Low-temperature noise in disordered systems in a wide temperature range

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Low-frequency noise with a near-$1/f$, $(1/\omega)$ spectrum is considered in structurally disordered materials and also in tunnel junctions and microjunctions with amorphous surrounds. It is shown that the cause of this noise in such systems can be the presence, in the disordered regions, of slowly relaxing excitations induced in the atomic system by the motion of atoms (or atom groups) in soft anharmonic two-well potentials. Such a model, previously used successfully to explain a number of thermodynamic and kinetic properties of glasses, is used to analyze the low-frequency noise spectrum in a wide temperature range in situations when the excitation relaxation is due either to tunneling or to activation. The estimates obtained for Hooge's constant are of the same order as the experimentally observed values.

INTRODUCTION

It is well known that in practically all systems there is present a low-frequency (lf) noise whose spectrum is close to $1/f$ (Ref. 1) (flicker noise). While the spectrum form is quite universal, the quantitative characteristics of the flicker noise differ substantially from system to system.

For charged-particle systems the correlator of the fluctuations of a quantity A in flicker noise is usually described by Hooge's empirical relation[2]

$$\langle(\delta A)^2\rangle_\omega/\langle A\rangle^2=2\pi\alpha/N\omega, \tag{1}$$

where N is the number of particles and α is called Hooge's constant. The values of α for various physical systems under various conditions range from 10^{-9} to 10^{-3}. Note that in a spatially homogeneous system proportionality to N^{-1} is tantamount to inverse proprotionality to the volume V_0.

A widely used hypothesis that explains this behavior was proposed by McWhorter[3] for the case of electron interactions with surface traps in semiconductors. It states that the scatterers have internal degrees of freedom with an exponentially broad scatter of the relaxation time.

This assumption seems natural and is apparently applicable to systems other than for which it was initially formulated. It is therefore appropriate to mention a number of brilliant experiments[4–6] performed on various small-area semiconducting (inversion channel of MOS transistor[4]) and non-semiconducting (tunnel junctions[5,6]) systems. A common feature of these experiments is that in systems with small geometric dimensions the low-frequency noise spectrum at low temperatures is not of the $1/f$ type, but comprises a Lorentzian line (or a sum of several such lines).[5] Increasing the temperature or the geometric dimensions causes the spectrum to assume a more complicated flicker form. The authors of Ref. 5 introduced the concept of an elementary fluctuator responsible for the individual Lorentz line, but did not specify its nature.

From our viewpoint, the exponentially wide scatter of the elementary-fluctuator relaxation time, needed for formation of flicker-type noise, is an inalienable property of disordered systems in which transitions between metastable states are due to tunneling or activation processes in which random barriers are surmounted. Favoring this assumption are, for example, experiments[7] demonstrating that introduction of a relatively weak disorder, which does not increase the resistance noticeably, increases the lf noise substantially.

The slowest processes in the systems of interest to us are connected, obviously, with motion of atoms or groups of atoms. Motion of this type is possible in structurally disordered materials, with the greatest degree of disorder reached in glasses. It has turned out that allowance for such motions is essential in principle for the understanding of the low-temperature properties of glasses; these were explained by Anderson, Halperin, and Varma[8] and by Phillips[9] by a two-level system (TLS) model formulated by them. According to this model, an atom or a group of atoms moves in an effective two-well potential, and transitions between the wells are by tunneling; the transition probability depends exponentially on the barrier strength.

The authors of Refs. 10 were the first to point out that TLS can act as sources of $1/f$ noise. A detailed theory of the corresponding phenomenon as applied to tunnel junctions and microjunctions, in both the normal and superconducting state, was developed by one of us,[11–14] and the estimates of Hooge's constant turned out to be realistic.

It must be noted, however, that the TLS concept in the form used in Refs. 10–14 is applicable only at sufficiently low temperatures (see Refs. 15–17).

One of the reasons is the broadening of the TLS levels by interaction with phonons. According to Ref. 15, the minimum relaxation time of a TLS with distance E between levels can be represented in the form

$$\tau_{min}=\hbar E_c^2/E^3, \tag{2}$$

where E_c is a certain characteristic energy $\sim$10–30 K. Thus, for $E\gtrsim E_c$ the spectrum of the most rapidly relaxing TLS is smeared out by collisions.

Another cause was indicated by Karpov, Klinger, and Ingat'ev,[16,17] who have shown that TLS are the result of soft atomic potentials. The characteristic distance W between the atom levels in such potentials is of the order 10–30 K. Therefore for $E\gtrsim W$ the spectrum is no longer two-level.

We wish to show in the present paper that the properties of low-frequency noise can be interpreted in the context of the notion of two-well potentials in the entire temperature range of practical interest, irrespective of the adequacy of the TLS model. The point is that contributions to lf noise are made by objects with long relaxation times ($\sim f^{-1}$), which

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can be achieved through high and wide barriers. The presence of more than one level in each well should in this case have practically no effect on the lf noise, since transitions between such "intrawell" levels take place in sufficiently short times. Obviously, over long times each well is described by characteristics averaged over the levels.

We shall see ultimately that the properties of elementary fluctuators with long relaxation times differ little from those considered in Refs. 10–14 in the framework of the TLS model. What is actually sensitive to the model is the distribution function of the relaxation times of the fluctuators and the temperature dependence of these times. The above group of questions is the subject of the present paper.

In the first section we formulate the problem. In the second we construct, on the basis of the model of soft potentials,[16,17] a theory of lf noise in bulky systems with structural disorder; this theory takes into account tunneling and activation processes. In the third section is considered 1f noise in tunnel junctions and microjunctions, and the connection between the results and the experimental data[5,6] are discussed.

1. FORMULATION OF PROBLEM

Consider a conductor in which the electrons are scattered by impurities or phonons, and which contains in addition defects with internal degree of freedom. Assuming that the relaxation times of these defects is much longer than the electronic quantum times, one can describe the electron kinetics by a Boltzmann kinetic equation in which account is taken of the integral of collisions with the indicated defects.[11–14] This integral has the standard form

$$\hat{I}_{Fl}=\sum_{\mathbf{p}',i}\{W_{\mathbf{pp}'}^{(i)}F_{\mathbf{p}}(1-F_{\mathbf{p}'})-W_{\mathbf{p}'\mathbf{p}}^{(i)}F_{\mathbf{p}'}(1-F_{\mathbf{p}})\}. \tag{3}$$

Here $F_{\mathbf{p}}$ is the electron distribution function, and $W_{\mathbf{pp}}^{(i)}$ is the probability of transition via scattering by the ith defect.

Assume now that the defect can exist in two configurations, 1 and 2, and the probabilities of filling these configurations are n_1 and $n_2=1-n_1$.

The probability of transition to the ith center can then be written in the form

$$W_{\mathbf{pp}'}^{(i)}=W_{\mathbf{pp}'}^{(i1)}n_1^{(i)}+W_{\mathbf{pp}'}^{(i2)}(1-n_1^{(i)}), \tag{4}$$

where $W_{\mathbf{pp}}^{(i1,2)}$ are the transition probabilities in configurations 1 and 2 respectively.

Let now an electric field be applied to the sample and produce an average current of density $\mathbf{j}_0$. We calculate the correction to the current for the fluctuations of the probability of filling the configurations. This can be done by iterating the kinetic equation in powers of the ratio $\delta\hat{I}_{Fl}/\hat{I}_0$, where

$$\delta I_{Fi}=\sum_{\mathbf{pp}',i}[W_{\mathbf{pp}'}^{(i1)}-W_{\mathbf{pp}'}^{(i2)}](F_{\mathbf{p}}-F_{\mathbf{p}'})\delta n_1^{(i)}, \tag{5}$$

and $\hat{I}_0$ is the nonfluctuating part of the collision operator. Note that we have confined ourselves here only to elastic scattering by the defects. The reason is that we are interested in defects with long relaxation times (see the analysis of a specific model of defects below).

Iterating the kinetic equation with respect to the ratio $\delta\hat{I}_{Fl}/\hat{I}_0$ as in Ref. 12, we get

$$\delta j=j_0\frac{l}{V_0}\sum_i(\sigma_1^{(i)}-\sigma_2^{(i)})\delta n_1^{(i)}, \tag{6}$$

where $\sigma_{1,2}^{(i)}$ are the effective scattering cross sections for the configurations 1 and 2, l is the mean free path of the electron, and V_0 is the sample volume.

As a result we have the following expression for the current-densities correlator:

$$\langle(\delta j)^2\rangle_\omega=j_0^2\frac{l^2}{V_0^2}\sum_i(\sigma_1^{(i)}-\sigma_2^{(i)})^2\langle(\delta n_1^{(i)})^2\rangle_\omega. \tag{7}$$

The object of the theory is thus the occupation-number correlator $\langle(\delta n_1)^2\rangle_\omega$. It is expressed in standard fashion in terms of the equilibrium occupation numbers n_{01} and their relaxation times τ:

$$\langle(\delta n_1^{(i)})^2\rangle_\omega=\frac{2}{\tau^{(i)}}\frac{n_{01}^{(i)}(1-n_{01}^{(i)})}{\omega^2+(1/\tau^{(i)})^2}. \tag{8}$$

We get ultimately

$$\frac{\langle(\delta j)^2\rangle_\omega}{j_0^2}\approx\left(\frac{l^3}{V_0}\right)^{2/3}\sum_i S_i\frac{\tau^{(i)}}{1+(\omega\tau^{(i)})^2}, \tag{9}$$

where

$$S_i=\frac{(\sigma_1^{(i)}-\sigma_2^{(i)})^2}{V_0^{4/3}}n_{01}^{(i)}(1-n_{01}^{(i)}) \tag{10}$$

is the "strength" of the ith fluctuator.

The procedure that follows consists essentially of summing the contributions of various fluctuators, for which we must determine the distribution functions of S_i and $\tau^{(i)}$.

2. CURRENT FLUCTUATION IN STRUCTURALLY DISORDERED CONDUCTORS

We assume that the elementary fluctuators are local defects described by two-well anharmonic potentials.

We describe the latter within the framework of the model of Refs. 16 and 17. According to this model a defect is an atom or a system of atoms for which the dependence of the potential energy on the configuration coordinate x is of the form

$$V(x)=\mathscr{E}\left[\eta\left(\frac{x}{a}\right)^2+t\left(\frac{x}{a}\right)^3+\left(\frac{x}{a}\right)^4\right], \tag{11}$$

where $\mathscr{E}$ is an energy on the order of atomic and a is a characteristic atomic spatial scale. The coefficients η and t are random quantities indicative respectively of the stiffness and asymmetry of the random potential. It is assumed that the distribution function $P(\eta,t)$ of the random quantities η and t can be factorized in the form $P(\eta,t)=P_1(\eta)P_2(t)$; typical plots of $P_{1,2}$ are shown in Fig. 1.

The form of the distribution functions reflects the following features of structurally disordered materials. The function P_1 is centered near 1, indicating that typical defects have rigidity of atomic order. The two-well potentials correspond to small absolute values of η in the region of the distribution "tail." It is shown in Refs. 18 and 19 that $P_1(\eta)$ should have near $\eta=0$ a singularity of type $|\eta|$ (see Fig. 1a). As to the function $P_2(t)$, it is known that it must be even and decrease with increase of $|t|$ (see Fig. 1b). The charac-

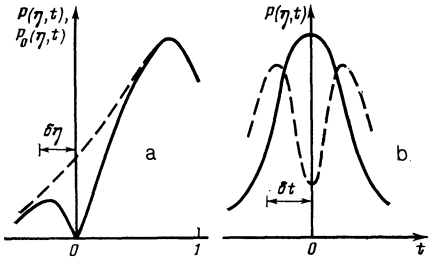

FIG. 1. Schematic forms of the distribution functions $P_1(\eta)$ (a) and $P_2(t)$ (b).

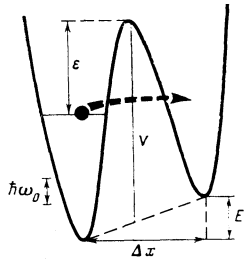

FIG. 2. Two-well potential with wide barrier.

teristic scales $\delta\eta$ and δt of the decrease of the functions P_1 and P_2 can be estimated from experimental data on the density of states and on the bond lengths in glasses: $\delta\eta = 0.1$–$0.2, \delta t \approx 0.3$–$0.5$. We shall use hereafter the expression

$$P(\eta, t) = |\eta| P_0(\eta, t), \tag{12}$$

where $P(\eta,t)$ is a monotonically decreasing function with scales $\delta\eta$ and δt.

We are interested in potentials corresponding to wells separated by wide barriers, meaning long relaxation times (Fig. 2).

In this situation the barrier height V_B is much larger than the distance between the E levels, which is determined in turn by the asymmetry of the potential. In this case

$$\Delta x = (2|\eta|)^{1/2}, \tag{13}$$

$$E = \mathscr{E}|t|\frac{(\Delta x)^3}{4} = \frac{1}{\sqrt{2}}\mathscr{E}|t||\eta|^{3/2}. \tag{14}$$

We shall find it convenient to use the characteristic dynamic parameters[17]

$$W = \mathscr{E}\eta_L^2, \quad \eta_L = \left(\frac{\hbar}{2ma^2\mathscr{E}}\right)^{1/3} \approx 10^{-2}.$$

Here

$$E = \frac{W|t|}{(2\eta_L)^{1/2}}\left(\frac{|\eta|}{\eta_L}\right)^{3/2}, \tag{15}$$

the barrier height is (see Fig. 2)

$$V_B = \frac{W}{4}\left(\frac{\eta}{\eta_L}\right)^2, \tag{16}$$

and its width is

$$\lambda = \int_{x_1}^{x_2} dx \left\{\frac{2m}{\hbar^2}[V(x) - E]\right\}^{1/2}$$

$$\approx \int_{x_1}^{x_2} dx \left[\frac{2m}{\hbar^2}V(x)\right]^{1/2} = \frac{2^{1/2}}{3}\left(\frac{|\eta|}{\eta_L}\right)^{3/2}, \tag{17}$$

where x_1 and x_2 are the coordinates of the minima of the potential. It is taken into account in (17) that the asymmetry of the potential is $E \ll V_B$ in the case of wide barriers and actual $E \approx T$.

The first question that we must answer is that the relation between the tunneling and activation. To this end we consider a transition from the left well to the right along the following path: activation to a level $(V - \varepsilon)$—tunneling on this level through a parabolic barrier—deactivation in the right-hand well (Fig. 2). We compare next this probability with the probability of activation through the barrier.

The probability of tunneling along the indicated path is $\exp(-2\lambda_\varepsilon)$, where

$$\lambda_\varepsilon = \frac{\pi}{2}\frac{\varepsilon}{W}\left(\frac{\eta_L}{|\eta|}\right)^{1/2}. \tag{18}$$

The gain in the activation probability on dropping to the level $V - \varepsilon$ is obviously $\exp(\varepsilon/T)$. Therefore the total probability of transitions of this type is

$$\sum_\varepsilon \exp\left[\frac{\varepsilon}{T} - \frac{\pi\varepsilon}{W}\left(\frac{\eta_L}{|\eta|}\right)^{1/2}\right]. \tag{19}$$

We see hence that when

$$T > T_a = \frac{W}{\pi}\left(\frac{|\eta|}{\eta_L}\right)^{1/2} \approx 0{,}32W\left(\frac{|\eta|}{\eta_L}\right)^{1/2} \tag{20}$$

the main process is activation, and for $T < T_a$ the transitions are determined by tunneling. Note than an estimate of this type can be obtained by comparing the arguments of the tunneling and activation exponentials ($2\lambda = V/T$), but in place of the number 0.32 we get in the estimate the number $3/8\sqrt{2} \approx 0.27$. We shall not distinguish between these estimates; we recognize only that at $T = T_a$ an abrupt transition takes place from tunneling to activation relaxation. Note that the presence of the abrupt transition and the estimate of T_a follow from the parabolicity of the potential $V(x)$ near the maximum.

We proceed now to estimate the relaxation times. Given the temperature T, the total interval of variation of η can be divided into two parts:

1) $|\eta| < \eta_T$, 2) $|\eta| > \eta_T$,

where

$$\eta_T = (128/9)\eta_L(T/W)^2. \tag{21}$$

For $\eta < \eta_T$ the principal role is assumed by activation, and the relaxation time can be represented in the form

$$\tau = \tau_0 \exp\left[\frac{W}{4T}\left(\frac{\eta}{\eta_L}\right)^2\right] = \tau_0 \exp\left[\frac{2^{3/2}}{3}\left(\frac{|\eta|}{\eta_T}\right)^{1/2}\left(\frac{|\eta|}{\eta_L}\right)^{3/2}\right]. \tag{22}$$

In the second region, the principal role is assumed by tunneling, and we have [according to (16)]

$$\tau = \tau_0 \exp\left[\frac{2^{3/2}}{3}\left(\frac{|\eta|}{\eta_L}\right)^{3/2}\right]. \tag{23}$$

In these expressions, $\tau_0 \sim \omega_0^{-1}$ is the period of the oscillations inside the well. We are interested in the calculation of the distribution functions of the variables E and τ from the known distribution function $P(\eta,t)$. We need therefore relations inverse to (22) and (23). Taking these equations into account, was well as relation (14), we have for the region 1

$$|\eta| = \eta_L \left(\frac{4T}{W}\right)^{1/2} \left(\ln\frac{\tau}{\tau_0}\right)^{1/2},$$

$$|t| = (2\eta_L)^{1/2} \frac{E}{W} \left(\frac{W}{4T}\right)^{1/4} \left(\ln\frac{\tau}{\tau_0}\right)^{-1/4}, \qquad (24)$$

and for the region 2

$$|\eta| = \eta_L \frac{3^{3/2}}{2} \left(\ln\frac{\tau}{\tau_0}\right)^{3/2}, \quad |t| = \frac{4}{3}\eta_L^{1/2} \frac{E}{W} \left(\ln\frac{\tau}{\tau_0}\right)^{-1}. \qquad (25)$$

The boundary between the regions corresponds here to the value

$$\tau = \tau_c(T) = \tau_0 \exp\{(8/3)^4 (T/W)^3\}. \qquad (26)$$

We point out that for an elementary fluctuator with a given value of η the crossover from tunneling to activation relaxation is at $T = T_a$. Observation of a crossover of this type was reported in Ref. 5. For most random fluctuators the crossover temperature depends on the observed noise frequency. Assuming, in accordance with (9) and (10), that the main contribution to flicker noise is made by fluctuators with $\tau \sim \omega^{-1}$, we find that the crossover from tunneling to activation relaxation occurs at

$$T_c = \left(\frac{3}{8}\right)^{4/3} W \left(\ln\frac{1}{\omega\tau_0}\right)^{1/3}.$$

Using (24) and (25) we get the following expressions for the distribution functions of E and τ:

For $\tau < \tau_c$

$$\mathcal{P}(E,\tau) = \frac{1}{\sqrt{2}} \frac{\eta_L^{5/2}}{W\tau} \left(\ln\frac{\tau}{\tau_0}\right)^{-5/4} \left(\frac{4T}{W}\right)^{7/4} \times P_0\left[-\eta_L\left(\frac{4T}{W}\right)^{1/2}\left(\ln\frac{\tau}{\tau_0}\right)^{1/2}, (2\eta_L)^{1/2}\frac{E}{W}\left(\frac{W}{4T}\right)^{3/4} \times \left(\ln\frac{\tau}{\tau_0}\right)^{-1/4}\right]. \qquad (27)$$

For $\tau < \tau_c$

$$\mathcal{P}(E,\tau) = \frac{8}{9}\left(\frac{3}{2^{3/2}}\right)^{4/3} \frac{\eta_L^{5/3}}{W\tau} \left(\ln\frac{\tau}{\tau_0}\right)^{-5/3} \times P_0\left[-\eta_L\left(\frac{3}{2^{3/2}}\right)^{2/3}\left(\ln\frac{\tau}{\tau_0}\right)^{2/3}, \frac{4}{3}\eta_L^{1/2}\frac{E}{W}\left(\ln\frac{\tau}{\tau_0}\right)^{-1}\right]. \qquad (28)$$

We calculate now the current fluctuations. In the expression for the current density

$$\frac{\langle(\delta j)^2\rangle_\omega}{j_0^2} = \frac{1}{4}\left(\frac{l^3}{V_0}\right)^{2/3} V_0 \int dE\, d\tau \mathcal{P}(E,\tau) \frac{S_0(E,\tau)}{\mathrm{ch}^2(E/2T)} \frac{\tau}{1+(\omega\tau)^2}, \qquad (29)$$

the quantity

$$S_0(E,\tau) = V_0^{-4/3}[\sigma_1(E,\tau) - \sigma_2(E,\tau)]^2$$

is determined by the change of the electron-scattering cross section for an atomic-particle displacement Δx (13). Substituting in (13) the expressions (24) and (25) for $\eta(T)$ and taking the smallness of η_L into account, we can conclude that $\Delta x/a \ll 1$ for all parameters of practical interest. This gives grounds for assuming that

$$\sigma_1 - \sigma_2 \approx \sigma_0 \frac{\Delta x}{a} = \sigma_0[2\eta(\tau)]^{1/2}.$$

Thus,

$$S_0 = V_0^{-4/3}\sigma_0^2(\Delta x/a)^2 = 2V_0^{-4/3}\sigma_0^2\eta(\tau), \qquad (30)$$

where the constant σ_0 is of the order of the geometric scattering cross section.

Analysis shows that the integration with respect to E is limited by the first factor, $E \lesssim T$. In fact, the characteristic values of τ are of the order of ω^{-1}, and at reasonable values of ω the second arguments of the function P_0 in (27) and (28) turn out to be small. Furthermore, in the logarithmic functions contained in the arguments of P_0, we can replace everywhere τ by ω^{-1}. We ultimately get

$$\frac{\langle(\delta j)^2\rangle_\omega}{j_0^2} \approx \left(\frac{l^3}{V_0}\right)^{2/3} S_0 \frac{\eta_L^{5/3} T}{W\omega} \Phi(\omega, T, \tau_0), \qquad (31)$$

where

$$\Phi = (T/WL^3)^{1/4} P_0[-\eta_L(4TL/W)^{1/4}, 0] \quad \text{for } \omega\tau_c \gg 1,$$
$$\Phi = \frac{2}{(3L)^{2/3}} P_0\left[-\eta_L\left(\frac{3L}{2^{3/2}}\right)^{2/3}, 0\right] \quad \text{for } \omega\tau_c \ll 1. \qquad (32)$$

We have put here $L = \ln(1/\omega\tau_0) \gg 1$.

We proceed to a discussion of the results. Bearing in mind a comparison with experiment, we introduce Hooge's constant α. Following tradition and Hooge's original paper,[2] we normalize the noise to the total number $N = n_e V_0$, of electrons in the sample, where n_0 is the electron density. We obtain then

$$\alpha = \frac{l^2 n_e}{2\pi} \langle(\sigma_1 - \sigma_2)^2\rangle \frac{T}{W} \eta_L^{5/3} \Phi. \qquad (33)$$

This expression can be rewritten in a different form for the case when the principal role is played by scattering from impurities having a density n_i and a scattering cross section σ_{tr}:

$$\alpha = \frac{n_e}{2\pi n_i^2} \frac{\langle(\sigma_1 - \sigma_2)^2\rangle}{\sigma_{tr}^2} \frac{T\eta_L^{5/3}}{W} \Phi. \qquad (34)$$

We see thus that in our model the Hooge constant α depends on temperature and is sensitive to the distribution function $P_0(\eta,0)$ of the fluctuators in the parameter η, namely, to the law governing its decrease at negative values of η with large moduli.

By way of illustration, we have carried out calculations for three specific $P(|\eta|,0)$ dependences. These dependences and the calculation results are listed in Table I, where $L = \ln(1/\omega\tau_0)$.

The frequency dependences of Hooge's constant α are the same as of the function Φ, and are logarithmic for most situations. However, for example in the case of a Gaussian function and in the activation relaxation regime, the main

TABLE I. The function $\Phi(\omega,T)$ for different models of the density of states.

No.	$P_0(\lvert\eta\rvert)/P_0(0)$	$\Phi(\omega,T)/P_0(0)$	
		$\omega\tau_c \gg 1$ activation	$\omega\tau_c \ll 1$ (tunneling)
1	$(1+\lvert\eta\rvert)^{-m}$	$\frac{1}{2^{1/2}}\left(\frac{4T}{W}\right)^{1/4}\frac{1}{L^{3/4}} \times\left[1+\eta_L\left(\frac{4T}{W}\right)^{1/2}L^{1/2}\right]^{-m}$	$\frac{2}{3^{2/3}}\frac{1}{L^{2/3}}\left[1+\eta_L\frac{3^{3/2}}{2}L^{2/3}\right]^{-m}$
2	$\exp\left(-\frac{\lvert\eta\rvert}{\eta_L}\right)$	$\frac{1}{2^{1/2}}\left(\frac{4T}{W}\right)^{1/4}\frac{1}{L^{3/4}} \times\exp\left[-\frac{\eta_L}{\delta\eta}\left(\frac{4T}{W}\right)^{1/2}L^{1/2}\right]$	$\frac{2}{3^{2/3}}\frac{1}{L^{2/3}}\exp\left[-\frac{\eta_L}{\delta\eta}\frac{3^{3/2}}{2}L^{2/3}\right]$
3	$\exp\left(-\frac{\eta^2+2\lvert\eta\rvert}{(\delta\eta)^2}\right)$ Gaussian centered about $\eta=1$	$\frac{1}{2^{1/2}}\left(\frac{4T}{W}\right)^{1/4}\frac{1}{L^{3/4}} \times\left(\frac{1}{\omega\tau_0}\right)^{-4\eta_L^2T/(\delta\eta)^2W} \times\exp\left[-\frac{2\eta_L}{(\delta\eta)^2}\left(\frac{4T}{W}\right)^{1/2}L^{1/2}\right]$	$\frac{2}{3^{2/3}}\frac{1}{L^{2/3}}\exp\left[-\frac{\eta_L^2}{(\delta\eta)^2}\frac{3^{4/3}}{4} \times L^{4/3}-\frac{\eta_L}{(\delta\eta)^2}3^{2/3}L^{2/3}\right]$

dependence is a power-law: $\alpha\propto\omega^{-s}$, where $s\approx(\eta_L/\delta\eta)^2(4T/W)$. The temperature dependences of α differ from the corresponding dependences of Φ by a factor T. In the tunneling regime $\alpha\propto T$. In the activation regime, however, the $\alpha(T)$ dependence should as a rule be nonmonotonic, a reflection of the nonmonotonic character of the barrier-height distribution function, which is a property of the soft-potential model. Indeed, the function $|\eta|P_0(\eta)$ is nonmonotonic, and $V_B=W(\eta/\eta_L)^2/4$. Note that this circumstance can explain also the presence of a relaxation absorption peak in amorphous materials at relatively high temperatures.[21]

Noise experiments at low frequencies offer thus in principle a choice between the three possibilities discussed above, and indicate, to a certain degree, the structure of the short-range order in disordered systems.

We present now a rough estimate of Hooge's constant. We assume here that the arguments of the function P_0 in (31) and (32) are small enough. The function $P_0(0,0)$ can then be connected, in order of magnitude, to the TLS density of state $\bar{P}$ known from low-temperature experiments by the relation (see Ref. 17)

$$P_0(0,0)\approx\bar{P}W/\eta_L^{3/2}. \tag{35}$$

Specifying $\bar{P}=10^{33}\,\mathrm{erg}^{-1}\cdot\mathrm{cm}^{-3}$, $n_e=10^{23}\,\mathrm{cm}^{-3}$, $l=10^{-6}$ cm, $W\approx30$ K, $T\approx300$ K, $\overline{(\sigma_1-\sigma_2)}\approx10^{-17}\,\mathrm{cm}^2$, $\omega=10^3\,\mathrm{s}^{-1}$, and $\tau_0=10^{-13}$ s, we obtain according to (34) and (31)

$$\alpha\approx10^{-3}.$$

This estimate offers evidence in favor of the lf noise mechanism discussed above.

3. SINGULARITIES OF NOISE CHARACTERISTICS IN TUNNEL JUNCTIONS AND MICROJUNCTIONS. DISCUSSION OF EXPERIMENTS

As a a rule the dielectric interlayer of a tunnel junction is disordered. One can therefore expect this interlayer to contain the considered defects with internal degree of freedom, which fluctuate with time. These fluctuations lead to modulation of the tunnel barrier, and hence of the tunneling probability.

We assume, as above, that the state of the defect is described by a two-well potential, and the equilibrium occupation numbers of the wells are n_1 and n_2; transitions from one well to another can be by activation or tunneling. Proceeding as in Refs 11–14, we express the correction $I^{(1)}$ to the tunnel current I_0 for the c ntribution of the centers of interest to us in the form

$$\frac{I^{(1)}}{I_0}=\frac{1}{A}\sum_i(\sigma_1^{(i)}n_1^{(i)}+\sigma_2^{(i)}n_2^{(i)}), \tag{36}$$

where A is the junction area. In contrast to the equations of the preceding sections, σ_1 and σ_2 denote here not the true scattering cross sections, but certain effective ones defined as

$$\sigma_{1,2}=\pi a^2\frac{\varkappa d}{W_0}\mathrm{Re}(V_q)_{1,2}, \tag{37}$$

where a is the distance between the atoms, U_0 the tunnel-barrier height, d its thickness,

$$\hbar\varkappa=[2mU_0-p_z^2]^{1/2}$$

the incident-particle momentum component perpendicular to the surface $W_0=\hbar^2\varkappa^2/2m$, and $(V_q)_{1,2}$ Fourier components ($q=|\mathbf{p}-\mathbf{p}'|/\hbar\approx p_F/\hbar$) of the defect components normalized to the unit cell volume.

As a result we get the following expression for the current fluctuations:

$$\frac{\langle(\delta I)^2\rangle_\omega}{I_0^2}\approx\sum_i S_i\frac{\tau^{(i)}}{1+(\omega\tau^{(i)})^2}, \tag{38}$$

where

$$S_i=\frac{(\sigma_1^{(i)}-\sigma_2^{(i)})^2}{A^2}n_0^i(1-n_0^i). \tag{39}$$

The rest of the calculation is exactly similar to the one above and leads, in the case of a large-area junction, to re-

placement of the factor $(l^3/V_0)^{2/3}S_0$ in (30) by $\langle(\sigma_1-\sigma_2)^2\rangle/A^2$.

The lf noise intensity is thus inversely proportional to the junction area. It must be noted here that in real junctions an effective contribution to the tunnel current is made by small sections of the junctions with minimum tunnel-barrier thickness. The effective junction area A can therefore turn out to be noticeably smaller than the geometric area.

Of particular interest are the characteristics of small-area tunnel junctions and of microjunctions. Interest in these objects is due to their small intrinsic capacitances, an important factor in a number of technical applications.

It may turn out here that only a few defects with internal degree of freedom are located in the junction region. The sums such as (8) and (38) contain then only a few terms. What is produced then is not a $1/f$ noise but a sum of Lorentzian contributions whose widths are determined by the times τ_i. In an investigation of an ensemble of junctions of different sizes one can expect, when the size is increased, the spectrum to change from a sum of Lorentzian functions (in the time scale such a noise is an assembly of several telegraph processes) to one of the $1/f$ type. A similar behavior can be expected also if the number fluctuators taking part in the considered phenomena is increased on account of an increase of the effective-energy region. This can be achieved either by raising the temperature or by increasing the junction voltage. The latter circumstances can be attributed to local heating as well as to direct interaction of the electrons with two-well defects.

It appears that the foregoing phenomena were observed in the experiments[5,6] on lf noise in tunnel junctions of area 10^{-9} cm^2 at various temperatures and junction voltages. When the temperature was raised in this experiment from 2 to 80 K a change was observed from a frequency spectrum corresponding to an assembly of several Lorentzians to a $1/f$ spectrum. The authors have attributed this behavior to the presence, in the junction region, of centers with broad relaxation-time spectra. They have investigated the temperature dependences of the relaxation times of individual fluctuators and observed a transition from tunneling to activation at $T=15$–20 K. This behavior agrees with the results of the theory above. A similar transition was observed also when the junction voltage was raised. If an attempt is made to estimate from the experimental results of Ref. 5 the cross section for the interaction between tunneling electrons and a fluctuator, using expression (38) and the junction geometric area for A, values 10^{-13}–10^{-12} cm^2 are obtained for σ. These values exceed the square of the tunnel-junction thickness and are therefore unreasonable. From our point of view, the effective junction area is much smaller than geometric. The reason is that the tunnel current depends exponentially on the barrier thickness, which can fluctuate along the barrier. Assuming as a rough estimate that the barrier height does not fluctuate and the thickness has a Gaussian fluctuation with a variance $\bar{\sigma}$, the ratio of the effective and geometric areas is of the order of $\exp(-\varkappa^2\bar{\sigma})$. Another cause of the increase of the effective cross section may be the increase of the barrier transparency by a contribution from the fluctuator potential. Allowance for this circumstance can increase the effective cross section by $\exp[2\max\{|V_{12}|r_0m/\hbar^2\varkappa\}]$, where r_0 is the effective radius of the fluctuator potential.

We see thus that the low-frequency noise of the junction depends unusually strongly on the presence, in the junction or in its vicinity, of defects with internal degrees of freedom. The noise should accordingly differ for different junctions. The distribution function of the corresponding Hooge constants can be determined with the aid of theory developed above.

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Note added in proof (29 November 1988). We have just learned that the result of a paper by K. S. Ralls and R. A. Buhrman [Phys. Rev. Lett. **60**, 2434 (1988)] (estimates of T_a, τ_0, V_b, and $\sigma_1-\sigma_2$) are in reasonable agreement with our present theory.

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