Critical problems in the theory of strong Langmuir turbulence; comparison of analytical and numerical models

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In this paper we give a numerical simulation of two-dimensional Langmuir turbulence. We show that the modulational instability is the main mechanism for pumping energy into the long-wavelength source region, and we determine the wave spectra and compare them with the results of analytical models. We discuss possible regimes for the absorption of Langmuir oscillations in the short-wavelength region of the Langmuir wave spectrum. We consider the relative role of conversion by sound and nucleation in the short-wavelength transfer of plasma oscillations.

1. INTRODUCTION

The phenomenological model of strong Langmuir turbulence[1,2] is based upon the analogy with hydrodynamic turbulence of an incompressible fluid.[3] As the elementary cell of Langmuir turbulence we take the caviton (a region of lowered plasma density) in which plasmons (the quanta of the Langmuir oscillations) are trapped within it, and which is the analog of a vortex in the incompressible fluid. The caviton collapse[4] guarantees the short-wavelength transfer of the plasmons trapped in the caviton, which proceeds down to the stage where such small scales are reached that any of the mechanisms for the dissipation of the plasma oscillation energy (Landau damping by resonance electrons or intersection of electron trajectories) become important. There is here also an analogy with hydrodynamic turbulence in which the nonlinear subdivision of the vortex scales proceeds until viscosity sets in.

As in hydrodynamic turbulence, we can distinguish in the spectrum of strong Langmuir turbulence three regions: the long-wavelength source region, the inertial range, where the energy is transferred through its scales by the collapsing cavitons, and the short-wavelength absorption region. The phenomenological model[1,2] describes the turbulence in all three regions, but although there are at the present time rather many papers[5–7] which essentially repeat the ideas of this model, the absence of a rigorous analytical theory makes it necessary to go to a numerical simulation to check the basic hypotheses on which the model is based. The present paper is devoted to the results of a numerical simulation of the hydrodynamic Zakharov equations[4] within the framework of the two-dimensional model of Langmuir turbulence which admits the collapse phenomenon. The crucial aspects of the model which were subjected to a check by numerical methods reduce to the following.

1. An elucidation of the mechanisms for the localization of the Langmuir oscillations in a caviton. Possible mechanisms are the modulational instability in the source region[1] and nucleation—the capture of plasmons in density wells which remain after the caviton of the preceding generation has been burned up.[6]

2. The determination of the effective collisional frequency characterizing the dissipation rate of the energy of the pumping wave into turbulence.

3. Finding by numerical simulation the inertial range, with a power-law turbulence spectrum which is a consequence of the transfer of Langmuir energy by collapsing cavitons.

4. To elucidate, for a plasma with weakly damped short-wavelength sound oscillations that accompany the modulational instability and the collapse, the relative roles of conversion via scattering by sound and via short-wavelength transfer of plasma oscillations.

We assume in the present paper that the distribution function of the resonance electrons remains unchanged and Maxwellian, and in that case simulation based upon the hydrodynamic equations cannot elucidate the relative roles of the two competing dissipation mechanisms—Landau damping which leads to the formation of a "tail" of accelerated electrons, and intersection of electron trajectories. An analysis of this problem in Ref. 8 has shown that even in the one-dimensional case for relatively weak pumping, $E_0^2/8\pi n_0 T \lesssim 10^{-2}$, the main dissipation mechanism is connected with the acceleration of "tails" of resonance electrons. It was also shown in Ref. 8 that for such weak pumping the more rigorous kinetic approach based upon an integration of the equations of motion of the separate particles is equivalent to the hydrodynamic approach supplemented by an equation describing the quasilinear deformation of the resonance particle distribution function. In this connection the problem of the two-dimensional numerical simulation of turbulence in the framework of the hydrodynamic Zakharov equations for the field and of a quasilinear equation for the resonance electrons is of interest. In particular, such a simulation enables us to determine the way the cavitons collapse in absorption-region scales and to choose one of the alternative models of this region which were proposed in Refs. 1 and 5 (*vide infra*).

2. TURBULENCE SOURCE REGION. INTEGRAL CHARACTERISTICS

We integrated numerically the set of hydrodynamic equations for the amplitude of the envelope of the field of the plasma oscillations

$$\mathbf{E}_l = {}^1/_2 \mathbf{E}(t, r)\exp(-i\omega_p t) + \text{c.c.}$$

and for the slow quasineutral density variations $\delta n(t,\mathbf{r})$ which occur under the action of the high-frequency pressure. This set of equations differs from the normal Zakharov equations[4] by the presence of a long-wavelength source

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$$\langle \mathbf{E}\rangle=\mathbf{E}_0=\text{const}$$

(the angle brackets indicate averaging over the volume of the plasma) and of absorption of the short-wavelength Langmuir and low-frequency oscillations. The corresponding set of equations has the form

$$\operatorname{div}\left\{\frac{1}{i\omega_p}\frac{\partial \mathbf{E}}{\partial t}-\frac{3}{2}r_D{}^2\,\nabla\operatorname{div}\mathbf{E}+\frac{1}{2}\frac{\delta n}{n_0}\mathbf{E}+\frac{1}{(2\pi)^3}\int\frac{\Gamma(\mathbf{r}-\mathbf{r}')}{i\omega_p}\mathbf{E}(\mathbf{r}')\,d\mathbf{r}'\right\}=0, \tag{1}$$

$$\frac{\partial^2\delta n}{\partial t^2}+\frac{2}{(2\pi)^3}\int\gamma(\mathbf{r}-\mathbf{r}')\frac{\partial}{\partial t}\delta n(\mathbf{r}')\,d\mathbf{r}'-\frac{T_e}{M}\Delta\,\delta n=\frac{1}{16\pi M}\Delta|\mathbf{E}|^2. \tag{2}$$

The Fourier transforms of the functions $\Gamma(\mathbf{r})$ and $\gamma(\mathbf{r})$ are the damping rates of the Langmuir and the low-frequency oscillations. We have neglected in the numerical simulation the acceleration of the electrons that are at resonance with the Langmuir oscillations and assumed that their distribution function is Maxwellian, so that the Fourier transform $\Gamma(\mathbf{k})$ of the function $\Gamma(\mathbf{r})$ has the form

$$\Gamma_{\mathbf{k}}=\left(\frac{\pi}{8}\right)^{1/2}\omega_p\frac{1}{k^3r_D{}^3}\exp(-1/2k^2r_D{}^2-{}^3/_2). \tag{3}$$

As to the damping rate of the low-frequency mode, we considered two cases—a non-isothermal plasma ($T_e \gg T_i$) with a weak damping

$$\gamma_{\mathbf{k}}=\left(\frac{\pi}{8}\right)^{1/2}\omega_p\frac{m}{M}\mathbf{k}r_D, \tag{4}$$

modeling the absorption of the low-frequency mode by the electrons, and an isothermal plasma ($T_e = T_i$) when the absorption of that mode by the ions is important

$$\gamma_{\mathbf{k}}=(\pi/8)^{1/2}\omega_p(m/M)^{1/2}\mathbf{k}r_D\{(m/M)^{1/2}+(T_e/T_i)^{3/2}\exp(-T_e/T_i)\}. \tag{4'}$$

We numerically integrated the set of Eqs. (1), (2) in the two-dimensional region $L_x = L_y = 404r_D$ with periodic boundary conditions and with the real mass ratio $M/m = 1836$. The initial conditions were modeled by random noise of the electric field and the density variations with the energies of the Langmuir and the low-frequency oscillations equal to, respectively, $W/nT\sim 10^{-4}$ and $W_s/nT\sim 10^{-8}$. For the solution we used a pseudo-spectral method with respect to both variables, with the number of harmonics equal to 128^2. The numerical model and the solution method are described in more detail in Refs. 9 and 10.

In the present section we consider in what follows the case of strong damping of the low-frequency mode.

We show in Fig. 1 the time dependence of the total energy of the oscillations,

$$W=\sum_{\mathbf{k}}\frac{|E_{\mathbf{k}}|^2}{8\pi}$$

obtained in the numerical simulation. The exponential growth of W with time in the initial stage of the process, which is caused by the flow of energy into turbulence from the pump changes into a quasi-stationary state in which this energy flow is compensated for by its flow along the spectrum as the result of the collapse of cavitons. The corresponding balance equation can be written in the form

$$dW/dt=\nu_{eff}E_0{}^2/8\pi-2\gamma_{eff}W. \tag{5}$$

In this equation ν_{eff} is the effective collisional frequency characterizing the rate of inflow of energy from the pump into the turbulence, and γ_{eff} is the characteristic growth rate of the caviton collapse, which can be written in the form

$$1/\gamma_{eff}=1/\gamma_1+1/\gamma_2, \tag{6}$$

where we have used the notation $1/\gamma_1$ for the time for a caviton to reach the self-similar regime and $1/\gamma_2$ for the characteristic time of the self-similar collapse. As a result of the collapse, the Langmuir oscillations trapped in the caviton become equal in scale to the absorption region and we can write the balance equation for ν_{eff} in the form

$$\nu_{eff}=2E_0{}^{-2}\sum_{\mathbf{k}}\Gamma_{\mathbf{k}}|E_{\mathbf{k}}|^2. \tag{7}$$

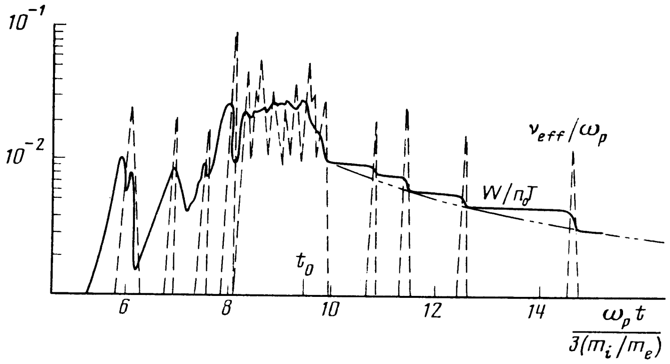

FIG. 1. Time dependence of the plasma-oscillation energy W/n_0T and of the effective collision frequency $\nu_{\text{eff}}/\omega_p$ in isothermal turbulence produced by pumping with an amplitude $E_0^2/8\pi n_0T = 1.5\times10^{-3}$. At the time $t_0\approx 9.7$ the pumping is switched off. The dash-dot curve shows the decrease in the energy W/n_0T according to Eq. (14).

The quantity ν_{eff} determined by this relation is also given in Fig. 1. The pulse nature of the time dependence of ν_{eff} is connected with the fact that the process of forming and burning up of collapsing cavitons is a discrete one.

The cavitons are formed over long-wavelength scales of the source region either due to the capture of plasma oscillations in the density well remaining after the burnout of the cavitons of the preceding generation (nucleation[6]) or due to the localization of plasma oscillations by the modulational instability.[1] The characteristic velocity of the latter process is of the order of the sound velocity $\gamma/k\sim (T/M)^{1/2}$ and it is accompanied by the appearance of self-consistent long-wavelength density fluctuations in which there occurs a balance between the high-frequency and the gas-kinetic pressure:

$$|\delta n_L|=\Big[\sum_{k<k_0}|\delta n_k|^2\Big]^{1/2}\approx W/T. \tag{8}$$

In this formula δn_L is the root-mean-square amplitude of the long-wavelength density fluctuations and the summation over k is over the scales of the source region $k < k_0$.

The characteristic wave numbers of the source region are determined from the relation

$$k_0 \approx \frac{1}{r_D}\left(\frac{|\delta n_L|}{3n_0}\right)^{1/2} \approx \frac{1}{r_D}\left(\frac{W}{3n_0T}\right)^{1/2} \qquad (9)$$

In the numerical simulation we have also observed a balance between the pressures of the low-frequency fluctuations and of the Langmuir oscillations (see Fig. 2). We also studied the frequency spectrum of the plasma oscillations for various values of the wavelength. We give in Fig. 3 the Fourier spectra of a high-frequency electric field for $k = \text{const}$. It is clear that in the long-wavelength region, $k < k_0$, an appreciable fraction of the plasmons have $\omega > \omega_p$ which corresponds to free plasmons produced as the result of the modulational instability, whereas in the nucleation mechanisms we do not have such plasmons. When the wave number increases the fraction of plasmons with $\omega < \omega_p$ which are trapped in the density well increases, and in the scales of the inertial range and the absorption region practically all plasmons are trapped in collapsing cavitons.

The results of the numerical simulation given in Figs. 2 and 3 confirm from our point of view the fact that in the overall balance of the turbulence the main mechanism for the localization of the long-wavelength plasma oscillations from the source region is connected with the modulational instability. One must also bear in mind that the sound waves producing the density well in which the plasmons are captured in the nucleation are formed at the concluding stage of the collapse, when as the result of the absorption of the Langmuir oscillations the balance between the gas-kinetic and the high-frequency pressures is violated and the excess gas-kinetic pressure is dumped in the form of sound waves emitted from the cavitons (see Ref. 2). There are therefore grounds for assuming that nucleation causes capture of sufficiently short-wavelength plasmons and this process cannot be dominating for the source region.

The existing analytical model of plasma turbulence starts from the assumption that the mechanism of the absorption of the pumping energy to produce turbulence is stochastic heating caused by the mixing of the phases of the plasmons when they are scattered by the long-wavelength density fluctuations. Correspondingly, the effective collisional frequency which characterizes the rate of energy dissipation is determined by the relation

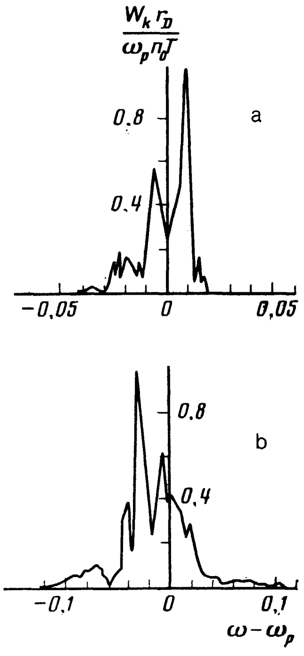

FIG. 3. Spectral frequency distribution of the energy of the Langmuir oscillations in an isothermal plasma for $k = \text{const}$, $\int W_k(\omega)d\omega dk = W$; a) long-wavelength source region, $kr_D = 0.05$, b) short-wavelength region, $kr_D = 0.25$.

$$\nu_{eff} \approx \omega_p |\delta n_L|/n_0. \qquad (10)$$

From this point of view the presence of the modulational instability of the pump is not the principal cause of dissipation; the modulational instability is in this case important only as a mechanism producing long-wavelength density fluctuations. To confirm this we performed the following numerical experiment: in a state of advanced turbulence the pump was decreased to a value below the threshold for the modulational instability, but nonetheless the dissipation of the pump continued with an effective frequency determined according to (10) by the level of the long-wavelength density fluctuations existing in the plasma. In the quasi-stationary turbulence state, when there is a balance of the gas-kinetic and the high-frequency pressures, the effective collision frequency is proportional to the energy of the oscillations and is determined by the following formula:

$$\nu_{eff} = \alpha\omega_p W/n_0T, \qquad (11)$$

where α is a numerical coefficient of order unity. The numerical simulation (see also Refs. 11 and 12) confirms the proportionality between the effective collision frequency and the energy and gives $\alpha \approx 0.7$ (see Fig. 2). The characteristic growth rate of the caviton collapse is determined by the modulational instability growth rate

$$\gamma_2 = \omega_p (mW/Mn_0T)^{1/2}.$$

We then get from the equation for the energy balance in the quasi-stationary state $dW/dt = 0$, using Eq. (11) for the effective collisional frequency, the following relation connecting the energies of the oscillations and of the pump:

$$2\lambda(W)(W/n_0T)^{1/2} = \alpha(M/m)^{1/2}E_0^2/8\pi n_0T; \qquad (12)$$

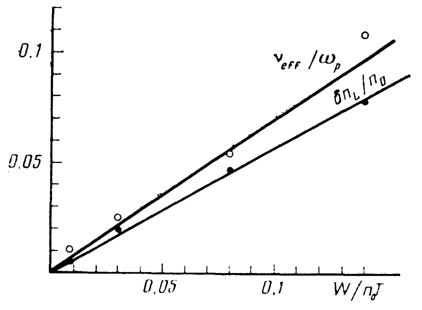

FIG. 2. The collision frequency ν_{eff}/ω_p (○) and the long-wavelength density perturbations (●) as functions of the energy of the plasma oscillations, obtained in the numerical simulation of isothermal turbulence. The straight lines correspond to the theoretical relations (11) and (8).

here we have used the notation

$$\lambda(W)=\gamma_1/(\gamma_1+\gamma_2).$$

The dependence of W on E_0^2, obtained by numerical simulation and is shown in Fig. 4, is the same as (12) for pump strengths $E_0^2/8\pi n_0T \lesssim 5\times10^{-3}$, provided $\lambda(W)=0.7(W/n_0T)^{1/2}$. We then have

$$W\approx{}^1/_2\left(\frac{M}{m}\right)^{1/2}E_0^2/8\pi. \tag{13}$$

The saturation of the growth of W with E_0^2 for large pumps is caused by the disappearance of the inertial interval and the turning-on of Landau damping directly in the source region (the "superstrong turbulence" regime[13]).

To verify that the short-wavelength transfer of Langmuir oscillations is caused only by the collapse, we performed a numerical experiment when the pump was switched off in a quasi-stationary state of turbulence. Figure 1 for $t>t_0$ illustrates the dynamics of the turbulence in that case. The switching-off of the pump leads according to (5) to a decrease of W with time. The steplike character of this decrease is a direct confirmation of the fact that the short-wavelength transfer of the energy of the Langmuir oscillations in the absorption region occurs solely through collapsing cavitons—each step corresponds to a process of formation, collapse, and burn-out of a caviton. Analysis of the spatial distribution of the field confirms this circumstance. The smoothed-out time dependence of W can with a good degree of accuracy be approximated by the formula

$$W=\frac{W_0}{1+{}^1/_2\omega_p(t-t_0)W_0/n_0T} \tag{14}$$

(t_0 is the moment the pump is switched off, W_0 the value of W at that time), which follows from Eqs. (5) for $E_0=0$ and the above mentioned value $\lambda(W)=0.7(W/n_0T)^{1/2}$.

3. INERTIAL RANGE AND ABSORPTION REGION

In the phenomenological model of the turbulence[1] it was postulated that, as in hydrodynamic turbulence, there exists an inertial range between the scales of the source region ($k_0r_D\sim(W/3n_0T)^{1/2}$) and of the absorption region ($k^a r_D\sim1$). When the low-frequency oscillations are strongly damped ($T_e=T_i$) the energy of the oscillations is transferred through the inertial range by the collapsing cavitons, as is confirmed in particular, by the numerical experiments with the switching-off of the pump which we described above. As in the case of hydrodynamic turbulence, Kolmogorov's hypothesis of a constant energy flux through the scales of this range is valid:

$$W_k\,dk/dt(k)=\text{const}, \tag{15}$$

where $dt(k)$ is the time for the spectral energy transfer through the wave-number interval k, $k+dk$. Assuming that the wave number of a plasmon captured by a collapsing caviton is determined by the characteristic reciprocal spatial scale of the caviton, $k\sim1/l$, we use the law of the self-similar caviton collapse, obtained in Ref. 4:

$$l\approx l_0[(t_0-t)/t_0]^{2/s},\quad \text{i.e.}\ dk\propto k^{1+s/2}\,dt, \tag{16}$$

where $s=1,2,3$ is the dimensionality of the collapsing caviton. In the case of two-dimensional turbulence ($s=2$) in which we are interested we then have the following spectrum of the Langmuir oscillations in the inertial range:

$$W_k\propto1/k^2. \tag{17}$$

An inertial turbulence range with a power-law spectrum is observed in numerical simulation for sufficiently low pumps: $E_0^2/8\pi n_0T\lesssim10^{-3}$ (see also Ref. 7). As an illustration we show in Fig. 5 the spectrum of the Langmuir oscillations which we obtained in the present work for $E_0^2/8\pi n_0T=10^{-4}$. One traces distinctly three sections of the spectrum—the source region with an approximately constant energy density, the inertial range with a power-law spectrum ($W\propto1/k^2-1/k^{2.5}$), and the absorption region where, in the considered case of a Maxwellian distribution function of resonance particles, the spectral density decreases exponentially with increasing wave number. For high pumps ($E_0^2/8\pi n_0T>10^{-3}$) the level determined from (13) is $W/n_0T\sim10^{-1}$, and there is essentially no inertial range in the turbulence spectrum, since for the damping rate (3) there appears appreciable damping already at $k^a r_D\sim\frac{1}{3}$.

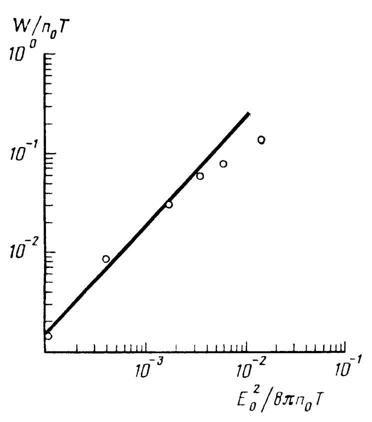

FIG. 4. The energy of the Langmuir oscillations W/n_0T as a function of the pumping amplitude $E_0^2/8\pi n_0T$. The points correspond to the results of the numerical simulation in an isothermal plasma, the straight line to Eq. (13).

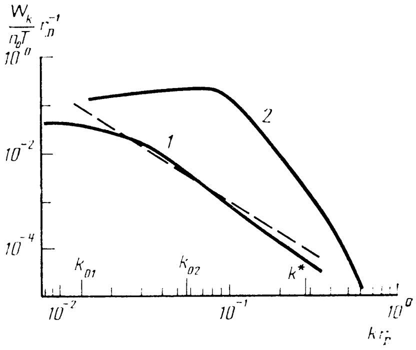

FIG. 5. The wave-number dependence of the spectral energy W_k in isothermal turbulence, $\int W_k dk=W$. The curves 1 and 2 correspond to pumps $E_0^2/8\pi n_0T=1\times10^{-4}$ and $E_0^2/8\pi n_0T=1.5\times10^{-3}$; k_{01} and k_{02} are the upper limits of the source region for these two cases, evaluated from Eq. (9); the quantity k^* corresponds to the maximum of the spectral absorption strength $\Gamma_k|E_k|^2$ and determines a characteristic wave number of the absorption region. The dashed line corresponds to the analytical expression $W_k\propto1/k^2$.

The absorption region can be described by a set of quasilinear equations which in the two-dimensional case have the following form:

$$\frac{\partial f}{\partial t}=\frac{\pi e^2}{M^2}\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left[\int_{\omega_p/v_\perp}^{\infty} dk\frac{|E_k|^2}{k^3}\frac{\omega_k^2}{(k^2v_\perp^2-\omega_k^2)^{1/2}}\frac{1}{v_\perp}\frac{\partial f}{\partial v_\perp}\right], \tag{18}$$

$$\frac{\partial |E_k|^2}{\partial t}+\frac{\partial}{\partial k}\left[|E_k|^2\frac{dk}{dt}\right]=2\Gamma_k|E_k|^2 \tag{19}$$

where

$$\Gamma_k=\frac{\pi\omega_k^2\omega_p^2}{2k^2}\int_{\omega_p/k}^{\infty} dv_\perp\frac{\partial f}{\partial v_\perp}\frac{1}{(k^2v_\perp^2-\omega_k^2)^{1/2}}$$

is the damping rate of the plasma oscillations, and the derivative dk/dt is determined by the law for the caviton collapse in the absorption region. In a quasi-stationary state the absorption is compensated for by the short-wavelength energy transfer caused by the collapse, and hence

$$\Gamma_k\sim\tau^{-1}(k),\quad \tau=t_0-t,$$

which is equivalent to the following distribution, sufficiently far ($kv_\perp\gg\omega_k$) from the tail:

$$f(v_\perp)\sim v_\perp^{-2}\tau^{-1}(v_\perp^{-1}). \tag{20}$$

Similarly, we get from the condition that the flux of resonance particles be constant,

$$J=\frac{1}{v_\perp}\int_{\omega_p/v_\perp}^{\infty} dk\frac{|E_k|^2}{k^3}\frac{\omega_k^2}{(k^2v_\perp^2-\omega_k^2)^{1/2}}\frac{1}{v_\perp}\frac{\partial f}{\partial v_\perp}=\text{const}$$

under the condition $kv_\perp\gg\omega_k$ the following wave spectrum:

$$|E_k|^2\sim k^{-3}\tau(k). \tag{21}$$

If, as was done in Ref. 2, we start from the hypothesis that due to the explosive nature of the collapse the burning-out of the main energy in the caviton proceeds only in the concluding stage of the collapse, and that the collapse law remains the same as in the inertial range $\tau(k)\sim 1/k^2$, we get from (20) and (21) the following formulae for the distribution function and the spectral energy density in the two-dimensional case:

$$f(v_\perp)\propto 1/v_\perp^4,\quad |E_k|^2\propto 1/k^5 \tag{22}$$

(in the three-dimensional case the analogous formulae have the form $f(v)\sim 1/v^{9/2}$, $|E_k|^2\sim 1/k^{9/2}$ (see Ref. 2)).

An alternative approach to the study of the spectra in the absorption region was proposed in Ref. 5. It is based upon the assumption of a constant number of collapsing cavitons through the scales of the absorption region $N_k k/\tau(k)=\text{const}$. If, moreover, one uses the self-similar law for the growth of the field in the center of the caviton $E_{\max}\sim 1/\tau(k)$, which follows from Eq. (2) for low-frequency density modulations and therefore is independent of the fact that the Langmuir oscillations are absorbed, one can write down the following equation for the spectral density of the energy of the oscillations:

$$|E_k|^2=N_kE_{max}^2/k^3\approx 1/k^3\tau(k). \tag{23}$$

This leads, together with (20) and (21) to the following results in the two-dimensional case considered by us for the collapse law:

$$\tau(k)\sim 1/k^3$$

and for the particle distribution function and the noise spectral density:

$$|E_k|^2\sim 1/k^6,\quad f(v)\sim 1/v^5 \tag{24}$$

(in the three-dimensional case, it would be $f(v)\sim 1/v^{7/2}$, $|E_k|^2\sim 1/k^{7/2}$). However, the assumption of the constancy of the flux of cavitons can, essentially, not be strictly justified as it contradicts the possibility of nucleation—the capture of plasmons in the density well remaining after the burn-out of the cavitons of the preceding generation. We therefore much prefer the approach used in Ref. 2. However, to check the collapse laws used in both approaches in the absorption region one needs a numerical simulation taking into account the deformation of the electron distribution function.

We have already mentioned above that a comparison of the two numerical models based on solving the hydrodynamic equations and on a direct modeling by the particle method, which was carried out in Ref. 8, showed that the hydrodynamical set of Zakharov equations supplemented by a quasi-linear equation for the resonance electron distribution function describes rather satisfactorily Langmuir turbulence in the short-wavelength scale absorption region even in the one-dimensional case.

4. THE ROLE OF SOUND IN THE TURBULENCE OF AN ISOTHERMAL PLASMA

The role of sound in the dynamics of plasma turbulence is connected with the fact that each density well remaining after the burn-up of the Langmuir oscillations is a source of short-wavelength sound waves. In an isothermal plasma ($T_e=T_i$) these oscillations are strongly damped due to the absorption of their energy by resonance ions, and the damping rate is then of the order of the sound frequency:

$$\gamma_s(k)\sim\omega_s(k)\approx\omega_p(m/M)^{1/2}kr_D.$$

However, in a non-isothermal plasma with hot electrons ($T_e\gg T_i$), when only the damping of the oscillations

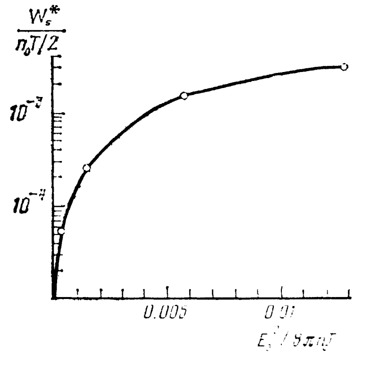

FIG. 6. The short-wavelength density perturbations W_s^* in non-isothermal turbulence as a function of the pumping amplitude.

TABLE I.

$\frac{E_0^2}{8\pi n_0 t}$	$\frac{I_{conv}}{\omega_p n_0 T}$	$\frac{\nu_{eff} E_0^2}{\omega_p 8\pi n_0 T}$	$\frac{1}{9}\sum_k \frac{\delta n_k^2}{n_0^2}\frac{1}{k^4 r_D^4}$
$1.5\cdot 10^{-3}$	$4.0\cdot 10^{-5}$	$1.2\cdot 10^{-4}$	0.54
$5.8\cdot 10^{-3}$	$3.0\cdot 10^{-4}$	$9\cdot 10^{-4}$	0.35

by electrons is important and the damping rate decreases to a value $\gamma_s \sim \omega_p mkr_D/M$, the buildup of sound becomes important. The stationary level of the energy of the short-wavelength sound oscillations W_s^*, determined from the balance between the energy flux into sound produced by the collapse and its absorption by resonance electrons, increases rather rapidly with increasing pumping amplitude[2]

$$W_s^* = \sum_{k>k_0} \delta n_k^2/n_0^2 \propto WE_0^2 \propto E_0^4. \tag{25}$$

In the numerical simulation such a fast increase in the energy of the short-wavelength sound oscillations with pumping amplitude was observed only for $E_0^2/8\pi n_0 T \lesssim 2\times 10^{-3}$ (see Fig. 6). For larger pumping the increase in W_s^* slows down, apparently because of the switching-on of new mechanisms for short-wavelength transfer of plasma waves. When these new mechanisms appear the collapse is stabilized[14] and the short-wavelength transfer of plasma waves is no longer accompanied by the creation of additional sound oscillations. On the whole the presence of short-wavelength sound affects the dynamics of the turbulence through two mechanisms. These mechanisms are conversion,[2] i.e., the scattering of plasma oscillations by the spectrum of the sound oscillations which appear as the result of the collapse and their transformation as the result of their scattering into short-wavelength ones which are absorbed by resonance electrons, and also nucleation,[6] that is, the capture of plasma oscillations in the density wells arising as the result of the modulation of the plasma density by sound waves and the subsequent collapse of the cavitons with plasmons which are then formed.

The conversion produces a flux of energy of plasma oscillations into the absorption region, equal to

$$I_{conv} = \frac{1}{9}\sum_{\mathbf{k},\varkappa} \Gamma_{\mathbf{k}} W_{\varkappa} \frac{|\delta n_{\mathbf{k}-\varkappa}|^2}{n_0^2 \mathbf{k}^4 r_D^4}. \tag{26}$$

The growth rate of the conversion given in Ref. 14 follows from (26) in the case when in the sound fluctuation spectrum there is no maximum in the short-wavelength ($k \approx k^a$) region and correspondingly the small $\varkappa$ region is important in the sum over $\varkappa$ in (26). We compare in the Table the energy flux of the plasma oscillations produced by conversion with the power absorbed from a pump $\nu_{eff} E_0^2/8\pi$. It follows from this comparison that it is just conversion which is the main channel for the short-wavelength transfer of plasmons, while the sound level is such that the characteristic conversion growth rate $I_{conv}/2W$ is of the order of the characteristic caviton collapse growth rate:

$$\gamma_{eff} = \lambda(W)\gamma_{mod}(W).$$

[From the preceding considerations it follows that the energy flux into turbulence $\nu_{eff} E_0^2/8\pi$ is equal to $2\lambda(W)\gamma_{mod} W$.] A faster conversion would lead to a complete suppression of the collapse which is impossible as it is just the caviton collapse which is the source of the sound oscillations. As to nucleation, this effect is essentially close to conversion and is based upon the capture of plasmons in density wells which are produced by the same density fluctuations by which they are scattered in conversion. Since, however, the short-wavelength acoustic turbulence is weak in the same sense as the parameter

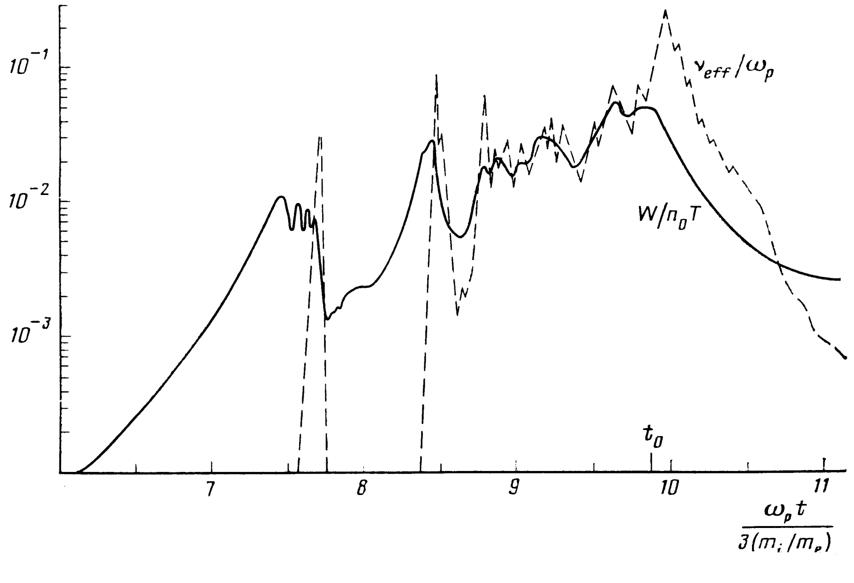

FIG. 7. Time-dependence of the energy of the plasma oscillations W/n_0T and of the effective collision frequency ν_{eff}/ω_p in non-isothermal turbulence produced by a pump of amplitude $E_0^2/8\pi n_0 T = 1.5\times 10^{-3}$. The quantity ν_{eff}/ω_p is determined by collapse only in the initial stage of the process. The further buildup of the sound turns on conversion, and ν_{eff}/ω_p becomes a continuous function of the time. At the time $t_0 \approx 9.9$ the pump was switched off.

$$\frac{1}{9}\sum_k \frac{|\delta n_k|^2}{n_0^2}\frac{1}{k^4 r_D^4} < 1$$

(according to the results given in the Table this parameter is less than unity), it is just the scattering of plasmons which is basic for turbulence. This is confirmed by the results of the numerical simulation of the turbulence under weak sound damping, which are given in Fig. 7. We show in that figure the function $W(t)$ for the case when the pumping of energy into turbulence is switched off at $t > t_0$. The switching-off of the pumping in the quasi-stationary regime leads to a monotonic, and not to a stepwise (as happened in the case $T_e \approx T_i$, see Fig. 1) decrease of W with time. In a non-isothermal plasma, therefore, i.e., when there are strong sound oscillations present, it is just the conversion, which is a continuous process, rather than collapse or nucleation of plasmons in collapsed cavitons, which is the basic channel for the transfer of plasma waves into the short-wavelength absorption region.

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