# Energy absorption in a ferrite beyond the kinetic instability threshold

A. Yu. Taranenko and V. B. Cherepanov

*Institute of Automation and Electrometry, Siberian Branch of the Academy of Sciences of the USSR, Novosibirsk*

(Submitted 12 September 1988)
Zh. Eksp. Teor. Fiz. **95**, 1810–1819 (May 1989)

Experimental and theoretical investigations were made of the nature of energy absorption by a ferrite at a high parametric pump power. The kinetic instability appeared at a pump power exceeding by 18–20 dB the parametric excitation threshold. This instability created nonequilibrium packets of secondary spin waves which were not in resonance with the pump. A theory of the interaction of secondary and parametric spin waves was developed. Measurements were made of the imaginary part of the nonlinear susceptibility of the investigated ferrite as a function of the excess of the pump power above the kinetic instability threshold in magnetic fields from 1040 to 1290 Oe. The predictions of the theory were compared with the experimental results.

## 1. INTRODUCTION

Discovery of an instability in a system of magnons far from equilibrium was reported in Ref. 1. This instability is called kinetic and it develops as a result of a reduction to zero of damping of long-wavelength magnons in the presence of a strong packet of parametrically excited spin waves. When the parametric pump power $P$ exceeds a certain threshold (critical) value $P_c$ which is 15–20 dB higher than the parametric excitation threshold, the number of secondary long-wavelength magnons rises strongly. This gives rise to electromagnetic radiation from the bottom of the spin-wave spectrum and an inflection in the dependence of the nonlinear susceptibility of a ferrite on the pump power, demonstrating a significant influence of secondary spin waves on parametric waves. Such kinetic instabilities in magnetic materials represent a fairly wide class of phenomena which occur at high excitation rates. A kinetic instability was discovered[2] on excitation of a spin wave by an antenna in a ferrite film. It is probable that kinetic instabilities develop also in antiferromagnets subjected to parametric excitation.[3–5]

We shall report an experimental and theoretical investigation of the mechanisms that limit the numbers of secondary spin waves in a ferrite under conditions of parametric excitation above the kinetic instability threshold. When the pump power exceeds the threshold value $P_c$ the number of secondary magnons rises exponentially until it becomes large and limited by nonlinear processes. The results of an experimental investigation of the spectrum of secondary spin waves emitted from the bottom of the spin wave spectrum show[1] that the frequency width of the packet $\Delta\omega_s$ is very small and amounts to several tens of the relaxation frequencies (a theoretical estimate of this width is given in Sec. 4). Equally narrow should be a packet of secondry spin waves of frequency $\omega_p - \omega_0$. A packet of parametrically excited spin waves is even narrower: its frequency width $\Delta\omega_p$ is always less than the damping $\gamma_p$ (Refs. 6 and 7). Consequently, beyond the kinetic instability threshold we can expect several packets of magnons coupled by four-particle interactions. The scattering of parametrically excited spin waves by a wider packet of secondary spin waves at frequencies near $\omega_p/2$ creates a new packet with a characteristic width $\Delta\omega_s \gg \Delta\omega_p$. Therefore, the behavior of a ferrite above the kinetic instability threshold is determined by the balance of magnons in four packets: a very narrow packet of parametrically excited spin waves, a packet of secondary spin waves localized in the region of the frequencies close to the bottom of the spin-wave spectrum $\omega_0$, another packet of secondary spin waves with frequencies close to $\omega_p - \omega_0$, and a packet of waves of the third generation of spin waves, formed as a result of scattering by secondary spin waves.

Section 2 describes an experimental investigation of the behavior of a ferrite beyond the kinetic instability threshold. Our experiments were carried out on high-quality samples of yttrium iron garnet (YIG) at room temperature. It was established in Ref. 1 that the kinetic instability threshold was minimal when the external constant magnetic field **H** was oriented along the [111] easy magnetization axis of the crystal. The threshold was then reached within the limits 1285 Oe $> H >$ 1040 Oe and outside this interval the threshold increased steeply. We measured the imaginary part $\chi''$ of the nonlinear susceptibility of this ferrite, representing the fraction of the incident power absorbed by a sample, as a function of the pump amplitude when the magnetic field was varied from 1050 to 1280 Oe. When the kinetic instability threshold was exceeded, the dependence of the susceptibility on the pump power had an inflection and, whereas below the threshold the imaginary part of the susceptibility increased, above the threshold it decreased, and vice versa. Section 3 describes a simple model of the mechanism of limitation of the kinetic instability, which is an approximation of the self-consistent interaction of packets of parametrically excited and secondary spin waves. It is assumed that the number of parametrically excited spin waves $n_p$ is frozen at the threshold level $n_p = n_p^c$ beyond the kinetic instability threshold and the relaxation frequency of parametrically excited spin waves increases on increase in the number of secondary spin waves $\gamma_p = \gamma_p(n_s)$. We can therefore calculate the nonlinear susceptibility of a ferrite near an instability threshold. In Sec. 4 we shall give a more detailed analysis of the processes of interaction and relaxation of parametrically excited spin waves, secondary spin waves, additional secondary spin waves, and third-generation spin waves. It is shown that the balance between the magnons belonging to these four packets determines the susceptibility of a ferrite far beyond the kinetic instability threshold. The experimental results are compared with the theoretical predictions in Sec. 5. It is

 0038-5646/89/051046-06$04.00  

shown that the self-consistent field approximation describes well the behavior of a susceptibility kink. A theory allowing for the scattering of parametrically excited spin waves, secondary spin waves, and third-generation spin waves agrees with the experimental results obtained in the range of fields $H \lesssim 1160$ Oe. In higher fields an increase in the pump power causes the susceptibility to reach a constant value, whereas the theory predicts a reduction in $\chi''$.

## 2. NONLINEAR SUSCEPTIBILITY OF THE INVESTIGATED FERRITE. EXPERIMENTS

1. Our experiments yielded the dependences of the imaginary part of the nonlinear susceptibility $\chi''$ on the pump power when a constant magnetic field was varied from 1281 to 1047 Oe. A pump pulse from a magnetron was applied, via a series of waveguide components, to a measuring section. The pump frequency was 9.37 GHz, the duration of the pump pulses was 100 $\mu$s, and the repetition frequency was 10 Hz. The measuring section consisted of a cavity resonator made of a standard waveguide supporting the $H_{011}$ oscillation mode. A ferrite sample was placed near the end wall of the resonator. The value of $\chi''$ was determined by measuring the reflection coefficient of the resonator $\Gamma = (P_{inc}/P_{refl})^{1/2}$, where $P_{inc}$ is the power of the radiation incident on the resonator and $P_{refl}$ is the reflected power. The susceptibility was calculated from[8]

$$4\pi\chi'' = \frac{1}{FQ_0}\left|\frac{r}{r_0} - 1\right|. \qquad (2.1)$$

Here $Q_0$ is the intrinsic $Q$ factor of the resonator; $F$ is the fill factor representing the degree of occupancy of the resonator by the ferrite; $r_0$ is the standing-wave ratio (SWR) of the resonator below the threshold of the parametric process ($P < P_{th}$); $r$ is the SWR of the resonator for a given supercriticality (express above the threshold), and is related to the reflection coefficient by $r = (1+\Gamma)/(1-\Gamma)$. The value of $Q_0$ was determined from the measured SWR. A precision attenuator employed in the SWR measurements, calibrated to within 0.1 dB, ensured that the relative error in the susceptibility measurements was 2%. The absolute value of $\chi''$ was subject to a much larger error of 20% because the ferrite and resonator parameters, such as the magnetization, volume, and $Q$ factor were not known sufficiently accurately. Therefore, in our graphs we plotted the relative values $\chi''/\chi''_{max}$. The absolute values of $\chi''_{max}$ depended weakly on the constant magnetic field and were within the range 0.15–0.21.

2. We investigated a single-crystal sphere of YIG with a diameter of 1.51 mm. A static magnetic field **H** was applied along the [111] easy magnetization axis. For this orientation of **H** the threshold was minimal.[1] The kinetic instability threshold was deduced from the appearance of a split in a pump pulse (see Ref. 1). The threshold kinetic instability power, reduced to the parametric excitation threshold, $p_c = P_c/P_{th}$, had minima in fields $H_1 = 1060$ Oe and $H_2 = 1260$ Oe. In the ranges $H < H_1$ and $H > H_2$ the threshold $p_c$ rose, whereas in the interval 1090 Oe $< H <$ 1220 Oe it was almost constant: $p_c \approx 90$ (19.5 dB). The threshold field for the parametric excitation $h_{th}$ varied from 0.96 Oe in a field $H = 1260$ Oe to 1.057 Oe in $H = 1050$ Oe ($P_{th} \propto h_{th}^2$).

The dependences of the imaginary part of the nonlinear susceptibility on the microwave power are plotted in Fig. 1 for magnetic fields $H = 1047$, 1122, 1192, 1232, and 1281

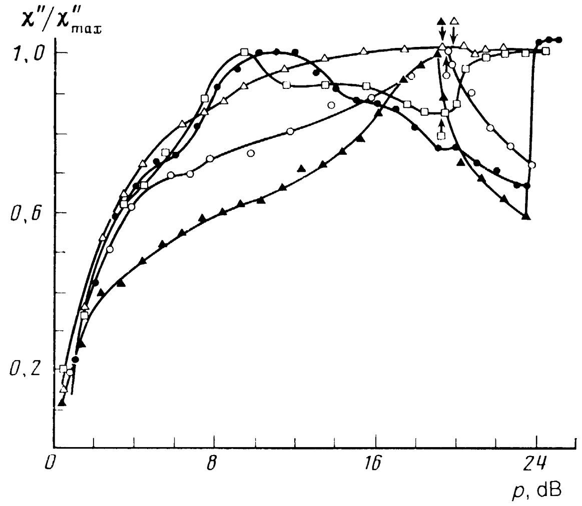


FIG. 1. Dependence of the imaginary part of the susceptibility on the pump power: ▲) $H = 1047$ Oe; ○) 1122 Oe; △) 1192 Oe; □) 1232 Oe; ●) 1281 Oe. The arrows identify the threshold kinetic instability powers.

Oe. In the interval $H \lesssim 1180$ Oe the susceptibility rose monotonically in the range $p < p_c$, whereas at $p = p_c$ there was a kink and the susceptibility fell in the range $p > p_c$. The maximum value of $\chi''$ occurred at $p = p_c$. In a narrow range of fields 1180 Oe $< H <$ 1192 Oe the derivative $\partial\chi''/\partial p$ was close to zero in the limit $p \to p_c - 0$. In this interval the kink of the dependence $\chi''(p)$ was undetectable and the susceptibility was nearly constant in the range $p > p_c$. When the field was $H > 1192$ Oe, the susceptibility reached its maximum at $p < p_c$ and this maximum shifted toward lower values of the supercriticality on increase in the magnetic field. In this range of fields the susceptibility again had a kink at $p = p_c$: $\partial\chi''/\partial p < 0$ when $p < p_c$ and $\partial\chi''/\partial p > 0$ when $p > p_c$. The magnitude of the kink increased on increase in the field. The susceptibility rose when $p > p_c$ and $H > 1192$ Oe and reached an approximately the same maximum value as for $p < p_c$. We were unable to detect a tendency to deviate from this maximum value on increase in the power right up to the maximum values $p \approx 25$ dB in the investigated range of magnetic fields.

In a magnetic field with the orientation **H**‖[111] and at relatively low pump powers $p \lesssim 10$–15 dB the amplitude of parametrically excited spin waves exhibited spontaneous oscillations. However, on approach to the kinetic instability threshold in fields $H < 1235$ Oe, these oscillations disappeared and they were observed only in a narrow range of fields from 1235 to 1290 Oe near the kinetic instability threshold. The average value of the susceptibility was determined in this range of fields.

## 3. SUSCEPTIBILITY OF A FERRITE FOR A SMALL EXCESS ABOVE THE KINETIC INSTABILITY THRESHOLD. APPROXIMATION OF A SELF-CONSISTENT INTERACTION BETWEEN PACKETS

The behavior of nonequilibrium magnons during the development of a kinetic instability in a ferrite is governed by the four-magnon scattering process. The magnon distribution function $n_{\mathbf{k}}$ far from the parametric resonance region (when $|\omega_{\mathbf{k}} - \omega_p/2| \gg \gamma_p$) is described by a kinetic equation.[9] If we ignore the thermal and elastic scattering of secondary spin waves by parametric spin waves, which do not alter the total numbers of either of the two waves, we find that the kinetic equation for the numbers of secondary spin waves becomes[10]:

$$\frac{\partial n_{\mathbf{k}}}{\partial t}+2(\gamma_{\mathbf{k}}{}^0-\Delta_{\mathbf{k}})n_{\mathbf{k}}$$
$$=4\pi\frac{v_0{}^2}{(2\pi)^6}\int|T_{\mathbf{k}123}|^2[(n_{\mathbf{k}}+n_1)\,n_2n_3-n_{\mathbf{k}}n_1(n_2+n_3)]$$
$$\times\delta(\omega_{\mathbf{k}}+\omega_1-\omega_2-\omega_3)\,\delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)\,d\mathbf{k}_1\,d\mathbf{k}_2\,d\mathbf{k}_3, \tag{3.1}$$

where $\gamma_{\mathbf{k}}^0$ is the magnon damping under thermodynamic equilibrium conditions,

$$\Delta_{\mathbf{k}}=\frac{v_0{}^2}{(2\pi)^5}\int|T_{\mathbf{k}123}|^2\,n_2{}^p n_3{}^p\delta(\omega_{\mathbf{k}}+\omega_1-\omega_p)$$
$$\times\delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)\,d\mathbf{k}_1\,d\mathbf{k}_2\,d\mathbf{k}_3 \tag{3.2}$$

is a negative contribution to the magnon damping due to the scattering by parametric spin waves. We have ignored here the spontaneous processes by assuming that $n_{\mathbf{k}}\gg 1$.

We can determine the dependence of the absorbed power on the pump amplitude above the kinetic instability threshold if we find the occupation numbers of secondary $n_{\mathbf{k}}^s$ and parametric $n_{\mathbf{k}}^p$ spin waves and the phase relationships governing the system of parametric spin waves. The correct allowance for the reaction of secondary on parametric spin waves is made in Sec. 4. However, when the excess above the kinetic instability threshold is small, the absorbed power can be determined subject to two assumptions: 1) the reaction of secondary on the parametric spin waves reduces to an additional contribution to the damping of the parametric waves, depending on the number of secondary waves; 2) the number of parametric spin waves $n_p$ is "frozen" at the threshold value $n_p^c$. These approximations are justified in Sec. 4.

In the approximation linear in the number of secondary spin waves, the damping of parametric spin waves is

$$\gamma_{p\mathbf{k}}=\bar{\gamma}_{\mathbf{k}}+2\nu_{\mathbf{k}}n_s,\qquad n_s=\frac{v_0}{(2\pi)^3}\int n_{\mathbf{k}}{}^s\,d\mathbf{k}, \tag{3.3}$$

$$\nu_{\mathbf{k}}=\frac{v_0}{(2\pi)^2}\int|T_{\mathbf{k},\mathbf{k}',\mathbf{k}+\mathbf{k}',0}|^2 n_{\mathbf{k}'}{}^p\delta(\omega_p-\omega_0-\omega_{\mathbf{k}+\mathbf{k}'})\,d\mathbf{k}'. \tag{3.4}$$

This contribution to the damping is due to the processes of decay of two parametric spin saves into two other magnons, one of which is in the instability region and the other carries away the excess energy or momentum, relaxing rapidly. Such processes give rise to a kinetic instability and reduce the number of parametric spin waves. The same coefficient $\nu_{\mathbf{k}}$ occurs in the expression for a negative correction to the damping of secondary spin waves:

$$\Delta=\frac{v_0}{(2\pi)^3}\int\nu_{\mathbf{k}}n_{\mathbf{k}}{}^p\,d\mathbf{k}=\nu n_p. \tag{3.5}$$

The number of parametric spin waves $n_p$ is related by a simple expression to the pump power[11]:

$$\gamma_p{}^2+(Sn_p)^2=|hV|^2. \tag{3.6}$$

Here, $S$ is the average value of the matrix element: $S_{\mathbf{k}\mathbf{k}'}\equiv T_{\mathbf{k},-\mathbf{k},\mathbf{k}',-\mathbf{k}'}$; $V$ is the coefficient of coupling to the pump radiation.

Since parametric spin waves are frozen at the threshold level, we did not determine the angular distribution of parametric spin waves and the average matrix element, but instead expressed them in terms of the threshold pump power $h_c^2$ for a kinetic instability and obtained explicit expressions for the damping of parametric spin waves $\gamma_p$ and the number of secondary spin waves:

$$\gamma_p=(|hV|^2-|h_cV|^2+\bar{\gamma}^2)^{1/2}=[\gamma_p{}^{02}(p-p_c)+\bar{\gamma}^2]^{1/2}, \tag{3.7}$$

$$\frac{n_s}{n_p}=\frac{\gamma_p-\bar{\gamma}}{2\gamma_s{}^0}=\frac{1}{2\gamma_s{}^0}\{[\gamma_p{}^{02}(p-p_c)+\bar{\gamma}^2]^{1/2}-\bar{\gamma}\}. \tag{3.8}$$

Here, $\gamma_p^0$ is the damping of parametric spin waves at the parametric instability threshold and $p=h^2/h_{\rm th}^2$ is the ratio of the pump power to the parametric instability threshold.

The experimentally determined imaginary part of the susceptibility $\chi''$ represents the ratio of the absorbed power to the power of the incident radiation. The power $W$ absorbed in a sample is governed by the rate of dissipation of the energy of parametric spin waves:

$$\chi''=\frac{2W}{\omega_ph^2}=\frac{2|V|^2}{|S|p}\left\{\left[p-p_c+\left(\frac{\bar{\gamma}}{\gamma_p{}^0}\right)^2\right]\left[p_c-\left(\frac{\bar{\gamma}}{\gamma_p{}^0}\right)^2\right]\right\}^{1/2}. \tag{3.9}$$

The real part of the susceptibility, representing the phase relationships in the system of parametric spin waves, is

$$\chi'(p)=\chi_c'(p_c/p), \tag{3.10}$$

where $\chi_c'$ is the real part of the susceptibility at the kinetic instability threshold.

The nature of relaxation of parametric spin waves below the kinetic instability threshold can vary considerably. If the wave vector of parametric spin waves is sufficiently small, the damping of these waves is independent of their number; $\bar{\gamma}=\gamma_p^0$; in the case of shorter waves a considerable contribution to the relaxation comes from nonlinear damping.[11] In the former case, we have

$$\chi''=\frac{2|V|^2}{|S|}\frac{(p-1)^{1/2}}{p}\quad\text{for}\quad p<p_c,$$
$$\chi''=\frac{2|V|^2}{|S|}\frac{[(p-p_c+1)(p_c-1)]^{1/2}}{p}\quad\text{for}\quad p>p_c; \tag{3.11}$$

in the latter case, we find that

$$\chi''=\frac{2|V|^2}{\eta}\frac{p^{1/2}-1}{p^{1/2}},\quad\gamma_p=\gamma_p{}^0+\eta n_p\quad\text{for}\quad p<p_c,$$
$$\chi''=\frac{2|V|^2}{\eta}\frac{p_c{}^{1/2}-1}{p^{1/2}},\quad\text{for}\quad p>p_c. \tag{3.12}$$

Hence, it is easy to obtain expressions for the jump in the derivative of the susceptibility $\partial\chi''/\partial p$ at $p=p_c$. In the case of the phase mechanism of limitation of the amplitude of parametric spin waves, we have

$$\frac{1}{\chi_c''}\Delta\left(\frac{\partial\chi''}{\partial p}\right)=\frac{1}{2}\frac{p_c}{p_c-1}\left(1+\frac{2}{p_c}-\frac{4}{p_c{}^2}\right)\approx\frac{1}{2}. \tag{3.13}$$

We have allowed for the fact that $p_c\approx 90\gg 1$. In the case when the nonlinear damping is strong, the susceptibility jump is

$$\frac{1}{\chi_c''}\Delta\left(\frac{\partial\chi''}{\partial p}\right)=-\frac{\eta^2-S^2}{2\eta^2}\frac{1}{p_c}. \qquad (3.14)$$

If $\eta > |S|$, this jump is negative. We can show that for any relationship between $S$ and $\eta$, the following condition is satisfied:

$$\Delta\left(\frac{\partial\chi''}{\partial p}\right)\Big/\left(\frac{\partial\chi''}{\partial p}\right)_{p_c-0}<0, \qquad (3.15)$$

i.e., the derivative $\partial\chi''/\partial p$ in the range $p<p_c$ is opposite in sign to the jump of the derivative at $p=p_c$.

## 4. DISTRIBUTION OF NONEQUILIBRIUM MAGNONS ABOVE THE KINETIC INSTABILITY THRESHOLD

We shall now analyze self-consistently the scattering of parametric spin waves on secondary spin waves and obtain an expression for the susceptibility in a wider range of supercriticalities (excess above the kinetic instability threshold). The result of scattering of parametric by secondary spin waves is a third packet of nonequilibrium magnons with frequencies $|\omega_{\mathbf{k}}-\omega_p/2|\lesssim\Delta\omega_s$. The width of this packet is fairly large: $\Delta\omega_s\gg\gamma_p$; therefore, the magnons in the third packet (group) are unrelated to the pumping. The distribution function of magr ɔns above the kinetic instability threshold

$$n_{\mathbf{k}}=n_{\mathbf{k}}^0+n_{\mathbf{k}}^p+n_{\mathbf{k}}^{s1}+n_{\mathbf{k}}^{s2}+n_t', \qquad (4.1)$$

where $n_{\mathbf{k}}^0$ is the distribution fur ion of thermal magnons, $n_{\mathbf{k}}^p$ is the corresponding function o ɔarametric magnons, $n_{\mathbf{k}}^{s1,2}$ is the corresponding quantity for secondary spin waves, and $n_{\mathbf{k}}^t$ for third-generation spin waves.

The distribution functions of secondary and third-generation spin waves can be found from the kinetic equation for magnons. The total numbers of secondary magnons are of the greatest interest in the present case:

$$n_{s1,2}=\frac{v_0}{(2\pi)^3}\int n_{\mathbf{k}}^{s1,2}\,d\mathbf{k},\qquad n_t=\frac{v_0}{(2\pi)^3}\int n_{\mathbf{k}}^t\,d\mathbf{k}. \qquad (4.2)$$

The equations describing these quantities can be obtained by integrating the kinetic equation in that part of the $\mathbf{k}$ space which is occupied by a suitable packet. In the case of secondary spin waves we have

$$n_{s2}=(\gamma_1+\gamma_2)\gamma_2^{-1}n_{s1},\qquad \gamma_1\gamma_2(\gamma_1+\gamma_2)^{-1}=\Delta. \qquad (4.3)$$

The equality (4.3) is based on two assumptions: neglect of elastic scattering of secondary spin waves by parametric spin waves, and the influence of scattering of secondary spin waves on one another. The first assumption is justified in Ref. 10. We shall now give grounds for the second assumption. A rough estimate of the frequency of scattering of secondary spin waves on one another gives $\Gamma_{ss}\propto(Tn_{s1})^2/\Delta\omega_s$. This estimate yields a very high value of the frequency of scattering of secondary spin waves. However, it should be noted that in the case of an extremely narrow packet of secondary spin waves we have $n_k^s\propto\delta(k-k_0)$ and the right-hand side of Eq. (3.1) vanishes, so that the scattering of secondary spin waves on one another does not result in the loss of these spin waves from the packet. It is shown in Ref. 10 that if $\gamma_{\mathbf{k}}^0-\Delta_{\mathbf{k}}<0$, the limitation of the kinetic instability occurs because of the collisional mechanism: secondary spin waves scattered by one another reach the part of the $\mathbf{k}$ space, where $\gamma_{\mathbf{k}}^0-\Delta_{\mathbf{k}}>0$ and are damped out there. Therefore, we can estimate the degree of freezing of the number of parametric spin waves by comparing the damping of secondary spin waves at the center and edges of a packet. The condition of validity of the second approximation is

$$|\gamma_{k_0+\Delta k}^0-\gamma_{k_0}^0|/\gamma_{k_0}^0\ll1, \qquad (4.4)$$

where $k_0$ is the wave vector at the center of a packet of secondary spin waves and $\Delta k$ is the width of this packet. An estimate of the packet width $\Delta k$ is difficult, but there is no doubt that $\Delta k\lesssim k_0$ and also that $|\gamma_{2k_0}^0-\gamma_{k_0}^0|\ll\gamma_{k_0}^0$ because in the case under discussion we have

$$\gamma_k^0-\gamma_{k=0}^0=(4\pi)^{-1}\frac{\Theta}{\Theta_c}\frac{\omega_m^2}{\omega_0}(ak). \qquad (4.5)$$

Here, $\Theta$ is the absolute temperature and $\Theta_c$ is the Curie point. We can easily show that if $k=k_0$, this correction amounts to $(10^{-2}\text{–}10^{-3})\gamma_{k_0}^0$. Therefore, we shall assume that the negative contribution to damping $\Delta$ of secondary spin waves, due to the influence of parametric and third-generation spin waves, compensates for the intrinsic damping $\gamma_s^0$ of secondary waves.

The third group of magnons is described by

$$\begin{aligned}\gamma_t^0n_t=\frac{v_0^3}{(2\pi)^8}\int|T_{1234}|^2\,[&n_1^tn_2^tn_3^p+2n_1^tn_3^pn_4^p\\&-2n_1^tn_2^t(n_3^{s1}+n_4^{s2})\\&+2(n_1^t+n_1^p)n_3^{s1}n_4^{s2}-2n_1^t(n_2^{s1}+n_2^{s2})n_3^p]\\&\times\delta(\omega_1+\omega_2-\omega_3-\omega_4)\\&\times\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4)\prod_{i=1}^4d\mathbf{k}_i.\end{aligned} \qquad (4.6)$$

The wave vector $k_0$ of secondary spin waves is very small: $k_0\ll k_p$ ($k_p$ is the wave vector of parametric spin waves), so that the distribution function of third-generation spin waves in the $\mathbf{k}$ space is almost exactly the same as the distribution function of parametric spin waves. We shall introduce the normalized distribution functions of parametric, secondary, and third-generation spin waves:

$$n_{\mathbf{k}}^p=n_p\varphi_{\mathbf{k}},\qquad n_{\mathbf{k}}^t=n_t\varphi_{\mathbf{k}},\qquad n_{\mathbf{k}}^{s1,2}=n_{s1,2}\psi_{\mathbf{k}}^{1,2}. \qquad (4.7)$$

Using these normalized functions we can rewrite the equations for the numbers of nonequilibrium magnons [Eqs. (4.3) and (4.6)] in the form

$$b(n_t+n_p)^2=\gamma_1\gamma_2(\gamma_1+\gamma_2)^{-1}, \qquad (4.8)$$

$$\begin{aligned}&\gamma_t^0n_t-an_t(n_t+2n_p)n_p+2bn_t(n_t+n_p)\\&\times(n_{s1}+n_{s2})-2c_1n_pn_{s1}^2-2c_2n_pn_{s2}^3+2d(n_p+n_t)n_{s1}n_{s2}=0,\end{aligned} \qquad (4.9)$$

where

$$a=\int R_{1234}\varphi_1\varphi_2\varphi_3\,dP,\qquad b=\int R_{1234}\varphi_1\varphi_2\psi_3^1\,dP,$$

$$c_{1,2}=\int R_{1234}\psi_1^{1,2}\varphi_2\psi_3^{1,2}\,dP,\qquad d=\int R_{1234}\varphi_1\psi_2^1\psi_3^2\,dP,$$

$$R_{1234}=\frac{v_0^3}{(2\pi)^8}|T_{1234}|^2\,\delta(\omega_1+\omega_2-\omega_3-\omega_4)$$

$$\times\delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4),\quad dP=\prod_{i=1}^4d\mathbf{k}_i. \qquad (4.10)$$

The interaction with secondary and third-generation spin waves also increases the damping of parametric magnons, since it displaces parametric waves from the region of resonance with the pump radiation. We then have

$$\gamma_p=\bar\gamma+an_t(n_t+2n_p)+2b(n_t+n_p)(n_{s1}+n_{s2})+2c_1n_{s1}^2+2c_2n_{s2}^2. \quad (4.11)$$

We can close the system of equations (4.8), (4.9), and (4.11) by using an equation from the $S$ theory,[11] which gives the total number of parametric spin waves [Eq. (3.7)], and allow for the fact that at the kinetic instability threshold the damping is the same for third-generation and parametric spin waves: $\gamma_t^0=\gamma_p$.

We shall now find the parameters $a$, $b$, $c$, and $d$. We shall assume that the distribution functions $\varphi_k$ and $\psi^{k\,1,2}$ are axially symmetric and that $\varphi_k$ is localized in the range of angles $|\theta_k-\pi/2|\lesssim\Delta\theta$ near the equator of a resonance surface $\theta_k=\pi/2$, whereas type-1 secondary spin waves are localized in the region where $k\sim k_0\ll k_p$ and $\theta\approx0,\pi$. We find from Eq. (4.1) that

$$a\approx\frac{1}{6\pi}\frac{2\langle|T_{kk'}|^2\rangle+\langle|S_{kk'}|^2\rangle}{k_pv_p}\ln\left(\frac{16\sqrt{2}}{\Delta\theta}\right), \quad (4.12)$$

where $v_p$ is the group velocity of parametric spin waves; $\langle|T_{\mathbf{kk}'}|^2\rangle$ and $\langle|S_{\mathbf{kk}'}|^2\rangle$ are the averaged (over the angle) values of the squares of the moduli of the matrix elements:

$$T_{\mathbf{kk}'}\equiv T_{\mathbf{k},\mathbf{k}',\mathbf{k},\mathbf{k}'},\qquad S_{\mathbf{kk}'}\equiv T_{\mathbf{k},-\mathbf{k},\mathbf{k}',-\mathbf{k}'}.$$

These quantities are easily expressed in terms of the angular Fourier harmonics of the matrix elements[11]:

$$\langle|T_{\mathbf{kk}'}|^2\rangle=T_0^2+2T_1^2+2T_2^2,\qquad \langle|S_{\mathbf{kk}'}|^2\rangle=S_0^2+S_2^2+S_{\bar 2}^2.$$

The parameter $b$ determines the negative contribution to the damping of secondary spin waves, investigated in detail in Ref. 1:

$$b=\frac{T^2}{2\pi\omega_m}\left(\frac{2\pi M}{H_c-H}\right)^{1/2}A^{-1/2},$$
$$A=4(\Omega_p^2+1)^{1/2}-3-6\Omega_0-[(\Omega_p-\Omega_0)^2+1/4]^{1/2}, \quad (4.13)$$
$$\Omega_p=\omega_p/\omega_m,\qquad \Omega_0=\omega_0/\omega_m,\qquad \omega_m=4\pi gM,$$

and $T=T_{\mathbf{k},\mathbf{k}',\mathbf{k}+\mathbf{k}',0}\approx0.7\pi g^2$ (see Ref. 1). Finally, the value of $c_1$ is related to the matrix element $T_1\approx-\pi g^2$ by the following expression[10]:

$$c_1\approx\pi\frac{T_1^2}{k_0v_p}\ln^2\left(\frac{v_pk_0}{\Delta\omega_s}\right), \quad (4.14)$$

$$c_2\sim a\ll c_1,\qquad d\sim b\ll c_1. \quad (4.15)$$

The largest of these coefficients is $c_1$ which contains a small denominator $k_0v_p$. The relationships (4.15) and (4.3) allow us to neglect in Eqs. (4.9) and (4.11) the terms proportional to $c_2$ and $d$.

For a small excess above the kinetic instability threshold the number of secondary spin waves is small. It follows from Eq. (4.9) that the number of third-generation spin waves is proportional to $n_{s1}^2$. If we confine ourselves to the terms linear in $n_{s1}$, we obtain the equations discussed in Sec. 3.

The condition (4.8) means that the total damping of secondary spin waves vanishes and it fixes the overall number of third-generation and parametric spin waves:

$$n=n_p+n_t=[\gamma_1\gamma_2/b(\gamma_1+\gamma_2)]^{1/2}. \quad (4.16)$$

When the excess above the kinetic instability threshold is not too high, the number of third-generation spin waves is small compared with the number of parametric spin waves, which makes it possible to simplify the system of equations (4.9), (4.11), and (3.7). When the main mechanism limiting the amplitude of parametric spin waves is nonlinear damping $\bar\gamma=\bar\gamma(n)$, we have

$$n_{s1}=\frac{1}{2}\left(\frac{\gamma_s^0}{b}\right)^{1/2}\left[\left(\frac{\gamma_p^0}{\gamma_s^0}\frac{p-p_c}{p_c^{1/2}}+\frac{b^2}{c_1^2}\right)^{1/2}-\frac{b}{c_1}\right],$$
$$n_p\approx n_p^c=(\gamma_s^0/b)^{1/2},\qquad (p-p_c)/p_c\ll1. \quad (4.17)$$

Since in this case the number of parametric spin waves is close to the threshold, the susceptibility is given by Eq. (3.13) derived earlier on the assumption that the number of parametric spin waves is frozen.

If the number of parametric spin waves is limited by the phase mechanism, we find that

$$n_{s1}\approx{}^1/_4\gamma_p^0(\gamma_s^0b)^{-1/2}(p-p_c),\qquad n_p\approx n_p^c=n. \quad (4.18)$$

In this case the susceptibility is again given by the expressions in Sec. 3. It should be pointed out that the distribution given by Eq. (4.18) exists in a narrow range near the kinetic instability threshold. Under the experimental conditions of Ref. 1 we have $p_c\sim100$ and $\gamma_p^0/\gamma_s^0\approx3$–6, so that $p-p_c\lesssim1$–3. At higher pump amplitudes the solution (4.18) does not exist and the number of parametric spin waves decreases abruptly; we then have to consider the other limiting case $n_p\ll n$. At moderately high pump powers, we obtain

$$n_p=\left\{\frac{\gamma_s^0}{b}\frac{p-[p^2-4\mu^2(p_c-\mu^2)]^{1/2}}{2(p_c-\mu^2)}\right\}^{1/2},\qquad \mu=\frac{\bar\gamma}{\gamma_p^0}\geqslant1. \quad (4.19)$$

If the value of $\bar\gamma$ is governed by nonlinear damping, then

$$n_p=n\left(\frac{p_c}{p}\right)^{1/2}=\left(\frac{\gamma_s^0p_c}{bp}\right)^{1/2},\qquad \chi''=\chi_c''\frac{p_c}{p}. \quad (4.20)$$

When the phase mechanism is the main factor responsible for the limitation of the amplitude of primary spin waves, we find that

$$n_p=\left(\frac{\gamma_s^0}{bp}\right)^{1/2},\qquad n_{s1}=\left(\frac{\gamma_p^0}{2c_1}\right)^{1/2}(p_cp)^{1/4},\qquad n_t\approx n,$$
$$\chi''=\chi_c''(p_c/p),\qquad p\gg\mu^2. \quad (4.21)$$

The more rigorous expression (4.19) describes the behavior of nonequilibrium magnons in the range $p\geqslant2p_c^{1/2}$.

We shall finally consider the limiting case of a large excess above the threshold. If the nonlinear damping mechanism is important, then

$$n_{s1}=\left(\frac{\gamma_p^0}{c_1}p\right)^{1/2}p^{1/4},\qquad n_p=\gamma_s^0(\gamma_p^0c_1)^{-1/2}p^{-1/4},$$
$$\chi''=\chi_c''\left(\frac{b}{c_1}\frac{\gamma_s^0}{\gamma_p^0}\right)^{1/2}p^{-1/4}. \quad (4.22)$$

For the phase mechanism of limitation of the amplitude of parametric spin waves, we obtain

$$n_{s1}=\left(\frac{\gamma_s^0}{c_1}\right)^{1/2}p^{1/4},\qquad n_p=\gamma_s^0(\gamma_p^0c_1)^{-1/2}p^{-1/4},$$
$$\chi''=\chi_c''\left(\frac{b}{c_1}\frac{\gamma_s^0}{\gamma_p^0}\right)^{1/2}p_cp^{-1/4}. \qquad (4.23)$$

Therefore, well beyond the kinetic instability threshold the imaginary part of the susceptibility falls proportionally to $p^{-1/4}$.

In the case when parametric and third-generation spin waves can coalesce with type-2 secondary spin waves, we have to allow for an additional contribution of these processes to the damping of magnons at the relevant frequency

$$\bar{\gamma}\to\bar{\gamma}+\eta_1 n_{s2},$$
$$\eta_1=\frac{v_0}{(2\pi)^2}\int|V_{\mathbf{k},\mathbf{k}',\mathbf{k}+\mathbf{k}'}|^2\psi_{\mathbf{k}'}{}^2\delta(\omega_\mathbf{k}-\omega_{\mathbf{k}'}-\omega_{\mathbf{k}+\mathbf{k}'})\,d\mathbf{k}'. \qquad (4.24)$$

Allowance for this circumstance reduces to the substitution $\gamma_1\to\gamma_1+\eta_1 n/2$ in all the expressions for the numbers of nonequilibrium magnons and for the susceptibility.

## 5. DISCUSSION OF RESULTS

The approximation of the self-consistent interaction of packets, developed in Sec. 3, predicts the behavior of the susceptibility in a narrow range of supercriticalities near the kinetic instability threshold. We shall therefore initially consider the behavior of a kink in the dependence of supercriticalities on the pump power. To compare the expression for the susceptibility $\chi''(p)$ of Eq. (3.10) with the experimental results we have to know the damping $\bar{\gamma}$ of parametric spin waves at the kinetic instability threshold, and this damping can be found by making certain model assumptions about the nature of relaxation of such waves. Roughly speaking, all the $\chi''(p)$ dependences obtained in different fields (Fig. 1) can be described by allowing for the dependence of the damping of parametric spin waves on their number $\gamma_p(n_p)=\gamma_p^0+\eta n_p$. The behavior of the susceptibility considered as a function of the nonlinear damping coefficient $\eta$ is discussed in detail in a review paper of Zakharov, L'vov, and Starobinets.[11] The experimentally determined susceptibility is described qualitatively by a coefficient $\eta(H)$ decreasing monotonically on increase in the magnetic field. If $H=1047$ Oe, then $\eta/S\approx 2$, whereas for $H=1190$ Oe, we have $\eta/S\approx 1$, and if $H=1280$ Oe, then $\eta/S\approx 0.6$. It follows from Eq. (3.9) that at the kinetic instability threshold the model of nonlinear damping in the range $p_c\gg 1$ yields the relationship

$$\left(\frac{\partial\chi''}{\partial\ln p}\right)_{p_c+0}\Big/\left(\frac{\partial\chi''}{\partial\ln p}\right)_{p_c-0}\approx\left[p_c\left(1+\frac{S^2}{\eta^2}\right)\right]^{1/2}. \qquad (5.1)$$

Under the experimental conditions we have $p_c^{1/2}\approx 10$, so that the slope of $\chi''(p)$ at the point $p=p_c$ is altered by an order of magnitude. Investigation of the relevant dependences (Fig. 1) confirms this relationship, except for $H=1281$ Oe. This is because the theory developed above is inapplicable to this case since in fields $H>H_2=1260$ Oe the wave vector of type-1 secondary spin waves differs from zero. In a field $H=1192$ Oe we find that $(\partial\chi''/\partial p)_{p_c-0}$ vanishes and, moreover, in the same field it follows from Eq. (5.1) that $(\partial\chi''/\partial p)_{p_c+0}=0$. A quantitative comparison of Eq. (5.1) with the experimental results is impossible because of the rough approximations about the nature of nonlinear relaxation of parametric spin waves and the scatter of the experimental points, which makes it difficult to calculate the derivative $\partial\chi''/\partial p$ in the immediate vicinity of $p_c$.

We shall now consider the behavior of $\chi''(p)$ when $(p-p_c)/p_c\sim 1$. In fields $H<1180$ Oe the dependence $\chi''(p)$ agrees well with the theoretical predictions in Sec. 4. Above the kinetic instability threshold the susceptibility falls monotonically. In the limit $p\to\infty$, it follows from Eq. (4.23) that $\chi''\propto p^{-1/4}$. An analysis of the experimental data at the maximum pump power, i.e., when the kinetic instability threshold was exceeded by 3–5 dB, gives $\chi''\propto p^{-\alpha}$, where the values of $\alpha$ are as follows:

| $H$, Oe | 1047 | 1062 | 1092 | 1122 |
|---|---|---|---|---|
| $\alpha$ | 0.35 | 0.3 | 0.25 | 0.25 |

We can see that the power exponent is close to 1/4. Its deviation from the theoretical value in lower fields is probably due to the fact that the pump power is insufficiently high to ensure that the susceptibility approaches the asymptotic law.

We have a pleasant duty to thank V. S. L'vov and G. A. Melkov for valuable discussions.

[1]A. V. Lavrinenko, V. S. L'vov, G. A. Melkov, and V. B. Cherepanov, Zh. Eksp. Teor. Fiz. **81**, 1022 (1981) [Sov. Phys. JETP **54**, 542 (1981)].
[2]P. E. Zil'berman, S. A. Nikitov, and A. G. Temiryazev, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 92 (1985) [JETP Lett. **42**, 110 (1985)].
[3]B. Ya. Kotyuzhanskiĭ and L. A. Prozorova, Zh. Eksp. Teor. Fiz. **86**, 658 (1984) [Sov. Phys. JETP **59**, 384 (1984)].
[4]B. Ya. Kotyuzhanskiĭ, L. A. Prozorova, and L. E. Svistov, Zh. Eksp. Teor. Fiz. **86**, 1101 (1984) [Sov. Phys. JETP **59**, 644 (1984)].
[5]V. S. Lutovinov, G. E. Fal'kovich, and V. B. Cherepanov, Zh. Eksp. Teor. Fiz. **90**, 1781 (1986) [Sov. Phys. JETP **63**, 1045 (1986)].
[6]I. V. Krutsenko, V. S. L'vov, and G. A. Melkov, Zh. Eksp. Teor. Fiz. **75**, 1114 (1978) [Sov. Phys. JETP **48**, 561 (1978)].
[7]V. S. L'vov and V. B. Cherepanov, Preprint No. 314 [in Russian], Institute of Automation and Electrometry, Siberian Branch of the Academy of Sciences of the USSR, Novosibirsk (1986).
[8]A. G. Gurevich, *Ferrites at Microwave Frequencies* [in Russian], Fizmatgiz, Moscow (1960).
[9]A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskii, *Spin Waves*, North-Holland, Amsterdam; Wiley, New York (1968).
[10]V. S. L'vov and V. B. Cherepanov, Zh. Eksp. Teor. Fiz. **81**, 1406 (1981) [Sov. Phys. JETP **54**, 746 (1981)].
[11]V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Usp. Fiz. Nauk **114**, 609 (1974) [Sov. Phys. Usp. **17**, 896 (1975)].

Translated by A. Tybulewicz