Intensity interferometry for fermions

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We consider the interference pattern (as a function of separation) that arises when fermions from two independent sources are recorded by two detectors; this is the analog of the Brown-Twiss effect for photons. We show that it is possible, under certain circumstances, to measure the angular dimensions of the sources emitting the fermions (which may be neutrinos, neutrons, electrons, etc.).

In 1956, Hanbury Brown and Twiss proposed a novel method, which they called "the intensity correlation method" (also known as intensity interferometry) for measuring the angular diameter of the stars nearest the earth.[1–3] Their correlation method has a number of advantages over the Michelson stellar interferometer. For example, phase distortions introduced by atmospheric turbulence, variations in the refractive index, and so on, have no effect on correlation measurements. Furthermore, the intensity interferometer enables one to utilize a much longer baseline that the Michelson interferometer, therby yielding higher instrumental resolution. Intensity interferometry was subsequently proposed as a means of measuring the scattering matrix[4] and for certain problems in spectroscopy—measuring optical linewidths, for example.[5]

All of the foregoing proposed applications of intensity interferometry were for instances in which the particles detected were bosons (photons). On the other hand, it has been shown[6] that these same effects come into play in a system of identical sources emitting fermions.[7] It would therefore be quite natural to suggest that an ensemble of fermion sources might exhibit behavior analogous to the Brown-Twiss effect.

In the present paper, we examine this effect for fermions, assuming for the sake of definiteness (and simplicity) that we are dealing with neutrinos. We consider two independent emitters separated by a distance D, with two detectors located a distance z_0 from the plane of the emitters (Fig. 1).

We shall assume throughout that direct interactions between the two sources, mediated by the radiation field, are negligible. For this to be so, we must have[7]

$$D \gg \pi/k, \tag{1}$$

where k is the absolute value of the particle wave number. The two detectors are turned on in unison, and can measure particle polarization independently. In the steady state, one measures the probability that the detectors simultaneously record particles, as a function of the distance d between detectors and the spin direction of each particle. We have shown previously[8] that the probability of simultaneous detection of two photons by two detectors is determined by the time-averaged probability of a two-photon state, and in the absence of any interaction between emitters via the radiation field, this reduces to the probability of a two-photon state of the system of two emitters (atoms) as the time for detection of the two photons tends to infinity, taking into consideration the photon production rates.

The two-particle fermion wave function can thus be cast in the form[8]

$$\psi(\mathbf{r}, \mathbf{r}'; t\to\infty) = \varphi(\mathbf{r}, \mathbf{r}')\chi(s_1, s_2), \tag{2}$$

where $\varphi(\mathbf{r},\mathbf{r}')$ and $\chi(s_1,s_2)$ are the space and spin wave functions, respectively. For two spin 1/2 particles, Eq. (2) may be written out explicitly as

$$\psi(\mathbf{r}, \mathbf{r}'; \infty) = 2^{-1/2}(\exp\{i\mathbf{k}_1(\mathbf{r}_1-\mathbf{R}_1)\}\exp\{i\mathbf{k}_2(\mathbf{r}_2-\mathbf{R}_2)\} + (-1)^S \exp\{i\mathbf{k}_1(\mathbf{r}_2-\mathbf{R}_1)\}\exp\{i\mathbf{k}_2(\mathbf{r}_1-\mathbf{R}_2)\})\chi_S, \tag{3}$$

where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the wave vectors of the first and second particle, and χ_S is the two-particle spin wave function, with $S = 1$ or 0. Making use of (3), we then obtain for the probability of simultaneous detection of two particles of given polarization by two detectors

$$W(\uparrow\uparrow) = W_0(1-\cos\theta),\ W(\downarrow\downarrow) = W_0(1-\cos\theta),\ W(\uparrow\downarrow) = 0, \tag{4}$$

where $\theta = k_1(|\mathbf{r}_1 - \mathbf{R}_1| - |\mathbf{r}_2 - \mathbf{R}_1|) + k_2(|\mathbf{r}_2 - \mathbf{R}_2| - |\mathbf{r}_1 - \mathbf{R}_2|)$ (see Fig. 1), and W_0 is the probability of simultaneous detection of two particles by the two detectors, independent of particle spin. The arrows inside the parentheses indicate the direction of particle polarization relative to the z axis, as determined by the detectors. For partially polarized particles, Eq. (4) takes the form

$$W(\uparrow\uparrow) = W_0(1-\cos\theta)(1/2+\xi)^2,\ W(\downarrow\downarrow) = W_0(1-\cos\theta)(1/2-\xi)^2,\ W(\uparrow\downarrow) = W_0(1/4-\xi^2). \tag{5}$$

For simplicity, we shall assume that the degree of polarization ξ is the same for the two sources.

The expression for the phase θ can be simplified (see Ref. 8) if we assume that $z_0 \gg D \gg d$; following some straightforward manipulation, we have

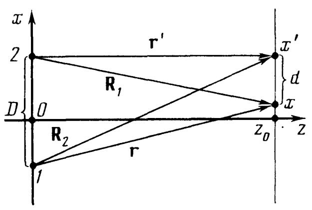

FIG. 1. Relative positions of fermion sources 1 and 2 and fermion detectors (in the plane $z = z_0$).

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$$\theta \approx kDd/z_0, \tag{6}$$

where $k \approx k_1 \approx k_2$.

Notice that for photons, the probability of simultaneous detection of two particles by the two detectors is virtually identical to Eq. (4), but with a plus sign in front of $\cos\theta$. From here on, for the sake of simplicity (but with no loss of generality in the final result), we shall assume that the particles emitted by the sources are unpolarized ($\xi = 0$).

Equations (5) were derived for point sources and detectors. Let us now consider a number of simpler situations, which are often more consonant with experimental conditions, in which the sources and/or detectors are of finite size. Averaging the correlation intensity over all phase differences, we obtain

$$\widetilde{W}_1 = W_0\left(1 - \frac{z_0}{2kdr_0}\cos k\frac{Dd}{z_0}\sin 2k\frac{r_0 d}{z_0}\right) \tag{7}$$

for two extended sources (with effective radius r_0) and point detectors (Fig. 2).

For the more general case in which both the sources and detectors are of finite size [with effective radii r_0 and x_0 respectively (Fig. 3)], the probability of detecting two fermions becomes

$$\widetilde{W}_2 = W_0(1 - J_2), \tag{8}$$

where

$$J_2 = \frac{z_0}{16kx_0r_0}\left[\mathrm{Si}\left(k\frac{D+2r_0}{z_0}(2x_0+d)\right) - \mathrm{Si}\left(k\frac{D+2r_0}{z_0}(d-2x_0)\right) + \mathrm{Si}\left(k\frac{D-2r_0}{z_0}(d-2x_0)\right) - \mathrm{Si}\left(k\frac{D-2r_0}{z_0}(d+2x_0)\right)\right],$$

and $\mathrm{Si}(x)$ is the sine integral.

In all of these instances, the expresions for the probability of simultaneous detection of two fermions retain oscillatory terms whose period is determined by

$$\Delta = k\frac{D\pm 2r_0}{z_0}(d \pm 2x_0), \tag{9}$$

and whose amplitude is proportional to $z_0/(kx_0r_0)$. One can estimate the order of magnitude of the parameters that determine the size of the effect ($\Delta \approx 1$) for two different cases.

Under typical laboratory conditions, $d \sim D \sim 10^{-3}$–10^{-4} cm and $z_0 \sim 10^1$–10^2 cm so the typical mean fermion energies for which the Brown-Twiss effect should be observable lie within a fairly broad range: 0.01 to 10^3 eV for electrons, and 1 to 10^6 eV for neutrinos.

In the other case, the objective is to measure the source angular diameter,

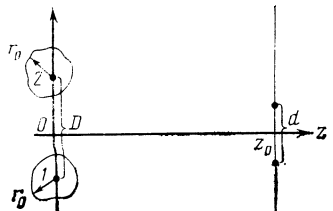

FIG. 2. Relative positions of extended fermion sources 1 and 2 and point fermion detectors (in the plane $z = z_0$).

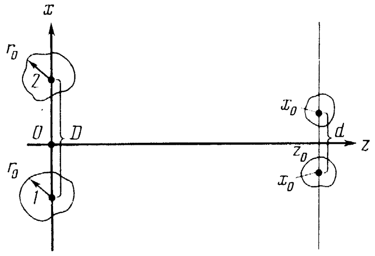

FIG. 3. Relative positions of extended fermion sources 1 and 2 and extended fermion detectors (in the plane $z = z_0$).

$$D/z_0 \approx 1/kd. \tag{10}$$

It follows from (10) that for a massless neutrino with momentum $k = 10^{12}$ cm^{-1} (corresponding to an energy $E_\nu \approx 20$ MeV), the limiting source angular diameter is $D/z_0 \approx 10^{-9}$.

Equations (7) and (8) share one further property which, in our opinion, is quite intriguing. Let us put $d = 0$; in other words, instead of two detectors, let us work with one, having a characteristic radius x_0. One can then easily show from Eq. (8) that the detection probability for simultaneous events in such a detector is

$$\widetilde{W}_3 = W_0\left(1 - \frac{z_0}{8kx_0r_0}\left[\mathrm{Si}\left(k\frac{D+2r_0}{z_0}2x_0\right) - \mathrm{Si}\left(k\frac{D-2r_0}{z_0}2x_0\right)\right]\right). \tag{11}$$

When the arguments of $\mathrm{Si}(x)$ are small, the entire expression in (11) tends to zero—that is, the probability no longer oscillates. For large values of the argument of $\mathrm{Si}(x)$, using the asymptotic expansion[9] in the form

$$\mathrm{Si}(x) \to \frac{\pi}{2} - f(x)\cos x - g(x)\sin x$$

where

$$f(x) \approx x^{-1}(1 - 2!/x^2 + \ldots),$$
$$g(x) \approx x^{-2}(1 - 3!/x^2 + \ldots),$$

the expression for the probability takes the form

$$\widetilde{W}_3 \to W_0\left[1 - \frac{z_0^2}{(2k_0x_0)^2r_0D}\left(\sin\frac{2kx_0D}{z_0}\times\sin\frac{4k_0x_0r_0}{z_0} + \frac{2r_0}{D}\cos\frac{2kx_0D}{z_0}\cos\frac{4kx_0r_0}{z_0}\right)\right]. \tag{12}$$

It is clear from Eq. (12) that the probability for simultaneously detecting two particles with a single extended detector still oscillates—i.e., the Brown-Twiss effect is feasible, even for a single detector.

Finally, let us consider the way in which this effect may have been manifested, in an almost exotic fashion, in the detection of the neutrinos from supernova 1987A on 23 February 1987[10–12]. For our analysis of this case, we employ Eq. (11), where $D \to 0$, and ultimately obtain a probability

$$\widetilde{W}_4 \approx W_0\left[1 - \frac{z_0}{8kx_0r_0}\left(\pi + 2\,\mathrm{Si}\left(k\frac{2r_0}{z_0}\cdot 2x_0\right)\right)\right]. \tag{13}$$

Bearing in mind that the neutron sphere produced by the collapse of the supernova progenitor star has a radius

$r_0 \sim 100$ km, that the star is at a distance $z_0 \approx 170$ light years, and that the sensitive volume of the detector has a radius $x_0 \sim 10$ m, we estimate the magnitude of the Brown-Twiss oscillation parameter to be $\Delta = \pi z_0/8kx_0r_0$. For massless neutrinos with mean energy $\overline{E}_\nu \approx 20$ MeV we obtain $\Delta \approx 1$–3, which suggests that the Brown-Twiss effect may actually have reduced the probability of detecting double events by some tens of percent.

We remark in conclusion that the Brown-Twiss effect for fermions should make it feasible to measure the angular size of distant sources in the universe, and in much the same way as for photons, it should facilitate the development of new methods of intensity interferometry for fermions.

[1]R. Hanbury Brown and R. Q. Twiss, *Nature* **177**, 27 (1956).
[2]R. Hanbury Brown and R. Q. Twiss, *Nature* **178**, 1046 (1956).
[3]R. Hanbury Brown and R. Q. Twiss, Proc. Roy. Soc. A **248**, 199 (1958).
[4]M. L. Goldberger and K. M. Watson, Phys. Rev. B **137**, 1396 (1965).
[5]D. T. Phillips, H. Kleiman, and S. P. Davis, Phys. Rev. **153**, 113 (1967)
[6]Yu. L. Dobrynin, V. V. Lomonosov, and G. V. Muradyan, Zh. Eksp. Teor. Fiz. **90**, 3 (1986) [Sov. Phys. JETP **63**, 1 (1986)].
[7]V. Ernst and P. Stehle, Phys. Rev. **176**, 1456 (1968).
[8]Yu. L. Dobrynin and V. V. Lomonosov, Zh. Eksp. Teor. Fiz. **92**, 769 (1987) [Sov. Phys. JETP **65**, 433 (1987)].
[9]I. S. Gradshtein and I. I. Ryzhik, *Table of Integrals, Sums, Series, and Products*, Nauka, Moscow (1971) [translated as Table of Integrals, Series, and Products, Academic Press, New York (1980)].
[10]J. N. Bachall and S. L. Glashow, *Nature* **326**, 476 (1987).
[11]J. N. Bachall, A. Dar, and T. Piran, *Nature* **326**, 135 (1987).
[12]K. Hirata *et al.* Phys. Rev. Lett. **58**, 1490 (1987).

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