Nonequilibrium superconductivity in a fluxon medium

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The nonequilibrium phenomena responsible for the development of threshold instability in long Josephson junctions are investigated by means of a multilayer tunnel structure fabricated from tin films separated by a dielectric barrier. It is demonstrated that the nature of the onset of the instability differs from the development of threshold instability in point tunnel junctions. The results are interpreted based on an analysis of a kinetic equation derived in the study that accounts for the features of fluxon motion.

INTRODUCTION

Nonequilibrium phenomena in superconductor tunnel junctions have stimulated a great deal of interest on the part of researchers.[1] These phenomena can be described by kinetic equations with a tunnel quasiparticle source which assumes a constant bias voltage across the junction. Experimental studies[2,3] have shown that the instability current I_I and the spatial distribution of the inhomogeneous state (IS) resulting from the narrow tunnel injection source are highly sensitive to weak magnetic fields. This pointed to the importance of the Josephson properties of tunnel structures for investigating nonequilibrium phenomena in low-resistance junctions. In Ref. 4 the formation mechanisms of the nonequilibrium inhomogeneous state in long Josephson junctions in which Josephson vortex (fluxon) flow occurs were investigated. It was demonstrated that the region in which the energy gap Δ is reduced in the presence of an inhomogeneous state depends on the direction of motion of the Josephson vortices. This led to the conclusion that the instabilities in tunnel junctions result from local quasiparticle injection due to the a.c. voltage produced by vortex motion. A more detailed investigation of these issues from both the experimental and theoretical viewpoint is of interest.

FORMULATION OF EXPERIMENT

The Sn–I–Sn–I–Sn (I represents the insulator) tunnel structures shown schematically in Fig. 1 were used in the experimental investigation of energy gap suppression. It was possible to analyze this effect both in the region where the vortices were generated and at their escape site where the amplitude of the a.c. voltage is at a maximum.[5] The 0.6×0.075 mm^2 quasiparticle injector formed by tin films S_1 and S_2 had a tunnel resistivity of $3\cdot10^{-5}$ Ω mm^2. The two detectors (detector d_1 was a S_2–I–S_3 tunnel junction, while detector d_2 was an S_2–I–S_4 tunnel junction) had a resistivity of $2\cdot10^{-3}$ Ω mm^2 while their dimensions (0.1×0.05 mm^2) were comparable to the distance the magnetic field penetrated into the low-resistance generator, $\lambda_J = 50$ μm. The $V_\Delta(V_g)$ relations were recorded on a different recorder in conjunction with the I-V characteristic of the generator. Here V_Δ is the bias voltage across the detector on the steepest section of its I-V characteristic at a fixed tunnel current, V_g is the bias voltage across the generator. The V_Δ voltage was amplified and then injected to the Y input of the recorder.

Figure 2 shows the $\delta V_\Delta(V_g)$ relation ($\delta V_\Delta = V_{\Delta_0} - V_\Delta$, V_{Δ_0} is the bias voltage across the detector with zero generator injection) at $T = 1.9$ K. It was possible to measure both the vortex density and the a.c. amplitude by applying an external magnetic field in the junction plane perpendicular to its long side 1. Figure 2 also shows the effect of the magnetic field on $\delta V_\Delta(V_g)$ (curves 1 and 2). The figure clearly indicates a threshold gap suppression effect that initially occurs at a generator bias voltage of $V_g = 2\Delta_0/3e$ and $V_g = \Delta_0/e$ (Δ_0 is the equilibrium value of the gap). The $\delta V_\Delta(V_g)$ relation indicates the virtual absence of the thermal energy gap suppression effect at voltages $V_g < 2\Delta/3e$ (Δ is the observed gap value). The onset of abrupt gap suppression at a bias voltage $V_g \approx 2\Delta/3e$ from a monotonic rise in the power applied to the junction can be related to the activation of the suppression mechanism which produces nonequilibrium quasiparticles of energies $\varepsilon > 2\Delta$.

We will consider the effect of the magnetic field on these phenomena. Figure 2 (curve 2) clearly indicates that an increase in external magnetic field H serves to reduce the gap suppression effect $V_g < 2\Delta_0/e$ and causes a rise in the voltage V_I at which the instability occurs. The $\delta V_\Delta(V_g)$ relation obtained for $H = 45$ Oe once again allows us to evaluate the possible thermal gap suppression effect. Since the I-V characteristic of the generator is already close to the quasiparticle characteristic in this magnetic field, the energy gap suppression resulting from heating cannot exceed the values of δV_Δ obtained in this magnetic field. When current flows in the direction shown in Fig. 1, and with the same magnetic field direction indicated in this figure, the changes in the energy gap described above are fixed by detector d_1 at the same time that the detector d_2 shows virtually no changes in the energy gap. Instability of the homogeneous, nonequilibrium superconducting state in the vicinity of the detector d_1 is observed when a bias voltage of $V_I \leqslant 2\Delta_0/e$ is achieved

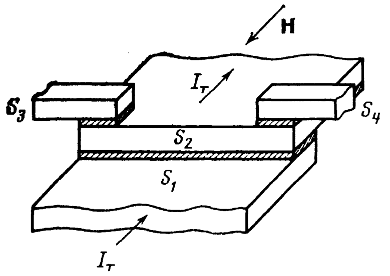

FIG. 1. Schematic representation of the Sn–I–Sn–I–Sn tunnel structure. The tin electrodes of the junction with different purposes are labeled S_i ($i = 1,2,3,4$).

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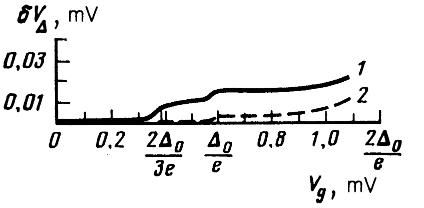

FIG. 2. The change in voltage δV_Δ corresponding to the energy gap at the detector plotted as a function of injector voltage V_g: 1—$H=0$, 2—$H=45$ kOe.

across the generator. The instability produces a sharp drop in the voltage across the generator and a substantial drop in δV_Δ. Analysis reveals that fluxons move in the direction of the detector d_1, i.e., in our case the instability appears where the fluxons reflect from the junction edge.

THEORETICAL MODEL

A theoretical analysis of these phenomena is possible if we take into account that the voltage across the junction is not constant when the vortices move and takes the form

$$eV(x,\ t)=V_{dc}(x)+V_{ac}(x)\cos[2V_{dc}(x)t], \tag{1}$$

where x is the distance; V_{dc} and V_{ac} is the d.c. bias voltage and the amplitude of the a.c. voltage produced by fluxon motion, respectively; for convenience these are measured in energy units; the time t is given in inverse energy units.

In the general case it is difficult to derive the tunnel source in a kinetic equation that describes the action of a uniform, nonstationary voltage on the junction, to say nothing of solving such an equation.

Here we consider a simplified model that we believe reflects the primary characteristics of the effect of fluxon motion on the nonequilibrium properties of the tunnel junction. The higher vortex velocity [above $V_{ac}(x)$] corresponds to a highly transparent long Josephson junction, while $V_{ac}(x)$ reaches its maximum near the junction edge (where the vortex velocity peaks).[5] In the "dirty" limit [by virtue of the inequality $\lambda_J \gg \{l_i,\ l_D,\ \xi(T)\}$, where l_i is the free path length between elastic collisions, l_D is the diffusion length, and $\xi(T)$ is the coherence length] a long Josephson junction—even one formed by relatively "clean" superconducting films—can be represented in terms of its nonequilibrium properties and with an accuracy sufficient for our purposes as a set of small (dimensions $L \lesssim \lambda_J$) Josephson junctions in parallel with the following uniform voltage injected to each junction:

$$V_i=V_{dc}+V_{ac}^{(i)}\cos(2V_{dc}t)$$

with different $V_{ac}^{(i)}$. We have therefore ignored the coupling of the separate junction regions here resulting from, for example, diffusion of nonequilibrium excitations. We consider the nonequilibrium effects in a small region of a Josephson junction of size $L \sim \lambda_J$. Consistent with this discussion the voltage

$$V=V_{dc}+V_{ac}\cos(2V_{dc}t),$$

generated in this region due to vortex motion is considered to be independent of position. The derivation of the tunnel source in the kinetic equation is based on the application of the Green-Keldysh function technique (see, for example, Refs. 6, 7) to the tunnel Hamiltonian formalism. In this approach to first order in the tunnel transparency the matrix tunnel self-energy part $\hat\Sigma$ takes the form

$$\hat\Sigma^{(1)R,A,K}(t,t')=\frac{1}{\pi\tau_T}\hat g_{(2)}^{R,A,K}(t,t'),$$
$$\hat\Sigma=\begin{pmatrix}\Sigma_1 & \Sigma_2\\ -\Sigma_2^+ & \bar\Sigma_1\end{pmatrix},\quad \hat g=\begin{pmatrix} g & f\\ -f^+ & \bar g\end{pmatrix}, \tag{2}$$

where $\Sigma_{1(2)}$ is the normal (anomalous) self-energy part, $g(f)$ is the normal (anomalous) electron Green-Keldysh function integrated over the kinetic energy, τ_T is the tunneling time, and the indices R, A or K label the corresponding Green's function and the self-energy part as retarded, advanced or correlated, while the superscripts in parentheses in (1) or (2) indicate whether the corresponding function belongs to the left or right film relative to the tunnel barrier. The tunnel source takes the form

$$Q_T=Q^{qp}+Q^{dc}, \tag{3}$$

where

$$Q^{qp}=g^K\Sigma_1^A-\Sigma_1^R g^K+g^R\Sigma_1^K-\Sigma_1^K g^A \tag{4}$$

accounts for tunneling of the quasiparticle excitations, while the Josephson and interferential currents can be obtained from

$$Q^{dc}=\Sigma_2^R f^{K+}-f^K\Sigma_2^{A+}-f^R\Sigma_2^{K+}+\Sigma_2^K f^{A+}\,. \tag{5}$$

In (4) and (5) the product of A and B is understood to be $AB=\int dt_1 A(t,t_1)B(t_1,t')$.

After using the expression (2) in (4), (5) and accounting for the gauge invariance of the wave functions[1)] from which

$$g(t,\ t')\to g(t-t')\exp\{i[\chi(t)-\chi(t')]\},$$
$$\bar g(t,\ t')\to \bar g(t-t')\exp\{-i[\chi(t)-\chi(t')]\},$$
$$f^+(t,\ t')\to f^+(t-t')\exp\{i[\chi(t)+\chi(t')]\},$$
$$f(t,\ t')\to f(t-t')\exp\{-i(\chi(t)+\chi(t')]\},$$

where

$$\chi(t)=\int_{-\infty}^{t} V(t)\,dt,\qquad V(-\infty)=0,\qquad \chi(t)=\chi_{(1)}(t)-\chi_{(2)}(t),$$

$\chi_{(1)}(t)$ or $\chi_{(2)}(t)$ is the phase of the electron wave function in the film of junction (1) or (2), we obtain the following expression for, e.g., Q^{qp}:

$$\begin{aligned}Q^{qp}(t,t')=\frac{1}{\pi\tau_T}\int dt_1\{[&g_{(1)}^{K}(t-t_1)g_{(2)}^{A}(t_1-t')\\ &+g_{(1)}^{R}(t-t_1)g_{(2)}^{K}(t_1-t')]\\ \times\exp[i(\chi(t')-\chi(t_1))]-[&g_{(2)}^{R}(t-t_1)g_{(1)}^{K}(t_1-t')\\ &+g_{(2)}^{K}(t-t_1)g_{(1)}^{A}(t_1-t')]\exp[i(\chi(t_1)-\chi(t)]\}.\end{aligned} \tag{6}$$

An analogous expression is easily obtained for Q^{dc} as well. If we recall the explicit form of $\chi(t)$ and the relation

$$\exp[i\alpha\cos\Omega t]=\sum_n J_n(\alpha/\Omega)\exp[in\Omega t]$$

[$J_n(x)$ is an nth order Bessel function], then by expressing the correlation function through the distibution function and substituting the explicit expressions in place of the functions $g^{R(A)}$ we can easily obtain the tunnel source in a form analogous to that of Refs. 6, 7.

Recalling, however, in this context the interpretation of our experimental results we limit the analysis to the simplest nonequilibrium effects of the threshold instability type[1] and, specifically, we ignore the effect of fluxon motion on the population imbalance effect between the electron and hole branches of the spectrum.[6] In this case we have the following expression from (6) for the tunnel quasiparticle source:

$$Q_T=8\tau_T^{-1}\Big\{\sum_{n=-\infty}^{\infty}J_n^2\Big(\frac{V_{ac}}{2V_{dc}}\Big)\sum_{k=-1}^{1}\rho_0^{(1)}(\varepsilon+kV_{dc}+2nV_{dc})+\sum_{n=1}^{\infty}J_n^2\Big(\frac{V_{ac}}{2V_{dc}}\Big)\times\rho_1^{(2)}(2nV_{dc}-\varepsilon)+\sum_{n=-\infty}^{\infty}J_n^2\Big(\frac{V_{ac}}{2V_{dc}}\Big)\rho_1^{(1)}(\varepsilon-2nV_{dc})+\sum_{n=1}^{\infty}J_n^2\Big(\frac{V_{ac}}{2V_{dc}}\Big)\sum_{k=-1}^{1}\rho_0^{(2)}(2nV_{dc}+kV_{dc}-\varepsilon)\Big\},\quad (7)$$

where the tunnel contact is assumed to be symmetrical, while

$$\rho_r^{(1)}(z)=\frac{z-(\Delta^2/\varepsilon)^r}{(z^2-\Delta^2)^{1/2}}[n(z)-n(\varepsilon)]\theta(z-\Delta)],$$
$$\rho_r^{(2)}(z)=\frac{z-(\Delta^2/\varepsilon)^r}{(z^2-\Delta^2)^{1/2}}[1-n(\varepsilon)-n(z)]\theta(z-\Delta),$$

r is the exponent in the "coherence factors" equal to 0 or 1, $n(\varepsilon)$ is the quasiparticle distribution function, and $\theta(x)$ is the Heaviside function.

Expression (7) for $V_{ac}=0$ becomes a well-known expression,[8] as we can easily see. For $V_{ac}\neq 0$ source (7) describes electron tunneling processes involving the photons (the terms with the multiplier k) and photon-induced Cooper pair-breaking. The index n represents the set of photons of frequency $\omega=2V_{dc}$ participating in this tunneling process. In our experimental conditions $V_{ac}/2V_{dc}\approx 0.1$–$1$ and hence the terms with $n=0, \pm 1$ make the primary contribution to the tunnel source. An approximate solution of this equation can be obtained by a method analogous to that of Ref. 9. Here we find that with vortex motion the narrow source condition obtains for an entire family of bias voltages satisfying the conditions

$$ekV_{dc}^{(k)}-2\Delta^{(k)}\ll\Delta,$$

where $k=2l+1\,(l=0,1,2,...)$ or $k=2m\,(m=1,2,...)$, unlike the case of $V_{ac}=0$ (Ref. 1), when only a single condition $eV_{dc}-2\Delta\ll\Delta$ exists. This causes Δ to be an ambiguous function of each $V_{dc}^{(k)}$ and can be written as (cf. Ref. 1)

$$\Delta^{(k)}(T)=\Delta_0^{(k)}(T)-\Delta_{(k)}^{(1)},$$

$$\Delta_k^{(1)}=\begin{cases}0, & V_{dc}<2\Delta_0^{(k)}/ek-\alpha^{(k)},\\ \frac{1}{2}(2\Delta_0^{(k)}-ekV_{dc}), & \frac{2\Delta_0^{(k)}}{ek}-\frac{\pi T_c\gamma_V^{(k)}}{2b\Gamma(T)k}<V_{dc}<\frac{2\Delta_0^{(k)}}{ek},\\ \pi T_c\gamma_V^{(k)}/2b\Gamma(T), & V_{dc}>2\Delta_0^{(k)}/ek,\end{cases}\quad (8)$$

where $\Delta^{(k)}(T)$ is the energy gap resulting from the onset of the kth instability; $\Delta_0^{(k)}(T)$ is the energy gap prior to the onset of the kth instability; $\Gamma(T)$ is the electron-phonon collision frequency; $B=0.77$; $b=7\zeta(3)/8\pi^2$; $\zeta(3)$ is the Riemann zeta function; and T_c is the critical temperature;

$$\alpha^{(k)}=\pi\Delta_0^{(k)}\gamma_V^{(k)}/\Gamma(T)Bk,\qquad \gamma_V^{(k)}=J_{k/2}^2(V_{ac}/2V_{dc}),$$
$$k=2m\quad(m=1,2,\dots),$$
$$\alpha^{(k)}=2\pi\Delta_0^{(k)}\gamma_V^{(k)}/1{,}25\Gamma(T),$$
$$\gamma_V^{(k)}=J_{(k-1)/2}^2(V_{ac}/2V_{dc})+J_{(k+1)/2}^2(V_{ac}/2V_{dc}),$$
$$k=2l+1\quad(l=0,1,2,\dots).$$

As we see the width of the ambiguity range Δ is proportional to $\gamma_V^{(k)}$ and is inversely proportional to k.

Figure 3 shows the qualitative dependence of the energy gap on the bias voltage across the tunnel junction V_{dc}. Clearly the onset of the kth instability causes an irregular suppression of the energy gap whose relative value is equal to

$$\delta\Delta^{(k)}/\Delta_0^{(k)}\approx\pi T_c\gamma_V^{(k)}/2\Delta_0^{(k)}b\Gamma(T)$$

and which diminishes with increasing k. We should note that there is a "universal" (compare to Ref. 1) dependence of $\Delta^{(k)}$ on V_{dc} in the instability range consistent with (8).

The jump $\delta\Delta^{(k)}/\Delta_0^{(k)}$ will therefore result from the development of instability attributable to a variety of tunneling processes at bias voltages: a) $V_{dc}^{(1)}=2\Delta/e$ ($k=1$, $l=0$, $n=0$), tunneling excluding the photons); b) $V_{dc}^{(2)}=\Delta/e$ ($k=2$, $m=1$, $n=1$, tunneling with Cooper pair breaking); c) $V_{dc}^{(3)}=2\Delta/3e$ ($k=3, l=1, n=1$ tunneling involving a photon), etc. The relative contribution of these processes with different k to the value of $\delta\Delta^{(k)}/\Delta_0^{(k)}$ is largely determined by the factor $\gamma_V^{(k)}$ which depends on $V_{ac}/2V_{dc}$. Thus, for example, for processes b) and c) ($k=2$ and 3, respectively) $\gamma_V^{(2)}$ and $\gamma_V^{(3)}$ are of the same order. This suggests that changes of $\delta V_\Delta(V_g)$ in the vicinity of bias voltages $V_g=2\Delta_0/3e$ and $V_g=\Delta_0/e$ are approximately identical. It can also be demonstrated that monotonic suppression of the energy gap will occur in the areas between the regions where V_{dc} changes sign, while inhomogeneous, stable nonequilibrium superconducting states with two energy gap values arise in the ambiguity regions.

As discussed above the theoretical analyses given here are valid for a small junction region of dimensions $L\lesssim\lambda_J$. The issue of fluxon effect on the formation of the inhomogeneous state in a long Josephson junction can in principle be analyzed using an expression for the tunnel source in the form of Eq. (6) if we recall that the inequality $qa\ll 1$ ($q_{\max}\sim\lambda_J^{-1}$) in the final analysis allows us to consider the

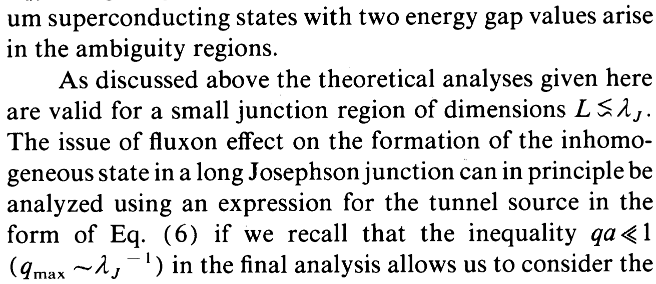

FIG. 3. The $\Delta(V_\Delta)$ relation accounting for fluxon motion for two different tunnel injection parameters $\tilde\alpha_1$ and $\tilde\alpha_2$ ($\tilde\alpha_1<\tilde\alpha_2$, $\tilde\alpha_i$ has a complex dependence on the junction parameters and the fluxon properties).

parameters $V_{dc}(x)$ and $V_{ac}(x)$ weakly dependent on the coordinate x. The kinetic equation in this case contains auxiliary terms that account for the spatial dependence of the distribution function, while the self-consistency equation will account for the dependence of the energy gap parameter on x (Ref. 10). However solving the problem of inhomogeneous state formation in a fluxon medium lies beyond the scope of the present article.

As suggested by our analysis, the fluxons are likely to have the greatest effect on the onset of the "threshold instability" near the edge of a long Josephson junction, where $V_{ac}(x)$ is maximized and consequently, where the probability of quasiparticle tunneling processes involving the fluxon-radiated photons is highest.

CONCLUSION

The present study has carried out a comparative analysis of experimentally observed phenomena and the proposed theoretical model and has established, both experimentally and theoretically, the energy gap suppression effect when $V_{dc} < 2\Delta_0/e$ holds; this effect is independent of the quasi-equilibrium thermal heating mechanism and is due rather to the injection of nonequilibrium quasiparticles together with photons. The experimentally observed sudden drop in the energy gap at generator bias voltages of $V_g = 2\Delta_0/3e$ and Δ_0/e can be attributed to the instability of the homogeneous superconducting state predicted by the theoretical model at these voltages. This theoretical model also explains the instability voltage V_I which is ordinarily below $2\Delta_0/e$ and which has not found an explanation in the present threshold instability theory.[10]

The analysis of nonequilibrium phenomena carried out in the present study accounting for the Josephson properties of low-resistance tunnel structures has identified a number of new effects such as nonequilibrium energy gap suppression at bias voltages of $V_{dc} < 2\Delta_0/e$, the possibility for the existence of an entire series of superconducting state instabilities in tunnel injection, together with variation in the effect of the tunnel injector.

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[1)] We note that the simple relations can be derived only when the inequality $qa \ll 1$ holds (q is the wave vector of the voltage irregularity, a is the interatomic distance).

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