Single-electron solitons in one-dimensional tunnel structures

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Static states and dynamic low-temperature processes in homogeneous one-dimensional structures made up of small-area tunnel junctions are considered. It is shown that effects of correlated single-electron tunneling produce in such structures topological solitons and antisolitons with charges $\pm e$. The form and intrinsic energy of a single soliton in a long structure are obtained, as are the energies of its interaction with an external electric field, with other solitons, and with edges of the structure. It is shown that for a finite number of transitions N the dependence of the density of a static soliton lattice on the external field exhibits hysteresis. Monte Carlo calculations of the soliton-lattice dynamics are carried out for the case of a nonzero external transport field. It is shown that narrow-band "single-electron" oscillations can take place in systems with sufficiently large N, with average frequency $f = \langle I \rangle / e$, where $\langle I \rangle$ is the average transport current.

1. INTRODUCTION

Several recent theoretical papers[1–4] predict a number of new effects that should be observed in systems of metal–insulator–metal (M–I–M) tunnel junctions of very small area at low temperatures. These effects have been jointly named correlated single-electron tunneling, since their common feature is establishment of a correlation, of one form or another, between the tunneling acts of individual electrons under the influence of their Coulomb interaction (see the review in Ref. 4).

The type of correlation depends substantially on the system considered. In a single tunnel junction through which a fixed current I is fed from the outside, a temporal correlation of successive acts should established, and by the same token periodic "single-electron" oscillations of the junction voltage V should set in, at a frequency[1]

$$f = I/e. \tag{1}$$

It is unfortunately very difficult to determine the current in experiment, in view of the influence of the capacitance between the current leads.[4] The prediction of the theory has therefore not yet been unambiguously confirmed. We point out, however, attempts made to attribute to just this effect (or an analogous effect for Cooper pairs[5]) unusual phenomena observed in scanning tunnel microscopy[6,7] and in two-dimensional granular structures near the percolation threshold.[8,9]

Such a reliable confirmation was also obtained[10,11] for another system—two series-connected tunnel junctions with a fixed voltage V across them (this fixing, on the contrary, presents no problems). In this system, however, only a mutual ("spatial") correlation of the acts of tunneling through the junction is realized, and there is no temporal autocorrelation of these acts in any of the junctions.[2]

Obviously, interest attaches therefore to systems in which both the temporal and the spatial correlations of single-electron tunneling can be observed under conditions when an external voltage V is applied. The simplest system of this type is a homogeneous chain (Fig. 1a) of $N \gg 1$ tunnel junctions with sufficiently small capacitance C and with tunnel conductance G (Ref. 4):

$$C \ll e^2/2T, \quad G \ll 2e^2/\pi\hbar, \tag{2}$$

where T is the temperature in energy units. Analysis of the processes in such a chain is in fact the task of the present paper.

As will be shown below, to obtain physically finite results account must be taken of electrostatic interaction not only between neighboring metallic electrodes of the system (an interaction characterized by the capacitances C) but also of other components of the electrostatic energy, i.e., other components of the matrix of the mutual capacitances C_{ij}. We assume in the present paper a very simple model in which, apart from $C_{i,i\pm1} = C$, only the diagonal elements of this matrix differ from zero, $C_{ii} = C_0$. This model is adequate enough for those system physical realizations used at present for experimental research. Recognizing that it is possible to measure the various external potentials at the end points of the chain, we arrive at its equivalent circuit shown in Fig. 1b.

2. BASIC RELATIONS

According to the general theory of correlated single-electron tunneling (see, e.g., Ref. 4) for systems satisfying the second relation in (2), a Markov approximation can be used, in which the probability of an individual tunneling act per unit time is fully determined by the difference between the free energies of the system before (F) and after (F') this event:

$$\Gamma = \frac{G}{e}\Phi\left(\frac{F'-F}{e}\right), \quad \Phi(U) \equiv \frac{U}{1-\exp(-eU/T)}. \tag{3}$$

For the system shown in Fig. 1b, the free energy can be writ-

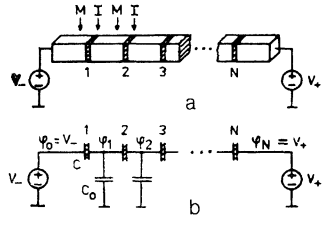

FIG. 1. Investigated one-dimensional system of N small-area tunnel junctions (a) and its equivalent circuit (b) M—metal, I—tunnel-thin insulator.

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ten in the form

$$F=\frac{C_0}{2}\sum_{i=1}^{N-1}\varphi_i^2+\frac{C}{2}\sum_{i=0}^{N-1}(\varphi_{i+1}-\varphi_i)^2-V_-Q_--V_+Q_+. \quad (4a)$$

Here φ_i are the electrochemical potentials of the electrodes and $Q_\pm$ are the total charges flowing through the external sources of the voltages $V_\pm$:

$$Q_+=C(\varphi_N-\varphi_{N-1})+em_+, \quad Q_-=C(\varphi_0-\varphi_1)+em_-, \quad (4b)$$

while $m_+(m_-)$ is the number of electrons that have tunneled into the interior of the chain through the outer right-hand (left-hand) junctions.

Using Eq. (5) jointly with the continuity equation, it is easy to express the potentials φ_i in terms of the electric charges $Q_i=Q_{0i}+en_i$ of the system metallic electrodes:

$$-C\varphi_{i-1}+(2C+C_0)\varphi_i-C\varphi_{i+1}=Q_i, \quad i=1,\ldots N-1. \quad (5)$$

The Q_{0i} depend both on the difference of the work functions of the electrode materials[12] and on the locations of the charged impurities in the dielectric layers.[4] Experiments[10] show that the diffusion of these impurities causes Q_{0i} to relax to values close to zero ($|Q_{0i}|\ll e$). We assume hereafter $Q_{0i}=0$.

For practical calculations it is easier to use not Eq. (4) itself but the simple expression that follows from it

$$F'-F={}^1/_2e[(\varphi_{i\pm1}+\varphi'_{i\pm1})-(\varphi_i+\varphi_i')], \quad (6)$$

where $(i\pm1)$ is the number of the site to which the electron tunnels from site i. It is sometimes convenient to use a different representation of the same quantity:

$$F'-F={}^1/_2e(\varphi_{i\pm1}^{(i)}-\varphi_i^{(i)})+e(\varphi_{i\pm1}^{(e)}-\varphi_i^{(e)}), \quad (7)$$

where $\varphi_i^{(e)}$ is an "external" potential that can be obtained from the system (5) by decreasing n_i by unity, while $\varphi_i^{(i)}$ is the "internal" potential, i.e., the solution of the system (5) for $Q_j=e\delta_{ji}$ and $\varphi_0=\varphi_N=0$ in the ith site. Accordingly, $\varphi_{i\pm1}^{(i)}$ is the solution of (5) for $Q_j=e\delta_{i\pm1,j}$ and $\varphi_0=\varphi_N=0$ in the $(i\pm1)$st site.

3. SINGLE-ELECTRON SOLITONS

The most important feature of the system (5) is the possibility[4] of existence, in a long chain of junctions, of solitary solutions in the form

$$\varphi_i=\pm\chi(i-j), \quad \chi(m)=\frac{e}{2C\,\mathrm{sh}\,\lambda}e^{-\lambda|m|}\equiv\frac{e}{C_{eff}}e^{-\lambda|m|}, \quad (8)$$

$$\lambda=\mathrm{arch}\,(1+C_0/2C), \quad (9)$$

$$C_{eff}=2C\,\mathrm{sh}\,\lambda=(C_0^2+4CC_0)^{1/2}. \quad (10)$$

This solution describes a topological soliton (antisoliton) with charge $\pm e$, formed in an initially electrically neutral system ($n_i\equiv0$) upon addition (or removal) of one electrode to (from) the electrode numbered j. This causes a substantial electric polarization of approximately $2M=2\lambda^{-1}$ neighboring transitions (Fig. 2) (in this sense, such a soliton can be called a classical polaron).

Using Eq. (4) it is easy to verify that the energy on a single soliton is positive and is equal to

$$E=e^2/4C\,\mathrm{sh}\,\lambda=e^2/2C_{eff}. \quad (11)$$

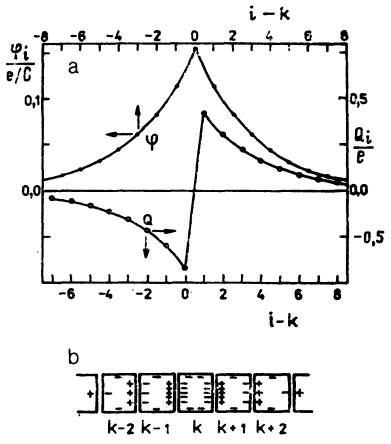

FIG. 2. Distribution of the electrostatic potentials φ_i of the system electrodes and of the charges $Q_i=C(\varphi_i-\varphi_{i-1})$ of its tunnel junctions, induced by a single-electron soliton, for $C_0/C=0.1$ (a), and the circuit of such a soliton (b).

Equations (8)–(11) show that in the case $C_0\ll C$ which is of greatest interest the single-electron soliton has a large "dimension" $2M\approx2(C/C_0)^{1/2}$ and an energy greatly exceeding the natural unit $e^2/2C$ (Refs. 1–4).

The soliton energy (11) is independent of its position (j) provided it is quite far from the edges of the chain or from other solitons. Otherwise, the interaction of these objects begins to manifest itself. For the interaction of two solitons we obtain from (4)

$$U_{ss}=\pm2Ee^{-\lambda|j_1-j_2|}, \quad (12)$$

where $j_{1,2}$ are the positions of the soliton centers. Solitons of like sign are repelled and of opposite sign attracted (this can lead to annihilation of a soliton–antisoliton pair).

Interaction of a soliton with the "passive" edge of a chain (in the absence of an external voltage) is described by the equation

$$U_{sp}=-Ee^{-2\lambda j}, \quad (13)$$

where j is the distance from the edge. It can be seen that the soliton is always attracted to the passive edge and tends to drop out of the chain. It is convenient to interpret Eq. (13) as the result of attraction between the soliton and its antipolar image at the edge of the chain; such an interpretation yields also a correct expression for the distribution of the potentials in this case:

$$\varphi_i=\chi(i-j)-\chi(i+j). \quad (14)$$

Finally, an external voltage V applied to the end of the chain causes an additional interaction

$$U_{sa}=\pm eVe^{-\lambda j}, \quad (15)$$

the sign of which corresponds to soliton repulsion from the edge of the chain if the sign of V is the same as that of the soliton charge.

The foregoing equations make it possible to draw a simple physical picture of the behavior of a long ($N\lambda\gg1$) chain. In the absence of external fields ($V_\pm=0$) and at low temperatures ($T\ll E$) only a state without solitons ($n_i\equiv0$, $\varphi_i\equiv0$) is stable. A weak field (e.g., $\varphi_0=V_-\neq0$) applied to

one end of the chain penetrates only to a depth $\sim M=\lambda^{-1}\ll N$. If, however, this field exceeds the threshold value

$$V_t=\frac{E}{e}(1+e^{-\lambda})=\frac{e}{2C}(e^{\lambda}-1)^{-1}, \tag{16}$$

it causes a successive injection of unipolar solitons, which begin to drift into the interior of the chain by virtue of their mutual repulsion.

Everything depends now on the value of the field at the other end of the structure. If $V_+ = V_-$, the soliton chains meet at the center of the structure and a certain distribution of unipolar solitons is established in it and constitutes in fact a one-dimensional Wigner crystal. Let us examine this important case in greater detail.

4. STATIONARY STATES

In the limit as $\lambda N\to\infty$ and $T\to 0$ the problem of the stationary state of our system reduces to the well known problem of equilibrium distribution of classical particles (in our case, single-electron solitons) with mutual repulsion over the sites (here—metallic electrons) of an infinite one-dimensional discrete lattice. Recall that for a fixed particle concentration $\langle n\rangle$ (per site) this distribution was obtained in Refs. 13 and 14, and the dependence of $\langle n\rangle$ on the total Gibbs energy H of one soliton (in our case $H=E-eU$, where $U=V_+=V_-$) was obtained in Ref. 15. This dependence takes the form of a devil's staircase with horizontal steps for all rational values of $\langle n\rangle = p/q$.

It follows from the general equations[15] that in our case the position of the left edge of the step with mutually prime p and q is given for $0\leqslant\langle n\rangle\leqslant 1$ by the equation

$$U_{\downarrow}=\frac{E}{e}\left\{1+2\sum_{i=1}^{\infty}e^{-\lambda r_i}[r_i(1-e^{-\lambda})+1]\right\}, \tag{17a}$$

where r_i are integers satisfying the relations

$$r_i\leqslant iq/p<r_{i+1}. \tag{17b}$$

The length of the step, however, depends in general only on q:

$$\Delta U_q=U_{\uparrow}-U_{\downarrow}=2\frac{E}{e}\frac{\operatorname{ch}\lambda-1}{\operatorname{ch}\lambda q-1}. \tag{17c}$$

It follows from Eqs. (17) that the steps do not overlap and cover jointly the entire horizontal axis (Fig. 3). The picture of the steps has translational symmetry with periods $\Delta\langle n\rangle=1$ and $\Delta U=2(E/e)\coth(\lambda/2)$; it is furthermore centrosymmetric with respect to the points $\langle n\rangle=k+\frac{1}{2}$ and $U=\Delta U(k+\frac{1}{2})$.

We call attention to the fact that the value $U_t=E/e$ at which soliton entry becomes energywise favored is always less than V_t [Eq. (16)] at which such an entry actually takes place. This points to the presence of "overheating" effects connected with the edge pinning of the solitons, in analogy with the situation for quantized magnetic-flux vortices in superconducting structures. Therefore the hysteresis-free dependence of $\langle n\rangle$ on U, shown in Fig. 3a, can be realized only for tunnel chains with smooth variation of the parameters at the edges.[18] In a homogeneous chain, on the other hand, the real dependences of $\langle n\rangle$ on U at $T=0$ exhibit

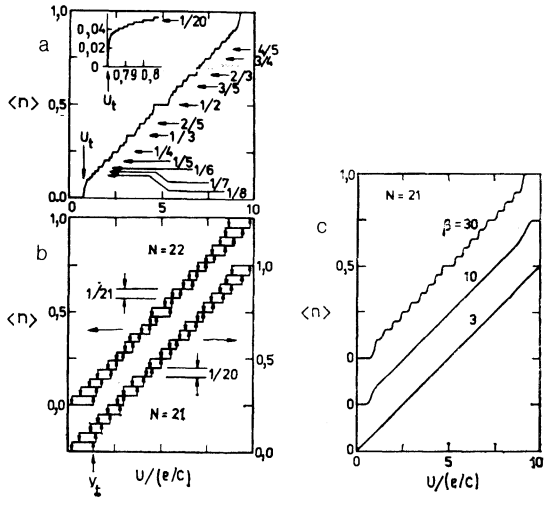

FIG. 3. Average soliton concentration $\langle n\rangle$ vs the external field U: equilibrium dependences for $N\to\infty$ and $T\to 0$ (a) and for finite T and n (c), and also nonequilibrium dependences for finite N at $T=0$ (b). For all curves $C_0/C=0.1$ and $\beta=e^2/2CT$.

hysteresis, i.e., they contain sections of metastable states.

We emphasize that a very deep analogy exists between single electron tunneling in the structure studied by us and vortices in a long homogeneous Josephson junction, which is described by a dynamic equation of the sine-Gordon type (see, e.g., Ref. 16). This analogy with the Josephson effect is valid also for other manifestations of correlated single-electron tunneling.[4] We note, however, that this analogy is only qualitative: single-electron tunneling is static even as $T\to 0$ and it cannot be described by any dynamic equation. An attempt made in Ref. 17 to derive such equations on the basis of qualitative considerations made most of the results there incorrect.

Calculation of the stability limits of these states for an arbitrary step of the devil's staircase is impossible even in the limit $\lambda N\to\infty$, since the complicated structure of the states at the sites with $\lambda j\gg 1$ (Refs. 13 and 14) is at any rate additionally reconstructed near the chain edges which determine in fact the stability. We have therefore carried out a number of Monte Carlo computer calculations.

In this method, which is suitable for any T, one obtains first for a fixed soliton configuration $\{n_i\}$, from the system of linear equations (5), aggregates of the values of $\{\varphi_i\}$ and $\{\varphi_i'\}$.[2)] Equations (3) and (6) are used next to determine the probabilities $\Gamma_i^{\pm}$ of all $2N$ possible independent events corresponding to tunneling of one electron in two directions through each of the N junctions. The probability of preserving the initial soliton configuration varies with time in this case like

$$P(t)=\exp\{-\Gamma_{\Sigma}(t-t_0)\},\qquad \Gamma_{\Sigma}\equiv\sum_{i=1}^{N}(\Gamma_i^{+}+\Gamma_i^{-}), \tag{18}$$

where t_0 is the instant of the preceding event that led to this configuration. Therefore if random numbers p are generated with a probability uniformly distributed over the segment [0, 1], the times t obtained from Eq. (18) give adequate random realizations of the instants of the subsequent tunneling. It remains then only to determine the number of this next event; this can be done, for example, by breaking up the interval [0,1] into segments proportional to $\Gamma_i^{\pm}$ and addi-

tionally selecting a random number on this segment. This is followed by replacing the soliton configuration with a new one corresponding to the segment in which the random number landed, and the entire process is repeated until stationary mean values set in. The described numerical method is quite economical; for example, for a chain with $N = 20$ the calculation of one tunnel event consumes only 10^{-2} s of microprocessor time of a relatively small computer.

Figure 3b shows the dependences of $\langle n \rangle$ on U obtained by this simulation for two close values of N (even and odd). Since the condition $\lambda N \gg 1$ is met for these values, the stability threshold of the step with $\langle n \rangle = 0$ agrees well with Eq. (16). It can be seen that the main difference from the limit $N \to \infty$ is the presence, in the staircase, of only steps corresponding to the values $\langle n \rangle = p/(N-1)$, and also the already discussed appreciable hysteresis section corresponding to the metastable states.

Finite values of the temperature lead to a finite lifetime τ_L of these states; according to (3), for small $T \ll E$ these times are of the order of $\tau_0 \exp\{-E/T\}$, $\tau_0 \approx C/G$. If the relaxation time used to calculate $\langle n \rangle$ is longer than τ_L, the dependences of $\langle n \rangle$ on U become single-valued even at small T (Fig. 3c). Its deviation from equilibrium (Fig. 3a) is then decreased. With rise of temperature the steps of the staircase become smoother, starting with those having the smallest width ΔU_q. For $T \gg E$ the steps vanish altogether and $\langle n \rangle$ tends to $U/\Delta U$; the state of the system corresponds then to constant intense thermal generation of soliton–antisoliton pairs and to their annihilation at all lattice sites.

5. DYNAMICS OF SOLITON STRUCTURES

If the values of V_+ and V_- are not equal, i.e., a longitudinal electric voltage

$$V = V_- - V_+ \tag{19a}$$

is applied to the chain, permanent motion of solitons along the chain of tunnel junctions can set in, so that a current with nonzero mean value $\langle I \rangle$ will flow through the chain. Figure 4a shows a number of typical I-V characteristics of the chain, obtained with a computer for the case $T \to 0$ and for different values of the average potential

$$U = (V_+ + V_-)/2. \tag{19b}$$

The curves show distinctly horizontal sections ($\langle I \rangle = 0$ for $|V| < V_t$) corresponding to the "Coulomb blockade" typical of all systems with correlated single-electron tunneling. The mean field U influences substantially the threshold value of V_t (Fig. 4c). U influences substantially the threshold value of V_t (Fig. 4c). It might seem that at $\lambda N \gg 1$ the stability limit of this stationary state relative to the entry (exit) of solitons on one edge of the chain should not depend on the value of the field on the other edge. The plot for each period would comprise then an envelope of $N-1$ broken lines:

$$V_t = \frac{U_p^{\uparrow} - U_p^{\downarrow}}{2} - \left| U - \frac{U_p^{\uparrow} + U_p^{\downarrow}}{2} \right|, \tag{20}$$

where $U_p^{\uparrow}$ and $U_p^{\downarrow}$ are the edges of the pth step on the plot of $\langle n \rangle$ against U (Fig. 3b). In Fig. 4c the envelope is the solid line.

The real values of V_t (points in Fig. 4c), however, lie on this envelope only in certain ranges of U, particularly at

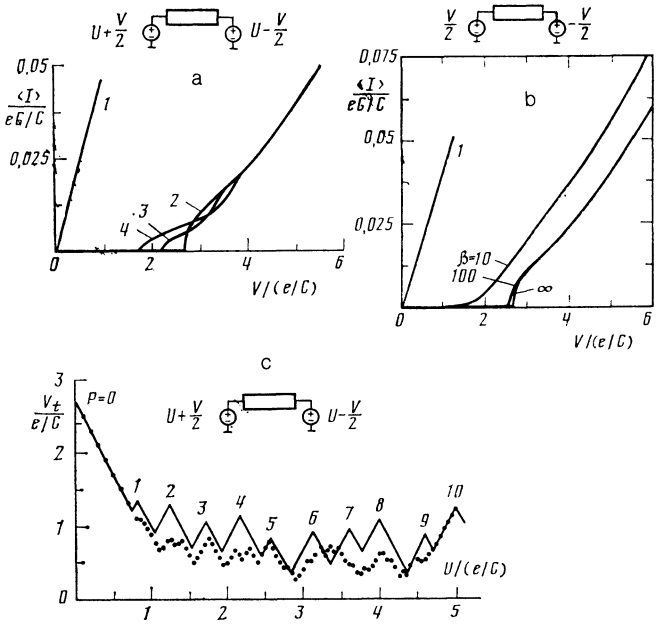

FIG. 4. Chain current-voltage characteristics: a) $T = 0$, *1*—$\langle I \rangle = GV/N$, *2*—$U/(e/C) = 0$, *3*—$U/(e/C) = 0.25$, *4*—$U/(e/C) = 0.5$; b)—$T \neq 0$, $N = 21$, $C_0/C = 0.1$, $\beta = e^2/2CT$, *1*—$\langle I \rangle = GV/N$; c) dependence of threshold voltage V_1 on the static field U at $T = 0$, $N = 21$, $C_0/C = 0.1$.

$U \approx k\Delta U$ and $U \approx (k + \frac{1}{2})\Delta U$. The reason is that most steps on Fig. 3b are degenerate, i.e., each corresponds to several soliton configurations having the same sum n_i. The stability limit of each configuration depends then on both fields $V_{\pm}$ even if $\lambda N \gg 1$, so that a nonzero longitudinal field lifts the degeneracy and leads to a more complicated $V_t(U)$ dependence.

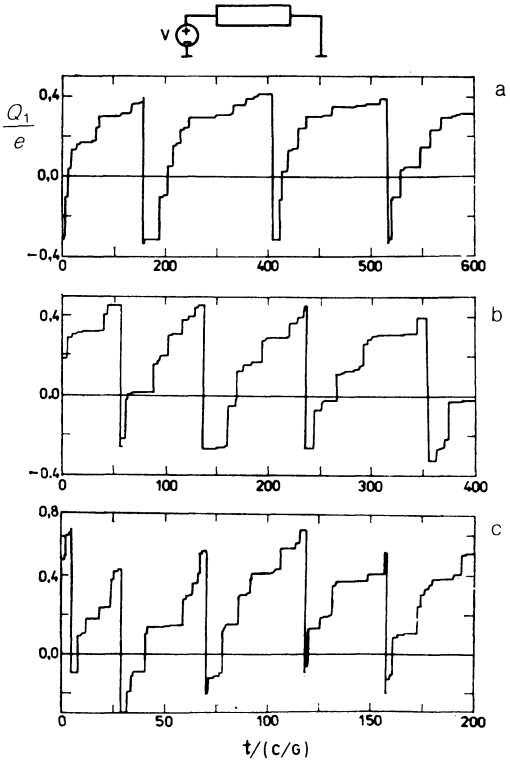

FIG. 5. Typical realizations of $Q_1(t)$ for chains with $N = 21$, $C_0/C = 0.1$, and $T = 0$ for various values of $V/(e/C)$: a—1.76, b—2.86, c—3.50.

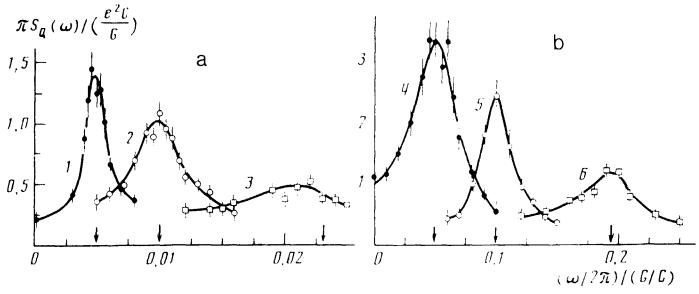

FIG. 6. Frequency spectra of Q_1 fluctuation: a—$N = 21$, $C_0/C = 0.1$, $T = 0$, $V/(e/C) = 1.76$ (*1*), 2.36 (*2*), 3.50 (*3*), $\langle I\rangle/(eG/C) = 0.05$ (*1*), 0.10 (*2*), 0.23 (*3*); b—$N = 32$, $C_0/C = 0.01$, $T = 0$, $V/(e/C) = 4.90$ (*4*), 5.47 (*5*), 6.80 (*6*), $\langle I\rangle/(eG/C) = 0.005$ (*4*), 0.010 (*5*), 0.019 (*6*). The arrows mark the theoretical position of the spectrum maximum.

Raising the temperature smears out the threshold and smooths out gradually the I-V characteristics (Fig. 4b). In the strong-field limit ($V \gg V_t$) the I-V characteristics always tend to the ohmic asymptote

$$\langle I\rangle = G\varepsilon, \quad \varepsilon = V/N.$$

The most important problem encountered if $\langle I\rangle \neq 0$ is the temporal coherence of the single-electron tunneling acts in the junctions of the chain (see the Introduction). Figure 5 shows examples of random realizations of the charge $Q_1(t) = C(\varphi_1 - \varphi_0)$ of one of the chain junctions. It can be seen that at not too large values of the current and temperature successive tunneling acts are substantially correlated. This correlation can be simply interpreted as the result of the Coulomb repulsion (12) of the solitons: at small excesses above the threshold (16), a soliton produced on one edge and drifting to the other suppresses, until it moves away a "distance" $\sim M = \lambda^{-1}$, the probability of creation of the next soliton. Thus, a quasi-equidistant chain of vortices is produced in the junction and drifts along the chain of the junction. In such an ordered motion, the frequency spectrum of the fluctuation (see Appendix 1) of Q_1,

$$S_Q(\omega) = \pi^{-1}\int_0^\infty K_Q(\tau)\cos\omega\tau\, d\tau,$$
$$K_Q(\tau) = \langle Q_1(t+\tau)Q_1(t)\rangle - \langle Q_1^2(t)\rangle \qquad (21)$$

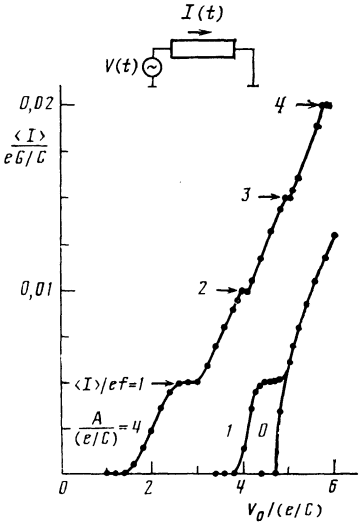

FIG. 7. Change of the IVC of a chain with $N = 32$ and $C_0/C = 0.01$ under the influence of an external signal of frequency $\omega/2\pi = 0.005(G/C)$ and amplitude A [see Eq. (23)].

(as well as of any other variable describing our system) acquires a relatively narrow peak (Fig. 6) with central frequency

$$\omega_s = 2\pi\langle I\rangle/e, \qquad (22)$$

corresponding to the "single-electron oscillations"[1,4] (the last equation follows simply from the continuity equation and from the assumption of a complete temporal correlation of the tunneling acts). Note that both computer simulation and analytic calculation show that, besides the simple translational motion of the soliton center, another process (having nearly the same probability even at $T = 0$), can occur in the system, namely, creation of a new soliton–antisoliton pair in front of the initial soliton, followed by annihilation of the antisoliton with the initial soliton. Multiple processes of this type can also occur.

The degree of correlation increases (and consequently the linewidth $\Delta\omega$ of the single-electron oscillations decreases) as the chain length N and the soliton dimension $2M = 2\lambda^{-1}$ increase (Figs. 5b and 6b). The minimum $\Delta\omega/\omega_s$ is reached for such currents when the distance between neighboring solitons is somewhat less than M, thereby ensuring their most effective interaction.

The single-electron oscillations (22) (just as all other narrow-band self-oscillations) can be synchronized by an external periodic (e.g., harmonic) signal at either the fundamental frequency ($\omega_s = \omega$) or harmonics and subharmonics ($n\omega_s = m\omega$). By virtue of the fundamental relation (22), quasi-horizontal steps appear on the IVC of the tunnel chain and are separated by intervals ΔI that are commensurate with the "quantum" $e\omega/2\pi$ (Refs. 1, 4, 5). An example of such a picture is shown in Fig. 7; it was obtained by assuming V_+ to be equal to zero and by varying V_- as[3)]

$$V_- = V + A\cos\omega t. \qquad (23)$$

This phenomenon is more promising for an experimental observation of single-electron oscillations than a direct observation of this process, which has a rather low power $P \lesssim 10^{-12}$ W at the transition parameters realized at present ($C \gtrsim 10^{-7}$ F, Refs. 10 and 11).

6. DISCUSSION OF RESULTS

We have considered the main properties of one-dimensional long chains of small-area tunnel junctions. The most interesting of these properties is undoubtedly the possibility of motion, in such chains, of quasiperiodic one-dimensional single-electron soliton structures, accompanied by generation of narrow-band oscillations with a central frequency (22), even if the external voltage (but not the current) is

fixed. It has been indicated many times (see, e.g., Ref. 4) that such a coherent process can result only from the coexistence, in one system, of a discrete transport of electrons through the tunnel junction, and a quasicontinuous charge transport in the system surrounding this junction. The question is—what ensures such a quasicontinuous transport in the investigated structure that consists of only tunnel junctions?

The answer is quite obvious from Fig. 2: the electric polarization of the chain by a single soliton extends over several (many, if $\lambda \ll 1$) tunnel junctions surrounding the charged electrode. If the soliton moves over to the next site, the induced charge of all the junctions (except the one through which the electron has tunneled) changes by an amount $\Delta Q \ll e$, i.e., quasicontinuously. Thus, for each chain junction the remaining junctions assume the role of a resistor that fixes the quasicontinuous current $I \approx \langle I \rangle$, accurate to a small quantity N^{-1} that is independent of the processes in this junction. Since the experimental observation of single-electron oscillations is hindered mainly just by the difficulty of determining the current, one can hope that the use of long homogeneous chains of junctions will permit observation and investigation of this new phenomenon, which is important both theoretically and practically.

This raises the important question of the influence of inevitable inhomogeneities of the transition parameters on the properties of the chain, and in particular on the degree of coherence of the one-electron oscillations. Although a quantitative investigation of this question is yet to be performed, simple quantitative arguments can be advanced in favor of the assumption that this coherence is not greatly suppressed by inhomogeneities.

In fact, the inhomogeneity of the parameters C and C_0 leads mainly to a substantial dependence of the soliton self-energy E on the position of its center. In particular, E will have the same deep minimum on the electrodes of a tunnel junction with a minimum capacitance, $C = C_{\min}$, and the soliton will be substantially delayed prior to tunneling through this junction. According to the result of Ref. 1, however, for an appreciable temporal correlation of successive acts of tunneling through this junction we need only that the current I be sufficiently well determined. This, obviously, can be achieved not only if C of the other junctions is much larger than $C_{\min}$, but even if relation (2) is not met for these junctions (it is necessary only that the second of these relations be met for a total chain conductance $G = 1/\Sigma_{i=1}^{N} G_i^{-1}$). Thus, even in strongly inhomogeneous media the coherence of single-electron oscillations should be preserved, at least in terms of some separate variables (for example, in the voltage on the junction with smallest C). The same consequences should follow also from the inhomogeneities of the junction tunnel conductances G.

The second unanswered but important question is the stability of our results to a decrease of the physical size of the electrodes. The point is that to raise the upper temperature boundary of the effects of correlated single-electron tunneling it is necessary to decrease this size as much as possible [for an ordinary transfer to the usual helium range—to values lower than 1000 Å, and in the nitrogen range to values of order 30—100 Å (Ref. 4)]. For these sizes, the finite time τ_e of the energy relaxation of an electron in a metallic electrode can become important (in our theory this time is assumed to be negligibly small compared with C/G). In addition, the discrete character of the energy levels of the metallic electrodes can come into play. With decrease of size these factors should undoubtedly cause a transition from effects of correlated single-electron tunneling (where the position of the electron at any instant of time is well determined) to effects of resonance tunneling in the produced superlattice of the tunnel junction (where the electron, conversely, is delocalized). A more exact determination of the boundary between these values, including allowance for the possible inhomogeneities of the structure, seems to us a very urgent problem.

APPENDIX

Calculation of the frequency spectra

The specific character of the variables describing the processes in our system (they are piecewise-constant random fluctuations of the time, see Fig. 5) calls for a nonstandard approach to the computer calculation of their frequency spectra.

We have actually used the equations

$$\tilde{S}_Q(\omega) = \pi^{-1} T^{-1} \int_0^T dt \int_t^{A+t} d\xi\, Q(t) Q(\xi) \cos[\omega(t-\xi)],$$

$$Q(t) = Q_1(t) - T^{-1} \int_0^T Q_1(t)\, dt.$$

The region $[t,\xi]$ breaks up into subregions in which $Q(t)Q(\xi)$ is constant, and in each of which the integration is carried out analytically, so that the spectrum calculation reduces to double summation. To improve the accuracy we carried out simple averaging of the functions $\tilde{S}_Q(\omega)$ over M realizations. However, even at the rather high values $T \approx 100/f_s$, $A \approx T/10$, and $M \approx 10$ the fiducial range of the calculation remained relatively large, $\Delta S/S \approx \pm 15\%$.

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[2)]In practice there is added to the initial vector $\{\varphi_i\}$ a vector $\{\Delta\varphi_i\} = \{\varphi_i' - \varphi_i\}$ that depends, according to Eq. (5) only on the increment of the vector $\{n_i\}$ that corresponds to the selected event. The aggregate of $2N$ vectors $\{\Delta\varphi_i\}$ is calculated by the tridiagonal inversion method and is stored in the computer memory.

[3)]This necessitated a modification of the described Monte Carlo method, wherein the selection of the random events was made at each time interval $\Delta t \ll 2\pi/\omega$.

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