Supersymmetric description of the thermodynamics of a two-dimensional Ising model with random exchange

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We show that the average thermodynamic potential of an Ising model with random exchange on a square lattice can be written as an integral over a superfield. We find that, if the disorder is small, the specific heat calculated in this way is finite at the transition point.

The study of the two-dimensional Ising model with random exchange is necessary for the solution of a number of problems in the physics of phase transitions, such as the effect of disorder on the critical indexes in the case when the heat-capacity index of the ordered system is zero,[1] and the problem of the lower critical dimensionality of a spin glass.[2] The main methods for studying this model are various modifications of the renormalization-group method in real space,[3] the accuracy of which is difficult to estimate, and in the case of small disorder the fermion representation of the partition function in conjunction with the replica method.[4–7] The most complete study of a model with small exchange fluctuations was carried out in Ref. 4, where a temperature dependence $C \propto \ln\ln|T-T_c|^{-1}$ of the specific heat was found and where the average of the spin pair correlator was evaluated. However, the average square of this correlator, found in Refs. 5 and 6, turns out to be less than the square of its average found in Ref. 4. In our opinion, the reason for this discrepancy is the irrenormalizability of the replica Hamiltonian for $n>2$ replicas.[4] The fact that the n-dependent expressions found in Refs. 4–7 by the renormalization-group method are valid only for $n = 1,2$, makes it impossible to extend them unambiguously to continuous $n \to 0$. It is therefore advisable to study this model without using the replica model.

We show in the present paper that one can write the thermodynamic potential of the Ising model on a square lattice, averaged over random exchange integrals, as an integral over a superfield with Grassmann and numerical components.[8] In the framework of such an approach we determine the temperature dependence of the heat capacity for the case of small exchange fluctuations. In contrast to the result obtained by the replica model[4,5,7] it turns out to be finite at the transition point in the supersymmetric formalism.

The Hamiltonian of the considered random Ising model on a square lattice has the form

$$H=-\sum_{\mathbf{x}}[J_1(\mathbf{x})s(\mathbf{x})s(\mathbf{x}+\mathbf{e}_1)+J_2(\mathbf{x})s(\mathbf{x})s(\mathbf{x}+\mathbf{e}_2)],$$

where $J_n(\mathbf{x})$ are random statistical independent exchange integrals with equal distribution functions, and $\mathbf{e}_n$ are the basis vectors of the unit cell.

We can obtain the expression for the corresponding partition function in the same way as in the case $J_1(\mathbf{x}) = J_2(\mathbf{x}) = \text{const}$,[9] by changing to a summation over single-loop diagrams with Kac-Ward factors and expressing their contribution in terms of the trace of the transition matrix in which the factors $\tanh(J_n(\mathbf{x})/T)$ are included. As a result the average thermodynamic potential is written in the form

$$F=-NT\ln 2-T\sum_{\mathbf{x}}\left\langle \ln\left(\operatorname{ch}\frac{J_1(\mathbf{x})}{T}\operatorname{ch}\frac{J_2(\mathbf{x})}{T}\right)\right\rangle -\frac{T}{2}\langle \ln\det(\hat{I}-\hat{A}\hat{B})\rangle,$$

$$\hat{A}=\begin{pmatrix}1 & \alpha^* & 0 & \alpha\\ \alpha & 1 & \alpha^* & 0\\ 0 & \alpha & 1 & \alpha^*\\ \alpha^* & 0 & \alpha & 1\end{pmatrix}\otimes\delta_{\mathbf{x},\mathbf{x}'}, \quad (1)$$

$$\hat{B}=\begin{pmatrix}\hat{\Delta}_1'\hat{t}_1 & 0 & 0 & 0\\ 0 & \hat{\Delta}_2'\hat{t}_2 & 0 & 0\\ 0 & 0 & \hat{t}_1\hat{\Delta}_1 & 0\\ 0 & 0 & 0 & \hat{t}_2\hat{\Delta}_2\end{pmatrix},$$

$$\alpha=\exp(i\pi/4),\quad \hat{\Delta}_n(\mathbf{x},\mathbf{x}')=\delta_{\mathbf{x}+\mathbf{e}_n,\mathbf{x}'},\quad \hat{\Delta}_n'(\mathbf{x},\mathbf{x}')=\hat{\Delta}_n(\mathbf{x}',\mathbf{x}),$$
$$\hat{t}_n(\mathbf{x},\mathbf{x}')=t_n(\mathbf{x})\delta_{\mathbf{x},\mathbf{x}'},\quad t_n(\mathbf{x})=\operatorname{th}(J_n(\mathbf{x})/T).$$

The angular brackets in (1) indicate averages over $J_n(\mathbf{x})$.

We shall in what follows consider only the last term in (1). Multiplying $\hat{I}-\hat{A}\hat{B}$ by a matrix $\hat{C}$ with $\det\hat{C}=1$, we can write it in the form

$$F_1=-{}^1/{}_2T\langle\ln\det(\hat{C}-\hat{D})\rangle, \quad (2)$$

$$\hat{C}=\begin{pmatrix}0 & \alpha & -1 & \alpha^*\\ \alpha^* & 0 & \alpha & -1\\ -1 & \alpha^* & 0 & \alpha\\ \alpha & -1 & \alpha^* & 0\end{pmatrix}\otimes\delta_{\mathbf{x},\mathbf{x}'},$$

$$\hat{D}=\hat{C}\hat{A}\hat{B}=\begin{pmatrix}0 & 0 & \hat{t}_1\hat{\Delta}_1 & 0\\ 0 & 0 & 0 & \hat{t}_2\hat{\Delta}_2\\ \hat{\Delta}_1'\hat{t}_1 & 0 & 0 & 0\\ 0 & \hat{\Delta}_2'\hat{t}_2 & 0 & 0\end{pmatrix}. \quad (3)$$

The matrix $\hat{C}-\hat{D}$ is Hermitean and, moreover, unitarily equivalent to an antisymmetric matrix; the corresponding unitary transformation is the same as for the matrix $\hat{C}$. Because of that, the matrix $\hat{C}-\hat{D}$ has $2N$ positive eigenvalues and $2N$ negative ones which have the same modulus so that the contribution to F_1 of the positive eigenvalues is equal to that from the negative ones [we assume that, for any $J_n(\mathbf{x})$, $\hat{C}-\hat{D}$ has no zero eigenvalues, at least until we take the limit as $N\to\infty$]. This enables us to write F_1 in the form[10]

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$$F_1=-T\oint_{C_+}\frac{d\lambda}{2\pi i}\ln\lambda\,\mathrm{Sp}\langle\hat{G}(\lambda)\rangle, \qquad (4)$$

$$\hat{G}(\lambda)=(\lambda\hat{I}-\hat{C}+\hat{D})^{-1}, \qquad (5)$$

where C_+ is a contour going around the positive eigenvalues of $\hat{C}-\hat{D}$ and $\ln\lambda$ is the main branch with a cut along the negative real axis.

For positive λ larger than the upper limit of the eigenvalues of $\hat{C}-\hat{D}$

$$\lambda>[\lambda_{max}(\hat{C}\hat{C}^+)]^{1/2}+[\lambda_{max}(\hat{D}\hat{D}^+)]^{1/2}=\sqrt{2}+1+\max_{x,n}|t_n(\mathbf{x})|$$

(Ref. 11) the resolvent $\hat{G}(\lambda)$ can be written as an integral over the superfield $\Psi_\alpha^i(\mathbf{x})$[8]:

$$G^{ij}_{\mathbf{z},\mathbf{z}'}(\lambda)=\int\prod_{\beta,y}d\Psi_\beta(\mathbf{y})\,d\overline{\Psi}_\beta(\mathbf{y})\,e^{\tilde{S}}\Psi_\alpha{}^i(\mathbf{z})\,\overline{\Psi}_\alpha{}^j(\mathbf{z}'), \qquad (6)$$

$$\tilde{S}=\sum_{\mathbf{x}}[\overline{\Psi}_\alpha(\mathbf{x})(\hat{C}_0-\lambda\hat{I})\Psi_\alpha(\mathbf{x})-t_1(\mathbf{x})L_1(\mathbf{x})-t_2(\mathbf{x})L_2(\mathbf{x})], \qquad (7)$$

$$C_0{}^{ij}=C^{ij}_{\mathbf{x},\mathbf{x}},\quad L_n(\mathbf{x})=\overline{\Psi}_\alpha{}^n(\mathbf{x})\Psi_\alpha^{n+2}(\mathbf{x}+\mathbf{e}_n)+\overline{\Psi}_\alpha^{n+2}(\mathbf{x}+\mathbf{e}_n)\Psi_\alpha{}^n(\mathbf{x}). \qquad (8)$$

Here the indexes i,j run from 1 to 4, and the index α numbers the Grassmann and numerical components. In contrast to (7), (8), there is no summation over the repeated index α in (6): it is satisfied for both the Grassman and the numerical components.

Averaging $\hat{G}(\lambda)$ with respect to $J_n(\mathbf{x})$ leads to replacing $\tilde{S}$ in (6) by

$$S=\ln\langle e^{\tilde{S}}\rangle=\sum_{\mathbf{x}}\{\overline{\Psi}_\alpha(\mathbf{x})(\hat{C}_0-\lambda\hat{I})\Psi_\alpha(\mathbf{x})-\langle t\rangle[L_1(\mathbf{x})+L_2(\mathbf{x})]\}+\sum_{k=2}^{\infty}\frac{(-1)^k}{k!}\langle\langle t^k\rangle\rangle[L_1{}^k(\mathbf{x})+L_2{}^k(\mathbf{x})], \qquad (9)$$

where $\langle t\rangle=\langle\tanh(J_n(\mathbf{x})/T)\rangle$, and $\langle\langle t^k\rangle\rangle$ are irreducible averages, e.g.,

$$\langle\langle t^2\rangle\rangle=\left\langle \mathrm{th}^2\frac{J_n(\mathbf{x})}{T}\right\rangle-\left\langle \mathrm{th}\frac{J_n(\mathbf{x})}{T}\right\rangle^2=\langle t^2\rangle-\langle t\rangle^2.$$

The thermodynamics of the random Ising model considered here is thus in the general case described by the supersymmetric non-polynomial action S. If we use this action to evaluate the pair correlator of the superfield $\Psi_\alpha(\mathbf{x})$ for large positive λ and if we analytically continue it to the vicinity of the positive poles, we can find from (4) the contribution to the averaged thermodynamic potential in which we are interested.

Writing the part of S which is quadratic in $\Psi_\alpha(\mathbf{x})$ in the form

$$\tilde{\Psi}_\alpha{}^l(\mathbf{p})=\frac{1}{2\sqrt{N}}\sum_{m,\mathbf{x}}i^{(l-1)(m-1)}e^{i\mathbf{p}\mathbf{x}}\Psi_\alpha{}^m(\mathbf{x}), \qquad (10)$$

we get

$$S_0=-\sum_{|p_n|<\pi}\overline{\tilde{\Psi}}_\alpha(\mathbf{p})\hat{\tilde{G}}_0^{-1}(\mathbf{p},\lambda)\tilde{\Psi}_\alpha(\mathbf{p}),$$

$$\hat{\tilde{G}}_0^{-1}(\mathbf{p},\lambda)=\lambda\hat{I}+\begin{pmatrix}\hat{v}_+(\mathbf{p})-t_c\hat{\sigma}_3 & \hat{v}_-(\mathbf{p})\\ \hat{v}_-(\mathbf{p}) & \hat{v}_+(\mathbf{p})+t_c^{-1}\hat{\sigma}_3\end{pmatrix},$$

$$\hat{v}_\pm(\mathbf{p})={}^1/_2\langle t\rangle\hat{\sigma}_3[\exp(-ip_1\hat{\sigma}_1)\pm\exp(ip_2\hat{\sigma}_2)],$$

where the $\hat{\sigma}_m$ are Pauli matrices and $t_c=\sqrt{2}-1$.

When $\langle t\rangle=t_c$ the quantities $\tilde{G}_0^{11}(0,\lambda)$ and $\tilde{G}_0^{22}(0,\lambda)$ have poles at $\lambda=0$. One may expect that for small $\langle\langle t^k\rangle\rangle$ in the vicinity $\langle t\rangle\approx t_c$ the components of the complete correlators $\langle\tilde{G}^{11}(0,\lambda)\rangle$, $\langle\tilde{G}^{22}(0,\lambda)\rangle$ will also have such poles, so that the singularity of F_1 in the ferromagnetic phase transition point is determined by the contribution from the long-wavelength fluctuations of the two-component superfield $\psi_\alpha^i(\mathbf{p})=\tilde{\Psi}_\alpha^i(\mathbf{p})$, $i=1,2$. We can obtain the effective action for $\psi_\alpha(\mathbf{p})$ with $p<1$ by integrating e^S over $\tilde{\Psi}_\alpha^3(\mathbf{p})$, $\tilde{\Psi}_\alpha^4(\mathbf{p})$, $\psi_\alpha(\mathbf{p})$ with $p>1$.

We consider here the simplest case, when one can neglect the contribution from such an integration to the effective action $S'(\psi_\alpha)$, and it is obtained from $S(\Psi_\alpha)$ of (9) by dropping the terms with the noncritical fields and expanding it with respect to $p<1$. Up to fourth order in $\psi_\alpha(\mathbf{p})$ we have

$$S'(\psi_\alpha)=-\sum_{p<1}\overline{\psi}_\alpha(\mathbf{p})\hat{g}_0{}^{-1}(\mathbf{p},\lambda)\psi_\alpha(\mathbf{p})+\frac{\langle\langle t^2\rangle\rangle}{4}\sum_{\mathbf{x}}[\overline{\psi}_\alpha(\mathbf{x})\hat{\sigma}_3\psi_\alpha(\mathbf{x})]^2, \qquad (11)$$

$$\hat{g}_0{}^{-1}(\mathbf{p},\lambda)=\lambda\hat{I}+\tau\hat{\sigma}_3+{}^1/_2\langle t\rangle(p_1\hat{\sigma}_2+p_2\hat{\sigma}_1),\qquad \tau=\langle t\rangle-t_c. \qquad (12)$$

We can obtain the conditions for the applicability of (11) from the requirement that the corrections to $\langle t\rangle$ and $\langle\langle t^2\rangle\rangle$ in $S'(\psi_\alpha)$, which result from the interaction with the critical fields, are small. The analytical continuation of the terms from perturbation theory in the vicinity of $\lambda=0$ give the estimates

$$\delta^{(k)}\langle t\rangle\sim\langle\langle t^k\rangle\rangle|\tilde{G}_0{}^{ii}(p\sim1,0)|^{k-1}\sim\langle\langle t^k\rangle\rangle\langle t\rangle^{1-k},\quad k\geqslant2,$$
$$\delta^{(k)}\langle\langle t^2\rangle\rangle\sim\langle\langle t^k\rangle\rangle|\tilde{G}_0{}^{ii}(p\sim1,0)|^{k-2}\sim\langle\langle t^k\rangle\rangle\langle t\rangle^{2-k},\quad k\geqslant3,$$

from which it follows that (11) is valid for distributions which for $\langle t\rangle\approx t_c$ satisfy the conditions

$$\langle\langle t^{k+1}\rangle\rangle\ll\langle t\rangle\langle\langle t^k\rangle\rangle,\quad k\geqslant1. \qquad (13)$$

These inequalities are satisfied for sufficiently narrow distributions $J_n(x)$, e.g., for a binary distribution with two positive values J and J', $|J-J'|\ll J+J'$, for arbitrary probabilities $p(J)=1-p(J')$. If, however, $JJ'\leqslant0$, (13) is not satisfied when $p(J)\neq0,1$.

We dropped in (11) terms of higher powers in ψ_α which are unimportant when we describe the critical behavior. Indeed, it follows from (12) that the scaling dimensionality of $\psi_\alpha(\mathbf{x})$ is equal to $\frac{1}{2}$ so that the dimensionality of $\langle\langle t^k\rangle\rangle$ equals[12] $2-k$ and we can drop terms with negative dimensionality for $k\geqslant3$.

The diagrams which are the first terms in perturbation-theory series and which determine the pair correlator $\hat{g}(\mathbf{p},\lambda)$ of the field $\psi_\alpha(\mathbf{p})$ and the ones which determine the complete interaction vertex $\Gamma_{ij,lm}(\lambda)$ for large positive λ are shown in the figure. We note the absence of diagrams with closed loops, the contribution from which vanishes because of the supersymmetry of S'.[8] Like the average resolvent $\langle\hat{G}(\lambda)\rangle$ of (5), the correlator $\hat{g}(\mathbf{p},\lambda)$, which is a linear combination of its components, is an analytical function of λ in the whole complex plane, except for singularities on the real axis when

$$|\lambda|<t_c^{-1}+\max_{\mathbf{x},n}|t_n(\mathbf{x})|.$$

One checks easily that each term of the perturbation theory series for $\hat{g}(\mathbf{p},\lambda)$ has the same analytical properties: the products of $\hat{g}_0(\mathbf{p},\lambda)$ of (12) in the corresponding integrals over the intermediate momentum are fractional-rational functions of λ with denominators reducible to the form

$$\Big[\tau^2+\frac{\langle t\rangle^2}{4}\sum_{i=1}^{k}x_ip_i^2-\lambda^2\Big]^k,\quad 0<x_i<1,$$

$$\sum_{i=1}^{k}x_i=1,\quad p_i^2<1,$$

by using a Feynman transformation, so that all terms in the series are analytic in λ for $\mathrm{Im}\,\lambda\neq0$ and have singularities on the real axis at $\tau^2<\lambda^2<\tau^2+\langle t\rangle^2/4$. The same also holds for the expansion of the complete vertex $\Gamma_{ij,lm}(\lambda)$. This last fact allows us to define alongside with $\hat{g}(\mathbf{p},\lambda)$ also the complete vertex for complex λ as a result of an analytical continuation of the perturbation-theory series. We shall in what follows consider $\hat{g}(\mathbf{p},i\mu)$ and $\Gamma_{ij,lm}(i\mu)$ for imaginary $\lambda=i\mu$ and we can find expressions for them using the renormalization-group method. To do this we introduce a cutoff for small momenta $\varkappa<p_i<1$ in the integrals which are the terms of the corresponding perturbation-theory series. The $\hat{g}(\mathbf{p},i\mu,\varkappa)$ and $\Gamma_{ij,lm}(i\mu,\varkappa)$ which are defined in this way are the same as the quantities in which we are interested when $\varkappa=0$. Neglecting the renormalization terms with p_n in (12) we put

$$\hat{g}^{-1}(\mathbf{p},i\mu,\varkappa)=\hat{r}(i\mu,\varkappa)+{}^1/_2\langle t\rangle(p_1\hat{\sigma}_1+p_2\hat{\sigma}_2).\qquad(14)$$

When the condition

$$\mathrm{Sp}\,\hat{r}\hat{r}^*\ll\langle t\rangle^2\varkappa^2\qquad(15)$$

holds, the renormalization-group equations, to lowest order in the invariant charge $u(i\mu,\varkappa)$,

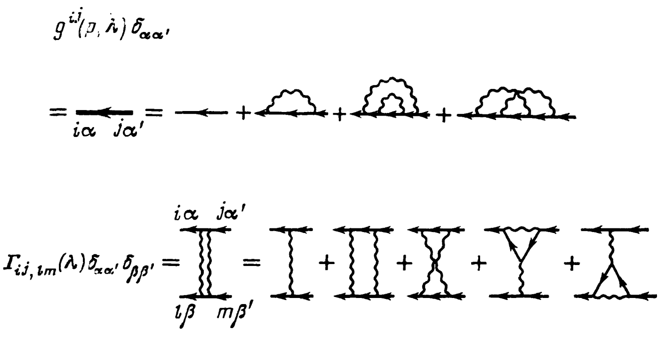
$$\Gamma_{ij,lm}(i\mu,\varkappa)={}^1/_4\pi\langle t\rangle^2u(i\mu,\varkappa)\hat{\sigma}_3^{ij}\hat{\sigma}_3^{lm}$$

have the form

$$\frac{d\hat{r}}{d\ln\varkappa}=-\frac{u}{2}(\hat{\sigma}_1\hat{r}\hat{\sigma}_1+\hat{\sigma}_2\hat{r}\hat{\sigma}_2),\quad\frac{du}{d\ln\varkappa}=2u^2.\qquad(16)$$

Using the boundary conditions at $\varkappa=1$:

$$\hat{r}_0=i\mu\hat{I}+\tau\hat{\sigma}_3\quad u_0=\langle\langle t^2\rangle\rangle/\pi\langle t^2\rangle\equiv\varepsilon,$$



we get from (16)

$$u(i\mu,\varkappa)=(\varepsilon^{-1}-2\ln\varkappa)^{-1},$$

$$\hat{r}(i\mu,\varkappa)=i\mu(1-2\varepsilon\ln\varkappa)^{1/2}\hat{I}+\tau(1-2\varepsilon\ln\varkappa)^{-1/2}\hat{\sigma}_3.\qquad(17)$$

One can show that there exist $\varkappa$ which satisfy (15) such that for them the contribution from the integrals over the momenta $p_i<\varkappa$ can be neglected, so that $u(i\mu,\varkappa)\approx u(i\mu,0)$, $\hat{r}(i\mu,\varkappa)\approx\hat{r}(i\mu,0)$. To do this we consider such a contribution to $\hat{r}$ to lowest order in u:

$$\delta\hat{r}=-u(i\mu,\varkappa)\hat{r}^*(i\mu,\varkappa)\ln[\langle t\rangle^2\varkappa^2/\mathrm{Sp}\,\hat{r}\hat{r}^*].$$

Since $u<\varepsilon$, (17), and $\langle t\rangle^2\approx t_c^2\ll1$ when $\tau\to0$ and $\varepsilon\ln t_c^{-2}\ll1$, (13), the value of $\varkappa$ that ensures smallness of $\delta\hat{r}$ and satisfies condition (15) can be defined by the equation

$$\mathrm{Sp}[\hat{r}(i\mu,\varkappa)\hat{r}^*(i\mu,\varkappa)]=\langle t\rangle^4\varkappa^2.\qquad(18)$$

One establishes easily that (18) has a solution $0<\varkappa<1$ when

$$\tau^2+\mu^2<\langle t\rangle^4,\qquad(19)$$

so that $\hat{g}(\mathbf{p},i\mu)$ is determined by Eqs. (14), (17), and (18) for such τ and μ.

We can express the singular part of the thermodynamic potential in terms of $\hat{g}(\mathbf{p},i\mu)$. To do this we use the analyticity of $\langle\hat{G}(\lambda)\rangle$ and transform the integration contour in (4) into a section of the imaginary axis $|\mu|<\mu_0$ and a semicircle $|\lambda|=\mu_0$, $\mathrm{Re}\lambda>0$. Since $\langle\hat{G}(\lambda)\rangle\sim\lambda^{-1}\hat{I}$ as $|\lambda|\to\infty$ and $\mathrm{Sp}\langle\hat{G}(\lambda)\rangle$ is an odd function of λ, we get as a result

$$F_1=\frac{T}{2}\lim_{\mu_0\to\infty}\Big(i\int_0^{\mu_0}d\mu\,\mathrm{Sp}\langle\hat{G}(i\mu)\rangle-\ln\mu_0\Big).$$

In view of the unitarity of the transformation (10) we get the singular part of F_1 through the substitution

$$\mathrm{Sp}\langle\hat{G}(i\mu)\rangle\approx N\int\frac{d\mathbf{p}}{(2\pi)^2}\mathrm{Sp}\,\hat{g}(\mathbf{p},i\mu),$$

while its singularity as $\tau\to0$ is determined by the contribution to the integral over μ from the vicinity of $\mu=0$ in which Eqs. (14), (17), and (18) are valid [see (19)]. Using (18) to change from an integration over μ to an integration over $\varkappa$ we get

$$F_1\approx\mathrm{const}-\frac{NT\tau^2}{6\pi\langle t\rangle^2\varepsilon}[8-9(1-2\varepsilon\ln|\tau|)^{-1/2}+(1-2\varepsilon\ln|\tau|)^{-3/2}],$$

whence we have for the singular part of the capacity

$$C \approx \frac{NT_c^2}{3\pi t_c^2 \varepsilon}\left(\frac{\partial\langle i\rangle}{\partial T}\right)^2 [8-9(1-2\varepsilon \ln|\tau|)^{-1/3}+(1-2\varepsilon \ln|\tau|)^{-4/3}].$$

When $\varepsilon \ln(1/|\tau|) \ll 1$, we have $C \propto \ln(1/|\tau|)$, and in the transition point, the heat capacity is finite, $C(T_c) \propto \varepsilon^{-1}$. In the case of a binary distribution with $|J-J'| \ll J+J'$ and probability $p(J) \equiv p$,

$$T_c(p) \approx 2[pJ+(1-p)J'] \ln^{-1}(\sqrt{2}+1),$$
$$\varepsilon = 4p(1-p)(J-J')^2/\pi T_c^2(p).$$

We note in conclusion that the legitimacy of the approach considered here depends decisively on the validity of the assumption that there are no zero eigenvalues of the matrix $\hat{C}-\hat{D}$ of (3) for any $J_n(\mathbf{x})$ and for finite N. Generally speaking, such an assumption is very natural as one may expect the appearance of a singularity in the thermodynamic potential only as $N \to \infty$, but its proof needs a more detailed investigation.

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