we shall consider the expression

$$a^{\lambda\lambda'}_{\mathbf{k}-\mathbf{k}'}(\omega)=\mathscr{G}_{\mathbf{k};\mathbf{p}}(\omega-\omega_\lambda)F^{\cdot\cdot}_{\mathbf{q}}\;\mathscr{G}^{\lambda'}_{\mathbf{p}-\mathbf{q};\mathbf{k}'}(\omega'-\omega_{\lambda'}). \qquad (7)$$

Calculation of the probability requires averaging over configurations of the square of the modulus of the scattering amplitude. We shall consider this expression in the specific case when $a^{\lambda\lambda'}_{\mathbf{k}-\mathbf{k}'}(\omega)$. We shall do this by representing the function being averaged in the form

$$\langle|a|^2\rangle=|\langle a\rangle|^2+\langle|\delta a|^2\rangle. \qquad (8)$$

The averaging restores the full symmetry of a crystal, so that the average value of the amplitude $\langle a\rangle$ is of the dipole-forbidden nature, exactly as in a perfect crystal.[14-16] Therefore, the contribution to the scattering cross section of the ground state of an exciton because of $\langle a\rangle$ is proportional to a small quantity $(\mathbf{k}-\mathbf{k}')^2a^2$; the contributions of all other states are suppressed by broadening of the exciton states in solid solutions. The observed enhancement of the scattering in the region of a resonance and its new properties in solid solutions should be described by the amplitude fluctuations δa.

We shall find later the average Green function $\langle\mathscr{G}^\lambda_{\mathbf{kq}}\rangle$ of an exciton. Since the averaging procedure does not affect the internal motion of an exciton, we shall drop the index λ. Bearing in mind the relationship between the Green function and the scattering matrix,[22,23] we shall write down $\langle\mathscr{G}^\lambda_{\mathbf{kq}}\rangle$ in the form

$$\langle\mathscr{G}_{\mathbf{kq}}(\omega)\rangle=\delta(\mathbf{k}-\mathbf{q})g_{\mathbf{k}}(\omega)+g_{\mathbf{k}}(\omega)\langle T_{\mathbf{kq}}(\omega)\rangle g_{\mathbf{q}}(\omega). \qquad (9)$$

As a result of averaging over the Gaussian distribution of the potential the problem reduces to that of a field theory characterized by φ^4 (Ref. 10). The averaging procedure restores the homogeneity of a crystal, so that

$$\langle T_{\mathbf{kq}}(\omega)\rangle=\delta(\mathbf{k}-\mathbf{q})T_{\mathbf{kk}}(\omega) \qquad (10)$$

and $T_{\mathbf{kk}}$ can be expressed in terms of the irreducible part of the scattering matrix $\tau_{\mathbf{kk}}$:

$$T_{\mathbf{kk}}(\omega)=\tau_{\mathbf{kk}}(\omega)[1-g_{\mathbf{k}}(\omega)\tau_{\mathbf{kk}}(\omega)]^{-1}. \qquad (11)$$

Therefore, $\langle\mathscr{G}\rangle$ can be represented in the form

$$\langle\mathscr{G}_{\mathbf{kq}}(\omega)\rangle=\delta(\mathbf{k}-\mathbf{q})G_{\mathbf{k}}(\omega), \qquad (12)$$

where

$$G_{\mathbf{k}}(\omega)=g_{\mathbf{k}}(\omega)[1-g_{\mathbf{k}}(\omega)\tau_{\mathbf{kk}}(\omega)]^{-1}. \qquad (13)$$

The imaginary part of $G_{\mathbf{k}}(\omega)$ found for the case $k\approx 0$ governs the normalized (to unity) coefficient of absorption of light at a frequency ω accompanied by the formation of a 1s exciton state:

$$\alpha(\omega)=\frac{1}{\pi}\frac{\Gamma_{00}(\omega)}{[\omega-\Delta_{00}(\omega)]^2+\Gamma_{00}^2(\omega)}, \tag{14}$$

where

$$\Gamma_{\mathbf{kk}}(\omega)=\operatorname{Im}\tau_{\mathbf{kk}}(\omega),\quad \Delta_{\mathbf{kk}}(\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\Gamma_{\mathbf{kk}}(z)\,dz}{\omega-z}. \tag{15}$$

Since the spectral dependence of the scattering cross section includes also the functions $G_{k\approx 0}(\omega)$ and $G_{k'\approx 0}(\omega')$, it follows that the knowledge of the spectral density of the ground state of an exciton makes it possible to use the experimental data on the absorption of light in an analysis of the scattering experiments. We shall show that the scattering experiments provide information on the behavior of the imaginary part of the scattering matrix $\Gamma_{00}(\omega)$ in the region of the ground state of an exciton when $\omega>0$. This function can also be reconstructed from the spectral density of the 1s state of an exciton. In fact, according to Ref. 24, we have

$$\Gamma_{00}(\omega)=\frac{1}{\pi}\frac{\alpha(\omega)}{r^2(\omega)+\alpha^2(\omega)},\quad \Delta_{00}(\omega)=\omega-\frac{1}{\pi}\frac{r(\omega)}{r^2(\omega)+\alpha^2(\omega)}, \tag{16}$$

where

$$r(\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\alpha(z)\,dz}{\omega-z}, \tag{17}$$

and energies are measured in an experimental spectrum in such a way that the first moment of the spectral density vanishes. In the limit of large negative energies we can calculate $\Gamma_{\mathbf{kk}}(\omega)$ explicitly and the result will be given later.

Using Eq. (12) and restoring the indices λ, we find that (apart from small corrections) the quantity $\langle a\rangle$ is described by

$$\langle a_{\mathbf{k}-\mathbf{k}'}^{\lambda\lambda'}(\omega)\rangle=G_{\mathbf{k}}^{\lambda}(\omega)F_{\mathbf{q}}^{\lambda\lambda'}G_{\mathbf{k}'}^{\lambda'}(\omega')\delta(\mathbf{q}-\mathbf{k}+\mathbf{k}'). \tag{18}$$

The deviation of the Green function from its value averaged over the configurations is given, in the first approximation, by

$$\delta\mathscr{G}_{\mathbf{kq}}=g_{\mathbf{k}}\delta t_{\mathbf{kq}}g_{\mathbf{q}},\quad \delta t_{\mathbf{kq}}=t_{\mathbf{kq}}-\delta(\mathbf{k}-\mathbf{q})\tau_{\mathbf{kk}}, \tag{19}$$

where $t_{\mathbf{kq}}$ is the quantum-mechanical scattering matrix for the specific configuration. Further complications of this expression, associated with inclusion of corrections for the average value of the scattering matrix, have the structure

$$g_{\mathbf{k}}\tau_{\mathbf{kk}}g_{\mathbf{k}}\delta t_{\mathbf{kq}}g_{\mathbf{q}}+g_{\mathbf{k}}\delta t_{\mathbf{kq}}g_{\mathbf{q}}\tau_{\mathbf{qq}}g_{\mathbf{q}}+\ldots \tag{20}$$

Summation of progressions gives the following expression for $|\delta a|^2$:

$$\begin{aligned}|\delta a_{\mathbf{k}-\mathbf{k}'}^{\lambda\lambda'}(\omega)|^2=|&G_{\mathbf{k}}^{\lambda}\delta t_{\mathbf{kq}}^{\lambda}G_{\mathbf{q}}^{\lambda}F_{\mathbf{q}-\mathbf{k}'}^{\lambda\lambda'}G_{\mathbf{k}'}^{\lambda'}+G_{\mathbf{k}}^{\lambda}F_{\mathbf{q}+\mathbf{k}}^{\lambda\lambda'}G_{\mathbf{q}}^{\lambda'}\delta t_{\mathbf{qk}'}^{\lambda'}G_{\mathbf{k}'}^{\lambda'}\\&+G_{\mathbf{k}}^{\lambda}\delta t_{\mathbf{kp}}^{\lambda}G_{\mathbf{p}}^{\lambda}F_{\mathbf{q}}^{\lambda\lambda'}G_{\mathbf{p}-\mathbf{q}}^{\lambda'}\delta t_{\mathbf{p}-\mathbf{q}:\mathbf{k}'}^{\lambda'}G_{\mathbf{k}'}^{\lambda'}|^2.\end{aligned} \tag{21}$$

A calculation of the quantity in Eq. (21) reduces to averaging of the products of the scattering matrices. The general properties of the fluctuation-induced contributions of the type described by Eq. (21), which are evident even before averaging, are the faster (compared with the contribution of the amplitude-average) fall away from a resonance and the vanishing in the absence of the random potential.

3. CALCULATION OF AVERAGED FUNCTIONS

We shall consider the task of calculating the average

$$\left\langle\prod_{\nu=1}^{L}t_{\mathbf{k}_\nu\mathbf{k}'_\nu}(\omega_\nu)\right\rangle \tag{22}$$

in the range of large negative values using the one-instanton approximation, i.e., in the approximation which is linear in respect of the density of the bound states. For each matrix we shall use the replica representation; right from the beginning it is convenient to separate one of the matrices and to use its frequency argument to introduce the following dimensionless variables:

$$\begin{aligned}t_{\mathbf{k}_\nu\mathbf{k}'_\nu}(\omega_\nu)=\omega_\mu\left(\frac{\hbar}{2M\omega_\mu}\right)^{3/2}&\lim_{n\to 0}Z_{n\nu}^{-1}\int\prod_{i=1}^{n}D[\Psi_{i\nu}]\\&\times\{\hat{J}_{1\nu}(\mathbf{k}_\nu)\hat{J}_{1\nu}(\mathbf{k}_\nu')\exp(-S^\nu)\}_{\beta_\nu=0}.\end{aligned} \tag{23}$$

The complete replica space consists of L orthogonal subspaces. The operator $\hat{J}$ acting on the function of S^ν is

$$\hat{J}_{1\nu}(\mathbf{k}_\nu)=\int d^3\mathbf{x}(2\pi)^{-3/2}\exp(i\mathbf{y}_\nu\mathbf{x})\Psi_{1\nu}(\mathbf{x})\frac{\partial}{\partial\beta_\nu(\mathbf{x})}, \tag{24}$$

$$x=\left(\frac{2M\omega_\mu}{\hbar}\right)^{1/2}r,\quad y_\nu=\left(\frac{\hbar}{2M\omega_\mu}\right)^{1/2}k_\nu,\quad \omega_\mu=|\omega_\mu|, \tag{25}$$

$$S^\nu=S_0^\nu+S_\nu',\quad S_0^\nu=\frac{1}{2}\int d^3\mathbf{x}\left[(\nabla\Psi_\nu)^2+\frac{\omega_\nu}{\omega_\mu}\Psi_\nu^2\right], \tag{26}$$

$$(\nabla\Psi_\nu)^2=\sum_{i=1}^{n}(\nabla\Psi_{i\nu})^2,\quad \Psi_\nu^2=\sum_{i=1}^{n}\Psi_{i\nu}^2, \tag{27}$$

$$S_\nu'=\frac{1}{2}\int d^3\mathbf{x}\left\{\frac{V(\mathbf{x})}{\omega_\mu}[\Psi_\nu^2(\mathbf{x})+2\beta_\nu(\mathbf{x})]\right\}, \tag{28}$$

$$Z_{n\nu}=\int\prod_{i=1}^{n}D[\Psi_{i\nu}]\exp(-S_0^\nu). \tag{29}$$

Averaging mixes the replica subspaces and gives rise to an interaction of the type

$$S(\beta)=-\frac{g_\mu}{4}\sum_{\nu,\nu_1}\int d^3\mathbf{x}(\Psi_\nu^2+2\beta_\nu)(\Psi_{\nu_1}^2+2\beta_{\nu_1})=-\frac{g_\mu}{4}J_4(\beta), \tag{30}$$

where

$$g_\mu=\frac{w^2}{2\omega_\mu^2}\left(\frac{2M\omega_\mu}{\hbar}\right)^{3/2}. \tag{31}$$

The total action after averaging is

$$S=\sum_\nu S_0^\nu+S(\beta=0),$$

i.e., it can be regarded as the sum of L independent functionals together with their cross interaction. The one-instanton approximation corresponds to the contribution Eq. (22) which is linear in respect of the density of bound states. We shall separate this contribution by finding and summing the contributions of all the saddle points in the functional of the action and calculate them independently. In finding the contribution of a saddle point with a number μ, which gives rise to a potential well with a level ω_μ located in the coordinate space at a point R^0_μ, we recall that all the other functional integrals differ from zero because of the cross interaction, i.e., because of fluctuations in the vicinity of this well with its center point R^0_μ irrespectively of the value of the energy argument of this or other matrix. Analysis of the case with more than one saddle point is outside the scope of the one-instanton approximation. In our calculations we shall adopt the method of Refs. 10–13.

After reversal of the sign of the constant g_μ, we shall separate the Kth term of the asymptotic series

$$\frac{1}{2\pi i}\oint\frac{dg}{g^{K+1}}e^{-S(\beta)}=\frac{(-1)^K}{K!}\exp[K\ln J_4(\beta)]. \tag{32}$$

The result of the action operator $\hat{J}$ is

$$\int\frac{d^3\mathbf{x}}{(2\pi)^{3/2}}\frac{4K}{J_4(0)}\sum_\mu \Psi_\mu^2(\mathbf{x})\Psi_{1\nu}(\mathbf{x})\exp(i\mathbf{y}_\nu\mathbf{x}). \tag{33}$$

The conditions for an extremum of the total action are given by the equations ($\beta=0$)

$$\frac{\delta S}{\delta\Psi_{i\nu}}=-\Delta\Psi_{i\nu}+\frac{\omega_\nu}{\omega_\mu}\Psi_{i\nu}-\frac{4K}{J_4(0)}\sum_\mu\Psi_\mu^2\Psi_{i\nu}=0. \tag{34}$$

We shall consider a saddle point with a number μ and assume that a nontrivial solution exists only for the μth subspace:

$$\begin{gathered}\Psi_{i\mu}=\left(\frac{4K}{I_4}\right)^{1/2}\Phi_{iC},\qquad \Phi_{iC}=u_i\Phi_C,\qquad u^2=1,\\ \Psi_{i\nu\neq\mu}=0,\qquad I_4=\int d^3\mathbf{x}\,\Phi_C^4(\mathbf{x}).\end{gathered} \tag{35}$$

The equation for Φ_c is

$$-\Delta\Phi_C+\Phi_C-\Phi_C^2\Phi_C=0. \tag{36}$$

The operators for quantum fluctuations in the μth subspace have the usual form[13]

$$\begin{gathered}M_L^\mu=[-\Delta+1-3\Phi_C^2(\mathbf{x})]\delta(\mathbf{x}-\mathbf{x}')+\frac{4\Phi_C^3(\mathbf{x})\Phi_C^3(\mathbf{x}')}{I_4},\\ M_T^\mu=[-\Delta+1-\Phi_C^2(x)]\delta(\mathbf{x}-\mathbf{x}'),\end{gathered} \tag{37}$$

and the corresponding determinants $D(1)$ and $D(1/3)$ are calculated after separation of collective variables and regularization,[12] using the eigenvalues and eigenfunctions of the equation

$$[-\Delta+1-3\lambda_n\Phi_C^2(\mathbf{x})]\Psi_n(\mathbf{x})=0. \tag{38}$$

Our calculations gave for $D(1/3)$ and $D(1)$ values amounting to 1.471 and 10.56 when 957 eigenvalues of Eq. (38) were included exactly; this result was in good agreement with that reported in Ref. 12. In the case of the remaining $(L-1)$ subspaces the fluctuation operators are

$$M_L^{\nu\neq\mu}=M_T^{\nu\neq\mu}=\left[-\Delta+\frac{\omega_\nu}{\omega_\mu}-\Phi_C^2(\mathbf{x})\right]\delta(\mathbf{x}-\mathbf{x}'). \tag{39}$$

A comparison of Eq. (39) with the expressions in Ref. 25 shows that the operator of longitudinal fluctuations does not appear in the latter case. It is clear from Eq. (39) that in all these subspaces, where $\nu\neq\mu$, the determinants cancel out for $n=0$. The corresponding functional integrals give preexponential functions in the calculation of which we shall use the eigenfunctions and the eigenvalues of the Schrödinger equation:

$$\left[-\Delta+\frac{E_\sigma}{\omega_\mu}-\Phi_C^2(\mathbf{x})\right]\Psi_\sigma(\mathbf{x})=0. \tag{40}$$

This equation has one bound state $\sigma=1s$, $E_{1s}=\omega_\mu$, and the wave function of this state is identical with the solution of the classical equation (36):

$$\Psi_{1s}(\mathbf{x})=\left(\frac{4}{I_4}\right)^{1/2}\Phi_C(\mathbf{x}).$$

Calculation of the functional integral for $\nu\neq\mu$ gives the usual expression for the scattering matrix

$$t^{\Phi}_{\mathbf{k}_\nu\mathbf{k}'_\nu}\left(\frac{\omega_\nu}{\omega_\mu}\right)=\omega_\mu^2\left(\frac{\hbar}{2M\omega_\mu}\right)^{3/2}\sum_\sigma\frac{J_\sigma(\mathbf{k}_\nu)J_\sigma(\mathbf{k}_\nu')}{\omega_\nu-E_\sigma+i\delta}, \tag{41}$$

where

$$J_\sigma(\mathbf{k}_\nu)=\int\frac{d^3\mathbf{x}}{(2\pi)^{3/2}}\exp(i\mathbf{y}_\nu\mathbf{x})\Phi_C^2(\mathbf{x})\Psi_\sigma(\mathbf{x}).$$

The results of the calculation of the functional integral for the μth subspace after summation of an asymptotic Borel series[10] can be written n the form

$$\begin{gathered}(2\pi)^3\delta\left(\sum_{\nu=1}^{L}(\mathbf{k}_\nu-\mathbf{k}_\nu')\right)\frac{C_{\mathbf{k}_\mu\mathbf{k}_\mu'}}{\Lambda^2}\int_0^\infty\frac{e^{-t}\,dt}{1+\Lambda t},\\ \Lambda=\frac{16\pi}{I_4}\left(\frac{\omega_0}{\omega_\mu}\right)^{1/2},\qquad \omega_0^{1/2}=\frac{w^2}{8\pi}\left(\frac{2M}{\hbar}\right)^{3/2}.\end{gathered} \tag{42}$$

The δ function appears as a result of integration with respect to three collective variables $\mathbf{R}^0_\mu$, which govern the position of the solution of Eq. (36) in the coordinate space. This solution relates the arguments of all the t matrices by a shared conservation law, because in all subspaces the quantum calculations fit the same solution of Eq. (36). We then have

$$\begin{gathered}C_{\mathbf{k}_\mu\mathbf{k}'_\mu}=\frac{\omega_\mu}{\pi^2}I_3(\mathbf{k}_\mu)I_3(\mathbf{k}_\mu')\left[\left(\frac{I_6-I_4}{3I_4}\right)^3\frac{D(1/3)}{D(1)}\right]^{1/2},\\ I_3(\mathbf{k}_\mu)=(2\pi)^{-3/2}\int d^3\mathbf{x}\exp(i\mathbf{y}_\mu\mathbf{x})\Phi_C^3(\mathbf{x})(4/I_4)^{1/2},\\ I_n=\int d^3\mathbf{x}\,\Phi_C^n(\mathbf{x}).\end{gathered} \tag{43}$$

The numerical values of these quantities are as follows:

$$I_3((0)=31{,}688(4/I_4)^{1/2}/(2\pi)^{3/2},\ I_4=75{,}585,\ I_6=659{,}928.$$

Analytic continuation of Eq. (42) to the range of negative coupling (interaction) constants, calculation of the pole contribution to the integral with respect to dt, and summation of the results of calculations obtained for all L saddle points give the final result which can be represented conveniently in the form

$$(2\pi)^3\sum_{\mu=1}^{L}\delta\Big(\sum_{\nu=1}^{L}(\mathbf{k}_\nu-\mathbf{k}_\nu')\Big)\int_{-\infty}^{0}\rho(z)\,dz$$

$$\times\Big[i\,\mathrm{Im}\,t^{\Phi}_{\mathbf{k}_\mu\mathbf{k}'_\mu}\Big(\frac{\omega_\mu}{z}\Big)\Big]_{1s}\prod_{\nu\neq\mu}t^{\Phi}_{\mathbf{k}_\nu\mathbf{k}'_\nu}\Big(\frac{\omega_\nu}{\omega_\mu}\Big).\qquad(44)$$

Here, $[\mathrm{Im}\,t^{\Phi}]_{1s}$ is the imaginary part of the pole contribution to Eq. (41) and $\rho(z)$ is the density of bound states in the one-instanton approximation:

$$\rho(\omega)=\rho_0(\omega)\frac{C}{g^2}\exp\Big\{-\frac{I_4}{16\pi}\frac{1}{g}\Big\},\qquad(45)$$

$$\rho_0(\omega)=\frac{1}{4\pi^2}\Big(\frac{2M\omega}{\hbar}\Big)^{3/2}\frac{1}{\omega},\qquad \omega=|\omega|,$$

$$C=\frac{I_4^2}{64\pi^2}\Big[\Big(\frac{I_6-I_4}{3I_4}\Big)^3\frac{D(1/3)}{D(1)}\Big]^{1/2},\qquad g=\frac{w^2}{8\pi\omega^2}\Big(\frac{2M\omega}{\hbar}\Big)^{3/2}$$

We can see from Eq. (21) that the expression for the scattering probability contains terms with pairs of complex-conjugate t matrices and with energy arguments differing only in the sign of the imaginary correction. These terms determine the probability and accurate calculation of their values requires introduction of finite damping of bound states $\gamma^\lambda(\omega)$. The poles of the complex-conjugate t matrices then shift relative to one another by an amount $2i\gamma^\lambda(\omega)$. Their contributions to Eq. (44) are additive, so that the sign of the damping governs also the sign of the pole contribution to the integral with respect to dt in Eq. (42).

The expression (44) can be used also to calculate the average value of one matrix. It is then imaginary, i.e., it governs the imaginary part of the pole term of the scattering matrix

$$\Gamma_{\mathbf{kk}}(\omega)=(2\pi)^3\int_{-\infty}^{0}\rho(z)\,dz\Big[\mathrm{Im}\,t^{\Phi}_{\mathbf{kk}}\Big(\frac{\omega}{z}\Big)\Big]\quad,\qquad(46)$$

where $\rho(z)$ is defined by Eq. (45).

4. CALCULATION OF THE SCATTERING EFFICIENCY

The first two terms in the square of the modulus on the right-hand side of Eq. (21) correspond to different sequences of the processes of the elastic scattering of an exciton and its interaction with a phonon. The third term also creates two contributions which are revealed by averaging and are combined with the first and second terms, respectively.

We shall give the results of the averaging for a specific sequence of the processes of the elastic scattering of an exciton and its interaction with a phonon (we shall assume that the elastic scattering occurs first) at a resonance on the exciting frequency scale.

In the range of localized exciton states the main contribution comes from multiple poles of the scattering matrix:

$$\langle\delta a^2_{\lambda\lambda_1\lambda_2\lambda}\rangle=\frac{\mathrm{Im}\,G_{\mathbf{k}}^{\lambda}(\omega)}{\gamma^\lambda(\omega)}\int d^3\mathbf{q}\,[M_{\lambda\lambda_1}(\mathbf{q}+\mathbf{k}-\mathbf{k}')\,G_{\mathbf{k}'}^{\lambda_1}(\omega')\,G_{\mathbf{k}'}^{\lambda_2}(\omega')$$

$$\times\ M_{\lambda_2\lambda}(-\mathbf{q}-\mathbf{k}+\mathbf{k}')\,]\,2\pi\delta(\omega-\omega'-\Omega^{LO}_{\mathbf{q}-\mathbf{k}'}).\qquad(47)$$

The total damping $\gamma^\lambda(\omega)$ of a bound exciton state in a fluctuation potential depends also on the state of the Rydberg series to which the exciton belongs:

$$M_{\lambda\lambda'}(\mathbf{q}+\mathbf{k}-\mathbf{k}')=\frac{\Psi_{1s}(\mathbf{q})F^{\lambda\lambda'}_{\mathbf{q}-\mathbf{k}'}}{D_{\mathbf{q}}^{\lambda}(\omega)}$$

$$+\int d^3\mathbf{p}\Big\{\frac{\Psi_{1s}(\mathbf{p})F^{\lambda\lambda'}_{\mathbf{q}-\mathbf{k}'}}{D_{\mathbf{p}}^{\lambda}(\omega)}[G^{\lambda}_{\mathbf{p}-\mathbf{q}+\mathbf{k}'}(\omega')\cdot$$

$$\times\ (t^{\Phi}_{\mathbf{p}-\mathbf{q}+\mathbf{k}',\mathbf{k}'}(\omega')-\delta(\mathbf{p}-\mathbf{q})\,\tau_{\mathbf{k}'\mathbf{k}'}(\omega'))\,]\Big\},\qquad(48)$$

$$D_{\mathbf{q}}^{\lambda}(\omega)=[1-g_{\mathbf{q}}^{\lambda}(\omega)\,\tau_{\mathbf{qq}}(\omega)\,],$$

$$\Psi_{1s}(\mathbf{q})=\frac{1}{(2\pi)^{3/2}}\int d^3\mathbf{x}\,e^{iy\mathbf{x}}\Psi_{1s}(\mathbf{x}).$$

Here, $y=(\hbar/2M\omega)^{1/2}q$; $\Psi_{1s}(\mathbf{q})$ is the normalized (to unity) wave function of a localized state of the center of mass of an exciton.

If ω is positive, we can use the fact that the square of the modulus of the first term in Eq. (21) includes a contribution to the existence of a pole $G_{\mathbf{q}}(\omega)$, which is of the form

$$\langle\delta a^2_{\lambda\lambda_1\lambda_2\lambda}\rangle=|G_{\mathbf{k}}^{\lambda}(\omega)|^2\int d^3\mathbf{q}\langle|\delta t_{\mathbf{kq}}^{\lambda}(\omega)|^2\rangle$$

$$\times|G_{\mathbf{q}}^{\lambda}(\omega)|^2F^{\lambda\lambda_1}_{\mathbf{q}-\mathbf{k}'}G_{\mathbf{k}'}^{\lambda_1}(\omega')\,G_{\mathbf{k}'}^{\lambda_2}(\omega')$$

$$\times F^{\lambda_2\lambda}_{-\mathbf{q}+\mathbf{k}'}2\pi\delta(\omega-\omega'-\Omega^{LO}_{\mathbf{q}-\mathbf{k}'}).\qquad(49)$$

In the limit of sufficiently large positive values of ω, when $(\hbar q^2/2M)>\mathrm{Im}\,\tau_{\mathbf{qq}}(\omega)$ (i.e., when the Ioffe–Regel' criterion is obeyed), we can make the substitution:

$$|G_{\mathbf{q}}^{\lambda}(\omega)|^2\approx\pi\delta\Big(\omega-\omega_\lambda-\frac{\hbar q^2}{2M}\Big)\Big/\Gamma^{\lambda}_{\mathbf{q}_0\lambda\mathbf{q}_0\lambda}(\omega),$$

$$\Gamma^{\lambda}_{\mathbf{q}_0\lambda\mathbf{q}_0\lambda}(\omega)=\mathrm{Im}\,\tau^{\lambda}_{\mathbf{q}_0\lambda\mathbf{q}_0\lambda}(\omega),\quad q_0^{\lambda}=\Big(\frac{2M|\omega-\omega_\lambda|}{\hbar}\Big)^{1/2}.$$

(50)

The function $\Gamma^{\lambda}_{q_0\lambda q_0\lambda}(\omega)$ represents the low-temperature probability of the elastic scattering of an exciton by a fluctuation potential, whereas at high temperatures we must include in $\Gamma^{\lambda}_{q_0\lambda q_0\lambda}(\omega)$ also the processes of the interaction with phonons.

Bearing in mind that for positive energies we have

$$\int d^3\mathbf{q}\,t_{\mathbf{kq}}^{\lambda}(\omega)\,t_{\mathbf{qk}}^{\lambda}(\omega)\,\pi\delta\Big(\omega-\omega_\lambda-\frac{\hbar q^2}{2M}\Big)=\mathrm{Im}\,t_{\mathbf{kk}}^{\lambda}(\omega),$$

(51)

we find after averaging that

$$\langle\delta a^2_{\lambda\lambda_1\lambda_2\lambda}\rangle=[\mathrm{Im}\,G_{\mathbf{k}}^{\lambda}(\omega)/\Gamma^{\lambda}_{\mathbf{q}_0\lambda\mathbf{q}_0\lambda}(\omega)]$$

$$\times\ [F^{\lambda\lambda_1}_{\mathbf{q}_0-\mathbf{k}'}G^{\lambda_1}_{\mathbf{k}'}(\omega')\,G^{\lambda_2}_{\mathbf{k}'}(\omega')\,F^{\lambda_2\lambda}_{-\mathbf{q}_0+\mathbf{k}'}]$$

$$\times\ 2\pi\delta(\omega-\omega'-\Omega^{LO}_{-\mathbf{q}_0+\mathbf{k}'}).\qquad(52)$$

The expressions (47), (48), and (52) give the dependences of the $1LO$ scattering probability on the frequency of the exciting light and on the parameters of a crystal in two parts of the spectrum important in experiments and located in the vicinity of the frequency of a resonance of the incident light with the ground state of an exciton. Under these conditions the main contribution to the scattering amplitude comes from the ground state of an exciton. We shall consider later this contribution in greater detail.

5. DISCUSSION OF RESULTS

The fluctuation terms in the scattering probability of Eqs. (47) and (52) describe, in contrast to Eq. (18), an allowed process, and (as shown below) ensure that the efficiency of the scattering per unit volume of a crystal increases in the region of a resonance, whereas the scattering described by the average value of the amplitude of Eq. (18) becomes weaker than in the case of a perfect crystal because of the broadening of the spectral density of an exciton. In fact, the absorption coefficient for the ground state of Eq. (14) has its maximum value at $\omega_{\max} = \Delta_{00}(\omega_{\max})$, which apart from a factor can be described by

$$\alpha(\omega_{max}) \sim 1/\pi\Gamma_{kk}(\omega_{max}). \qquad (53)$$

The quantity $\Gamma_{kk}(\omega_{\max})$ is close in value to the half-width of an absorption band. As soon as the broadening of the exciton states due to the random potential exceeds the homogeneous damping $\gamma_{kk}(\omega)$ (or the longitudinal—transverse splitting in the case of the ground state), the cross section for the scattering in a resonance with these states becomes weaker by a factor $[\gamma_{kk}(\omega)/\Gamma_{kk}(\omega)]^2$, compared with a perfect crystal. Therefore, the cross section described by the average amplitude of Eq. (18) reaches a value comparable with that for a perfect crystal only when the exciting frequency shifts by $\delta\omega \gg \Gamma_{kk}(\omega_{\max})$ from a resonance.

In the range $\omega < 0$ the fluctuation contribution to the cross section is given by Eqs. (47) and (48). When $|\omega|$ is sufficiently high, Eq. (48) is dominated by the term with t^{Φ} which does not contain any additional small factors. The results of the calculations can be represented in the form

$$\frac{2\pi g_{LO}^2}{\gamma^{1s}(\omega)} \frac{\Gamma_{kk}(\omega)}{[\omega-\Delta_{kk}(\omega)]^2+\Gamma_{kk}^2(\omega)} J_{LO}(\omega)\left(\frac{I_3(\mathbf{k}')\Omega_0^{LO}}{\omega-\Omega_0^{LO}}\right)^2, \qquad (54)$$

where the integral

$$J_{LO}(\omega)=\left(\frac{4}{I_4}\right)^2\left(\frac{2M|\omega|a^2}{\hbar}\right)^{3/2}$$
$$\times\int\frac{d^3\mathbf{y}}{(2\pi)^3}\left|\int d^3\mathbf{x}\,\Phi_C^2(\mathbf{x})e^{i\mathbf{y}\mathbf{x}}\right|^2 |F_{\mathbf{q}}^{1s1s}|^2,$$
$$y=q(\hbar/2M|\omega|)^{1/2} \qquad (55)$$

gives the dependence of the strength of the Fröhlich interaction on the depth ω of a bound state. This dependence is plotted in Fig. 1.

The above expression describes the scattering involving the states of excitons localized in isolated fluctuation wells each of which contains one level. Such wells are the most probable and they determine the asymptote of the density of states in the fluctuation potential.[8-10,12] In the case of systems with a weak exciton–phonon interaction at temperatures which are low compared with the localization energy, an exciton can only escape by radiative recombination.

Equation (54) is derived on the assumption that the

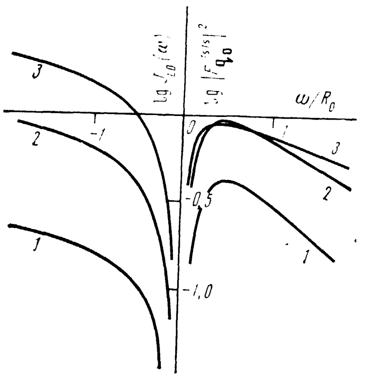

FIG. 1. The dependence of the integral $J_{LO}(\omega)$ on the depth of a localized exciton state is shown on the left ($\omega < 0$). The dependence of the form factor of the exciton-phonon interaction on ω is shown on the right ($\omega > 0$); $q_0 = (2M\omega/\hbar)^{1/2}$, and R_0 is the binding energy of an exciton. Curves 1–3 correspond to $m_v/m_c = 2$, 4, and 8, respectively.

nature of the broadening of an exciton state in a well corresponds to a weak exciton–phonon interaction similar to an assumption made earlier in Ref. 5. Under this condition a level broadens into a band consisting of a narrow zero-phonon line and its acoustic phonon wing, whereas the interaction with the optical phonons shifts the whole pattern by frequencies which are multiples of the optical phonon frequency. The half-width of the zero-phonon line $\gamma^{1s}(\omega)$ for an isolated well is a quantum limit which is zero because decay processes involving phonons are impossible. We can therefore expect the low-temperature half-width to be limited by the radiative processes. This picture is supported qualitatively by the observed[5] strong increase in the quantum efficiency of the luminescence emitted due to localized excitons. The existence of a narrow zero-phonon line is essential for the appearance of narrow *LO* lines in the scattering spectrum. When the temperature is increased, such a narrow zero-phonon line in the spectrum of a localized exciton broadens and is transferred to the acoustic wing. This should be accompanied by "quenching" of narrow *LO* scattering lines and their replacement by wide bands due to simultaneous participation of optical and acoustic phonons.

A large parameter which governs an increase in the scattering in the region of localized states of an exciton, compared with a perfect crystal, is the ratio $\Gamma_{kk}(\omega)/\gamma^{1s}(\omega)$. The scattering reaches an asymptote described by Eq. (18) when this ratio becomes less than or of the order of unity.

It should be noted that the dependence of the second factor in Eq. (54) on ω can be deduced from independent measurements of the absorption coefficient for the 1*s* state of an exciton. Therefore, simultaneous determination of $\alpha(\omega)$ and of the scattering cross section gives new information on $J_{LO}(\omega)/\gamma^{1s}$ and, consequently, new information about the dependence of γ^{1s} on the depth ω of a bound state.

In the limit of strong absorption of the incident light, when the scattering length is governed by the depth of penetration of light into a crystal, i.e., by the reciprocal of the absorption coefficient, the dependence of the cross section is identical with the dependence of J_{LO}/γ^{1s} apart from the denominator $(\omega - \Omega_0^{LO})^{-2}$.

In the $\omega > 0$ range the scattering is described by Eq. (52). Near the ground state of an exciton, if we assume that $\lambda = 1s$ in all cases, we find that

$$\frac{2\pi g_{LO}{}^2}{\Gamma_{\mathbf{q}_0\mathbf{q}_0}(\omega)}\frac{\Gamma_{\mathbf{kk}}(\omega)}{[\omega-\Delta_{\mathbf{kk}}(\omega))^2+\Gamma_{\mathbf{kk}}{}^2(\omega)}|F_{\mathbf{q}_0}^{1s1s}|^2\left(\frac{\Omega_0{}^{LO}}{\omega-\Omega_0{}^{LO}}\right)^2 \tag{56}$$

Here, $q_0=(2M\omega/\hbar)^{1/2}$; $\Gamma_{\mathbf{q}_0\mathbf{q}_0}(\omega)$ represents the total probability of the elastic scattering of an exciton in its ground state and of its interaction with phonons. The formal similarity of Eqs. (54) and (56) is worth noting.

The range of validity of Eq. (56) is limited by the inequality $\Gamma_{\mathbf{q}_0\mathbf{q}_0}(\omega)<\omega<\Omega_0^{LO}$, which follows from restrictions imposed on the transformation (50) and from the fact that we are considering a contribution at resonance with the incident light. This is the strong absorption region where for $\omega\gtrsim e^4\mu/2\hbar^3\varepsilon_0^2$ the absorption is governed by transitions to excited states of an exciton and its continuous spectrum. Consequently, the depth of penetration of light into a crystal is small and approximately constant. The general nature of the frequency dependence of the scattering cross section is governed by two resonance denominators which are shifted by the phonon frequency. Figure 1 shows the calculated dependence of the form factor $|F_{\mathbf{q}_0}^{1s1s}|^2$ on ω. If we eliminate from the experimental dependence the integrated scattering intensity in this region $|F_{\mathbf{q}_0}^{1s1s}|^2$ and the last factor in Eq. (56), we obtain the function $\alpha(\omega)/\Gamma_{q_0q_0}(\omega)$.

Thermal quenching of the scattering process in the range $\omega>0$ begins, as demonstrated by Eq. (56), at temperatures sufficiently high so that the temperature-dependent phonon contribution to $\Gamma_{\mathbf{q}_0\mathbf{q}_0}(\omega)$ becomes comparable with the contribution of the elastic scattering and with the temperature-independent quantum limit of the phonon contribution.

Additional information on the matrix describing the scattering of an exciton by the fluctuation potential in the range $\omega>0$ can be obtained by comparing the intensities of the scattering characterized by the emission of one and two *LO* phonons. The calculation method described above provides an opportunity of finding also the probability of multiphonon processes. If we restrict ourselves to the range $\Gamma_{\mathbf{q}_0\mathbf{q}_0}<\omega<\Omega_0^{LO}$, we find that

$$\frac{\sigma_{1LO}(\omega)}{\sigma_{2LO}(\omega+\Omega_0{}^{LO})}\approx\frac{\Gamma_{\mathbf{kk}}(\omega)}{\Omega_0{}^{LO}}\frac{\Omega_0{}^{LO}}{2\Gamma_{\mathbf{kk}}{}^{LO}(\omega)}\left(\frac{\Omega_0{}^{LO}+\omega}{\omega}\right)^2, \tag{57}$$

where

$$\Gamma_{\mathbf{kk}}^{LO}(\omega)=\pi g_{LO}{}^2\Omega_{LO}\frac{a^3}{(2\pi)^3}\int d^3\mathbf{q}|F_{\mathbf{k-q}}^{1s1s}|^2\delta\left(\omega-\frac{\hbar q^2}{2M}\right).$$

In the calculation of Eq. (57) it is assumed that a two-phonon process is excited at a frequency $(\omega+\Omega_0^{LO})$. It is clear from Eq. (57) that the ratio $\Gamma_{\mathbf{kk}}(\omega)/\Omega_0^{LO}$determines the effectiveness of the scattering of an exciton by the fluctuation potential in the 1*LO* resonant scattering process. A comparison with Eq. (16) shows that $\Gamma_{\mathbf{kk}}(\omega)$ can also be found from the data on the spectral density of the 1*s* state of an exciton.

Considering the influence of fluctuations of the composition of a solid solution on the process of the scattering involving *LO* phonons, we restricted ourselves to the electron subsystem on the assumption that the influence of fluctuations of the composition on the phonons is weak at least in that part of the lattice spectrum which participates in the scattering processes, i.e., fluctuations influence weakly the long-wavelength optical phonons. In fact, in the case of solutions of III-V and II-VI compounds investigated so far the lines of one- and many phonon scattering processes are almost as narrow as in the case of perfect crystals. The high intensities of these lines compared with the other scattering processes demonstrates that the mechanism responsible for their appearance in the spectra of solid solutions is the Fröhlich interaction, as in the case of perfect crystals. This is supported also by the selection rules which are obeyed by the intensities of these lines. The additional (compared with perfect crystals) broadening of the *LO* scattering lines of solid solutions is small compared with the *LO–TO* splitting $(\Omega_0^{LO}-\Omega_0^{TO})$, which governs the average value of the Fröhlich interaction constant for each *LO* mode in a crystal (here, *TO* denotes transverse optical phonons). Hence we can assume that the fluctuation correction to the Fröhlich interaction constant should also be small.

An analysis shows that investigations of the scattering processes involving longitudinal optical phonons together with a determination of the spectral density of the ground state of an exciton can extend greatly the range of information which can be obtained about exciton states in disordered solid solutions.

The authors regard it as their pleasant duty to thank A. S. Ioselevich for a detailed discussion of a number of topics considered above.

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Translated by A. Tybulewicz