# Scattering of electrons by an orderable system of charges

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An analysis is made of a semiconductor in which impurity centers are resonant donors with an electron energy level lying  $E_D$  above the bottom of the conduction band. At high impurity concentrations only a small proportion of the centers is ionized and the charge of these centers is compensated by degenerate electrons with the Fermi energy equal to  $E_D$ . In view of the finite width of a resonant level the charge may be transferred from center to center and the positions of ionized impurities are correlated in space because of the Coulomb repulsion between them. The charges tend to form a Wigner lattice against the negative background of electrons. It is shown that at sufficiently high temperatures the mobility of electrons is limited by thermal vibrations of the Wigner lattice of charges, whereas at low temperatures it is limited by the nonideal nature of this lattice associated with the random distribution of impurities. Such a situation is encountered experimentally in HgFeSe solid solutions where iron impurity centers act as resonant donors.

(1)

# **1.INTRODUCTION**

The scattering of electrons by ionized centers in semiconductors is usually described assuming that the distribution of ionized impurities in space is random. This leads to the Brooks–Herring expression for the mobility in a degenerate electron gas<sup>1</sup>:

where

$$\mu_{0} = \frac{3\pi}{2} \frac{\hbar^{3} \varkappa^{2}}{m^{2} e^{3}} \frac{n}{N_{I}} = \frac{0.2n}{N_{I}} \left( \frac{m_{0}}{m} \varkappa \right) \mathrm{cm}^{2} \cdot \mathrm{V}^{-1} \cdot \mathrm{s}^{-1}$$

 $\mu = \mu_0 \left[ \ln \left( 1 + \xi_0 \right) - \xi_0 / (1 + \xi_0) \right]^{-1}, \quad \xi_0 = 4k_F^2 r_0^2,$ 

 $m_0$  and *m* are the masses of a free electron and of an electron in the conduction band, respectively;  $\varkappa$  is the permittivity;  $r_0$ is the screening radius of a degenerate electron gas;  $k_f$  is the Fermi momentum; *n* and  $N_I$  are, respectively, the density of electrons and the concentration of the scattering centers.

In some cases it is necessary to allow for correlation in the distribution of the centers. It can appear, for example, during growth or annealing of crystals.<sup>2</sup> Under these conditions Eq. (1) is generally invalid.

However, even in the case of a random distribution of the impurity centers the spatial distribution of the charged impurities may be correlated.<sup>3</sup> This problem becomes particularly important in the case of the  $Hg_{1-x}Fe_x$  Se system.<sup>4-9</sup> In this case the impurity centers of iron replacing the mercury atoms act as donors with an electron energy level located  $E_D \approx 230$  meV above the bottom of the conduction band. At low iron concentrations all the impurity centers give up electrons to the conduction band and become charged. However, when the iron concentration is  $x = x^* \approx 3 \times 10^{-4}$ , the Fermi level of electrons becomes comparable with the energy of a resonant donor and then the density of electrons ceases to rise on increase in x and remains equal to  $n \approx 5 \times 10^{18} \text{ cm}^{-3}$ . Only some of the centers remain charged and the fraction of such centers is given by KN = n, where  $K = x^*/x$ ; N is the total concentration of the impurity centers;  $N = \mathcal{N}x$ : $\mathcal{N} = 1.77 \times 10^{22}$  cm<sup>-3</sup> is the concentration of the sites in the mercury sublattice.<sup>5</sup>

The quantity K is analogous to the degree of compensation in the case of an ordinary semiconductor. An important distinction from the latter case is that instead of randomly distributed point compensating acceptors the charge of the ionized donors becomes neutralized by electrons distributed uniformly in space.

It has been established experimentally that cooling increases the mobility of an electron gas, governed by the scattering on ionized centers, and for some values of x this mobility is considerably higher than the theoretical value deduced using the Brooks-Herring expression. A qualitative explanation of this effect was given by Mycielski.<sup>7</sup> The finite width  $\Gamma$  of a resonant donor level means that electrons can be transferred from donor to donor right down to the lowest temperatures. The spatial distribution of charged donors then corresponds to a minimum of the energy of the electrostatic interaction of charges with one another. If the level width  $\Gamma$  is very small and the electron mobility is high, we can assume that this mobility is governed by the static distribution of charged impurities. At T = 0 the positive charges tend to be located far from one another because of repulsion. In the range  $x \ge x^*$  such charges form a Wigner lattice against a negative background of electrons. The lattice regarded as an ordered structure does not scatter carriers, in agreement with the Bloch theorem, and this increases the mobility of electrons as temperature is lowered.

However, to the best of our knowledge, a quantitative theory of the mobility in an electron gas has not yet been developed for the case of a strong correlation in the spatial distribution of charged centers. Our aim is to fill this gap. We shall show that at sufficiently high temperatures the mobility is limited by thermal vibrations of the Wigner lattice of charges, whereas at low temperatures it is limited by the nonideal nature of this lattice associated with a random distribution of the iron atoms. In the sections below we shall provide a qualitative analysis of this problem, obtain quantitative results, and compare these results with the experimental data.<sup>5</sup>

# 2. QUALITATIVE ANALYSIS

There are three temperature ranges in which the temperature dependence of the mobility is due to physically different types of ordering of ionized centers. When the temperature T is much higher than the energy of the Coulomb interaction between charges separated by the average distance,  $e^2n^{1/3}/\kappa$ , the probability of filling all the donors with electrons is constant (it depends weakly on the environment) and close to 1 - K. The charge correlation then resembles correlation in the case of a slightly nonideal plasma. The correlation (screening) radius  $r_s$  is described by 10

$$r_{s} = [\kappa T/4\pi e^{2}n(1-K)]^{\frac{1}{2}}.$$
 (2)

The factor 1 - K in the denominator of Eq. (2) reflects the fact that at low values of K the concentration of the screening carriers is equal to n and in the limit  $K \rightarrow 1$  it is equal to (1 - K)n. It should be noted that the electron screening radius is greater than  $r_s$  on condition that  $T < E_F$ , which is assumed below.

The charge correlation function can be calculated by analogy with Ref. 10, which will be done in the next section. The mobility calculated using this correlation function is

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μ

$$\begin{aligned} u &= \mu_0 \left[ \ln \left( 1 + \xi_s \right) - K \xi_s / (1 + \xi_s) \right]^{-1}, \\ \xi_s &= 4k_F^2 r_s^2 = T/T_0 \left( 1 - K \right), \\ T_0 &= (9\pi)^{-1/2} e^2 n^{1/2} / \kappa \approx 32 \text{ K.} \end{aligned}$$
(3)

In this temperature range the mobility rises logarithmically as a result of cooling, because of reductin in the correlation radius. The main terms in Eqs. (1) and (3) differ simply by the replacement of  $r_0$  with  $r_s$ .

Equation (3) is valid if  $T \ll T_0$ . On the other hand, if  $T \ll T_0$ , the system of charged donors is a strongly correlated Wigner liquid. According to Ref. 11, this liquid does not crystallize at any temperature because the distribution of the donor centers is random. However, we are interested only in the short-range order in the distribution of the charged centers, which is assumed to differ little from the crystalline order even when  $T \ll T_0$  and  $x \gg x^*$ . Thermal motion causes the charges to wander between impurity centers which are in the vicinity of a certain site fixed by the positions of the neighboring charges. If  $T \ll T_0$ , the amplitude of such random-walk motion is small compared with the average distance between the charges. We can then assume that the scattering of electrons is by dipoles. The negative pole of each of them corresponds to a site in an ideal (at a given point) Wigner lattice, whereas a positive pole corresponds to a charged impurity center which wanders in the vicinity of a given site. In the next section we shall show that the mobility limited by a scattering system of dipoles with randomly oriented moments can be described by

$$\mu = \mu_0 \left[ 4Dk_F^2 \overline{r^2}(T) \right]^{-1}, \tag{4}$$

where  $\overline{r}^2(T)$  is the average value of the square of the distance between the charges in the dipole and D as a numerical coefficient. It is assumed that the moments of the neighboring dipoles are uncorrelated. We shall consider this approximation later.

In the calculation of  $r^2(T)$  we have to consider two temperature ranges. At relatively high temperatures the quantity  $(\vec{r}^2)^{1/2}$  is considerably larger than the average distance between the impurity centers. In this case the discrete distribution of the positive charges between the donors can be ignored and we can assume that such charges form a Wigner lattice (without a short-range order) against the negative background. It should be mentioned that this lattice does not

exhibit acoustic vibrations of frequency exceeding  $\Gamma/\hbar$ , since  $\hbar/\Gamma$  is the time in which the charge is transferred from one impurity center to another. If  $\Gamma \ll T$ , as assumed below, then acoustic branches can be ignored completely and the value of  $\overline{r}^2(T)$  can be calculated assuming that the positive charge under consideration moves in a static potential of other charges forming a Wigner lattice. We then find that

$$\overline{r^2}(T) = \int d^3 \mathbf{r} r^2 \exp\left[-\frac{U(r)}{T}\right] / \int d^3 \mathbf{r} \exp\left[-\frac{U(r)}{T}\right], \quad (5)$$

where

$$U(r) = Cne^2 r^2 / \varkappa \tag{6}$$

is the potential created at a distance r from a lattice site by the negative background of electrons and by the Wigner positive charge lattice. The constant C depends weakly on the nature of the Wigner lattice. We can determine it by replacing a Wigner-Seitz cells with spheres of appropriate volume. The sphere radius R is governed by  $\frac{4}{3}\pi R^{3}n = 1$ . We shall assume that the charge is distributed uniformly inside a sphere. In calculating the potential near the center of the sphere we can ignore the charges of the neighboring spheres, because they are electrically neutral. Such calculations give Eq. (6) with  $C = 2\pi/3$ . Using Eqs. (6) and (5), we obtain

$$\overline{r^{2}}(T) = \frac{9}{4\pi} \frac{\kappa T}{ne^{2}} = \frac{3}{4\pi} \left(\frac{3}{\pi}\right)^{\prime h} \frac{T}{T_{0}} n^{-\frac{\eta}{h}},$$
(7)

whereas Eqs. (7) and (4) yield the mobility in the investigated rang of temperatures. It should be noted that  $\overline{r^2} = 9r_s^2$  in the limit  $K \rightarrow 0$ .

An important feature of the above derivation is the neglect of correlation along the directions of the dipole moments at neighboring lattice sites. Strictly speaking, this is invalid, because Eq. (7) can be readily used to demonstrate that the energy of the dipole-dipole interaction is also of the order of T. However, we shall assume that this simply alters the numerical value of C in Eq. (6). A calculation of the change in this value should in our opinion be carried out on a computer.

It is clear from Eq. (7) that the quantity  $(\overline{r}^2)^{1/2}$  becomes comparable with the average distance between the impurities given by  $N^{-1/2}$  when  $T \sim T_0 K^{1/2} \equiv T$ . At lower temperatures we have to allow for the fact that the spatial distribution of the charged centers corresponds to the energy minimum. At low K this distribution can be established as follows. Let us consider an ideal Wigner lattice and assume that only those impurity centers become charged which are closest to each of the sites in the ideal lattice. We then find that  $(\overline{r}^2)^{1/2}$  can be derived from the short-range order distribution. This gives the following expression:

$$\overline{r^2}(T \leq T^*) = r_{min}^2 = (3/4\pi)^{\frac{4}{3}} \Gamma(\frac{5}{3}) N^{-\frac{4}{3}}.$$
(8)

The substitution of Eq. (8) into Eq. (4) gives the results for the mobility at temperatures  $T \leq T^*$ . We can see that in this temperature range the mobility is independent of temperature and that it rises on increase in x as  $x^{2/3}$ . This is due to the fact that on increase in the impurity concentration the random nature of the distribution of impurity atoms in space becomes gradually less important.

# **3. QUANTITATIVE THEORY**

In this section we shall derive expressions for the mobility valid in various temperature ranges. In the above expressions the electron spectrum is assumed to be parabolic. However, in describing experiments we have to go beyond this approximation because, firstly, HgFeSe is a zero-gap semiconductor with inverted bands ( $E_g < 0$ ,  $E_g = -0.22 \text{ eV}$ ) and, secondly, the Fermi level  $E_f \approx E_p \approx 230 \text{ meV}$  is comparable with the separation  $\Delta = 0.45 \text{ eV}$  to the spin-orbit-split energy band. <sup>5-6</sup> The energy band structure of the compound is described by the Kane Hamiltonian and the mobility related to the scattering by the random potential with the impurities can be described by<sup>12</sup>

$$\mu = e\tau/m,$$
  
$$\tau^{-1} = \frac{2\pi}{\hbar} \Omega \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \langle \overline{|V_{\mathbf{k}-\mathbf{k}_0}|^2} \rangle (1 - \cos \varphi)$$
  
$$\times \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}_0}) \left( 1 + 4A \sin^2 \frac{\varphi}{2} + 4B \sin^4 \frac{\varphi}{2} \right), \qquad (9)$$

where  $|k_0| = k_F$ ;  $m = \hbar^2 k_F [(\partial \varepsilon / \partial k)_{\varepsilon = e_F}]^{-1}$  is the effective mass of an electron at the Fermi level, which is independent of temperature in the range  $T \leq 100$  K under discussion, where  $E_F$  does not vary with T (Ref. 5); is the angle between k and  $k_0$ ;  $\Omega$  is the normalized volume;  $|\overline{V_q}|^2$  is the square of the modulus of the Fourier component of the random potential of charged impurities and the bar above denotes averaging with respect to time. The time dependence of the random potential is related to slow wandering (random walk) of charges between the impurity atoms. The angular brackets denote averaging over the coordinates of all the impurity centers. The appearance of the last factor in the expression for  $\tau^{-1}$  [Eq. (9)] is related to the matrix nature of the Kane Hamiltonian. The constants A and B depend on the energies  $E_f$ ,  $E_g$ , and  $\Delta$ ; for our values of the parameters<sup>12</sup> they are A = -0.57 and B = 0.34. In the case of a simple band we have A = B = 0.

## a) Range where $T \gg T_0$

In this temperature range the scattering is on Gaussian fluctuations of the charged impurity concentration. The size of these fluctuations is greater than the average distance between the charges. Therefore, the fluctuations are described by a smooth function  $\xi(\mathbf{r},t)$ , representing a local deviation of the concentration of charged impurities from the average value at a moment t. We then find that

$$V(\mathbf{r},t) = \frac{e^2}{\varkappa} \int d^3\mathbf{r}' \frac{\xi(\mathbf{r}',t)}{|\mathbf{r}-\mathbf{r}'|}, \quad \langle \overline{|V_q|^2} \rangle = \frac{1}{\Omega} \left(\frac{4\pi e^2}{\varkappa q^2}\right)^2 L(\mathbf{q}),$$
(10)

where

$$L(\mathbf{q}) = \int d^3 \mathbf{r} \exp[i\mathbf{q}(\mathbf{r} - \mathbf{r}')] \langle \overline{\xi(\mathbf{r}, t)\xi(\mathbf{r}', t)} \rangle \qquad (11)$$

is the correlation function of fluctuations of the charged center concentration.

If we can ignore the correlation in the distribution of the charged impurities, i.e., if we assume that  $\overline{\langle \xi(\mathbf{r},t)\xi(\mathbf{r}',t)\rangle}$ =  $N_1\delta(\mathbf{r}-\mathbf{r}')$ , then Eq. (9) readily yields the usual Brooks-Herring expression obtained allowing for the Kane nature of the spectrum.<sup>12</sup>

The correlation function of Eq. (11) is calculated in

exactly the same way as in the case of an impurity energy band in a lightly doped and weakly compensated semiconductor, considered by Uzakov and Éfros.<sup>10</sup> We shall carry these calculations as follows. Fluctuations of the concentration of the iron impurity centers are assumed to be frozen and fixed when a sample is prepared. The concentration of the charge centers, however, fluctuates at each point in time since electrons in thermal motion are transferred from one center to another. The averaging with respect to time can be replaced with the averaging over an ensemble and the time average of the quantity  $\xi(\mathbf{r}',t)\xi(\mathbf{r}',t)$  can be obtained for a fixed distribution of the impurity centers in the form of the functional integral

$$\overline{\xi(\mathbf{r},t)\xi(\mathbf{r}',t)} = \int D\xi\xi(\mathbf{r})\xi(\mathbf{r}')\exp\left[-\frac{R_{min}\{\xi\}}{T}\right] / \int D\xi\exp\left[-\frac{R_{min}\{\xi\}}{T}\right], \quad (12)$$

where  $R_{\min}{\{\xi\}}$  is the minimum work that has to be done to produce a fluctuation of  $\xi(\mathbf{r})$  for a fixed configuration of impurities:  $R_{\min} = H - T\Delta S$ , where the Hamiltonian H and the change in the entropy  $\Delta S$  are due to a fluctuation of  $\xi(\mathbf{r})$ are of the form<sup>10</sup>

$$H = \frac{e^2}{2\kappa} \int d^3 \mathbf{r} \, d^3 \mathbf{r}' \, \frac{\boldsymbol{\xi}(\mathbf{r}) \boldsymbol{\xi}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}, \qquad (13)$$
$$\Delta S = -\frac{1}{2N(1-K)} \left[ \frac{1}{K} \int d^3 \mathbf{r} \, \boldsymbol{\xi}^2(\mathbf{r}) - 2 \int d^3 \mathbf{r} \, \boldsymbol{\xi}(\mathbf{r}) \, \boldsymbol{\eta}(\mathbf{r}) \right]^{\mathsf{T}}$$

and it depends on the frozen fluctuations of the impurity concentration  $\eta(\mathbf{r}) = N(\mathbf{r}) - N$ , where  $N(\mathbf{r})$  is the local concentration of impurities and we have  $\langle N(\mathbf{r}) \rangle = N$ .

A calculation of the average described by Eq. (12) with the aid of Eq. (13), which is fully analogous to that described in Ref. 10, shows that the result depends quadratically on the fluctuations  $\eta(\mathbf{r})$ . This is related to the validity of the Gaussian statistics in the case of the fluctuations  $\xi(\mathbf{r})$ and  $\eta(\mathbf{r})$  when  $T \gg T_0$ . Therefore, in calculating the average of Eq. (12) for a distribution of impurities it is necessary to specify the correlation properties of the function  $\eta(\mathbf{r})$ . We shall assume that a distribution of the impurity centers in space is random:

$$\langle \eta(\mathbf{r})\eta(\mathbf{r}')\rangle = N(1-x)\delta(\mathbf{r}-\mathbf{r}').$$
 (14)

The appearance of the factor 1 - x in Eq. (14) is due to the fact that the impurity centers are located solely at the lattice sites, i.e., that the distribution is fully ordered when x = 1. Substituting Eq. (13) into Eq. (12), integrating, and then applying Eq. (14), we obtain

$$L(\mathbf{q}) = n \frac{q^2}{q^2 + r_{\bullet}^2} \left[ 1 - \frac{K}{1 + q^2 r_{\bullet}^2} \right], \tag{15}$$

where a correlation radius  $r_s$  is described by Eq. (2). Substituting Eqs. (15), (11), and (10) into Eq. (9) and integrating, we find that the mobility is given by

$$\mu = \mu_{0} (\alpha_{0} + 4A \alpha_{A} + 4B \alpha_{B})^{-1},$$
  

$$\alpha_{0} = \ln(1 + \xi_{s}) - \frac{K\xi_{s}}{1 + \xi_{s}}, \quad \alpha_{A} = 1 - \frac{1 + K}{\xi_{s}} \ln(1 + \xi_{s}) + \frac{K}{1 + \xi_{s}},$$
  

$$\alpha_{B} = \frac{1}{2} - \frac{1 + K}{\xi_{s}} - \frac{K}{\xi_{s}(1 + \xi_{s})} + \frac{1 + 2K}{\xi_{s}^{2}} \ln(1 + \xi_{s}), \quad (16)$$

where  $\xi_s$  is defined earlier in Eq. (3),  $\mu_0$  is defined in Eq. (1),  $N_1 = n$ , and *m* is the effective mass at the Fermi level:  $m = 0.061m_0$  (Ref. 4). This gives  $\mu_0 = 4.74 \times 10^4$  $\text{cm}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$ , because x = 29.7 (Ref. 12).

In the calculation of Eq. 15 no allowance is made for the screening by free electrons, which is justified if  $T < E_f$ . In the parabolic approximation, i.e., when A = B = 0, Eq. (16) leads to Eq. (3). It is important to point out that in the derivation of Eq. (15) we assume that fluctuations of charged  $\xi(\mathbf{r})$  and neutral  $\eta(\mathbf{r})$  centers are Gaussian and smooth functions of the coordinates. The range of validity of this approximation corresponds to the condition  $1 - K \ll 1$ , i.e., to  $T \gg T_0$ . In the case when  $nr_{s^2} \gg 1$ , this approximation is valid if the more stringent condition  $T \gg T_0(1 - K)^{-1/2}$ , is satisfied, as demonstrated in Ref. 10. The correlation function of Eq. (10) differs from that obtained in Ref. 10 only because the charge of the centers is not compensated by acceptors but by electrons.

#### b) Range where $T^* \ll T \ll T_0$ and $K \ll 1$

We shall assume that at these temperatures the shortrange order in the distribution of charges corresponds to a Wigner crystal. The electron wavelength  $\lambda \sim n^{-1/2}$  exceeds the characteristic deviation of a positive charge from a site in a Wigner crystal [Eq. (7)]. Therefore, if we ignore the dipole-dipole interaction, we find that the scattering is governed by a system of randomly oriented and randomly distributed dipoles with a concentration *n*. The mobility can be calculated using Eq. (9) and substituting there Eq. (10), but we have to replace the Coulomb potential with the potential of the dipole averaged over the random orientations<sup>13</sup> when the characteristic distance between the charges in the dipole is given by Eq. (7), and we have to assume that  $L(\mathbf{q}) = n$ . Then, the square of the modulus of the matrix element of Eq. (10) is

$$\langle |\overline{V_{\mathbf{q}}}|^2 \rangle = n \left(4\pi e^2 / \varkappa q^2\right)^2 q^2 \overline{r^2}(T) / 3. \tag{17}$$

Substituting Eq. (17) into Eq. (9) and integrating, we find that the mobility described by Eq. (4), where  $\overline{r^2}$  is given by Eq. (7) and

$$D = \frac{1}{3}(1 + 2A + 4B/3). \tag{18}$$

We can see from Eqs. (16), (4), (7), and (2) that Eqs. (16) and (4) are matched parametrically at the boundary of validity of  $T^2 \sim T_0$ . Moreover, Eq. (16), which is valid when  $T \ge T_0$ , in the limit when  $T \ll T_0$  and  $K \to 0$  is identical (apart from a factor) with Eq. (4) valid when  $T \ll T_0$ . This can be demonstrated by comparing Eqs. (10), (15), and (17) for the squares of the modulus of the matrix element. It is clear from these equations that formally the dependences (4) and (16) are exactly identical if in the latter we replace  $\xi_s$  with  $3\xi_s$  and assume that K = 0.

### c) Range where $T \leq T^*$ and $K \ll 1$

As pointed out already, when  $[\overline{r^2}(T)]^{1/2}$  of Eq. (7) becomes comparable with the average distance between the impurity centers at low temperatures, the mobility ceases to depend on temperature. We then obtain Eq. (4) where  $\overline{r^2}$  should be replaced by Eq. (8). The coefficient *D* is still described by Eq. (18).

We have thus been able to describe the scattering of electrons by an orderable system of ionized centers in all the parametrically different temperature ranges. We shall compare our results with the experimental data of Ref. 5 by deriving an interpolation expression for the mobility which matches Eqs. (16) and (4) that are valid in different temperature ranges.

#### 4. COMPARISON WITH EXPERIMENTS

The interpolation expression for the mobility, valid at all temperatures  $T \ll T_0$ , can be obtained from Eq. (5) for  $\overline{r^2}$ (T) where the lower limit of integration with respect to r should be  $r_{\min}$  of Eq. (8). This means that Eq. (7) is substituted in Eq. (4) and the former should be multiplied by

$$W = \frac{2}{3} \frac{\Gamma({}^{5}/_{2}, x_{min})}{\Gamma({}^{3}/_{2}, x_{min})} = \begin{cases} 1, & x_{min} \to 0 & (T \gg T^{*}), \\ {}^{2}/_{3} x_{min}, & x_{min} \to \infty & (T \ll T^{*}), \end{cases}$$
(19)

where

$$x_{min} = \frac{2}{3} \pi r_{min}^2 e^2 n / \chi T = \frac{3}{2} \pi^2 \int_{-\infty}^{\infty} \Gamma(5/3) T^* / T$$

and  $\Gamma(a,x)$  is an incomplete gamma function.

We now have to match the expressions for  $\mu(T)$  at  $T \gg T_0$  and  $T \ll T_0$  in the region of  $T \sim T_0$ . We can do this using Eq. (16), when instead of  $\xi_s$  we have to substitute  $\tilde{\xi}_s$ :

$$\tilde{\xi}_{\bullet} = \xi_{\bullet} \frac{1 + 3\beta W T_{\bullet}/T}{1 + \beta T_{\bullet}/T} = \begin{cases} \xi_{\bullet}, & T \gg T_{\bullet}, \\ 3W \xi_{\bullet}, & T \ll T_{\bullet} \end{cases}$$
(20)

( $\beta$  is a fitting parameter). Therefore, for all temperatures  $T < E_F$  we now have an interpolation expression for the mobility  $\mu(T,\beta)$  governed by the scattering on ionized centers.

The scattering by ionized (charged) impurities competes with the temperature-independent scattering by neutral centers. The functional dependence of the scattering in the latter case on the composition is<sup>14</sup>

$$\mu_{\rm H}(x) = \mu^*/x(1-x), \qquad (21)$$

where  $\mu^*$  is regarded as a fitting parameter to be used in the



FIG. 1. Theoretical (continuous curves) and experimental<sup>5</sup> temperature dependences of the electron mobility in Hg<sub>1-x</sub>Fe<sub>x</sub>Se samples of the following compositions: 1)  $x = 10^{-3}$ ; 2)  $5 \times 10^{-4}$ ; 3)  $3 \times 10^{-3}$ ; 4)  $5 \times 10^{-3}$ ; 5)  $10^{-2}$ ; 6)  $3 \times 10^{-2}$ ; 7)  $5 \times 10^{-2}$ . Contribution of the phonon scattering was subtracted from the experimental values in accordance with Ref. 12.



FIG. 2. Dependence of the fitting parameter  $\mu^*$  of Eq. (21) on the degree of compensation  $K = x^*/x$ .

theory. In a comparison with the experimental results, we shall use the following interpolation:

$$\mu_{int}(x,T) = [\mu^{-1}(T,\beta) + \mu_{\pi}^{-1}(x)]^{-1}, \qquad (22)$$

introducing two fitting parameters  $\beta$  and  $\mu^*$ . Here,  $\mu(T,\beta)$  is the mobility due to the scattering by a system of ionized iron center which becomes orderable when the temperature is varied. It is given by Eq. (16), where  $\xi_s$  should be replaced by  $\xi_s$  of Eq. (20).

Figure 1 shows the experimental<sup>5</sup> and theoretical [Eq. (22)] temperature dependences of the mobility for seven samples with compositions in the range  $5 \times 10^{-4} \le x \le 5 \times 10^{-2}$ . The experimental values are corrected for the phonon scattering, which is subtracted in accordance with Ref. 12. The best agreement between the theory in experiment corresponds to the fitting parameters

 $\beta = 0.02$  and  $\mu^*$ ; the dependence of the latter on the compensation parameter  $K = x^*/x$  is shown in Fig. 2. We can see that the theoretical curves describe satisfactorily the experimental results.

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