Fluctuations of the hopping conductance of one-dimensional systems

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An expression is obtained for the distribution function of the hopping conductance of a long one-dimensional chain of localized states. The random oscillations of the resistance of the chain with change of the position of the Fermi level are studied, and the correlator of the logarithms of the resistance, which is a quantitative characteristic of the oscillations, is calculated.

1. INTRODUCTION

Interest in the study of the conductance of one-dimensional systems, the electronic states in which are localized at zero temperature, has grown in recent years after these systems were obtained experimentally.[1] They were obtained using silicon MIS structures, in which, by means of profiled doping by acceptors, a narrow (~ 0.1 μm) strip of n-type semiconductor, in contact with the oxide, was isolated. Because of the presence of neighboring p-type regions, situated along the entire strip on both sides of the strip, space-charge regions were created inside the strip, so that almost all the donors within the strip were ionized. The electrons, however, were localized only on donors situated in a much narrower electrically neutral strip, the width of which, according to the estimates, amounts to 100–150 Å. The measurement carried out in Ref. 1 showed that the conductance of this strip increases exponentially with increase of the temperature T. But the dependence of the conductance on the gate voltage displays random oscillations about the average value. For samples of length $L \sim 10$ μm the amplitude of the oscillations reached one and a half orders at temperatures $T \sim 0.1$ K and fell rapidly with increase of temperature.

The first model designed to explain the dependences that had been observed was proposed in Ref. 2. Lying at its basis was the assumption that the passage of current in the strip is realized by tunneling of electrons from one contact to the other with virtual capture at localized states (LS) with levels near the Fermi level (resonance tunneling). Change of the gate voltage leads to change of the position of the Fermi level, and hence to replacement of the localized states through which the electrons tunnel. Since the transmission coefficient depends on the LS energies and on the arrangement of the LS about the middle of the strip, this rearrangement leads to oscillations of the conductance of the sample.

An alternative model was proposed in Ref. 3. In that paper it was assumed that the passage of current is determined by tunneling hops of electrons between localized states with emission and absorption of phonons. In this case, with each pair of LS we can associate a certain equivalent resistance linking them. With change of the position of the Fermi level some of the resistances increase exponentially and others fall, so that their contributions to the resistance of the sample are redistributed. Since the principal contribution to the resistance of the sample is made by a few of the largest resistances (those which are not shorted by smaller resistances connected in parallel with them), this redistribution is accompanied by oscillations of the resistance of the sample.

The comparison of the results of Refs. 2 and 3 undertaken in Ref. 4 showed that the first of the suggested mechanisms can determine the conductance of the system only at very low temperatures: $T \lesssim 10$ mK. The experimental situation corresponded to temperatures $T \gtrsim 25$ mK, and so the authors of Ref. 5 invoked the mechanism of Ref. 3 to interpret the experimental results. The experiments performed made it possible to estimate sufficiently accurately such parameters of the samples investigated as the decay length of the LS wave function, and also the one-dimensional density of states. On the other hand, the number of conductance maxima that were observed over the entire range of variation of the Fermi level was extremely large, and therefore the problem of the theoretical study of the fluctuations of the hopping conductance of a one-dimensional chain of localized states is timely. This problem is solved in the present paper. In Sec. 2 the general approach proposed in Ref. 6 is used to calculate the distribution function of the hopping conductance of the chain. Earlier, in Ref. 7, this function was found by computer modeling for two specific values of the chain length L. In addition, in Ref. 7 arguments are given that make it possible to estimate the position of the maximum, and the width, of the distribution function for arbitrary values of L. The analytical expression obtained in the present paper for the distribution function describes with great accuracy the results of the computer modeling in Ref. 7. In Secs. 3 and 4 a quantitative theory of the oscillations of the resistance of a one dimensional chain upon variation of the Fermi level is constructed (in Ref. 3 these oscillations were studied by means of computer modeling). The dependence of the characteristic amplitude and period of the oscillations on the temperature and chain length is obtained, and the correlator of the logarithms of the resistance for different positions of the Fermi level is calculated (this correlator is a quantitative characteristic of the oscillations).

2. DISTRIBUTION FUNCTION OF THE RESISTANCE OF A LONG ONE-DIMENSIONAL CHAIN

We shall consider a one-dimensional chain of randomly positioned localized states and of length L. To calculate its hopping conductance we must associate with each pair of localized states i and j a resistance R_{ij} linking them[8]:

$$R_{ij}=R_0 \exp\left[\frac{2|x_i-x_j|}{a}+\frac{|\varepsilon_i-\mu|+|\varepsilon_j-\mu|+|\varepsilon_i-\varepsilon_j|}{2T}\right], \tag{1}$$

where x_i and ε_i are the coordinate and energy of the ith site, a is the decay length of the LS wave function, μ is the position

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of the Fermi level, and R_0 is the pre-exponential factor. After this the problem reduces to calculating the total resistance of the equivalent electric circuit. For a chain of infinite length this problem was first considered in Ref. 9, in which it was shown that the resistivity of the chain is determined by "breaks"—exponentially rare segments within which there are no LS with energies close to the Fermi level (such a break is shown schematically in Fig. 1a). If breaks were absent, the resistance of the chain at low temperatures would be determined by the Mott formula, which, in the one-dimensional case, has the form[8]

$$R=R_0 \exp[(\beta T_0/T)^{1/2}], \quad T_0=1/ga, \tag{2}$$

where g is the density of localized states and β is a numerical coefficient. Despite the fact that breaks are exponentially rare, it is they which determine the total resistance of the chain, since their resistance exponentially exceeds the Mott resistance (2). We note that such a situation is specific only to the one-dimensional case, since in this case the current lines cannot "go round" the breaks.

It is clear that the resistance of a break is determined by its shape and size. We denote by $\rho(u)$ the concentration of breaks with a specifed resistance $R_0 \exp(u)$ ($u \gg 1$) in a chain of infinite length. Since a break is a region free from localized states on the (x,ε) plane (see Fig. 1a), the probability of its formation is proportional to $\exp(-gA)$, where $A \gg 1/g$ is the area of the region (the phase volume). The concentration $\rho(u)$ is determined by the regions of that shape for which the area A is a minimum for the given resistance $R_0 \exp(u)$ of this region. Using formula (1), it is easy to show that this condition is satisfied by a rhombus with diagonals of lengths $au/2$ and $2uT$, respectively (Fig. 1b). In fact, as can be seen from formula (1), the resistance between any two LS positioned symmetrically on opposite sides of the rhombus (Fig. 1b) is the same and equal to $R_0 \exp(u)$, so that any decrease of the area of the rhombus (the dashed curve on Fig. 1b) leads to a decrease of the total resistance. Since the area of the rhombus is $A = aTu^2/2$, for the required quantity $\rho(u)$ we obtain

$$\rho(u)=\rho_0 \exp\left(-\frac{gTau^2}{2}\right), \tag{3}$$

where ρ_0 is the pre-exponential factor and can be estimated as $\rho_0 \sim gTu$ (the concentration of LS in the energy band uT). Since breaks are exponentially rare and do not overlap, the chain can be replaced by a system of series-connected breaks of different types, and all the other resistances can be replaced by low-resistance connecting wires. The resistance of the chain is then equal to

$$R=R_0 L \int_0^\infty du\, \rho(u) e^u. \tag{4}$$

Taking into account that the integrand has a sharp maximum at $u = (gTa)^{-1} = T_0/T$, we have

$$R=R_0 \frac{L}{a}\left(\frac{T_0}{T}\right)^{1/2} e^{T_0/2T}, \tag{5}$$

where T_0 is determined by formula (2).

The expression obtained differs from the result of Ref. 9 in having a numerical coefficient of 1/2 instead of 1/4 in the argument of the exponential. This is connected with the fact that optimization with respect to the shape of the break was not carried out in Ref. 9.[1)]

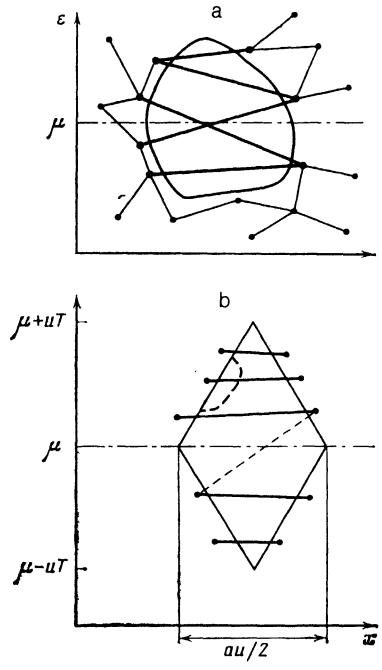

FIG. 1. (a) Schematic representation of a break in a one-dimensional chain. The points correspond to localized states; (b) a break of optimal shape. The LS pairs that determine the total resistance of the break are connected by straight-line segments.

The formula (5) is applicable only for a sufficiently long chain, since in its derivation it was in fact assumed that the number of optimal breaks with $u = T_0/T$ over the length of the chain is large, i.e., $L\rho(T_0/T) \gg 1$. The latter condition can be represented conveniently in the form of a strict inequality $\nu > 1$, where the parameter ν is determined by the following relation[2)]:

$$\nu=\frac{\ln(L\nu^{1/2}/a)}{|\ln[\rho(T_0/T)/\rho_0]|}=\frac{2T}{T_0}\ln\left(\frac{L\nu^{1/2}}{a}\right). \tag{6}$$

For $\nu < 1$, optimal breaks are not to be found in a typical chain. The logarithm of the resistance of the chain in this case will be determined by a few breaks, with the largest resistance, from amongst all those present in the chain. Quantitatively, this corresponds to the fact that in the integral (4) the upper limit must be replaced by u_f, where the quantity u_f is determined from the condition $L\rho(u_f) \sim 1$, whence $u_f = T_0 \nu^{1/2}/T$, where ν is determined by the expression (6) (that such a simple expression is obtained for u_f is due to the choice of definition of the parameter ν). As a result, we have

$$\ln(R/R_0)=u_f=\frac{\nu^{1/2}T_0}{T}\approx\left\{2\frac{T_0}{T}\ln\left[\frac{L}{a}\left(\frac{T}{T_0}\right)^{1/2}\ln^{1/2}\left(\frac{L}{a}\right)\right]\right\}^{1/2}. \tag{7}$$

The expression obtained agrees with the result of Ref. 7 to within the numerical coefficient, which was not found in Ref. 7.

It is clear that for a sufficiently small length L (when $\nu < 1$) there is a random spread of values of the resistance

between different chains, so that one can speak only of the distribution function of the logarithm of the resistance over the chains. The expression (7) determines the position of the maximum of the distribution function. To find the form of the distribution function it is necessary to take into account the contributions to the resistance from all the principal breaks present in the chain. The important point is that these contributions are independent. This is sufficient to make it possible to use the approach developed in Ref. 6, in which was studied the distribution function of the conductance of an arbitrary barrier of finite area, the local transparency of which experiences random exponential scatter. Just as the resistance of the chain is determined by the sum of the resistances of the segments with anomalously large resistance, the conductance of this barrier is equal to the sum of the conductances of the "punctures"—local regions with anomalously large transparency. In effect, the barrier and the chain are mutually dual, i.e., quantitative descriptions of the two systems can be reduced to each other by formal replacement of the resistance by the conductance and of series connection by parallel connection.

The distribution function of interest to us is defined as follows:

$$f(Q)=\left\langle \delta\left(Q-\ln\sum_i \Delta n_i e^{u_i}\right)\right\rangle , \quad Q=\ln(R/R_0), \tag{8}$$

where Δn_i is the number of breaks of the ith kind in the chain, and $R_0\exp(u_i)$ is the resistance of such a break. The averaging in formula (8) is performed over all possible values of $\Delta n_i = 0, 1, 2, \ldots$. This averaging was performed in Ref. 6, and for $f(Q)$ an expression was obtained which, in application to a chain, can be represented in the form

$$f(Q)=\frac{e^Q}{2\pi}\int_{-\infty}^{\infty} dt \exp\left\{ite^Q+L\int_0^\infty du\,\rho(u)\left[\exp(-ite^u)-1\right]\right\}, \tag{9}$$

where the function $\rho(u)$ is defined by formula (3).

This expression is analyzed in exactly the same way as was done in Ref. 6. It turns out that the distribution function has different forms in three exponentially wide ranges of variation of the chain length L, or, in other words, in three ranges of variation of the parameter ν (see formula (6)). For $\nu<1$, when in a typical chain there is no optimal break, we have

$$f(Q)=\frac{e^\Delta}{\pi}\int_0^\infty dx \exp\left(-x_{\nu^{1/2}}\cos\frac{\pi\nu^{1/2}}{2}\right) \times\cos\left(xe^\Delta-x^{\nu^{1/2}}\sin\frac{\pi\nu^{1/2}}{2}\right), \tag{10}$$

$$\Delta=Q-\frac{\nu^{1/2}T_0}{T}, \tag{11}$$

i.e., the function f is centered near the value $Q=\nu^{1/2}T_0/T$ in accord with the result (7). The expression (10) is simplified substantially for $\nu\ll 1$ and takes the form

$$f(Q)=\nu^{1/2}\exp(-\nu^{1/2}\Delta-e^{-\nu^{1/2}\Delta}). \tag{12}$$

It can be seen from this expression that the width of the distribution function for $\nu\ll 1$ is of the order of

$$\delta Q\sim\frac{1}{\nu^{1/2}}=\left(\frac{T_0}{2T}\right)^{1/2}\ln^{-1/2}\left[\frac{L}{a}\left(\frac{T}{T_0}\right)^{1/2}\ln^{1/2}\left(\frac{L}{a}\right)\right] \tag{13}$$

and increases with lowering of the temperature and increase of L. The ratio of the width δQ to the position (7) of the maximum is found to be of the order of $\ln^{-1}(LT^{1/2}/aT_0^{1/2})$ and depends weakly on the temperature.

Another particular case in which the integral in formula (10) can be calculated analytically is the case $\nu=1/4$. For this we have

$$f(Q)=\frac{1}{2\pi^{1/2}}\exp\left[-\frac{\Delta}{2}-\frac{1}{4}e^{-\Delta}\right]. \tag{14}$$

If the parameter ν lies in the interval $1<\nu<4$, in a typical chain there are many optimal breaks, so that the position of the maximum of $f(Q)$ is given by formula (5). However, the distribution function is not Gaussian (although it is exponentially narrow), since its width is determined by entirely different breaks, the number of which in a typical chain is of order unity. The distribution function takes a Gaussian form only for $\nu>4$.

Figure 2 shows graphs of the function $f(Q)$ for the values $\nu_1=0.2$ and $\nu_2=0.25$. These values of ν are chosen because they correspond to the chain lengths and temperature for which the distribution function was found in Ref. 7 by numerical modeling. We determined the values of ν_1 and

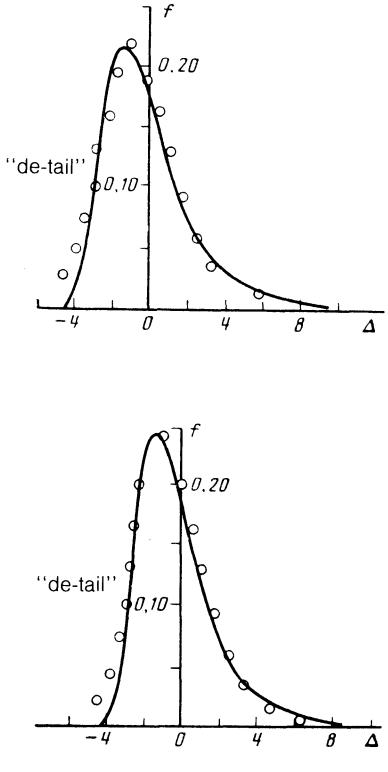

FIG. 2. Distribution function of the logarithm of the resistance of a one-dimensional chain of LS. The solid curves represent the results of calculations using formula (10) for (a) $\nu=0.2$ and (b) $\nu=0.25$; the circles show results of the numerical modeling from Ref. 7.

ν_2 by comparing the numerical values given in Ref. 7 for the variance $D = \overline{(\Delta^2)} - (\overline{\Delta})^2$ with the analytical expression

$$D = \frac{\pi^2}{6}\left(\frac{1}{\nu} - 1\right), \tag{15}$$

which can be obtained with the aid of formula (10). The results of the calculations of Ref. 7 are shown in Fig. 2 by circles. It can be seen that they practically coincide with the curves plotted from formulas (12) and (14). We note, however, that if we use formula (6) to determine the values of ν_1 and ν_2, substituting into it the values of T and L from Ref. 7 we obtain $\nu_1 = 0.29$ and $\nu_2 = 0.34$, i.e., values greater by a factor of approximately 1.4. In our opinion, this is due to the fact that the formula (3) for $\rho(u)$—the concentration of breaks with a given resistance $R_0 \exp(u)$—is asymptotic in the parameter $u(T/T_0)^{1/2} \gg 1$. The relative correction to the expression (6) for the parameter ν is, consequently, of the order of $\delta\nu/\nu \sim (T_0/Tu^2)^{1/2}$. On the other hand, $u \sim (T_0/T)\nu^{1/2}$, so that $\delta\nu/\nu \sim (T/T_0\nu)^{1/2} \sim \ln^{-1/2}(L/a)$. This correction decreases very slowly with increase of L, so that, although in Ref. 7 the value of ν_2 corresponds to $L/a = 180$, the ratio $\delta\nu/\nu$ is not sufficiently small. On the other hand, the form of the distribution function is universal and does not change when the parameter ν is modified as a result of corrections in the logarithm of $\rho(u)$.

3. RANDOM OSCILLATIONS OF THE CHAIN RESISTANCE

The qualitative pattern of the oscillations of the resistance of a chain upon variation of the position of the Fermi level can be clarified conveniently with the aid of Fig. 3a. As already pointed out above, the principal contribution to the resistance is made by breaks which on the (x,ε) plane can be depicted in the form of voids having the shape of a rhombus with sides of slope $d\varepsilon/dx = T/a$, with one of the diagonals of the rhombus coinciding with the Fermi level μ. We shall assume that the rhombus is symmetric about the straight line $\varepsilon = \mu_0$. For $\mu = \mu_0$ it is "balanced", i.e., the same resistance corresponds to all the LS pairs shown in Fig. 1b. It is easy to see that when the Fermi level is displaced upward or downward the balance is destroyed. For $\mu = \mu_2 > \mu_0$ (more precisely, $\mu_2 - \mu_0 \gtrsim T$), the resistance will be determined by the LS pairs situated in the upper half of the rhombus, while for $\mu = \mu_1 < \mu_0$ it will be determined by the LS pairs located in the lower half of the rhombus. In both cases the resistance of the rhombus will fall as $\exp(-|\mu - \mu_0|/T)$. Thus, the resistance of the rhombus passes through a maximum at $\mu = \mu_0$, as shown in Fig. 3b. We see, therefore, that for those values of μ for which a given rhombus can determine the resistance of the entire chain, the rhombus undergoes an internal switch from the "upper" to the "lower" resistances. After the switching, the resistance of the break falls with increase of μ and, at a certain $\mu = \mu_0 + \delta\mu_c$, becomes smaller than the resistance of another break, centered at $\mu = \mu_0 + 2\delta\mu_c$, the resistance of which, for $\mu < \mu_0 + 2\delta\mu_c$, increases with increase of μ. Upon this switching the total resistance of the chain passes through a minimum (Fig. 3b). With further increase of μ the resistance of the new break passes through a maximum, and so on. Thus, the oscillations of the resistance of the chain are a sequence of internal and mutual switchings of different breaks.

The characteristic period $\delta\mu_c$ of the oscillations can be

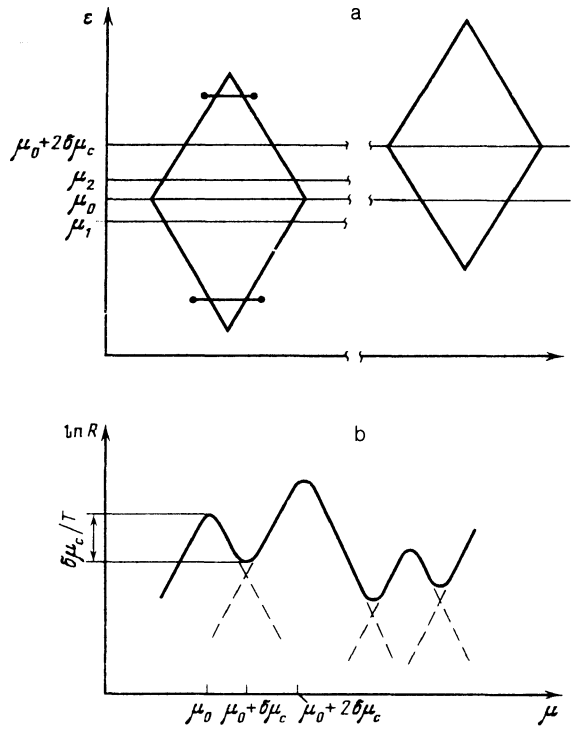

FIG. 3. (a) Scheme clarifying internal and mutual switchings of breaks upon variation of the Fermi level; (b) oscillations of the logarithm of the resistance of a chain upon variation of the position of the Fermi level; the dashed lines show the variation of the resistance of individual breaks.

estimated as follows. The resistance of the chain for each position of μ is determined principally by two competing breaks, with their representative rhombuses shifted relative to each other along the energy axis by a certain random quantity $2\delta\mu_c$. Far from the point of their mutual switching the characteristic difference of the logarithms of the resistances of these breaks is of the order of $\delta\mu_c/T$. On the other hand, it is obvious that this difference should coincide in order of magnitude with the width δQ of the distribution function [formula (13)], whence

$$\delta\mu_c \sim (T_0T)^{1/2}/\ln^{1/2}\left[\frac{L}{a}\left(\frac{T_0}{T}\right)^{1/2}\ln^{1/2}(L/a)\right] \sim \frac{T}{\nu^{1/2}}. \tag{16}$$

It can be seen that the period increases with increase of the temperature. The expression (16) differs from that given in Ref. 3 by the larger logarithm in the denominator.

The above qualitative picture of the oscillations of the chain resistance is rather crude, inasmuch as we assumed that different rhombuses differ from each other only by their position on the energy axis. From this one might conclude that each minimum on Fig. 3b is located exactly midway between the two neighboring maxima. In reality, however, the sizes of the rhombuses have a random spread, such that the fluctuations of their resistances at the points of the maxima are of the order of the width of the distribution function. Allowance for this circumstance leads to the result that the mutual location of the minima and maxima is random.

An important conclusion from the above analysis is the fact that for any position of the Fermi level the dependence $\ln R(\mu)$ is of the activation type: $d\ln R(\mu)/d\mu = \pm 1/T$. In

other words, there should be no horizontal sections (plateaux) on the pattern of the oscillations (Fig. 3b). This conclusion is not obvious, because, as can be seen from formula (1), for any pair of LS with energies ε_1 and ε_2 the dependence of the resistance linking them on μ has a plateau in the interval $\varepsilon_1 < \mu < \varepsilon_2$. In our case, however, the resistance of the chain is determined by specific configurations of localized states—rhombuses, and pairs of localized states located on opposite sides of the Fermi level (in Fig. 1b they are linked by a dashed straight line) make no appreciable contribution to the resistance of the rhombus. This conclusion agrees with experiment[5] and with the results of computer modeling.[3] A quantitative characteristic of the random oscillations, making a comparison with experiment possible, is the correlator of the logarithms of the resistance for different positions of the Fermi level. This correlator is calculated in the following section.

4. CORRELATOR OF THE LOGARITHMS OF THE RESISTANCE

As shown in Ref. 6, the correlator of the logarithms of the resistance

$$K(\mu_2-\mu_1)=\overline{\ln R(\mu_1)\ln R(\mu_2)}-(\overline{\ln R})^2 \tag{17}$$

can be expressed in terms of the pair density of breaks $\rho_{\mu_1\mu_2}(u_1,u_2)$, defined as follows:

$$\rho_{\mu_1\mu_2}(u_1,u_2)=\left\langle \delta\left(\ln\frac{\mathscr{R}(\mu_1)}{R_0}-u_1\right)\delta\left(\ln\frac{\mathscr{R}(\mu_2)}{R_0}-u_2\right)\right\rangle . \tag{18}$$

The averaging in (17) is performed over an ensemble of chains of LS with a given length L. The angular brackets in (18) denote averaging over all possible configurations of localized states forming a break, and $\mathscr{R}(\mu)$ is the resistance of a break for a given position of the Fermi level μ. The quantity $\rho_{\mu_1\mu_2}(u_1,u_2)$ is the dimensionless concentration of breaks for which the logarithm of the resistance is equal to u_1 for $\mu=\mu_1$ and u_2 for $\mu=\mu_2$ in a chain of infinite length. This quantity can be represented conveniently in the form of the product

$$\rho_{\mu_1\mu_2}(u_1,u_2)=\rho(\bar{u})P_{\bar{u}}(\delta u,\delta\mu), \tag{19}$$

where $\bar{u}=(u_1+u_2)/2$, $\delta u=u_2-u_1$, $\delta\mu=\mu_2-\mu_1$, and the function $\rho(u)$ is given by formula (3). In the case $\nu<1$ that we are considering, the expression for the correlator (17) in terms of the function P can be written in the following form:

$$K(\delta\mu)=\frac{\pi^2}{6}\left(\frac{1}{\varphi^2(\nu)}-1\right)-\frac{1}{\varphi(\nu)}\int_0^\infty\frac{d\nu}{\nu}\ln\left(\frac{J_{\delta\mu}(\nu)}{J_{\delta\mu}(0)\,(1+\nu)^{\varphi(\nu)}}\right), \tag{20}$$

$$J_{\delta\mu}(\nu)=\int_{-\infty}^{\infty}P_{u_f}(\delta u,\delta\mu)\,(e^{-\delta u/2}+\nu e^{\delta u/2})^{\varphi(\nu)}d\delta u, \tag{21}$$

where the quantity $u_f(\nu)$—the average logarithm of the resistance of the chain—is determined by formula (7). The function $\varphi(\nu)$ is defined as $\varphi(\nu)=-d(\ln\rho(u))/du|_{u=u_f}$. Using the formulas (3) and (7) we obtain $\varphi(\nu)=\nu^{1/2}$. Thus, to determine the correlator it is necessary to calculate the pair density of breaks (18).

First of all, we note that in calculating the resistance of a break we can disregard pairs of LS with energies lying in a narrow band of width $\delta\mu$ near the average position $\bar{\mu}=(\mu_1+\mu_2)/2$ of the Fermi level, since the characteristic values of $\delta\mu$ are found to be much smaller than the size u_fT of the rhombus (Fig. 1b). In addition, as already noted in the preceding section, we can neglect pairs of LS located on opposite sides of the Fermi level (the dashed straight line in Fig. 1b). Taking into account what has been said, we can represent the resistance of a break for the Fermi-level positions $\mu=\mu_1$ and $\mu=\mu_2$ in the form

$$\begin{aligned}\mathscr{R}(\mu_1)&=R_0[\Sigma^+e^{-\delta\mu/2T}+\Sigma^-e^{\delta\mu/2T}]^{-1},\\ \mathscr{R}(\mu_2)&=R_0[\Sigma^+e^{\delta\mu/2T}+\Sigma^-e^{-\delta\mu/2T}]^{-1},\end{aligned} \tag{22}$$

where Σ^+ and Σ^- are the inverse resistances of the upper ($\varepsilon>\bar{\mu}$) and lower ($\varepsilon<\bar{\mu}$) halves of the rhombus depicting the break, calculated at $\mu=\bar{\mu}$. Substituting (22) into (18), we rewrite the expression for the pair density of breaks as follows:

$$\rho_{\mu_1\mu_2}(u_1,u_2)=\int_{-\infty}^{\infty}ds\int_{-\infty}^{\infty}dt\,\delta[\ln(e^{-s-\delta\mu/2T}+e^{-t+\delta\mu/2T})+u_1]\delta[\ln(e^{-s+\delta\mu/2T}+e^{-t-\delta\mu/2T})+u_2]\langle\delta(\ln\Sigma^+-s)\delta(\ln\Sigma^--t)\rangle. \tag{23}$$

The random quantities Σ^+ and Σ^- appearing in the last two factors in (23) can be expressed in terms of the energies and coordinates of localized states belonging to opposite halves of the rhombus. Therefore, the average of the product of δ-functions decomposes into a product of averages, after which the double integral (23) is easily calculated. As a result, we have

$$\rho_{\mu_1\mu_2}(u_1,u_2)=\frac{\tilde{\rho}(s_0)\,\tilde{\rho}(t_0)}{2\,\mathrm{sh}(\delta\mu/T)}\exp(s_0+t_0-u_1-u_2), \tag{24}$$

where we have introduced the auxiliary function $\tilde{\rho}(s)=\langle\delta(\ln\Sigma^+-s)\rangle$, and s_0 and t_0 are the values of s and t for which the arguments of the first two δ-functions in (23) vanish:

$$\begin{aligned}s_0&=\bar{u}-\ln\left[\mathrm{sh}\left(\frac{\delta\mu}{2T}-\frac{\delta u}{2}\right)\Big/\mathrm{sh}\frac{\delta\mu}{T}\right],\\ t_0&=\bar{u}-\ln\left[\mathrm{sh}\left(\frac{\delta\mu}{2T}+\frac{\delta u}{2}\right)\Big/\mathrm{sh}\frac{\delta\mu}{T}\right].\end{aligned} \tag{25}$$

The function $\tilde{\rho}(s)$ is the dimensionless concentration of "triangular" breaks for which the equivalent resistance of the pairs of LS with $\varepsilon>\bar{\mu}$ is equal to $\exp(s)$. On the other hand, the function $\rho(u)$ (3) is the concentration of breaks having the shape of a rhombus, which is composed of two such triangles. Therefore, the functions $\tilde{\rho}$ and ρ are connected by the relation $\tilde{\rho}(u)=(\rho(u)/\rho_0)^{1/2}=\exp(-gTau^2/4)$. Using this relation, and also substituting (25) into (24), we obtain

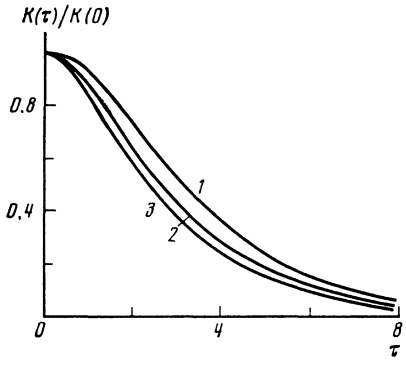

FIG. 4. Correlator of the logarithms of the resistance for different positions of the Fermi level; curves 1, 2, and 3 correspond to the values $\nu^{1/2} = 0.9, 0.5, 0.2$.

$$\rho_{\mu_1\mu_2}(u_1, u_2) = \frac{2\rho(\bar{u})}{\rho_0}\left(2\,\mathrm{sh}\,\frac{\delta\mu}{T}\right)^{1-\bar{u}T/T_0}\left(\mathrm{ch}\,\frac{\delta\mu}{T} - \mathrm{ch}\,\delta u\right)^{\bar{u}T/2T_0-1},$$

$$|\delta u| < \frac{\delta\mu}{T}. \tag{26}$$

Comparing (26) with (19), for the function $P_{\bar{u}}(\delta u, \delta\mu)$ appearing in the expression for the correlator we obtain

$$P_{\bar{u}}(\delta u, \delta\mu) = C\left(\mathrm{ch}\,\frac{\delta\mu}{T} - \mathrm{ch}\,\delta u\right)^{\bar{u}T/2T_0-1},$$

where the constant C does not depend on δu, and, therefore, as can be seen from the formulas (20), (21), does not affect the form of the correlator. The final expression for the correlator can be written conveniently in the form

$$K(\tau) = \frac{\pi^2}{6}\left(\frac{1}{\nu} - 1\right) - \frac{1}{\nu^{1/2}}\int_0^\infty \frac{dv}{v}\ln\left(\frac{J_\tau(v)}{J_\tau(0)(1+v)^{\nu^{1/2}}}\right), \tag{27}$$

$$J_\tau(v) = \int_{-1/2}^{1/2} dt\left(\mathrm{ch}\,\frac{\tau}{\nu^{1/2}} - \mathrm{ch}\,\frac{2\tau t}{\nu^{1/2}}\right)^{\nu^{1/2}/2-1}(e^{-\tau t/\nu^{1/2}} + ve^{\tau t/\nu^{1/2}})^{\nu^{1/2}}, \tag{28}$$

where we have introduced the notation

$$\tau = \frac{\delta\mu}{T}\nu^{1/2}. \tag{29}$$

A straightforward analysis of the formula (27), (28) shows that the characteristic scale of the variation of the correlator $K(\tau)$ is $\tau \sim 1$ both for $\nu \ll 1$ and for $\nu \lesssim 1$, corresponding to a scale $\delta\mu_c \sim T/\nu^{1/2}$. This result agrees with the characteristic period (16) of the oscillations of the logarithm of the resistance which was obtained by a qualitative analysis. The result of a numerical calculation of the correlator $K(\tau)$ for different values of ν is presented in Fig. 4.

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[1)]For the expression (5) to be applicable it is necessary that the width E of the energy band (in the vicinity of the Fermi level) within which the density of states g does not depend on the energy be greater than the quantity T_0. The condition $E > T_0$ implies that the average distance between neighboring LS, equal to $1/gE$, is smaller than a—the radius of localization of the LS wavefunction. This leads to the result that at large distances the LS wavefunction will fall off as $\exp[-x/(a+\delta a)]$, where $\delta a \sim aI/T_0$, I being the overlap integral of two LS situated at a distance a. For $I \sim E \sim T_0$ we have $\delta a \sim a$.

[2)]The introduction of the parameter ν in the argument of the logarithm in (6) is not important for the fulfillment of the inequality $L\rho(T_0/T) \gg 1$. However, this particular definition of ν turns out to be convenient in what follows.

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