Quasimomentum spectral asymmetry and the anomalous magnetic properties of an orbital antiferromagnet

A. A. Gorbatsevich
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Antiferromagnetics with an asymmetric quasimomentum spectrum are examined. Spectral asymmetry occurs in all magnetics whose symmetry permits an energy invariant that is odd with respect to the quasimomentum components. A gauge-invariant scheme is developed for calculating the magnetic susceptibility of the inhomogeneous phase of a magnetic with an asymmetric spectrum and it is demonstrated that the inhomogeneity enhances the diamagnetism. The diamagnetic susceptibility can assume arbitrarily large absolute values. The ground state of a superconducting antiferromagnetic with an asymmetric spectrum is a state with a superconducting phase gradient, while in a closed superconducting ring the state has spontaneous current flow.

1. INTRODUCTION

The familiar theorem of band theory concerning the spectral symmetry of elementary excitations with respect to the quasimomentum $\mathbf{k}$ applies to crystals whose symmetry group contains an inversion of time t. In magnetics t-invariance is broken, and if magnetic ordering applies to the orbital subsystem the spectrum may have quasimomentum asymmetry: $E(\mathbf{k}) \neq E(-\mathbf{k})$. From the microscopic view point such a symmetry requires a polar direction in the magnetic given by the vector $\mathbf{T}$ which changes sign upon time inversion (the product $\mathbf{Tk}$ remains invariant in this case). The vector $\mathbf{T}$ is permitted in 31 of a total of 122 magnetic classes.[1,2] The macroscopic physical meaning of the vector $\mathbf{T}$ is the toroidal moment density.[3] The toroidal moments represent a third independent family of electromagnetic multipoles (together with the electrical and magnetic multipoles).[4] The vector $\mathbf{T}$ characterizes antiferromagnetic ordering while the density of the magnetic moment may be strictly equal to zero for $\mathbf{T} \neq 0$. The microscopic mechanism responsible for the quasimomentum spectral asymmetry of the charge carriers are discussed in Sec. 2 of the present study. Such asymmetry exists in spin antiferromagnetics with $\mathbf{T} \neq 0$ solely as a measure of the spin-orbital interaction, and the spectral asymmetry parameter has relativistic smallness. The spectral asymmetry in orbital antiferromagnetics (OAF) is due to strong Coulomb interaction.

The magnetic properties of OAF are of special interest. The paramagnetic component of the magnetic susceptibility in a toroidal OAF is suppressed due to the specific poloidal topology of the current contours.[5] The possibility of a strong diamagnetic response of the inhomogeneous phase of OAF was discussed in Refs. 3, 6. These studies also proposed phenomenological models of diamagnetism. The primary problem in a microscopic analysis involved the difficulty of achieving gauge invariance of the calculations in the inhomogeneous phase. A gauge-invariant scheme for calculating the magnetic susceptibility of an inhomogeneous OAF within the framework of perturbation theory in the spectral asymmetry parameter is developed in Sec. 3. It is demonstrated that asymmetric spectral deformation causes the system energy to rise in a magnetic field, i.e., it corresponds to a diamagnetic response. Diamagnetism grows with increasing inhomogeneity. However the problem of determining possible limits of the range of the dimagnetic susceptibility makes it necessary to work outside the framework of perturbation theory.

The vector $\mathbf{T}$ is analogous to the vector-potential of a magnetic field in its symmetry transformation properties. This analogy helps to achieve an exact calculation of the magnetic susceptiblity of an inhomogeneous OAF with an arbitrary asymmetry parameter value. An OAF model is formulated in Sec. 4; in this model the inhomogeneity of the spectral asymmetry parameter plays the role of the effective magnetic field. (A similar situation exists in ^{3}He–A where the analogy of the order parameter inhomogeneity in the texture and the magnetic field allows calculation of the excitation spectrum and identification of the nature of the anomalous contribution to the current[7,8]). The magnetic susceptibility of an inhomogeneous OAF with an asymmetric spectrum in a weak external magnetic field in the model proposed in Sec. 4 coincides with the differential magnetic susceptibility of a system with a zero asymmetry parameter in a true magnetic field equal to the effective field which is related to the inhomogeneity of the asymmetry parameter. If the system satisfies the condition for the de Haas-van Alphen effect the differential diamagnetic susceptibility may, as we know,[9] assume arbitrarily large absolute values. The results of Sec. 4 therefore strictly prove the possibility of an anomalously strong diamagnetism of the spatially-inhomogeneous phase of OAF.

Another physical example in which the analogy of the toroidal vector $\mathbf{T}$ and the vector-potential of the magnetic field is manifested in the macroscopic effects is examined in Sec. 5. This section demonstrates that in a superconducting OAF whose symmetry allows a toroidal moment the expression for the superconducting current contains an additive term. The ground state of such a superconductor is a state with a superconducting phase gradient (the phase generation affect), while in a closed superconducting ring the ground state has spontaneous current flow. The total magnetic moment of such a ring in zero magnetic field is at most one-half of the flux radiation. If the antiferromagnetic transition occurs at a lower temperature than the superconducting transition a singularity of the temperature derivative of the upper critical field will exist at this point.

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2. MAGNETIC ORDERING AND QUASIMOMENTUM SPECTRAL ASYMMETRY

The conditions for the development of an OAF and its properties have been investigated most thoroughly and consistently within the framework of the two-band exciton dielectric (ED) model.[10,11] The Hamiltonian of the exciton dielectric model contains a one-electron part $\hat{H}_0$ and the interaction Hamiltonian $\hat{H}_{\text{int}}$:

$$\hat{H}=\hat{H}_0+\hat{H}_{int}. \tag{1}$$

The one-electron part takes the form of an ordinary two-band Hamiltonian:

$$\hat{H}_0=\begin{pmatrix} \varepsilon_1(\hat{\mathbf{k}}) & \tilde{W}(\hat{\mathbf{k}}) \\ \tilde{W}^*(\hat{\mathbf{k}}) & \varepsilon_2(\hat{\mathbf{k}}) \end{pmatrix}. \tag{2}$$

Here $\varepsilon_{1(2)}(\mathbf{k})$ is the dispersion law in band 1 and (2) $\tilde{W}(\mathbf{k})$ is the interband hybridization. The ED model examines bands with near-congruent dispersion laws $\varepsilon_1 \approx -\varepsilon_2$. The Hamiltonian $\hat{H}_{\text{int}}$ describes Coulomb interelectron interactions.

In the mean field approximation the interaction Hamiltonian $\hat{H}_{\text{int}}$ takes the form

$$\hat{H}_{int}=\begin{pmatrix} 0 & \hat{\Delta}(\mathbf{r}) \\ \hat{\Delta}^+(\mathbf{r}) & 0 \end{pmatrix}, \tag{3}$$

where $\hat{\Delta}$ is the matrix of the self-consistent potential (SCP) in spin space. The SCP (the order parameter) is analogous to the ordinary crystal potential. However unlike the latter the SCP has a complex spin and phase structure. In the general case $\hat{\Delta}$ contains four components[12]:

$$\hat{\Delta}=\Delta_R^s+i\Delta_I^s+\boldsymbol{\sigma}(\Delta_R^t+i\Delta_I^t). \tag{4}$$

Here $\boldsymbol{\sigma}$ is the vector consisting of the Pauli matrices, $\Delta_{R,I}^s$ are the singlet real and imaginary components of the order parameter, $\Delta_{R,I}^t$ are the triplet real and imaginary components. The singlet real order parameter Δ_R^s describes charge ordering (the charge density wave), the triplet real parameter Δ_R^t describes the spin antiferromagnetic ordering of the band electrons (the spin density wave), and the singlet imaginary parameter Δ_I^s corresponds to orbital antiferromagnetism, while the ordering chracterized by the triplet imaginary order parameter Δ_I^t can be interpreted as relating to the onset of the local spin current density.[13] Each of the four types of ordering may arise given certain specific relations between the microscopic parameters of model (1), (2) (see Ref. 14 for a more detailed discussion).

In addition to the order parameter symmetry the macroscopic symmetry of the ordered phase is also determined by the symmetry of the wave functions of electrons producing the SCP. Information on the wave function symmetry is contained in the matrix hybridization element $\tilde{W}(\mathbf{k})$ in the Hamiltonian (2).

For hybridization we will limit the analysis to an approximation that is

$$\tilde{W}(\mathbf{k})=i\mathbf{P}\mathbf{k}/m, \tag{5}$$

where $i\mathbf{P}$ is the interband matrix element of the momentum, which is nonzero when the wave functions in bands 1 and 2 have different parities. The spectrum of the homogeneous OAF ($\Delta_I^s(\mathbf{r})\equiv\Delta(\mathbf{r})=\text{const}$) takes the form

$$E(\mathbf{k})=\pm(\varepsilon^2(\mathbf{k})+(\mathbf{P}\mathbf{k}/m+\Delta)^2)^{1/2} \tag{6}$$

(here we have set $\varepsilon_1=-\varepsilon_2=\varepsilon$). The spectrum (6) has quasimomentum asymmetry $E(\mathbf{k})\neq E(-\mathbf{k})$ due to the quasimomentum-linear term $2\mathbf{P}\mathbf{k}\Delta/m$ under the radical in (6). The product $\mathbf{P}\Delta_I^s$ therefore functions as the polar t-odd vector $\mathbf{T}$ in the model (1)–(3).

The magnetic ordering that produces quasimomentum spectral asymmetry in the ED model has an orbital nature and the asymmetry parameter Δ_I^s is determined by strong Coulomb interaction. However a magnetic phase transition of a purely orbital nature evidently represents a rather exotic situation. Ordinarily the violation of t-invariance that accompanies the magnetic transition results from ordering in the crystal spin subsystem. The quasimomentum spectral asymmetry arises in this case as a measure of the spin-orbital interaction. Spin ordering of collectivized (band) electrons is described within the framework of the ED model by the triplet real order parameter Δ_R^t. The spin-orbital interaction induces an imaginary parameter Δ_I^s which results in spectral asymmetry in a crystal without an inversion center and a nonzero Δ_R^t, as noted in Ref. 12 and demonstrated in Ref. 15.

We will consider the spectral asymmetry mechanism in a magnetic with localized spins. Without an inversion center, spin-orbital interaction eliminates the spin degeneracy of the energy for the prescribed quasimomentum direction (see, for example, Ref. 16):

$$\varepsilon_\sigma(\mathbf{k})\neq\varepsilon_\sigma(-\mathbf{k}). \tag{7}$$

The symmetry of the total spectrum with respect to inversion of the quasimomentum is conserved in this case:

$$\varepsilon_\sigma(\mathbf{k})=\varepsilon_{-\sigma}(-\mathbf{k}). \tag{8}$$

The action of the ordered localized spins of the band electrons is described by the exchange field. The exchange field shifts the spectral branches ε_σ and $\varepsilon_{-\sigma}$ corresponding to the different spin projections in the opposite direction with respect to energy, which violates relation (8) and results in quasimomentum spectral asymmetry $\varepsilon_\sigma(\mathbf{k})\neq\varepsilon_{-\sigma}(-\mathbf{k})$. The spin separation and the related quasimomentum spectral symmetry breaking may be caused not only by the exchange magnetic field but also by an ordinary magnetic field. Specifically such a situation occurs in Te,[17] where the asymmetric shift is 1.7 meV in a field of $H=40$ kGauss.

This analysis shows that in a system without an inversion center the quasi-momentum spectral asymmetry of the band electrons is the sole consequence of magnetic ordering. Photoemission of electrons with angular resolution can be used for direct experimental observation of spectral asymmetry. An experiment carried out using this technique on antiferromagnetic chromium revealed quasi-momentum spectral asymmetry in the Cr surface region[18] which included an isolated polar direction normal to the surface and no inversion center. Chromium represents a typical band antiferromagnetic with a thin density wave described by the triplet real order parameter.[19] Hence the observed[18] spectral asymmetry is most naturally attributed to the induction of the imaginary singlet order parameter discussed above. It is important to remember that without an inversion center the triplet real order parameter will induce, as a measure of spin-

orbital interaction, both a singlet real SCP (the order parameter) that is nondiagonal with respect to the band indices, together with a diagonal intrinsic energy part[15] that has quasimomentum asymmetry. The separation of these two contributions to spectral asymmetry is arbitrary in some sense and depends on the specific selection of the basis of the wave functions.

3. GAUGE INVARIANCE AND THE MAGNETIC SUSCEPTIBILITY OF AN INHOMOGENEOUS OAF

The exact expression for the free energy density of a system with Hamiltonian (1), (2) in the mean-field approximation takes the form

$$F=-2\frac{\Theta}{V}\sum_{\mathbf{k}}\ln 2\operatorname{ch}\frac{E(\mathbf{k})}{2\Theta}+\frac{1}{V}\int\frac{\Delta_\alpha^2(\mathbf{r})}{|g_\alpha|}d\mathbf{r}. \tag{9}$$

Here Θ is the temperature, V is the system volume, the index α designates one of the four order parameter components (4), g_α is the effective interaction constant for the order parameter Δ_α. We will assume a fixed number of charge carriers n. We will assume that hybridization $\tilde{W}$ and the imaginary singlet order parameter Δ_I^s are both nonzero. The imaginary parameter $\Delta_I^s\neq 0$ can form from a natural transition to the OAF state or can be induced from spin-orbit interaction in the spin AF (Sec. 2). In the latter case

$$\Delta_I{}^s=\lambda_{so}d^t, \tag{10}$$

where λ_{so} is the effective spin-orbital interaction constant, d^t is the triplet spin parameter (the parameter Δ_R^t functions as such a parameter in the band spin AF, while the antiferromagnetic vector $\mathbf{L}$ functions as such a parameter in localized electron antiferromagnetics).

An inhomogenous spectral asymmetry parameter $\Delta(\mathbf{r})\neq\text{const}$ is the most interesting case in the investigation of the magnetic properties of OAF.[3,6] The following are among the primary reasons for inhomogeneity of the parameter $\Delta(\mathbf{r})$:

1) Formation of a superlattice of the inhomogeneous Larkin-Ovchinnikov-Fulde-Ferrel (LOFF)-state type[20,21]; such an order parameter inhomogeneity occurs in chromium[22];

2) proximity effects where the order parameter diminishes near the interface of the ordered and disordered phases;

3) a composition variation that will result in a spatial alteration of the microscopic parameters of the Hamiltonian (the bandgap, the effective masses and the effective interaction constant) on macroscopic scales; the order parameter will experience corresponding changes as a function of the model microparameters;

4) the influence of defects that may induce the OAF state localized in the vicinity of the defect.[23]

The primary technique that makes it possible to achieve gauge invariance of the calculations in model (1)–(3) involves reducing the sum of all nongauge terms in the expressions for the physical quantities to a total derivative.[5,11] Total derivatives vanish when summed over the occupied states due to the periodicity of the dispersion law in the Brillouin zone. An exact expression for the dispersion law can be obtained in isolated cases only in an inhomogeneous system ($\Delta(\mathbf{r})\neq\text{const}$). The energy levels $E(\mathbf{k})$ in (9) in zero field are determined by eigenvalue equations:

$$\begin{aligned}&\varepsilon_1(\hat{\mathbf{k}})U_{\mathbf{k}}(\mathbf{r})-i(\Delta(\mathbf{r})+W(\hat{\mathbf{k}}))V_{\mathbf{k}}(\mathbf{r})=E(\mathbf{k})U_{\mathbf{k}}(\mathbf{r}),\\&\varepsilon_2(\hat{\mathbf{k}})V_{\mathbf{k}}(\mathbf{r})+i(\Delta(\mathbf{r})+W(\hat{\mathbf{k}}))U_{\mathbf{k}}(\mathbf{r})=E(\mathbf{k})V_{\mathbf{k}}(\mathbf{r}),\end{aligned} \tag{11}$$

in which the potential for the wave functions U and V is used as the order parameter. We have employed a purely imaginary expression $\tilde{W}(\mathbf{k})=iW(\mathbf{k})$, $W(\mathbf{k})=W^*(\mathbf{k})$ for the hybridization (11) consistent with (5). We transform the system to permit an exact solution in a class of reflectionless potentials. We consider the quasi-one-dimensional bare dispersion law:

$$\varepsilon_{1,2}(\mathbf{k})=\pm v_F k+\mu+t_{1,2}(\mathbf{k}_\perp), \tag{12}$$

where k is the quasimomentum from the Fermi surface ε_F in the direction x, $\mathbf{k}_\perp$ is the transverse quasimomentum, μ is the shift of the Fermi level due to doping. We also assume hybridization is dependent solely on the transverse quasimomentum: $W(\mathbf{k})\equiv W(\mathbf{k}_\perp)$. Consistent with the selection of the choice direction we assume that the spectral asymmetry parameter is also dependent solely on the coordinate x: $\Delta(\mathbf{r})\equiv\Delta(x)$.

We carry out a canonical transform in Eqs. (11)

$$U_{\mathbf{k}}(\mathbf{r}),V_{\mathbf{k}}(\mathbf{r})\to U_k(x),V_k(x)\exp\{i(v_F^{-1}\tilde{t}(\mathbf{k}_\perp)x-\mathbf{k}_\perp\mathbf{r}_\perp)\},$$

$$\tilde{t}(\mathbf{k}_\perp)=\frac{t_1(\mathbf{k}_\perp)-t_2(\mathbf{k}_\perp)}{2}$$

and go over to the functions

$$u_k,v_k=2^{-1/2}(U_k\pm V_k).$$

Equations (11) in the new designations take the form

$$\begin{aligned}&iv_F\nabla u_k(x)+i(\Delta(x)+W(\mathbf{k}_\perp))u_k(x)\\&\quad=(E(\mathbf{k})+\mu-t(\mathbf{k}_\perp))v_k(x),\\&iv_F\nabla v_k(x)-i(\Delta(x)+W(\mathbf{k}_\perp))v_k(x)\\&\quad=(E(\mathbf{k})+\mu-t(\mathbf{k}_\perp))u_k(x),\\&t(\mathbf{k}_\perp)=1/2(t_1(\mathbf{k}_\perp)+t_2(\mathbf{k}_\perp))\end{aligned} \tag{13}$$

and make it possible to express one wave function through another in local form. We first set the hybridization equal to zero ($W=0$). Equations (13) are thus reduced to a purely one-dimensional eigenvalue problem $E(k)=E(\mathbf{k})+\mu-t(\mathbf{k}_\perp)$ allowing an exact solution in the class of reflectionless finite-band potentials.[24,25] We consider an order parameter $\Delta_0(x)$ of the type

$$\Delta_0(x)=-\Delta_1\operatorname{sn}(x\Delta_1/\varkappa v_F,\varkappa). \tag{14}$$

The spectrum of one-dimensional motion $E(k)$ in potential (14) contains two forbidden bands $(-E_+,-E_-)$ and (E_-,E_+), while the wave functions and state density $\rho(E)$ take the form[24,25]

$$\begin{aligned}&v_E{}^0(x)=\left(\frac{h(x)+b(E)}{L\langle h(x)\rangle+b(E)}\right)^{1/2}\exp\left\{\frac{i}{v_F}\int_0^x\frac{2R^{1/2}(E)}{h(y)+b(E)}dy\right\},\\&\rho(E)=\frac{1}{L}\frac{dN}{dE}=\left|\frac{E\langle h\rangle+b(E)}{\pi R^{1/2}(E)}\right|,\quad\langle\ldots\rangle=\frac{1}{L}\int_{-L/2}^{L/2}\ldots dx;\end{aligned} \tag{15}$$

here L is the length of the system along x,

$$h(x)=\Delta_0^2(x)-v_F\nabla\Delta_0(x),\quad b(E)=2E^2-E_+^2-E_-^2,$$
$$R(E)=E^2(E^2-E_-^2)(E^2-E_+^2).$$

The parameters Δ_1 and $\varkappa$ in (14) are expressed through the spectral boundaries: $\Delta_1=E_+-E_-$, $\varkappa=(E_+-E_-)/(E_++E_-)$.

The quasimomentum k is determined in accordance with Ref. 24 as

$$k(E)=\left\langle\frac{2R^{1/2}(E)}{v_F(h(x)+b(E))}\right\rangle. \tag{16}$$

Unlike the Peierls case[25] the portions of spectrum (12) related by time inversion ($\mathbf{k}$ and $-\mathbf{k}$) belong to the same band ε_i. Hence compared to Ref. 25 all energy levels in two-band model (2), (12) are doubly degenerate. Hybridization can be taken into acount in perturbation theory by selecting the basis (14), (15) as a zeroth approximation. The expansion parameter here is the ratio

$$W(\mathbf{k}_\perp)/\max(\Delta,\Theta)\ll 1,$$

which designates the existence of energy surfaces opening in the direction of $\mathbf{k}_\perp$, accounting for the quasi-one-dimensional nature of the intraband component of the bare dispersion law (12) ($|t|\ll\varepsilon_F$). With these assumptions the quasimomentum linear approximation is not suitable for hybridization (5) since the energy denominator do not allow convergence of the expression for the physical quantities with large values of $|\mathbf{k}_\perp|$. It is therefore necessary to explicitly account for the periodicity of all parameters of the Hamiltonian in the direction $\mathbf{k}_\perp$ with the reciprocal lattice period $\mathbf{G}_\perp$

$$W(\mathbf{k}_\perp+\mathbf{G}_\perp)=W(\mathbf{k}_\perp),\quad t(\mathbf{k}_\perp+\mathbf{G}_\perp)=t(\mathbf{k}_\perp). \tag{17}$$

We will now calculate the magnetic response of an inhomogeneous OAF. The free energy functional of the model with quasi-one-dimensional spectrum (12) takes the form

$$F=-2\Theta\int\frac{d\mathbf{k}_\perp}{S}\int dE\rho(E)\ln 2\operatorname{ch}\frac{E+t(\mathbf{k}_\perp)-\mu}{2\Theta}\cdot$$
$$+\frac{1}{L}\int\frac{\Delta^2(x)}{|g|}dx; \tag{18}$$

here S is the cross-sectional area of the elementary cell. We will assume a weak magnetic field and applicability of perturbation theory in the vector potential. We will select the following gauge for the vector potential

$$\mathbf{A}(\mathbf{r})=(0,A(x),0),\quad A(x)=A_qe^{iqx}+\text{c.c.}$$

The equations for the wave functions of the system in a magnetic field are obtained from (13) by the substitution $\mathbf{k}_\perp\to\mathbf{k}_\perp-e\mathbf{A}/c$. The magnetic susceptibility is determined both by the shift of the one-particle energy levels E, Eq. (18), and the change in the order parameter in the linear field approximation.[26] The asymmetry parameter is strictly determined by the spin ordering structure in induced OAF, Eq. (10). The asymmetry parameter can be assumed to be independent of the field in the linear field approximation. We will show that it is also possible to ignore the change in the order parameter in a field in the case of intrinsic OAF ordering in the model with a quasi-one-dimensional spectrum and open energy surfaces. The self-consistency equation for the order parameter appears as

$$\Delta(x)=gG_{12}(\mathbf{r},\mathbf{r})\equiv g\ \overset{\mathbf{r}\quad\mathbf{r}}{\underset{1\quad 2}{\longrightarrow}}, \tag{19}$$

Here $G_{12}(\mathbf{r},\mathbf{r}')$ [$G_{12}(\mathbf{r},\mathbf{r})=G_{12}(x,x)$] is the interband anomalous Green's function of Eqs. (1), (13). An equation equivalent to (19) is obtained by minimizing the functional (18) in the order parameter $\Delta(x)$. When expanding the self-consistency equation into a series in the vector potential it is important to account for both the nondiagonal perturbation component,

$$W\left(\mathbf{k}_\perp-\frac{e}{c}\mathbf{A}\right)-W(\mathbf{k}_\perp)\approx W^{(1)}+W^{(2)}+\dots,$$
$$W^{(1)}=A\frac{\partial}{\partial k_y}W(\mathbf{k}_\perp),$$
$$W^{(2)}=\frac{1}{2}A^2\frac{\partial^2}{\partial k_y^2}W(\mathbf{k}_\perp), \tag{20}$$

and the diagonal component,

$$t^{(1)}=A\frac{\partial}{\partial k_y}t(\mathbf{k}_\perp). \tag{21}$$

The self-consistency equation takes the following form to first order in the field and the transverse diversion components

$$\Delta^{(1)}=\int\frac{d\mathbf{k}_\perp}{S}[G_{11}^0\otimes W^{(1)}\otimes G_{22}^0-G_{12}^0\otimes W^{(1)}\otimes G_{12}^0]$$
$$=\int\frac{d\mathbf{k}_\perp}{S}\frac{\partial W(\mathbf{k}_\perp)}{\partial k_y}[G_{11}^0\otimes A\otimes G_{22}^0-G_{12}^0\otimes A\otimes G_{12}^0]. \tag{22}$$

Here $G\otimes A\otimes G=G(x,x')A(x')G(x',x)dx'$. The term containing $t^{(1)}$ on the right side of (22) is dropped since it vanishes by virtue of $t^{(1)}(\mathbf{k}_\perp)=-t^{(1)}(\mathbf{k}_\perp)$. In (22) G_{ij}^0 is the Green's function of the zeroth approximation in the transverse dispersion formulated in the basis of functions $U_k(x)$ and in $V_k(x)$ in Eq. (13) and independent of transverse motion. Hence the expression under the integral sign in Eq. (22), represents the total derivative and it vanishes upon integration with respect to the transverse momentum $\mathbf{k}_\perp$. The field source in the self-consistency equation in an arbitrary order of perturbation theory in $W(\mathbf{k}_\perp)$ vanishes analogously when $t(\mathbf{k}_\perp)$ is ignored. The first nonvanishing term in the field source is the cross term in W and t. It contains the diagrams

$$\overset{1\quad 1}{\longrightarrow}W^{(1)}\overset{2\quad 2}{\longrightarrow}t(\mathbf{k}_\perp)\overset{2\quad 2}{\longrightarrow}$$
$$+\ \overset{1\quad 1}{\longrightarrow}W(\mathbf{k}_\perp)\overset{2\quad 2}{\longrightarrow}t^{(1)}\overset{2\quad 2}{\longrightarrow} \tag{23}$$

and W and t obtained from the diagrams by substitution together with the indices of the Green's functions. When $A=\text{const}$ the sum of diagrams (23) reduces to the total derivative

$$\left[A\frac{\partial}{\partial k_y}[W(\mathbf{k}_\perp)t(\mathbf{k}_\perp)\right]G_{11}^0\otimes G_{22}^0\otimes G_{22}^0,$$

which ensures gauge invariance of the self-consistency equation. However for $A(x)\neq\text{const}$

$$G_{11}^0\otimes G_{22}^0\otimes A\otimes G_{22}^0\neq G_{11}^0\otimes A\otimes G_{22}^0\otimes G_{22}^0,$$

the sum of diagrams (23) no longer reduces to a total deriva-

tie and a source that is linear in the magnetic field appears in the self-consistency equation. Therefore the change in the order parameter in the linear magnetic field approximation is a second-order effect in the transverse dispersion components when $t \sim W$. Moreover the energy level shift will occur as early a the first order in W. Hence it is possible to ignore the change in the order parameter in the field in the quasi-one-dimensional model with open Fermi surfaces. Perturbation theory in $t(\mathbf{k}_\perp)$ is not suitable in the model with an isotropic spectrum and a closed Fermi surface.[6,26,27] It is therefore not possible to explicitly provide gauge invariance of the calculations in the inhomogeneous phase by isolation of the total derivatives in the model with an isotropic spectrum.[26,27] The contributions to the energy from the induced component Δ and the energy level shift in the model from Ref. 26 are of the same order of magnitude,[27] which substantially complicates the calculations but does not alter the qualitative physical picture of the diamagnetic response.

Having determined the shift of energy levels E in the magnetic field from equations (13) we can demonstrate that the contribution to the free energy F, Eq. (18), that is a second-order contribution in the field is determined by terms containing solely the component $W^{(1)}$ of the perturbation (20). The terms containing the component $W^{(2)}$ reduce to a total derivative and vanish upon integration. As a result for the magnetic subsceptibility χ' we have

$$\chi' = -\frac{\partial^2 F(B)}{\partial B^2}\Big|_{B=0}$$
$$= -\frac{\partial^2}{\partial q^2}\Big\{\int \frac{d\mathbf{k}_\perp}{S}\Big(\frac{\partial W(\mathbf{k}_\perp)}{\partial k_y}\Big)^2 \int dE\, dE'\, \rho(E)\rho(E')$$
$$\times \frac{|\langle E|\Delta_0(x)e^{iqx}|E'\rangle|^2}{E^2-E'^2}\frac{1}{E}\Big(\mathrm{th}\frac{E-\mu}{2\Theta}+\mathrm{th}\frac{E+\mu}{2\Theta}\Big)\Big\}. \quad (24)$$

The structure of the expression in parentheses in (24) is similar to the formula for the change in system energy in the second order of perturbation theory which, as we know,[28] reduces the energy. However due to the multiplier $1/E$ which is negative for the occupied states the expression in braces in (24) is generally strictly nonnegative. Determination of the sign of terms proportional to q^2 in (24) requires additional study.

We will consider the matrix element in (24). Due to the periodicity of $\Delta_0(x)$ of (14) we have

$$\langle E|\Delta_0(x)e^{iqx}|E'\rangle = \sum_{n=0,\pm 1} P_n(E,E',G_0)\,\delta(k-k'-q-nG_0), \quad (25)$$

where

$$P_n(E,E',G_0) = \sum_{m\geqslant 0}[f^s(E,E',(m-n)G_0) - f^s(E,E',(m+n)G_0)$$
$$+ f^a(E,E',(m-n)G_0) + f^a(E,E',(m+n)G_0)]\Delta(m),$$
$$\Delta(m) \equiv \Delta(mG_0) = \frac{1}{l}\int_0^l \Delta_0(x)\sin(mG_0x)\,dx.$$

Here k is the quasimomentum of (16) ($E' \equiv E(k')$), $G_0 = 2\pi/l$ is the reciprocal lattice period [for the finite band spectrum $E(k) \neq E(k+G_0)$, as follows from (13) and (14)]. In (25) f^s and f^a are the symmetric and antisymmetric components of the expansion of the product of the Bloch multipliers $v_E(x)e^{ikx}$ and $v_{E'}{}^*(x)e^{-ik'x}$ in the reciprocal lattice periods:

$$f^s(E,E') = f^s(E',E), \quad f^a(E,E') = -f^a(E',E).$$

Subject to (25) the expression for the magnetic susceptibility χ' is represented as a sum over the reciprocal lattice periods:

$$\chi' = -\frac{\partial^2}{\partial q^2}\sum_n\Big\{\int \frac{d\mathbf{k}_\perp}{S}\Big(\frac{\partial W(\mathbf{k}_\perp)}{\partial k_y}\Big)^2 \int dE\, dE'\rho(E)\rho(E')\frac{1}{E}$$
$$\times \frac{|P_n(E,E',G_0)|^2}{E^2-E'^2}\Big(\mathrm{th}\frac{E-\mu}{2\Theta}+\mathrm{th}\frac{E+\mu}{2\Theta}\Big)\Big\}. \quad (26)$$

It is possible to limit the analysis to a term with $n=0$ in the sum (26) accurate to $\Delta_1/(E(k+G_0)-E(k)) \ll 1$ in a highly inhomogeneous system where the period l is small while the reciprocal period G_0 is substantial ($E(k+G_0)-E(k) \gg \Delta_1$). Here the matrix element $P_0(E,E',G_0)$ of (25) contains solely the component asymmetric in $E'-E$, and its expansion in powers of q begins with the term linear in q. Hence, ignoring exhaustive search processes, we see that the expansion in powers of q of the expression in braces in (26) begins immediately with the term quadratic in q which also determines the susceptibility that is diamagnetic in this case. We note that the lack of a zeroth-order term in the expansion of the matrix element (25) in powers of q (ignoring exhaustive search processes) is a direct consequence of the inhomogeneity of order parameter (14). For $\Delta = \text{const}$ the matrix element is weakly dependent on energy and on q, while the susceptibility (26) is determined by the expansion of the energy denominator in q and is paramagnetic, as we can show.

This section has therefore employed a quasi-one-dimensional exactly soluble model to demonstrate that the strongly-inhomogeneous phase of a system with an asymmetric spectrum is diamagnetic. The hybridization serves as a small parameter in this model. It is therefore not advisable to obtain specific estimates of the diamagnetic susceptibility (26) which contains this same small parameter.

4. PSEUDOMAGNETIC FIELD IN AN INHOMOGENEOUS OAF

We consider the initial Hamiltonian (1)–(3) with an inhomogeneous spectral asymmetry parameter

$$\Delta_I{}^s(\mathbf{r}) = \Delta(\mathbf{r}) \neq \text{const}.$$

We employ linear approximation (5) for hybridization. The macroscopic symmetry of the system is characterizd by the polar t-odd vector $\mathbf{T}$ formed by the combination of microscopic parameters $\mathbf{T} \sim \mathbf{P}$. From the symmetry viewpoint the vector $\mathbf{T}$ is entirely analogous to the vector-potential of the magnetic field, although the vector $\mathbf{T}$ does not directly enter into the Hamiltonian nor microscopic equations of motion (11). We will show that the Hamiltonian (1)–(3) allows reduction to a model in which the spectral asymmetry parameter plays a role that is entirely analogous to the vector-potential of the magnetic field. We consider the anisotropic intraband component of the dispersion law:

$$\varepsilon_i(\mathbf{k})=\varepsilon_i(k_x, k_z)+\varepsilon_i(k_y), \quad i=1, 2, \tag{27}$$

where the origin in the first term is selected so that the average over the Brillouin zone satisfies $\langle \varepsilon_i(k_y)\rangle = 0$. We assume that the intraband dispersion in the y direction can be ignored compared to the hybridization dispersion:

$$|\varepsilon_i(k_y)| \ll |Pk_y|/m. \tag{28}$$

We note that diamagnetism of an inhomogeneous OAF grows with increasing $|\partial W(\mathbf{k}_\perp)/\partial k_y|$ consistent with the formula obtained in the preceding section for magnetic susceptibility (26). It is therefore likely that the portions of the spectrum for which the approximation (5) is applicable will make the dominant contribution to the diamagnetism, while relation (28) corresponds to enhancement of the trend toward diamagnetism. We then obtain the reduced Hamiltonian:

$$\hat{H}_{red}=\begin{pmatrix} \varepsilon_1(\hat{k}_x, \hat{k}_z) & i\left(\frac{1}{m}P\hat{k}_y+\Delta(\mathbf{r})\right)+\tilde{\Delta} \\ -i\left(\frac{1}{m}P\hat{k}_y+\Delta(\mathbf{r})\right)+\tilde{\Delta} & \varepsilon_2(\hat{k}_x, \hat{k}_z) \end{pmatrix}. \tag{29}$$

Here the intraband components $\varepsilon_i(k_x, k_z)$ may be random functions of the momenta k_x, k_z characterizing both the semiconductor and the semimetallic intrinsic dispersion law. We have included in (29) the potential $\tilde{\Delta}$ representing the components of the order parameter (4) that differ from the imaginary singlet component for generality. The spectral asymmetry parameter $\Delta(\mathbf{r})$ enters into Hamiltonian $\hat{H}_{\text{red}}$ (29) in the same manner as the vector potential of the magnetic field running along the y axis. Redesignating the asymmetry parameter:

$$\Delta(\mathbf{r})=\frac{P}{m}\frac{e}{c}A_{eff}(\mathbf{r}), \quad (\mathbf{A}_{eff}=(0, A_{eff}, 0)),$$

we find that the Hamiltonian

$$\hat{H}_{red}\{\Delta\}\equiv\hat{H}_{red}\{A_{eff}\}$$

coincides with the Hamiltonian of a general two-band model in the effective magnetic (pseudomagnetic) field B_{eff}:

$$\mathbf{B}_{eff}=\text{rot}\,\mathbf{A}_{eff}=\left(-\frac{c}{e}\frac{m}{P}\nabla_z\Delta(\mathbf{r}), 0, \frac{c}{e}\frac{m}{P}\nabla_x\Delta(\mathbf{r})\right). \tag{30}$$

In order to determine the properties of an inhomogeneous OAF described by the reduced Hamiltonian (29) we can therefore use the well-developed methods of calculating the macroscopic characteristics of a homogeneous system in an external magnetic field. The true magnetic field $\mathbf{B}$ given by the vector-potential $\mathbf{A}$ ($\mathbf{B}=\text{curl}\,\mathbf{A}$) enters into Hamiltonian (29) by the additive substitution $\mathbf{A}_{\text{eff}}\to\mathbf{A}_{\text{eff}}+\mathbf{A}$ The magnetic susceptibility here is determined by the joint action of the true and pseudomagnetic fields as though a single magnetic field $\mathbf{B}_\Sigma=\mathbf{B}_{\text{eff}}+\mathbf{B}$ were present in the system. Since expression (30) for B_{eff} contains the multiplier c in the numerator, the pseudomagnetic field value may be quite substantial on the scale of the true magnetic field. The magnetic susceptibility of OAF (29) with an inhomogeneous spectral asymmetry parameter $\Delta(\mathbf{r})$ in a weak magnetic field coincides with the differential magnetic susceptibility of a homogeneous system with $\Delta=0$ in an external magnetic field of $\mathbf{B}_{\text{eff}}$.

An interesting situation occurs when the Fermi level ε_F is located in the energy range corresponding to the forbidden band of the Hamiltonian (29) with $B_{\text{eff}}=0$. Given the large value of B_{eff} at sufficiently low temperatures $\Theta\ll\mu_B B_{\text{eff}}$ (μ_B is the Bohr magneton) the system will satisfy de Hass-van Alphen effect conditions, while the diamagnetic differential susceptibility χ', as we know,[9] may assume arbitrarily large absolute values. Unlike the standard formulation of the de Haas–van Alphen effect which considers a homogeneous field, the pseudomagnetic field $\mathbf{B}_{\text{eff}}$ (30) related to the spectral asymmetry parameter $\Delta(\mathbf{r})$ is fundamentally inhomogeneous. Such a situation more closely corresponds to the de Haas-van Alphen effect in the presence of Schanberg domains.[9] However, as demonstrated in Ref. 29, if the characteristic scale q^{-1} of the inhomogeneity of the asymmetry parameter $\Delta(\mathbf{r})$ substantially exceeds the magnetic length $\lambda=ck_F/eB_{\text{eff}}$ (k_F is the Fermi momentum), it is sufficient to limit the analysis to the zeroth order in the expansion of $\chi'(q)$ in powers of q, $\chi'(0)$ ($\chi'(q)\approx\chi'(0)+q^2\chi_1+\ldots$), i.e., to calculate the differential susceptibility in a a homogeneous field B_{eff}. According to the general theory of the de Haas-van Alphen effect[9,30] the magnetic susceptibility contains a continuous component $\bar{\chi}'$ and an oscillating component $\tilde{\chi}'$ in the magnetic field. The smooth component is smaller than the oscillating component by the factor

$$\bar{\chi}'/\tilde{\chi}'\sim(\mu_B B_{eff}/\varepsilon_F)^{1/2}\ll 1.$$

The oscillating susceptibility component $\tilde{\chi}'$ consists of a sum of the field-periodic components with multiple periods. The term with the lowest period makes the primary contribution to the sum; for this term we have[29]

$$\tilde{\chi}'=a\cos(\varepsilon_F/\mu_B B_{eff}),$$

$$a=\left(\frac{e}{c}\right)^{1/2}\frac{\varepsilon_F{}^2 m}{2^{1/2}\pi^{5/2}B_{eff}^{1/2}}\frac{\varkappa}{\text{sh}\,\varkappa}, \quad \varkappa=\frac{2\pi^2\Theta}{\mu_B B_{eff}}, \quad B_{eff}\approx B_\Sigma. \tag{31}$$

For identical values of B_{eff} correspondng to the positive half-period of the cosine in (31) the susceptibility $\tilde{\chi}'$ is paramagnetic while it is diamagnetic for the other values corresponding to the negative half-period. However the thermodynamic stability condition limits the paramagnetic susceptibility to $\tilde{\chi}<1/4\pi$. (Refs. 9, 29). Assuming $M\ll B_{\text{eff}}$ holds we will write an expression for the induced moment corresponding to the susceptibility (31):

$$M=a'\sin k(H+4\pi M-B_{eff}) \quad (B=H+4\pi M), \tag{32}$$

where $k=\varepsilon_F/\mu_B B_{\text{eff}}{}^2$, $a'=a/k$. Figure 1 provides a plot of the function $M(H)$ of Eq. (32) for $4\pi a'k>1$ ($a>1/4\pi$). The origin is given by the pseudomagnetic field B_{eff} in the

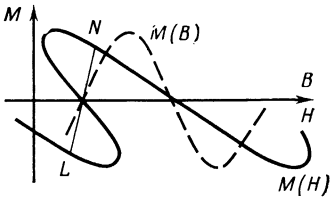

FIG. 1.

argument of the sine of (32). On the segment of negative slope $\chi' > 1/4\pi$ holds. In the ambiguity range the change in the mean moment of the specimen $M(H)$ over the segment LN corresponds to the energy minimum; the boundaries of this segment (the points L and N) are determined by the thermodynamic area rule.[9] The slope of the segment approaches vertical as the demagnetization factor decreases. The system decomposes into two types of domains on the LN segment within which the magnetization assumes values corresponding to points L and N. In a weak field H the susceptibility $\chi = \chi'/(1 - 4\pi\chi')$ ($\chi = \partial M/\partial H$) is determined by the slope of the graph $M(H)$ at the origin. If the coordinate origin lies within the domainization range LN the susceptibility is paramagentic due to the change in the ratio of the specific volumes of the two types of domains occurring from the change in field H (for $a > 1/8$ the system decomposes into domains across the entire positive half-period of function (31). Here the local susceptibility of the domains corresponding to the slope of $M(H)$ at points L and N is diamagnetic. Hence measurements over times less than the domain relaxation time show a diamagnetic type of system response.) If B_{eff} given by the spatial distribution of the asymmetry parameter $\Delta(\mathbf{r})$ is such that the coordinate origin on the $M(H)$ graph lies outside the domainization range the susceptibility is diamagnetic. The susceptibility of the diamagnetic phase tends toward the susceptibility of an ideal diamagnetic, $\chi' \to -\infty$ ($\chi \to -1/4\pi$), in the limit $a \to \infty$ (the actual existence of the diamagnetic domains proves that the coefficient a may attain large values.) It is important, however, to take into account that formula (31) was obtained ignoring charge carrier scattering by impurities and other defects and assuming a homogeneous pseudomagnetic field. Both the scattering and the inhomogeneity of the pseudomagnetic field will blur the discrete levels of charge carrier motion in the plane perpendicular to the pseudomagnetic field and will diminish the susceptibility. The drop in susceptibility is described by the Dingle factor, $\exp(-\tau^{-1}/\mu_B B_{\text{eff}})$, where τ^{-1} is the characteristic energy scale of level broadening.

5. THE PHASE GENERATION EFFECT IN A SUPERCONDUCTING OAF

From the formal viewpoint the vector $\mathbf{T}$ characterizing the transformational properties of an OAF with an asymmetric spectrum is analogous to not only the vector-potential but also the current density $\mathbf{j}$. However the existence of current whose density is simply proportional to the vector parameter $\mathbf{T}$ ($\mathbf{j} \sim \mathbf{T}$) rather than its coordinate derivative is forbidden by the gauge invariance of the expressions for the energy and current.[6] The current density is related to the parameter $\mathbf{T}$ by the relation

$$\mathbf{j}(\mathbf{r}) = c \operatorname{rot}\operatorname{rot}\mathbf{T}(\mathbf{r}), \tag{33}$$

which determines the relation of the current to the density of the toroidal moment.[4,11] Gauge symmetry breaking occurs in superconductors. Specifically this is manifested as the combination $\nabla\varphi - 2e\mathbf{A}/c$ (φ is the phase of the superconducting order parameter) functioning as the gauge-invariant quantity; this combination may appear explicitly in the macroscopic relations. An investigation of the properties of an orbital aniferromagnetic is of interest in this case.

We can write the following expression for the superconducting current in a magnetic[31]:

$$\mathbf{j}_c = \eta\left(\nabla\varphi - \frac{2e}{c}\mathbf{A} - e\xi\mathbf{T}\right). \tag{34}$$

Here ξ is the coefficient having the dimensions of the inverse velocity. The coefficient η in (34) is proportional to the superconducting electron density ($\eta \sim n_c \sim |\Delta_c|^2$, Δ_c is the superconducting order parameter).

The magnetic symmetry allowing existence of a t-odd polar vector can be achieved by both spin and orbital ordering. However it is possible to demonstrate even on the macroscopic level that the additive term in expression (34) for the current is related to magnetic ordering of an orbital nature. A term in the form of a Lifshitz invariant corresponds to the auxiliary contribution to the current in the expression for the free energy (34) near the superconducting transition temperature:

$$\delta\mathcal{F} = i\varkappa\mathbf{T}\left(\Delta_c\left(\nabla - i\frac{2e}{c}\mathbf{A}\right)\Delta_c^* - \text{c.c.}\right). \tag{35}$$

The orbital nature of this contribution is obvious since all spin parameters enter into the interaction energy between the system and the electromagnetic field soley in combination with the magnetic field and not the vector potential as is the case in (35). A specific calculation in two-band ED model (1)–(3) yields the following expression for the auxiliary contribution to the superconducting current (34):

$$\mathbf{j}_{c\tau} = e\eta\xi\mathbf{T} = \frac{7}{8}\zeta(3)\frac{ek_F|\Delta_c|^2}{\pi^4\Theta^2}\mathbf{P}\Delta_I^s,$$

where $\zeta(n)$ is the Riemann zeta function.

We investigate the macroscopic consequences of the auxiliary contribution in the expression for the superconducting current (34). We obtain an equation for the superconducting phase from relation (34) and the continuity condition:

$$\nabla^2\varphi(\mathbf{r}) = e\xi \operatorname{div}\mathbf{T}(\mathbf{r}),$$

from which it follows that the auxiliary contribution to the superconducting current causes a phase multiplier dependent on the coordinate to appear in the order parameter

$$\Delta_c(\mathbf{r}) = |\Delta_c| \exp\left\{ie\xi\int \mathbf{T}_\parallel(\mathbf{r}')\,d\mathbf{r}'\right\}. \tag{36}$$

Here $\mathbf{T}_\parallel(\mathbf{r})$ represents the irrotational field component given by the vector $\mathbf{T}(\mathbf{r})$ ($\operatorname{curl}\mathbf{T}_\parallel(\mathbf{r}) = 0$; when $\mathbf{T}(\mathbf{r}) = \text{const}$ it is necessary to set $\mathbf{T}_\parallel = \mathbf{T}$). We first consider the case where there is no rotational field component $\mathbf{T}_\perp(\mathbf{r}) = \mathbf{T}(\mathbf{r}) - \mathbf{T}_\parallel(\mathbf{r})$ (more precisely, $\operatorname{curl}\mathbf{T}_\perp(\mathbf{r}) = 0$). Here the phase multiplier in (36) totally balances the auxil-

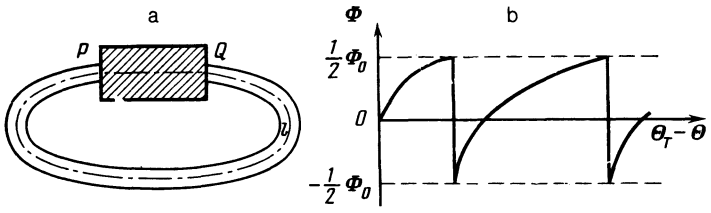

FIG. 2.

iary contribution to the current, and the ordinary Meissner effect occurs in the system with magnetic field damping on the scale of the London depth of penetration λ_L.

The appearance of the coordinate-dependent phase of the order parameter (36) can be observed in the following experiment. We terminate the ends of the superconductor (36) with an ordinary superconductor (Fig. 2). We consider a closed ring l in which $\mathbf{j}_c = 0$. Such a ring exists due to the Meissner effect. Integrating expression (34) over the ring l we find that the total magnetic flux Φ through the ring attains values of

$$\Phi=\Phi_0\left\{n-\frac{e\xi}{2\pi}\int_P^Q \mathbf{T}_\parallel(\mathbf{r})\,d\mathbf{r}\right\}; \tag{37}$$

here the flux quantum is $\Phi_0 = \pi c/e$; n is an integer. The current I corresponds to the magnetic flux Φ:

$$I=c\mathscr{L}^{-1}\Phi, \tag{38}$$

where $\mathscr{L}$ is the inductance of the ring. It follows from (37) and (38) that if the integral has a value other than $2\pi n$ in (37) the ground state of the superconducting ring (Fig. 2) is a state with spontaneous current flow.

References 32, 33 have examined spontaneous current states attained in weak-link superconducting rings. Reference 32 investigated a Josephson junction ring containing magnetic impurities and demonstrated that spontaneous current arises if the matrix element of the tunnel junction with spin inversion is greater than the matrix element of an ordinary junction without spin inversion and, more, the product of the critical Josephson current and the ring inductance exceeds a certain critical value. Reference 33 analyzed an *S–F–S*-junction formed by an interlayer of a normal ferromagnetic metal. A current state will exist in such a system if the thickness of the interlayer assumes values lying within certain specific intervals, while the product $I_c\mathscr{L}$, as in Ref. 32, is sufficiently great. Spontaneous current vanishes in the transition to the multidomain interlayer. The current states of Ref. 32, 33 and therefore achieved in rather rigid conditions.

The phase-generation effect in the superconducting antiferromagnetic (34), (36) manifests an essentially volumetric nature. The spontaneous current (38) will arise, unlike Ref. 32, 33, with any ring dimensions (and any dimensions of the antiferromagnetic specimen) and also independent of the structure of the magnetic superconductor/vacuum boundary where screening currents are concentrated.

The value of the number n in (37) is determined from the minimum ring energy $E = \Phi^2/2\mathscr{L}$. We then find from (37) that the magnetic flux flowing through the ring in the ground state is at most one-half of the flux quantum:

$$\max|\Phi|\leqslant{}^1/_2\Phi_0. \tag{39}$$

If the magnitude of vector $\mathbf{T}$ is sensitive to the value of any controlled system parameter such as temperature, the temperature dependence of the flux Φ will oscillate. When the flux reaches $\frac{1}{2}\Phi_0$ the value of n in (37) will change by unity and the flux magnitude will jump. The value of n characterizing the ground state is determined by the following relation:

$$n=[2\tau]-[\tau], \qquad \tau=\frac{e\xi}{2\pi}\int_P^Q \mathbf{T}_\parallel(\mathbf{r})\,d\mathbf{r},$$

where $[\tau]$ is the integer part of τ. Figure 2,b, shows the magnetic flux plotted as a function of temperature for the case where the vector $\mathbf{T}$ appears from a second order phase transition at temperature Θ_T.

It is important to note that a reason independent of the phase-generation effect exists for spontaneous current to appear in the ring shown in Fig. 2 even if the antiferromagnetic whose symmetry is characterized by the vector $\mathbf{T}$ is of a purely spin nature. The problem is that in such an antiferromagnetic the jump of the tangential component of vector $\mathbf{T}$ at the surface facing inward on the ring will induce surface currents according to Eq. (33) which relates the current to the density of the toroidal moment; these currents will produce a surface magnetic moment $\mathbf{M}_S$ which will make a contribution to the total flux Φ. The magnetic moment $\mathbf{M}_S$ is determined by Maxwell's equation and as a result will have relativistic moments compared to the orbital contribution to (37). The electromagnetic formation mechanism of the surface contribution to the flux will be easily differentiable from the phase generation effect in experiment. The strict quantization condition $\Phi = n\Phi_0$ will hold for a flux of electromagnetic nature, while the flux level with changes in the system parameters will vary in jumps only and will remain constant in the range of parameters corresponding to the given optimum value of n (compare to Fig. 2,b).

We will consider the rotational vector component of $\mathbf{T}$: $\operatorname{curl}\mathbf{T}_\perp(\mathbf{r})\neq 0$. Applying the rotation operation to relation (34) we obtain the London equation

$$\operatorname{rot}\Lambda^2\mathbf{j}_c=-c^{-1}\mathbf{B}-{}^1/_2\xi\operatorname{rot}\mathbf{T}. \tag{40}$$

Here $\Lambda^{-2}=2e\eta$, $\lambda_L=\Lambda c/(4\pi)^{1/2}$ is the London penetration depth. If the coefficient ξ in (34), (40) is normalized so that the vector $\mathbf{T}$ coincides with toroidal moment (33), Maxwell's equation takes the form

$$\operatorname{rot}\mathbf{B}=\frac{4\pi}{c}\mathbf{j}_c+4\pi\operatorname{rot}\operatorname{rot}\mathbf{T}. \tag{41}$$

We obtain a relation for determining the density of the superconducting current from (40) and (41):

$$\nabla^2\mathbf{j}_c-\lambda_L^{-2}\mathbf{j}_c=\frac{c}{\lambda_L^2}\operatorname{rot}\operatorname{rot}\mathbf{T}+\frac{c^2\xi}{\lambda_L^2}\operatorname{rot}\operatorname{rot}\mathbf{T}. \tag{42}$$

The rotational component of the vector $\mathbf{T}$ is therefore the source for the superconducting currents. The first term on the right side of equation (42) describes the contribution to the source of the magnetic induction associated with the macroscopic inhomogeneity of the density of toroidal moment (33). This term has a purely electromagnetic nature, exists in both orbital and spin magnets with a toroidal moment, and plays a role analogous to the term curl $\mathbf{M}$ in the equation

$$\nabla^2\mathbf{j}_c-\lambda_L^{-2}\mathbf{j}_c=c\lambda_L^{-2}\operatorname{rot}\mathbf{M},$$

characterizing the distribution of the superconducting screening currents in an inhomogeneous ferromagnetic.[34] The second term on the right side of (42) is due to direct interaction of the superconducting and toroidal order parameters and has no analog in superconducting ferromagnetics. Its contribution substantially exceeds that of a term of electromagnetic nature since $c\xi\sim c/v_F\gg 1$.

The interaction of the superconducting order parameter and the rotational components of the orbital antiferromagnetic order parameter results in a unique temperature dependence of the rotational critical magnetic field. Let the superconducting transition temperature Θ_c lie above and close to the antiferromagnetic transition temperature Θ_T so that the Ginzburg-Landau expansion can be used. We write the linearized equation for the superconducting order parameter

$$\frac{1}{2m}\left(-i\nabla-\frac{2e}{c}\mathbf{A}(\mathbf{r})-e\xi\mathbf{T}(\mathbf{r})\right)^2\Delta_c=-\alpha\Delta_c, \tag{43}$$

where $\alpha=a(\Theta-\Theta_c)$, $\mathbf{A}(\mathbf{r})=(0,xH,0)$. Without antiferromagnetic ordering ($\mathbf{T}=0$) and setting the minimum eigenvalue ω_c of the operator on the left side of equation (43) equal to the coefficient α we obtain the usual expression for H_{c2}:

$$H_{c2}^0=-mc\alpha/e\sim\Theta_c-\Theta. \tag{44}$$

It is necessary to account for the shift of the eigenvalue ω_c to the extent of the parameter $\mathbf{T}$ below the antiferromagnetic transition temperature Θ_T. The longitudinal component of the vector $\mathbf{T}$ is eliminated from the eigenvalue equation (43) by the gauge transformation (36). We can limit the analysis to a linear approximation in $\mathbf{T}_\perp(\mathbf{r})=\mathbf{T}(\mathbf{r})-\nabla\varphi(\mathbf{r})$ in the vicinity of Θ, where $\varphi(\mathbf{r})$ is the phase of order parameter (36). The shift of the eigenvalue in this case is determined by the integral:

$$\delta\omega_c=\frac{\xi eH}{mSc\lambda}\int xT_{\perp y}(\mathbf{r})\exp\left\{-\left(\frac{x-x_0}{\lambda}\right)^2\right\}d\mathbf{r}. \tag{45}$$

Here $\lambda=(c/eH)^{1/2}$ is the magnetic length and S is the cross-sectional area of the sample in the plane $x=\text{const}$. The integral (45) is a function of the position of the "orbital center" x_0 with the given coordinate relation $\mathbf{T}_\perp(\mathbf{r})$. At various points in space, nucleation will therefore occur at different magnetic field levels. Strictly speaking this nucleation mechanism is different from the ordinary pattern of the onset of superconductivity when $H=H_{c2}$. We will use a different designation—H_{cT}—for the magnetic field value at which the nucleus of the superconducting phase localized at the inhomogeneity $\mathbf{T}_\perp(\mathbf{r})$ initially appears. The nucleus first forms in the vicinity of the point $x_0=x_{0\,\min}$ which minimizes the integral (45). We will isolate the amplitude $\mathbf{T}$ and the coordinate multiplier $t(\mathbf{r})$ in the parameter $\mathbf{T}_\perp(\mathbf{r})$:

$$\mathbf{T}_\perp(\mathbf{r})=\mathbf{T}t(\mathbf{r}),\quad \max|t(\mathbf{r})|=1.$$

Since $T\sim(\Theta_T-\Theta)^{1/2}$, we have for H_{cT}:

$$H_{cT}=H_{c2}^0+f(\Theta)(\Theta_T-\Theta)^{1/2}, \tag{46}$$

where $f(\Theta=\Theta_T)\neq 0$. The radical term in (46) causes a singularity in the temperature dependence $dH_{cT}/d\Theta$:

$$\frac{dH_{cT}}{d\Theta}\approx\begin{cases}-amc/e, & \Theta>\Theta_T\\ -amc/e-\dfrac{f(\Theta)}{(\Theta_T-\Theta)^{1/2}}, & \Theta<\Theta_T\end{cases}$$

Here we have ignored the derivatve $df/d\Theta$ which enters into the expression for $dH_{cT}/d\Theta$ with the small multiplier $(\Theta_T-\Theta)^{1/2}$ when $\Theta<\Theta_T$. When the direction of the magnetic field is reversed, the position of $x_{0\,\min}$ changes, although the sign of the singularity is conserved.

6. CONCLUSION

The results of the present study show that magnetics with an inversion center have a number of unusual magnetic properties attributable to the quasimomentum spectral asymmetry of the elementary excitations. The spectral asymmetry is due to violation of the invariance with respect to the time inversion in the orbital subsystem of the crystal and is most strongly expressed in magnetics of an orbital nature. In these magnetics the spectral asymmetry parameter is determined by the Coulomb interaction and is quite large. In spin magnetics the asymmetry parameter is due to spin-orbital interaction and is always nonzero if only the symmetry group of the crystal allows energy invariants to exist that are odd with respect to the quasimomentum components. The difference between the orbital and spin magnetics in this respect is a purely quantitative difference. These effects will be observed in all magnetics with symmetry of this type.

The interaction of the charge carriers producing the asymmetric spectrum with the external magnetic field in classical physics corresponds to the diamagnetic precesion of the orbital current rings.[3,6] The system inhomogeneity increases the rigidity of the rings and suppresses the paramagnetic component of the response, thereby enhancing the diamagnetism. In principle the diamagnetic susceptibility can attain arbitrarily large absolute values. A specific calculation of the magnetic susceptibility in the range of large asymmetry parameter values which permits anomalous diamagnetism was carried out in Sec. 4 for a model with a metallic-type spectrum. Moreover the diamagnetic response is a common property of inhomogeneous magnetics with an asymmetric spectrum as indicated by calculations employing perturbation theory in the spectral asymmetry parameter (Sec. 3). Hence the model from Sec. 4 should be consid-

ered one possible physical picture of strong diamagnetic response.

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