Critical behavior of the 1/f noise in percolation systems

A. E. Morozovskiĭ and A. A. Snarskiĭ

Institute of Metal Physics, Academy of Sciences of the Ukrainian SSR, Kiev
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An analysis is made of the composition dependence of the relative density of the 1/f noise φ^e in macroscopically inhomogeneous composites above, below, and at the percolation threshold. A relationship is established between the critical exponents k and k', on the one hand, and the critical exponents of the conductivity and correlation length, on the other. Two new critical exponents are introduced and situations are identified in which the terms of φ^e described by these exponents may play the dominant role. A comparison is made with the experimental results.

INTRODUCTION

The density of the $1/f$ noise[1] is one of the important characteristics of a medium. Much work has been done on this noise in homogeneous and inhomogeneous (macroscopically and microscopically) media. Considerable attention has been given recently to the density of the $1/f$ noise in macroscopically inhomogeneous media near the percolation threshold and in particular to its dependence on the composition.

The relative noise density can be defined as follows (see, for example, Refs. 2 and 3)

$$\varphi_R \equiv \frac{S_R}{R^2} = \frac{\langle \delta R \delta R \rangle}{R^2}, \tag{1}$$

where $\langle \delta R \delta R \rangle$ is the mean-square fluctuation of the resistance; S_R is the real value of the intensity of the $1/f$ noise measured for a constant external current; R is the resistance of the whole. We have[3]

$$\frac{\langle \delta R \delta R \rangle}{R^2} = \frac{\langle \delta U \delta U \rangle}{U^2} = \frac{\langle \delta I \delta I \rangle}{I^2}, \tag{2}$$

where U and I are the voltage and current in a sample. In macroscopically inhomogeneous two-phase systems the basic problem is as follows: knowing the noise density for pure phases (highly conducting φ_1 with a conductivity σ_1 and poorly conducting φ_2 with a conductivity σ_2), we have to find the composition dependence of the noise density for the whole sample. Much work has been done on this problem. The noise density was found in Ref. 4 for the two-phase materials discussed in Ref. 5. The behavior of the noise density in percolation systems, i.e., in the case of a large difference between the conductivities of the phases near the percolation threshold, is considered in Refs. 2, 3, and 6–11. These investigations demonstrated that the composition dependence of the effective noise density near the percolation threshold can be written as follows:

$$\varphi^e_+ \propto (p-p_c)^{-k}, \quad \varphi^e_- \propto (p_c-p)^{-k'}, \tag{3}$$

where p is the concentration of the highly conducting phase; p_c is the percolation threshold[12]; φ_+ and φ_- are the values of the noise density to the right ($p > p_c$) and left ($p < p_c$) of the percolation threshold; k and k' are the corresponding critical exponents.

It therefore follows that the composition dependence of the noise density near the percolation threshold is governed, in accordance with Eq. (3), by the critical exponents k and k' and our main task will be to determine these exponents.

We shall use the model of a "weak link"[13–14] to find the relationship between the "noise" critical exponents k and k', on the one hand, and the critical exponents of the effective conductivity, on the other, and we shall derive the following (compared with the parameter $\sigma_2/\sigma_1 \ll 1$) terms in Eq. (3); introduction of two new critical exponents w and w' will be used in a comparison with the results of numerical modeling and of experiments.

The composition dependence of the noise density of a macroscopically inhomogeneous system can be found using, for example, the dependences proposed in Ref. 6 for the lattice models:

$$\varphi_R = \sum_i \left(\frac{R_i}{R}\right)^2 \varphi_{R_i}, \qquad \varphi_R = \sum_i \left(\frac{R}{R_i}\right)^2 \varphi_{R_i}, \tag{4}$$

where φ_R is the noise density in a system consisting of resistances R_i; R is the total resistance; φ_{R_i} is the noise density of the resistance R_i; the first relationship is written down for the resistances connected in series and the second for those connected in parallel.

In the case of a homogeneous sample with dimensions $L_1 \times L_2 \times L_3$ the expressions given by Eq. (4) lead to

$$\varphi_R = \frac{a_0^3}{L_1 \times L_2 \times L_3} \varphi_0, \tag{5}$$

where φ_0 is the noise density in a sample of dimensions $a_0 \times a_0 \times a_0$.

It should be pointed out that in calculation of the noise density in an inhomogeneous medium it is insufficient to know the total resistance of the medium, since φ^e naturally represents that part of a sample where the current is flowing. This becomes particularly clear if we write down φ^e, in the same way as in Ref. 5 (we can easily demonstrate the identity of this definition with that introduced above):

$$\varphi_{V_0}{}^e = \frac{1}{V_0} \frac{\langle C(\mathbf{r}) [\sigma(\mathbf{r}) (\mathbf{E}(\mathbf{r}))^2]^2 \rangle}{(\sigma^e \langle \mathbf{E}(\mathbf{r}) \rangle^2)^2}, \tag{6}$$

where $\varphi^e_{V_0}$ is the noise density in a sample of volume V_0; C is a quantity related to the noise density φ_V in a region of volume V by the expression $C = \varphi_V V$; $\sigma(r)$ is the electrical conductivity; $\mathbf{E}(\mathbf{r})$ is the electric field intensity; V_0 is the volume of the sample.

We can therefore determine the noise density if we know not only the spatial distribution of the phases, but the distributions of the electric field and of the Joule heat. Clear-

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ly, in the case of randomly inhomogegeous media such a problem is generally insoluble.

However, there are randomly inhomogeneous media which behave in many respects in a universal manner, namely, strongly inhomogeneous media near the percolation threshold. Numerical methods were used in Refs. 15 and 16 to show that in strongly inhomogeneous media the bulk of the Joule heat is evolved near the percolation threshold in regions in the form of thin long bridges ($p > p_c$) connecting "thick" parts of a metallic cluster (base) and thin layers of a poorly conducting phase ($p < p_c$) between the final metallic clusters (see also Ref. 17).

We can use Eq. (6) if we know at least approximately the analytic expressions for the distributions of the electric fields and currents in strongly inhomogeneous media. This can be done employing a model of randomly inhomogeneous media. The first model capable of describing the flow of a current in a medium with $p > p_c$ was that proposed by Skal and Shklovskiĭ[18] (see also Ref. 19). The main assumption of this model is that the current flows only along single-strand percolation channels forming a framework of an infinite cluster. In the two-dimensional case the distance between the nodes of a network forming an infinite cluster rises faster than the length of such single-strand channels[12] and we have to allow for more complex elements of the structure. The importance of layers or spacers in the description of the conduction process in the case when $p < p_c$ was considered in Ref. 20. A specific geometry of weak points, in the form of a bridge and a spacer (Fig. 1), was proposed in Refs. 13 and 14. According to this model, in the three-dimensional case we have[13,14]

$$\frac{s_b}{l_b L} \propto |\tau|^t, \quad \frac{L^2 l_b}{s_b l_s} \propto |\tau|^{-(t+q)}, \quad \frac{s_s}{l_s L} \propto |\tau|^{-q}, \tag{7}$$

where $\tau = p_c - p$; t and q are the critical exponents of the conductivity; s_b is the characteristic area of a bridge; s_s is the characteristic area of a spacer.

In the two-dimensional case, we have

$$\frac{b_b}{l_b} \propto |\tau|^t, \quad \frac{L l_b}{b_b l_s} \propto |\tau|^{-(t+q)}, \quad \frac{b_s}{l_s} \propto |\tau|^{-q}, \tag{8}$$

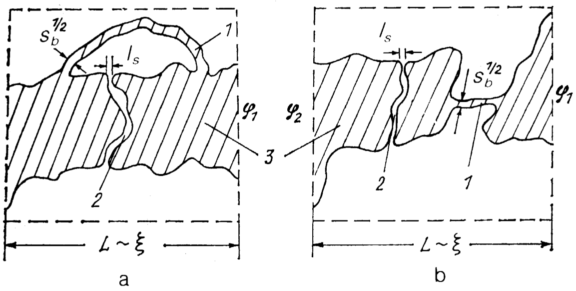

FIG. 1. Model of the structure of a medium near the percolation threshold: a) $p > p_c$, the shaded region represents a highly conducting phase showing a bridge (1) of thickness $s_b^{1/2}$ in three-dimensional space and b_b in the two-dimensional case, and a spacer (2) of a poorly conducting phase of thickness l_s and of area s_s, as well as a base (3) with a characteristic size ξ; b) $p < p_c$ here the number 2 represents a spacer of a poorly conducting phase of thickness l_s and of area s_s. In both figures the individual metallic clusters are not shown and neither are the islands of a poorly conducting phase within the bases, dead ends, etc.; L^3 is the characteristic volume containing just one bridge and one spacer. In the "weak link" models it is assumed that this volume is sufficiently representative for the description of the transport properties of the medium as a whole and that the sample consists of such volumes.

where b_b is the width of a bridge, and we have $s_b \propto a_0^2$, $l_s \propto a_0$, and a_0 is the characteristic size of the "cubes" of which the randomly inhomogeneous medium is assumed to be composed.

The relationships (7) and (8) describe the main assumptions of the "weak-link" models and are selected so that the effective conductivity σ^e depends on the composition in accordance with the following expressions deduced from percolation theory[21]:

$$\begin{aligned} \sigma^e(p) &\approx \sigma_1 |\tau|^t \left\{ A_0 + A_1 \frac{\sigma_2}{\sigma_1} |\tau|^{-(q+t)} + \ldots \right\}, \\ & p > p_c \\ \sigma^e(p) &\approx \sigma_2 |\tau|^{-q} \left\{ B_0 + B_1 \frac{\sigma_2}{\sigma_1} |\tau|^{-(q+t)} + \ldots \right\}, \\ & p < p_c, \end{aligned} \tag{9}$$

where A_0, A_1, B_0, and B_1 are constants of the order of unity.

If $\sigma_2/\sigma_1 \ll 1$, the second terms (and, consequently, all the other terms) are usually ignored, but there are physical situations where the main role is played by the second term.[22] We shall show that a similar situation can occur also in the noise density problem.

A description of the structure of a strongly inhomogeneous medium near the percolation threshold by means of the model of Ref. 18 and by the "weak link" model is not rigorous and requires further refinement,[23,24] but nevertheless we can describe a whole range of phenomena (see, for example, Refs. 20 and 24–26).

CALCULATION OF THE COMPOSITION DEPENDENCE OF THE NOISE DENSITY

We shall use the models described in Refs. 13 and 14 to calculate the noise density. If $p > p_c$ the main voltage drop ($\sigma_1 \gg \sigma_2$) is across a bridge and a spacer (Fig. 1a). Using Eq. (7) we can deduce from Eq. (6) ($C_1 = \varphi_1 a_0^3$, $C_2 = \varphi_2 a_0^3$) the expression

$$\varphi^e \propto \frac{\varphi_1 j_1^2 E_1^2 V_b/V + \varphi_2 j_2^2 E_2^2 V_s/V}{\sigma_e^2 E^4} \frac{a_0^3}{V_0}, \tag{10}$$

where j_1 and E_1 are the current and the field in the bridge; j_2 and E_2 are the current and the field in the spacer; $V_b = a_0^2 l_b$ is the volume of the bridge; $V_s = a_0 s_s$ is the volume of the spacer; $V \propto \xi^3$; $\xi \propto |p - p_c|^{-\nu}$ is the correlation length; ν is the critical exponent governing the behavior of the correlation length; E is the average field in a sample; φ_1 and φ_2 are the values of the noise density for highly conducting and poorly conducting phases.

We shall use $\Delta\varphi$ for the difference between the potentials across the relevant dimensions (Figs. la and lb); then, assuming that the voltage drop across the base can be ignored, we find that

$$j_1 \approx \frac{\Delta\varphi}{R_1 a_0^2}, \quad E_1 \approx \frac{\Delta\varphi}{l_b}, \quad j_2 \approx \frac{\Delta\varphi}{R_2 s_s}, \quad E_2 \approx \frac{\Delta\varphi}{a_0}, \tag{11}$$

where $R_1 \approx l_b/\sigma_1 a_0^2$ is the resistance of a bridge and $R_2 \approx a_0/\sigma_2 s_s$ is the resistance of a spacer.

Substituting Eq. (11) into Eq. (10), and using Eq. (7) we obtain (in the expressions that follow we omit V_0 since it is independent of τ):

$$\varphi^e_+ \propto \varphi_1 |\tau|^{t-4\nu} + \varphi_2 \left(\frac{\sigma_2}{\sigma_1}\right)^2 |\tau|^{-q-2(\nu+t)}, \quad p > p_c. \tag{12}$$

We can calculate similarly the quantity φ^e for the two-dimensional case. The form of φ^e remains the same, but the power exponent of τ changes. Writing down φ^e in its general form [see Eq. (3)]

$$\varphi^e_+ \propto \varphi_1|\tau|^{-k}+\varphi_2\left(\frac{\sigma_2}{\sigma_1}\right)^2|\tau|^{-w}, \tag{13}$$

we find that the critical exponents to the right ($p>p_c$) of the percolation threshold obey the following relationships:

$$k=2\nu(d-1)-t,\quad w=q+2t+2\nu. \tag{14}$$

We shall now consider the case when $p<p_c$. To the left of the percolation threshold the general expression for φ^e can be written as follows:

$$\varphi^e_- \propto \varphi_2|\tau|^{-k'}+\varphi_1\left(\frac{\sigma_2}{\sigma_1}\right)^2|\tau|^{-w'},\quad p<p_c. \tag{15}$$

Calculations similar to those described above give

$$k'=2\nu-q,\quad w'=2q+t+2\nu(d-1). \tag{16}$$

DISCUSSION

The expressions for φ^e to the right ($p>p_c$) and left ($p<p_c$) of the percolation threshold were obtained above for values of the concentration p that are on one hand quite close to the percolation threshold $p_c|\tau|\ll 1$ and on the other outside the smearing region Δ (Ref. 21), where in the case of the problem of conduction in the absence of a magnetic field we have

$$\Delta=(\sigma_2/\sigma_1)^\alpha,\quad \alpha=1/(t+q).$$

Since the noise density is governed entirely by the distributions of the currents and fields in an inhomogeneous medium, the smearing interval $\Delta\varphi$ of the noise density coincides with the smearing interval Δ of the conductivity.

Substituting Δ instead of τ into Eq. (14) or (15), we obtain

$$\varphi_0^e\sim\varphi_1\left(\frac{\sigma_2}{\sigma_1}\right)^{[t-2\nu(d-1)]/(t+q)}+\varphi_2\left(\frac{\sigma_2}{\sigma_1}\right)^{(q-2\nu)/(t+q)} \tag{17}$$

It should be noted that the second terms in $\varphi^e_{\pm,0}(\tau)$ make a considerable contribution to the noise density, at least in the region of the percolation threshold (in the smearing interval). For example, if these second terms are ignored, then the following physically obvious inequality cannot be satisfied:

$$\varphi_+^e(|\tau|\ll\Delta)\approx\varphi_-^e(|\tau|\ll\Delta).$$

In the two-dimensional case ($\nu_2=t_2=q_2=\frac{4}{3}$) we have

$$\varphi_0^e(d=2)\propto(\varphi_1+\varphi_2)(\sigma_2/\sigma_1)^{-1/2} \tag{18}$$

and we obtain an expression for the noise density in what are known as the Dykhne media (i.e., two-dimensional media with geometrically equivalent distributions of the phases present in equal concentrations).[28] It is shown in Ref. 28 that the distributions of the Joule heat are the same in both phases and this is clearly related to the fact [Eq. (18)] that φ_1 and φ_2 occur symmetrically in φ^e_0 for such media:

$$\varphi_0^e(\varphi_1,\varphi_2)=\varphi_0^e(\varphi_2,\varphi_1).$$

It follows from Eq. (17) [$t-2\nu(d-1)<0$, $q-2\nu<0$] that the stronger the inhomogeneity (σ_1/σ_2) of the system the higher the noise density.

We shall now consider the behavior of the noise density outside the smearing interval in the simplest case when the second terms in Eqs. (13) and (14) can be ignored. In this case the expressions φ^e_+ ($\tau>0$) and φ^e_- ($\tau<0$) assume the form known from the literature.[2,3,6–11] The critical exponents k and k' were calculated earlier by the method of renormalization group in real space,[3] which gave $k_2=1.339$; in Ref. 10 it was found that $k'_2=1.339$ and $k'_3=0.660$, whereas calculation by the transfer matrix method reported in Ref. 8 gave $k_2=1.2$. For comparison, we shall substitute the known values of the critical exponents of the conductivity t and q and of the correlation length ν, which are $t_2=q_2=\nu_2=\frac{4}{3}$, $t_3=1.8$, $q_3=0.98$, and $\nu_3=0.9$, so that after such substitution in Eqs. (14) and (16), we obtain $k_2=k'_2\approx1.33$, $k'_3=1.8$, and $k'_3=0.82$.

The limits on both sides are obtained for k and k' in Ref. 7:

$$d\nu+1-2\zeta_R\leqslant k\leqslant d\nu-\zeta_R,\quad d\nu+1-2\zeta_G\leqslant k'\leqslant d\nu-\zeta_G, \tag{19}$$

where ζ_R and ζ_G are the critical exponents representing the average values of the resistance and conductance over a distance equal to the correlation length ξ: $\zeta_R=t-(d-2)\nu$, $\zeta_G=q+(d-2)\nu$. Substituting ζ_R and ζ_G in Eq. (19) we find that the upper limits in Eq. (19) coincide with k and k' of Eqs. (14) and (16). The values of k and k' from Eqs. (14) and (16), coinciding with the upper limit, are nearly exact (or perhaps even identical with the exact values), as demonstrated by the results of a comparison of Eqs. (16), (14), and (19) with the values of k and k' obtained by the $6-\varepsilon$ expansion method,[11] which gives the exact values of the critical dimensionality in Percolation theory $d_c=6$ and the corrections in terms of a small parameter $\varepsilon=6-d$. According to Ref. 11, we have $k_6=2$ and $k(d)\approx2-0.181\varepsilon$. Substituting $\nu=0.5+0.06\varepsilon$, $t=3-0.238\varepsilon$ (Ref. 11) into Eq. (14), we obtain $k\approx2-0.178\varepsilon$, which ensures a good agreement, and if $d=d_c=6$ the exact agreement with k of Ref. 11. The lower limit of k in Eq. (19) gives even in the zeroth order in ε $k_6=12$, which is an order of magnitude different from the exact value. Therefore, k of Eq. (14) and k' of Eq. (16) obtained -above are in satisfactory agreement with the available data.

It is worth noting also Ref. 29, reporting a determination of the noise density in mixtures of a carbon powder and wax near the percolation threshold ($p>p_c$) of this system, which gave

$$\varphi_+^e\sim|\tau|^{-a},\quad a=5\pm1. \tag{20}$$

The results reported in Refs. 2, 3, and 6–9 are insufficient to account for the critical exponent $a=5\pm1$: for example, it follows from Eq. (19) in Ref. 2 that $1.01\leqslant k_3\leqslant1.57$. The value of a can be apparently understood only on the basis of a fuller description which is based on the "weak link" model. As a rule the noise density is inversely proportional to the conductivity $\varphi_1/\varphi_2\sim\sigma_2/\sigma_1$. In this case if $\Delta\ll\tau\ll\Delta^m$, where $m=(t+q)/(q+3t-2\nu)$ (which is always possible in the case of a sufficiently strong inhomogeneity since $0 < m < 1$), the second term in φ^e_+ is greater than the first and, therefore, the noise density is governed by the critical exponent w and not by k. Estimates of w give $w_3 = 2t_3 + q_3 + 2v_3 \approx 6.4$ in the three-dimensional case and $w_2 \approx 6.7$ in the two-dimensional case; this is in satisfactory agreement with the experimental results that yield $a_3 = 5 \pm 1$ (Ref. 29) and $a_2 = 6.27 \pm 0.08$ (Ref. 30).

In the description of the distribution of the current in a randomly inhomogeneous medium it is assumed that we can ignore the imaginary part of the conductivity. It is obvious that this is possible only in the low-frequency limit [see, for example, Eq. (22) in Ref. 31].

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