Observation of Kosterlitz–Thouless phase transition in the composite superconductor (NbTi)–Cu

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Results are reported of an experimental investigation of the resistive behavior of a composite superconductor carrying a current perpendicular to the superconducting filaments. The sample resistance exhibits in this case, depending on the temperature and on the measurement current, a number of peculiarities, and in particular a two-step transition to the superconducting state. On the basis of an analysis of the laws governing these peculiarities, a model is developed for topological Kosterlitz–Thouless phase transitions in bulk systems. Topological defects of a new type, current-stimulated excitations, are considered. The deduced empirical relations scale with $\varepsilon = I/I_c$. A correlation is established between the characteristic values for two- and three-dimensional systems.

1. INTRODUCTION

Of great importance in the study of fundamental problems of phase-transition theory is a detailed analysis of the correlation between the dimensionality of a system and the character of the ordering. Particular attention to this question is paid in the classical model of a two-dimensional lattice of generalized spins localized in one plane (2*D*-*XY* model). In 2*D* systems (thin films, superfluid films, planar magnets, and other), spontaneous ordering with onset of long-range order (LO) of the usual type is possible at all temperatures $T > 0$.

At the same time, Kosterlitz and Thouless (K–T) predicted[1] a new phase transition of the order–disorder type, connected with establishment of a quasi- or topological long-range order (TLO). A feature of this transition in a 2*D* spin lattice is that the spin–spin correlation function changes at a finite temperature $T_{\rm KT}$ from exponential to a power-law function of the distance r:

$$\langle S(r)-S(0)\rangle \propto \exp\left[-r/\xi(T)\right] \quad \text{for} \quad T>T_{\rm KT}, \tag{1a}$$

$$\langle S(r)-S(0)\rangle \propto r^{-\eta(T)} \quad \text{for} \quad T<T_{\rm KT}, \tag{1b}$$

where the correlation length

$$\xi(T) \propto r_0 \exp\left[B/(T-T_{\rm KT})\right]^{1/2} \tag{1c}$$

determines the dimension of the fluctuation in the high-temperature phase, the critical exponent $\eta(T)$ determines the interaction strength at $T < T_{\rm KT}$, and r_0 is the generalized-spin lattice parameter or the vortex-core dimension. Other elementary excitations are, besides spin waves that permit in 2*D* systems only a TLO of type (1b), pairs of coupled vortex structures of opposite sign separated by a distance characterized by a corresponding coherence length ξ_-. At large distances ($r \gg \xi_-$) the field of these vortices is cancelled out and has no effect on the long-range order in the spin system. The cause of the critical behavior is the fact at $T > T_{\rm KT}$ the appearance of free vortices becomes energywise favored, and these disrupt even the TLO, meaning in fact a qualitative change of the macroscopic properties of the entire system, accompanies by anomalous effects. Halperin and Nelson (H–N) have shown[2] that such a model can be used to describe the properties of this superconducting films and of a planar network of weakly coupled superconducting elements.

Most experiments were performed to date on thin high-resistivity films, which were considered in analogy with an electrically neutral superfluid film. It turned out, however, that this experimental approach is greatly complicated by the insufficient homogeneity of the samples and by a large number of other problems encountered in their preparation and measurement.

Another group of experiments was performed on large regular 2*D* constructs of weakly coupled superconducting elements, which permit the problem to be studied in the discrete-lattice variant.[3–5] A characteristic feature of the resistive behavior of such structures is a two-step transition to the superconducting state. As the temperature is lowered the resistance first decreases jumpwise to approximately 20–70% of the normal value, owing to onset of superconductivity in the islands. The subsequent gradual decrease of the resistance in a rather wide temperature interval is due to the appearance of the proximity effect. The final resistance jump to zero is identified on all these investigations as the K–T phase transition.

A similar two-step transition, but in the temperature dependence of the alternating magnetic susceptibility, was observed in an investigation of a composite superconductor consisting of finely dispersed NbTi in a copper matrix.[6] In light of the aforementioned problems, such composite superconductors are highly interesting model materials. Their cross section is a 2*D* construct of alternating superconducting and normal metals, identical to a regular planar lattice of *S–N–S* junctions. Besides the ease of their preparation (commercial technical-grade material is used), such samples have a perfect structural homogeneity along the *Z* axis perpendicular to the 2*D* cross section in the *XY* plane. This excludes the influence of various types of edge effects that are difficult to control in films. At the same time, in practice such samples have a thickness $d_0 \gtrsim 0.1$ mm, much larger than the coherence length in the normal metal. Consequently, these quasi-two-dimensional superconductors can be regarded as three-dimensional constructs (with strong anisotropy of the binding energy), the experimental study of which is a most vital problem.

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The first direct measurements of the transverse resistance in a composite finely dispersed superconductor were performed on systems consisting of a large number of NbTi filaments in a copper matrix.[7-9] It was established later that the transverse resistance and the susceptibility in a magnetic field parallel to the filaments have similar temperature dependencies, undergoing a two-step superconducting transition.[10] Analysis of the variation of the current-voltage characteristics (IVC) of such samples with temperature and with magnetic field has revealed in bulk samples, for the first time, a phase transition with features of the K–T transition.[11,12]

We report here the results of a detailed investigation of the resistive behavior of a multifilament superconductor carrying a current perpendicular to the superconducting filaments. We present, on the basis of new experimental data, an analysis of the critical behavior of the system near the low-temperature transition.

2. MEASUREMENTS AND RESULTS

2.1. Measurement procedure and experimental IVC

The measured samples were prepared from a technical composite superconductor. The initial conductor of 1 mm diameter consisted of 1045 Nb–Ti (NT-50) filaments in a copper matrix and had a filling factor 0.45 in the central part. In cross section, the conductor consisted of clusters, each containing 19 superconducting filaments of $\sim 20\ \mu$m diameter, with spaces of order 3–4 μm between the filaments in the cluster and $\sim 20\ \mu$m between neighboring clusters. A piece of this conductor was ground on two sides in the longitudinal direction to form plane parallel surfaces, after which the sample was cut in the form of a narrow bar transverse to the conductor. The samples were ~ 0.9 mm long and had a cross section $\sim 0.5 \times 0.2$ mm. The sample volume contained approximately 15 clusters and accordingly about 300 superconducting filaments. Magnetoresistance measurements have shown that these samples can be regarded, to first order, as an almost hexagonal structure of *S–N–S* junctions, with lattice parameter $r_0 = 20 \pm 2\ \mu$m.[12]

Tinned copper wires, serving as current and potential leads, were soldered to the end faces of the sample in such a way that the measurement-current was perpendicular to the superconducting filaments. The main results were obtained at temperatures below the superconducting transition of the solder, to obviate corrections for the solder resistance. The voltage from the sample was read with an R-341 nanovoltmeter feeding a printer. The measurement current and the temperature were recorded simultaneously. The voltage ranged from zero to $\sim 0.7\ \mu$V and was recorded accurate to ± 3 nV ($= 0.2$ arb. units). This corresponded to a measurement current from 0 to ~ 500 mA, which was accurate to ± 0.1 mA. The IVC of the samples were plotted point by point in the absence of an external magnetic field and at fixed values of the temperature. To exclude the influence of the thermoelectric power, the measurements were made at two current directions. The temperature ranged from 15 to 1.5 K and was kept constant accurate to $\gtrsim 0.001$ K. The results were reproducible within the scatter of the experimental points, but when the temperature was lowered relaxation effects became more and more pronounced, so that the time for equilibrium to set in after switching the measurement current reached 30 min at $T = 1.5$ K. Nothing like this was observed in control measurements on comparable copper samples (in which case the time for equilibrium was $\lesssim 1$ min), showing that simple overheating of the sample was not the cause of the anomaly.

Figure 1a shows typical experimental IVC in log-log scale for one and the sample: similarity of the curves at high currents was accompanied at low currents by a strong temperature dependence that changed qualitatively between curves 2 and 3, as manifested by the vanishing of the linear section [we take the temperature of ths transition to be $T_{c0} = 5.2 \pm 0.1$ K (Ref. 11)] The superconducting transition temperature of the niobium–titanium filaments was $T_{cs} = 9.5$ K.

The temperature dependence of the transverse resistance below T_{cs} is shown explicitly in Fig. 2 for various densities of the measurement current. It can be seen that an increase of the current density increases the width of the lower transition to the nonresistive state (transverse resistivity $< 10^{-9}\ \Omega\cdot$cm) and at $j = 3.2 \cdot 10^2$ A/cm^2 the transition vanishes completely in the considered temperature range.

The results are very similar to the analogous data for 2*D* systems,[4,13,14] where the nonlinear behavior of the IVC is usually related to the slope α of the curves as $U \propto I^{\alpha(T)}$.

The corresponding analysis of our curves for different temperatures $T < T_{c0}$ is shown in Figs. 1b and 1c, where α was determined at each point of the experimental curve. (The values of the normalizing current I_{KT}, the meaning of which will be made clear below, are indicated in Fig. 1 by upward arrows.) A strong dependence of α on the current is observed, with extrapolation $\alpha \to \infty$ as $I \to 0$, in contrast to experiments with film samples, where $\alpha = \text{const} \geqslant 3$ as $I \to 0$.

2.2. Coupling energy and critical temperatures

We assume that the main properties of our sample are governed by its two-dimensional symmetry. We have then a planar grid of superconducting islands that interact with one another with a coupling energy[15]

$$E = -E_y(T) \cos \Delta\varphi_{1,2}, \tag{2a}$$

where $\Delta\varphi_{1,2}$ is the phase difference of the order parameter of two islands. Such a junction is characterized by a superconducting current[15,16]

$$i = i_c(T) \sin \Delta\varphi_{1,2}, \tag{2b}$$

where $i_c(T) = 2eE_y(T)/\hbar$ is the critical current of an isolated *S–N–S* junction in the absence of thermal fluctuations and cannot be determined directly from experiment.[17] In large regular structures this quantity is renormalized, by the fluctuations of individual quasispins at large distances, to the critical current i_0 of the Ginzburg–Landau (G–L) theory, and by the vortical fluctuations to the critical current i_s of the H–N theory (i.e., $i_c > i_0 > i_s$) measured in experiment with nonlinear effects neglected[1-3,17,18] (the quasi-spin is here the complex order parameter $|\psi_j| \exp(i\varphi_j)$ of the individual islands[19]).

Near T_{cs} the critical current through a single *S–N–S* junction of thickness d obeys the relation[20]

$$i_c(T) \propto \xi_N(T)\ (1 - T/T_{cs})^2 \exp[-d/\xi_N(T)], \tag{3a}$$

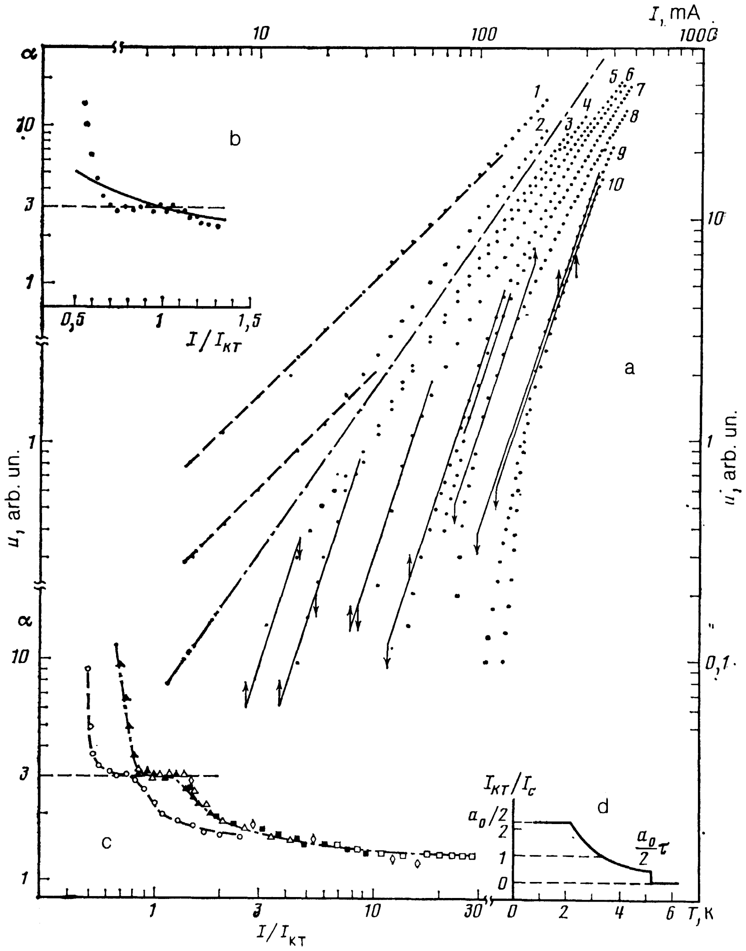

FIG. 1. Experimental IVC: a) 1—$T = 7.1$ K; 2—5.7 K; 3—4.5 K; 4—4.2 K; 5—3.5 K; 6—3.0 K; 7—2.5 K; 8—2.0 K; 9—1.7 K; 10—1.5 K. Solid lines—$\alpha = 3$, dashed—$\alpha = 1$, dash-dot—boundary; $\downarrow I_c$; $\uparrow I_{KT}$; b) solid line—$\alpha(I) = 1 + 1/2\eta(I)$, η from Eq. (11d), $T = 1.5$ K; dashed line—$\alpha = 3$; c) ▲—$T = 1.7$ K; ○—2.0 K; △—2.5 K; ■—3.0 K; ◇—3.5 K; □—4.2 K; d) temperature dependence of critical current I_{KT} normalized to I_c.

where ξ_N is the coherence length in the normal metal,[10]

$$d/2\xi_N \gg 1, \quad \xi_N = \hbar v_F / 2\pi k_B T. \tag{3b}$$

These conditions are not well satisfied in our measurements (e.g., $T = 2$ K$d/2\xi_N \approx 3$ and $T/T_{cs} \ll 1$), but it has been established empirically that Eq. (3a) without the exponential factor describes well enough the experimental data in the considered temperature region $T < T_{cs}$ (Ref. 11), i.e.,

$$I_c = \frac{X_0}{T}\left(1 - \frac{T}{T_{cs}}\right)^2, \quad X_0 = 0.252 \ \text{A} \cdot \text{K} \tag{4}$$

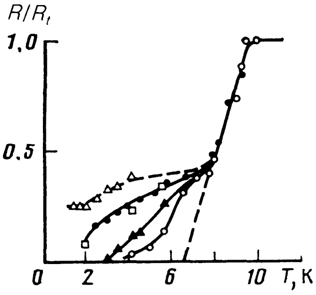

FIG. 2. Temperature dependence of transverse resistance for different currents (dashed line—linear extrapolation of the high-temperature values of $R(T)$ to $R \to 0$): ●—sample No. 1, $j = 120$ A/cm^2; all other points—sample No. 3, $j = 120$ A/cm^2 (□), $j = 40$ A/cm^2 (▲), $j = 16$ A/cm^2 (○), and $j = 320$ A/cm^2 (△).

(we assume hereafter that to first order the critical current of the entire system, disregarding thermal fluctuations, is $I_c(T) = Ni_c(T)$, where N is the number of superconducting filaments in the sample cross section).

Equation (4) yields the effective temperature of the system:

$$\tilde{T} = \frac{I_c(T_c)}{I_c(T)} T \propto \frac{T}{E_v(T)} \tag{5}$$

with normalization $\tilde{T}_c = T_c$; (a more accurate value of T_c is given below).

Let us analyze our results with respect to the existence of a correlation length of type (1c), replacing T by $\tilde{T}$. If it is assumed that to first order the sample resistance depends only on the concentration n_f of free vortices, and $n_f \propto \xi^{-2}$, we obtain on the basis of (1c) (Refs. 2 and 17)

$$R = R_\infty \exp[-b_0(\tilde{T}/T_c - 1)^{-1/2}], \tag{6}$$

where it is expected that $b_0 \equiv 2(B/T_c)^{1/2}$ is of the order of unity[21,22] and does not depend on the measurement current in the limit $I \to 0$ (Refs. 2 and 5). The resistance of our samples, however, defined as $R \equiv U/I$, is a strong function of the measurement current with extrapolation $R \to 0$ as $I \to 0$ and at $T < T_{c0}$ (Figs. 1 and 2). We consider therefore, formally for the time being, Eq. (6) for various given values $I =$ const and determine $R_\infty = \lim_{I \to \infty} R(I)$ (we shall obtain later a more accurate value of R_∞ using (8) and extrapolating

$\ln R(I^{-1})$ to $I^{-1}\to 0$ on Fig. 4). Equation (6) allows T_c to be determined directly from the experimental $R(I,T)$, in the form

$$\delta_R(\tilde{T}) \equiv \Delta\tilde{T}/b_0 T_c = [\ln(R/R_\infty)]^{-2}, \quad \Delta\tilde{T} = \tilde{T} - T_c, \tag{7a}$$

recognizing that

$$\lim_{\tilde{T}\to T_c} \delta_R(\tilde{T}) = 0. \tag{7b}$$

The experimental values of $\delta_R(T)$ for different measurement-current levels are shown in Fig. 3a. It can be seen that, the strong dependence of δ_R on I notwithstanding, all curves tend to zero at $T_c = (1.7 \pm 0.5)$ K.

From Eq. (7a) and the experimental values of the resistance we obtain (Fig. 3b)

$$b_0 = b_0(I) \propto 1/I. \tag{7c}$$

Starting from a set of values of $R(I,T)$, we can obtain the $\tilde{T}(T)$ dependence from Eqs. (7) or else from Eq. (5) with allowance for (4): the corresponding results are shown in Fig. 3c. Linear extrapolation to $\tilde{T} = 0$ gives the critical temperature T_{cI} for which $E_y \to \infty$, meaning spontaneous ordering of the entire system with onset of a long-range order than no longer depends on the measurement current. The suggested phase transition at T_{cI} is due to the fact that the sample behaves as a homogeneous three-dimensional superconductor at $T < T_{cI}$ and as a discrete $2D$ system above T_{cI}. Consequently $T_{cI} = 0$ for an ideal 2D system. In our samples such a transition takes place at $T_{cI} \approx 1$ K (Fig. 3c). This result agrees with the coherence-length estimate $2\xi_N(T_{cI}) \approx d$, i.e., the cylinders thickened by the proximity effect are short-circuited at T_{cI} (a direct study of the proximity effect in NbTi–Cu yields[23] $\xi_N(1...0.5\text{ K}) \approx 1...2$).

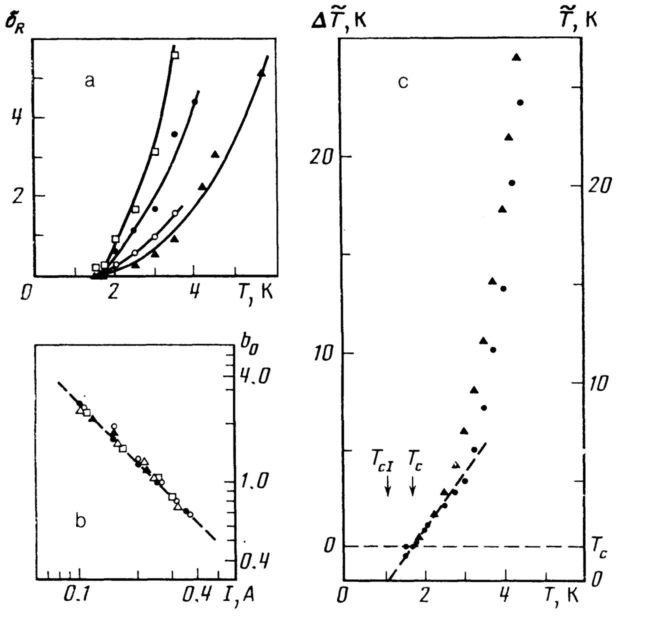

FIG. 3. a) Temperature dependence of δ_R near T_c for different values of the current (A): □—0.25, ●—0.20, ○—0.15, ▲—0.10; b) dependence of the parameter b_0 on the current: ●—$T = 2.5$ K; ○—$T = 3.5$ K; △—$T = 4.2$ K; □—$T = 4.5$ K; ▲—$T = 7.1$ K; dashed line—$b_0 \sim 1/I$; c) dependence of $\tilde{T}$ (and $\Delta\tilde{T}$) on the temperature T: ●—T as given by Eqs. (4) and (7); ▲—T as given by experiment in accordance with Eq. (9); dashed line—$\tilde{T}(T)$ approximation at $T \lesssim T_c$.

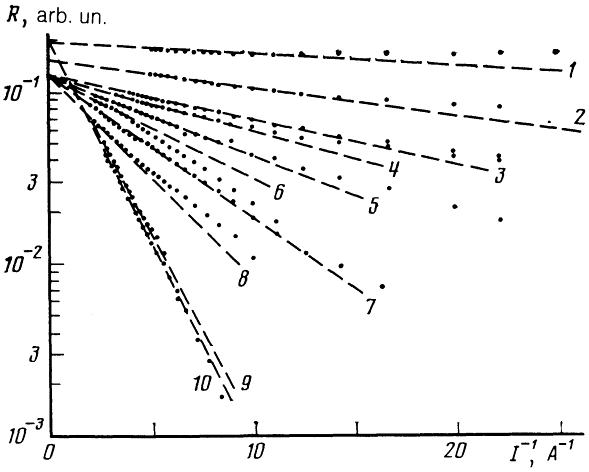

FIG. 4. Dependence of $\ln R$ on I^{-1} for various temperatures: 1—7.1 K; 2—5.7 K; 3—4.5 K; 4—4.2 K; 5—3.5 K; 6—3.0 K; 7—2.5 K; 8—2.0 K; 9—1.7 K; 10—1.5 K; dashed curves—$\ln R \sim I^{-1}$.

2.3. Influence of current on a vortex structure

Reduction of the experimental data on the basis of Eq. (6) points to a possible existence of a critical temperature T_c and of a corresponding correlation length of type (1c), if it is assumed that b_0 is a function of the current. This result is apparently due to the fact that at $I \neq 0$ the coupled vortex pairs can be dissociated not only by thermal excitation, but also by Lorentz forces.[13] For a more accurate description of $R = R(I)$ we rewrite our results, in accordance with (6) and (7c), in the form $\ln R \propto I^{-1}$ (Fig. 4). This reveals the following distinctive features:

1) all the results are well described by the equation

$$R(I) = R_\infty \exp(-b_R/I); \tag{8}$$

2) at $T_c < T < T_{c0}$ the parameter $b_R(T)$ varies at a certain mean value of the current in such a way that $R_\infty = \text{const}$ in this temperature region;

3) at $T = T_c$ a noticeable jump of R_∞ is observed and the b_R break vanishes.

These results become understandable if the free-vortex density n_f is expressed in the form

$$n_f = {}^1/_2(n_{fI} + n_{fT}), \tag{9a}$$

$$n_{fI} = \exp(-a_1/I), \tag{9b}$$

$$n_{fT} = \exp(-a_2/I), \tag{9c}$$

where n_{fT}, n_{fI}, and n_f so normalized that max $n_f = 1$. We determine a_1 and a_2 from the general premise that the dependencies of R and b_0 on the measurement current are the consequence of a certain current-stimulated activation of the free vortices.

Under strong-coupling conditions ($K_0 \equiv E_y(T)/k_B T \gg 1$, K_B is the Boltzmann constant) this is the only possibility of exciting a noticeable quasiparticle concentration n_f. The influence of the electric current reduces to the fact that it produces a phase gradient that lowers greatly the effective energy barrier to the onset of fluctuations. In the Coulomb-gas model this corresponds to dipole dissociation by an electric field.[24] One can imagine two processes, direct excitation by the current, (9b), and indirect, i.e., breaking, by the Lorentz force, of the vortex pairs already existing in the form of thermal fluctuations, (9c). Taking the preceding results into account, we assume that the second process (n_{fT}) is characterized by the same correlation length as the K–T transition [(1c) and (1b) with allowance for (7c)]. Consequently, $a_2 \sim \tau$, where $\tau \equiv (\widetilde{T}/T_c - 1)^{-1/2}$. At the same time, the effect of the applied field ε should depend on the binding energy $E_y(T)$, i.e., a_1 and a_2 should be proportional to $E_y(T)$ and to $I_c(T)$. A similar assumption follows from the scale invariance of the Coulomb gas,[25] where $|\varepsilon| \equiv I/I_c$. Summarizing the foregoing, we obtain

$$a_1 = a_{01} I_c(T), \tag{9d}$$

$$a_2 = a_{02} I_c(T) \tau. \tag{9e}$$

We have then $n_{fI} \ll n_{fT}$ for $\widetilde{T} \gg T_c$, so that we can verify (9c) directly on the basis of experimental data and determine the parameter a_{02}, taking into account only the values of $R(I)$ for small I: $a_2(T) = I \ln(R_\infty/2R)$. The results of such an analysis are shown in Fig. 5 in the form $a_2(T)/\tau \propto I_c$ and confirm assumption (9c). From this result and from (9a)–(9c) we determine $a_1(I_c)$. Figure 5 shows that Eqs. (9) are satisfied in the entire range of temperature (or of I_c) with high accuracy, and

$$a_{01} = a_{02} = a_0, \tag{9f}$$

i.e.,

$$R = \frac{1}{2} R_\infty \left[\exp\left(-\frac{a_0 I_c}{I}\right) + \exp\left(-\frac{a_0 I_c}{I}\tau\right) \right], \quad T_c < T < T_{c0}. \tag{9g}$$

For the slopes of the straight lines in Fig. 4 ($b_R \equiv \partial \ln R / \partial(I^{-1})|_{I\to\infty}$ we get from (9)

$$b_R = \begin{cases} \dfrac{a_0}{2} I_c(T)(1+\tau), & T_{c0} > T > T_c \\ a_0 I_c(T), & T \leqslant T_c \end{cases} \tag{9h}$$

Equation (9f) emphasizes the similarity of the two activation processes, with $a_0 = 4.6$. Another interesting result is connected with the jump of R_∞ at the temperature T_c, such that R_∞ increases below T_c (Fig. 4). It can be assumed that this effect is a consequence of the qualitative restructuring in the energy spectrum of the coupled vortex pairs at $T = T_c$, below which there are no thermal fluctuations of this type and consequently $n_{fT} = 0$ for all I.

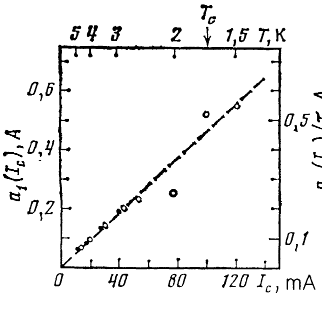

FIG. 5. Determination of a_1 and a_2 from the experimental data: ●—a_2/τ, ○—a_1; dashed line—approximation in accord with (9).

The results make it possible to refine the value of T_c by optimizing Eqs. (9); this yields $T_c = 1.71 \pm 0.01$ K.

2.4. Behavior of resistance at $I \to 0$

The total system resistance due to the motion of the free vortices is determined by their concentration n_f and mobility μ (Refs. 2 and 17).

$$R \propto \mu n_f. \tag{10a}$$

If it is assumed that the vortex motion is due to thermally activated diffusion between the lattice sites, we can determined the mobility in deep potential wells ($K_0 \gg 1$) by using the results of Ref. 26, and obtain

$$\mu(I) \sim R_\infty(I) \sim \begin{cases} [1-(I_c/I)^2]^{1/2}, & I > I_c \\ =0, & I \leqslant I_c \end{cases}. \tag{10b}$$

The $\mu(I)$ dependence becomes decisive for the behavior of $R(I)$ at low currents.

With account taken of (10b), Eq. (9g) agrees well with the experimental data at $T_{c0} > T > 2.5$ K in the entire measurement-current interval, while at lower temperatures there are observed noticeable deviations from (10b), which point possibly to a certain collective mechanism of vortex motion.[11,12] by suitably transforming Eqs. (9a) and (10a) we determine the normalized vortex mobility $c_d = \mu(I)/\mu$ $(I \to \infty)$ for the entire temperature range with the aid of the experimental values of $R(I,T)$. The result is shown in Fig. 6a for several values of the parameter I/I_c, and illustrates the qualitative changes in the behavior of the system at $T = T_{c0}$ and $T \lesssim 2.5$ K. In the latter case the singularity of c_d becomes more pronounced the smaller the current. At $T > T_{c0}$ the effective energy barrier is abruptly lowered and $\mu(T)$ is constant.[2,17] In first-order approximation we have[11] at $T \gtrsim T_{c0}$

$$R = R_N [n_{f0} + (1 - n_{f0}) n_{fI}], \tag{10c}$$

where R_N is proportional to the normal resistance of one S–N–S junction, and

$$n_{f0} \sim r_0^{-2} \exp[-b_1 (T/T_{c0} - 1)^{-1/2}] \equiv \xi_+^{-2}.$$

This is the known concentration of the thermal fluctuations in the H–N model, and is determined by a corresponding correlation length of type (1c) in which no current excitations are taken into account. The experimental results yield an estimate $b_1 = 2.5...3$, which agrees with the K–T theory.[21] For a reliable description of the behavior of the resistance in this temperature region we need, however, more detailed measurements, which must be compared with the H–N model with allowance for the current fluctuations, the influence of which on the $R(T)$ behavior with increase of temperature terminates at $T_w = 7.5...8$ K (Fig. 2). Linear extrapolation of the values $R(T)$ at $T_{cs} > T > T_w$ to $R \to 0$ determines the G–L transition temperature T_c^0 (Ref. 2). In this case $T_c^0 = (6.4 \pm 0.2)$ K (Fig. 2). Consequently, the general behavior of the resistance at $T > T_{c0}$ and at small currents agrees qualitatively with the H–N theory. It must be noted here that the critical region $T_c^0 - T_{c0}$ is much broader than can be expected from the corresponding estimates of Refs. 2, 17, and 27.

Neglecting the current excitations, the contribution of

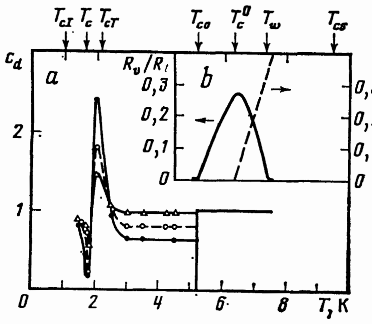

FIG. 6. a) Temperature dependence of normalized vortex mobility $c_d\mu(I)/\mu(I\to\infty)$ for different current values; solid line $\varepsilon<1$, ●—$\varepsilon=1.5$; ○—$\varepsilon=2.0$; △—$\varepsilon=4.0$. b) Components of the resistance near T_c^0.

the activated vortices (R_v) to the total resistance as a function of temperature can be estimated by subtracting from the experimental $R(T)$ for $I\to 0$ the linear approximation of the contribution of the fluctuations of individual quasi-spins (R_s) at $T>T_c^0$ (Ref. 2). The result is shown schematically in Fig. 6b and correlates with an analogous analysis of the behavior of the resistance in the form $dR(T)/dT$ (Ref. 28). A direct confirmation of this interpretation is the fact that the resistance-oscillation amplitude in weak magnetic fields is $\Delta R(T)\propto dR/dT$. The applicability of the vortex model to the results of our measurements was confirmed by observation of quantum oscillations of this type in current excitations of n_{fI} (Ref. 12).

3. DISCUSSION OF RESULTS

3.1. The correlation length ξ and the critical exponent η

The foregoing analysis of the IVC shows that, starting with the K–T transition concept, we can describe the behavior of $R(I,T)$ of our system with the aid of the correlation lengths ξ_+, ξ_{TI} and ξ_I (where $\xi_{TI}\propto n_{fT}^{-1/2}$ and $\xi_I\propto n_{fI}^{-1/2}$). The K–T transition has been theoretically studied so far only for the limit $I\to 0$ or $l_c/r_0\gg 1$ ($l_c=l_c(I)\sim 1/I$ is the minimum size of the vortex pairs dissociated by the measurement current), with neglect of the renormalizing effect of the current excitations.[2,13,17,18] In this limit the mobility of free vortices in film systems is $\mu=\text{const}\neq 0$, so that in accordance with (10a) the behavior of $R(T,I)\equiv U/I$ determines the concentration n_f of the free vortices as a function of the temperature and the current. In this case one can obtain from the experimental data the temperature dependence of the correlation length ξ_+ and of the critical exponent η of the K–T transition:

$$\begin{gathered}\xi_+\sim n_{f0}^{-1/2}\sim R^{-1/2}\quad (T>T_{\rm KT}),\\ \eta={}^1\!/_2(\alpha-1),\quad \alpha(T,\ I)=1+d(\ln n_f)/d(\ln I),\\ n_f=n_f(I)\sim R(I)\quad (T<T_{\rm KT}).\end{gathered}\tag{11a}$$

For our bulky samples these quantities can in principle not be determined directly for $I\to 0$, in view of (10b) (in particular, at $I<I_c$ it does not follow from $R=0$ that $n_f=0$). We therefore extrapolate Eqs. (9), which were obtained for $I>I_c$ into the region $I_c>I\to 0$. With account taken of (10) and (11a), we obtain

$$\eta(I,T)=\frac{\varepsilon_0}{2}\left[1-\frac{1-\tau}{\exp[-(1-\tau)/\varepsilon_0]+1}\right]^{-1},$$
$$\varepsilon_0\equiv I/a_0I_c\equiv\varepsilon/a_0.\tag{11b}$$

Thus,

$$\eta(I\to 0,\ T<T_{c0})=0,\quad \alpha(I\to 0,\ T>T_{c0})=1,$$

i.e., the critical temperature differs from the K–T transition temperature $T_{\rm KT}$ by the fact that the known (for $2D$ systems) universal jump of α (or η) at $T_{\rm KT}$, when $\alpha=1$ for $T_{\rm KT}^+$ and $\alpha=3$ for $T_{\rm KT}^-$ (or $\eta=\frac{1}{4}$ for $T_{\rm KT}^-$),[29] becomes infinite in this case $\alpha=1$ for T_{c0}^+ and $\alpha=\infty$, while $\eta=0$ for T_{c0}^-. This behavior is a feature of $3D$ superconductors, for which $R=0$ at behavior is a feature of $3D$ superconductors for which $R=0$ at $T<T_{c0}$ when $0<I<I_c$ and $\eta=0$. For a better understanding of the physical meaning of (11b) we introduce, in accordance with the two vortex activation processes (9), two critical exponents η_{TI} and η_I, each of which determines the order in the system when the other process (9) or (11a) is neglected:

$$\eta_{TI}=\varepsilon_0/2\tau,\tag{11c}$$

$$\eta_I=\varepsilon_0/2.\tag{11d}$$

Consequently $\eta_{TI}=\eta_I$ for $\tau(T_{cT})=1$ ($T_{cT}\approx 2.2$ K), so that for $T_{cT}<T<T_c$ the topological order is disrupted mainly as a result of the breaking of the thermally activated pairs by the current. As $T\to T_c$ we have $\eta_{TI}\to 0$ and according to (11d) the order of the system at $T<T_c$ no longer depends on temperature explicitly, but only via $I_c(T)$.

It follows from (11b) and (11c) that the critical exponent of the system, and by the same token also the value of the universal jump, are at $T=T_{c0}$ linear functions of the current (Fig. 7):

$$\eta(T_{c0}^-,\ \varepsilon_0)=\begin{cases}\varepsilon_0/2\tau_{c0}, & \varepsilon_0\to 0,\quad \tau_{c0}\equiv\tau(T_{c0})\\ \varepsilon_0/(1+\tau_{c0}), & \varepsilon_0\to\infty\end{cases}.\tag{12a}$$

For average current values Eq. (11b) is transformed in the broad interval $(1-\tau_{c0})<\varepsilon_0<\infty$ into

$$\eta(T_{c0}^-,\ \varepsilon)=\eta_{\rm KT}^0+\varepsilon_0/(1+\tau_{c0}),\quad \eta_{\rm KT}^0\equiv 0{,}25\mp 0{,}02,\tag{12b}$$

$\eta_{\rm KT}^0$ practically coincides with the value obtained in Ref. 29 for a $2D$ system at $T=T_{\rm KT}$ in the zero-current approxima-

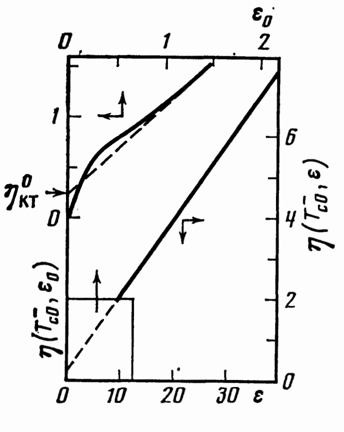

FIG. 7. Dependence of the critical exponent η on ε (or ε_0) at $T=T_{c0}^-$: solid lines—from Eq. (11b), dashed—from (12b), $\eta_{\rm KT}^0\equiv\frac{1}{4}$.

tion. If the result (12b) is not assumed to be fortuitous, it constitutes an expression for the universal jump (allowing for $I \neq 0$) in $2D$ systems. Assuming for $2D$ systems the same processes of free-vortex activation by current, we obtain with the aid of (11a) the concentrations, corresponding to (9), of the free vortices for $2D$ systems, by solving the inverse problem relative to n_f (Ref. 11):

$$n_{fI}{}^{2D}=n_{fI}, \tag{13a}$$

$$n_{fT}{}^{2D}=(1+2\eta_{KT}\tau/\varepsilon_0)^{-1/2\eta_{KT}}\neq n_{fT}, \tag{13b}$$

where η_{KT} is the critical exponent in the K–T theory. This empirical result contains a theoretical prediction for $I\to 0$, in the form of a particular case: $n_{fT}^{2D}(I\to 0)\propto I^{\alpha-1}(U\propto I^{\alpha})$. This is a weighty argument in favor of the analysis proposed by us. The strong deviation of the $\eta(I)$ dependence as $I\to 0$ for our sample from the behavior of a $2D$ system obviously demonstrates its characteristic $3D$ properties with spontaneous ordering and with establishment of long-range order at $T\leqslant T_{c0}$ (Fig. 7).

If Eqs. (11) for the critical exponent η describe approximately correctly the real situation in this system, it is necessary to make more precise the physical meaning of the quantities η (or α) and ξ in different regions of variation of the current $I>0$.

The parameter $\eta(I)$ determines the dimensionless binding energy K_0 renormalized, with account of the fluctuations, to the value K_R at a distance $l_c(I)$:

$$K_R=K_R(l_c,\ T)=(2\pi\eta)^{-1} \quad \text{or} \quad \alpha=1+\pi K_R, \tag{14a}$$

and also the value of the coherence length[13,18,21]:

$$\xi_-=r_0\exp[1/(1/\eta-4)] \quad \text{for} \quad T<T_{c0}; \tag{14b}$$

ξ_- characterizes the mean distance between vortices coupled into pairs and is similar to the analogous quantity in an ordinary superconductor. We can therefore define an energy gap $\Delta\propto 1/\xi_-$ such that at $T<T_{c0}$ the free vortices can become excited only at an energy $\gtrsim\Delta$. At $T\gtrsim T_{c0}$ we have $\Delta=0$ and the system is characterized by a certain equilibrium concentration of vortices coupled into pairs, if the distance between them is smaller than or equal to ξ_+. The quantity ξ_+ characterizes the size of the ordered-phase fluctuations. If $l_c>\xi_+$, then $K_R(l_c,\ T>T_{c0})=0$ and $R(I)\sim\xi_+^{-2}=\text{const}$, while $\alpha=1$ (curves 1 and 2 in Fig. 2a). An increase of the current leads to $l_c<\xi_+$ and the behavior of $\eta(I)$ [or $\alpha(I)$] yields information on the spatial renormalization of the vortex interaction (14a).[13] There is at present no detailed theoretical analysis of the behavior of $\alpha(I)$, but it is clear that $K_R(I)$ should be a monotonically increasing function of the current ($I\propto 1/l_c$) in the absence of an appreciable renormalizing effect of current excitations, and a monotonically decreasing one in the inverse situation.

At $T<T_{c0}$ it is reasonable to assume that the value of η determined from (11a) has the meaning of a critical exponent of the entire system in accordance with (14b), so long as $\eta(I)<\frac{1}{4}$, i.e., $\xi_-<\infty$. For $\eta>\frac{1}{4}$, at the same time, the physical meaning of this quantity in (14a) remains the same so long as $l_c<\xi_-(I)$. Consequently $\eta(I_{KT})=\frac{1}{4}$ [or $\alpha(I_{KT})=3$] determines the critical current I_{KT} for which $\Delta\to 0$, and the free vortices produced by the current induce a topological phase transition so that $K_R=0$ at large distances if $I/I_{KT}>1$. The values of I_{KT} obtained with the aid of (11c) and (11d) are marked on Fig. 1a for various temperature by upward arrows. The temperature dependence of the critical current I_{KT} normalized to I_c is shown in Fig. 1d and indicates a similar role of the temperature and current as parameters that determine the topological order of the system. This is also confirmed by Eqs. (11) and (14), where in place of the usual $K_0\propto I_c/k_BT$ we have $K_R\propto 1/\eta\propto I_c/I$, while at intermediate values of I and T we have a superposition of the thermal and current fluctuations.

The critical current I_{KT} is a characteristic parameter that determines the influence of the measurement current on the topological order. Figure 1c, as well as Eq. (11), shows that the behavior of α (and by the same token of K_R) scales with this parameter. It follows from Figs. 1b and 1c that at $I\lesssim I_{KT}$ the experimental points deviate noticeably from the approximation (11), and at $I\approx I_{KT}$ there exists a section $\alpha(I)\approx\text{const}=3$ followed by an abrupt increase $\alpha(I)\to\infty$ at $I\lesssim I_{KT}/2$ (for clarity, straight lines with slope $\alpha=3$ are superimposed on the experimental IVC in Fig. (2a). It appears that the empirical equations (9) must be made more precise in the "critical" region $I\approx I_{KT}$. As $I\to 0$, however, a deviation from (9) can be due also to the zero vortex mobility (10). The characteristic behavior $\alpha=3$ is thus observed at $I_{KT}>I_c$ and becomes more and more pronounced as the temperature is lowered (see Fig. 1).

3.2. Overall picture

The overall picture of our result is shown schematically in Fig. 8. The vertical lines separate the temperature regions in which the influence of the measurement current on the macroscopic properties of the system differs qualitatively and in which the LO and the TLO are disturbed (mainly) by different quasiparticles:

$T>T_{cs}$—normal electrons in islands;

$T=T_{cs}$—superconducting transitions of Nb–Ti islands (filaments); onset of a structure of "quasi-spins" interacting via the proximity effect.

$T_w<T<T_{cs}$—fluctuations of individual quasi-spins that are correlated at close distances;

$T=T_w$—short-range order sufficient for the onset of the first stable vortex formations;

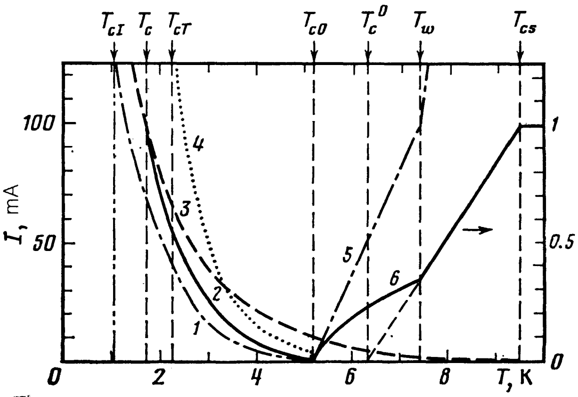

FIG. 8. Phase diagram describing the behavior of the system in various temperature and current ranges. Temperature dependence of the resistance as $I\to 0$. The marked parameters are explained in the text: 1—I_m, 2—I_s, 3—I_c, 4—I_{KT}, 5—I_R, 6—R_0/R_n

$T_c^0 < T < T_w$—ordering of quasispins in the form of free vortices at short distances, and of correlated fluctuations at long ones;

$T = T_c^0$—all the quasispins are ordered in the form of a neutral $2D$ gas of free vortices, formation of first stable pairs of bound vortices at short distance;

$T_{c0} < T < T_c^0$—free and pairwise-bound vortices at long and short distances, respectively;

$T = T_{c0}$ — $3D$—$3D$ analog of K–T transition with a characteristic difference in the value of the universal jump which is a strong function of the current; the behavior of the system as $I \to 0$ recalls the usual transition of a $3D$ structure into a state with LO;

$T_{cT} < T < T_{c0}$—as $I \to 0$ all the vortices are bound into pairs (topological dipoles) whose interaction with one another can lead to formation of more complicated defects (e.g., to topological quadrupoles);

$T_c < T \leqslant T_{cT}$—the renormalizing effect of thermally activated topological defects becomes insignificant, the order of the system is determined predominantly by direct current excitation (n_{fI}). The T_{cT} transition is presumedly due to the fact that at $T \leqslant T_{cT}$ and as $I \to 0$ all the topological dipoles are correlated with one another in the form of multipoles of higher order;

$T = T_c$—the renormalizing effect of the thermal excitation vanishes, i.e., $I_s = I_c$;

$T_{cI} < T < T_c$—the order in the quasi-spin system does not depend (explicitly) on temperature, but is determined only by the current excitations n_{fI};

$T \leqslant T_{cI}$—joining of the Nb–Ti islands thickened by the proximity effect, and by the same token vanishing of the discrete lattice structure; establishment of LO of the superconducting phase over the entire sample.

The qualitative changes in the behavior of the system at various critical temperatures and as $I \to 0$ will be called topological phase transitions. As shown by the systematization above, each topological phase transition can be set in correspondence with an individual type of topological defect, the onset or vanishing of which causes the given transition. In the considered range of measurement currents, the macroscopic properties of the samples at $T < T_{cI}$ correspond to homogeneous $3D$ superconductors. The same result is obtained at $T > T_{cI}$ for $I < I_m$. Here, however, the behavior of the system depends on its dimension l_0 in the XY plane, which determined the critical current I_m that characterizes the appearance of the first free vortices, i.e., $l_c(I_m) = l_0$. Starting from the relation $l_c \propto K_R T/I$ (Ref. 13) and using Eqs. (11) and (14a), we obtain $I_m \propto (a_{\min} T/l_0)^{1/2}$, where $a_{\min} = \min[a_1, a_2]$. According to (10b), the free vortices produced at $I_m < I < I_c$ and $T < T_{c0}$ are immobile, or else have an extremely low mobility; this can produce in this region metastable states with anomalously long relaxation times when the external parameters are varied. We have mentioned above that we have observed similar relaxation effects. They will be analyzed in detail in forthcoming papers. At $I_c < I < I_{\rm KT}$ we have a definite concentration of free vortices with finite mobility (i.e., $R \neq 0$), but these still have little effect on the TLO of the system so long as $\xi_- < \infty$. At $I = I_{\rm KT}$ the renormalizing effect of the current excitations leads to $\xi_- \to \infty$ and by the same token to violation of the TLO. The function $I_{\rm KT}(T)$ in Fig. 4d can consequently be interpreted as the critical line of a topological transition of the K–T type, caused by the measurement current with account taken of the thermal fluctuations. The behavior of $R_0(T)$ as $I \to 0$ on Fig. 8 agrees with the H–N theory, and as the measurement current is increased topological dipoles that are stable at $T_{c0} < T < T_c^0$ and $I < I_R$ begin to dissociate (i.e., $\xi_+ = l_c(I_R)$, see the deviation from Ohm's law in Fig. 1a—curves 1 and 2), or more high-energy fluctuations are excited at $T > T_c^0$ sand $I > I_R$.

We conclude by examining the necessary premises for the applicability of the K–T model to superconducting systems.[2] The characteristic parameter is the effective depth $\lambda_{\rm eff} \equiv 2\lambda_L^2/d_0$ of penetration of the transverse magnetic field. Here λ_L is the London penetration depth and is connected with the screening currents between the superconducting islands. In the limit of small wave vectors we have[19]

$$\lambda_L \approx \frac{\Phi_0}{2\pi}\left[\frac{5}{2\pi z(1-f)k_B T K_R}\right]^{1/2},$$

where z is the number of nearest neighbors and f is the filling factor (for our samples, $z = 6$ and $f = 0.45$). Taking Eqs. (11a) and (14a) into account we obtain

$$\lambda_{\rm eff} \sim (\eta/T) = [2T(\alpha-1)]^{-1},$$

where η is determined at large distances. Consequently $\lambda_{\rm eff} \to \infty$ as $\alpha \to 1$ (or $I \to I_{\rm KT}$) in analogy with the behavior of the coherence length $\xi_- = \xi_-(I)$. By the same token our results indicate that the renormalizing effect of the current-induced topological excitations transfers the characteristic system parameters $\lambda_{\rm eff}$ and ξ_- from the $3D$ regime ($\lambda_{\rm eff}$, $\xi_- \ll d_0$) to the $2D$ regime ($\lambda_{\rm eff}$, $\xi_- \gg d_0$), and this justifies the application of the main premises of the K–T model to bulky samples. The $\alpha(I)$ plots shown in Fig. 1b can be physically interpreted as a manifestation of the measurement-current-induced change of the effective dimensionality of the system (i.e., from $\alpha(I \to 0) \to \infty$, which corresponds to a transition from $3D$ to $2D$ as $I \to I_{\rm KT}$). The quantity $I/I_{\rm KT}$ in Fig. 1b and 1c, which is a feature of a transition to two-dimensional properties, agrees with the estimate $\xi_-(I) \approx l_0$ with allowance for (11) and (14b) ($l_0 = 0.2$ mm and $I/I_{\rm KT} = 0.9$ for our sample). A theoretical analysis of effective-dimensionality variation between $0D$ and $3D$ regimes of critical phenomena as a function of temperature is known for the disordered $3D$ structures of Josephson junctions.[30]

Our experimental study of regular systems points out the important role of allowing for current excitations when intermediate stages of the $0D$–$1D$–$2D$–$3D$, transition are considered, particularly the $2D$ properties.

4. CONCLUSION

Our results show that topological phase transitions, which are similar to K–T transition, and the associated $2D$ topological defects, are observed also in bulky samples and alter substantially their macroscopic properties in a wide temperature range. The observed topological phase transitions explain and systematize the behavior of the IVC in the entire temperature region below T_{cs}, and are characterized by their own specific topological defects.

The main features of the transition of a system into the resistive state can be described by starting from the K–T model, with account taken of the renormalizing effects of the current excitations which induce, at $I \approx I_{\rm KT}$ and $T < T_{c0}$

with increase of current, an analogous topological transitions that transfers the $3D$ propertiers of a bulky sample into the $2D$ regime. All the empirical relations obtained by us, which scale the quantity $\varepsilon = I/I_c$ in accordance with the $2D$ Coulomb-gas model, have a clear physical meaning and can serve as a guide for further theoretical developments. The universal jump at T_{c0} exhibits new characteristic properties, depending on the measurement current and hence on the effective dimensionality of the system.

We have shown that in measurements similar to ours the degree of correlation, and hence the critical exponent, are smoothly regulated by the current. This permits, in particular, a direct experimental study of the physical properties of substances as functions of the topological order, and a direct verification of the corresponding theoretical models.

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