Electrons on surfaces of cryogenic crystals: scattering by lattice vibrations

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(Submitted 21 July 1988)
Zh. Eksp. Teor. Fiz. **95**, 698–708 (February 1989)

A study is reported of the scattering of electrons, moving above the surfaces of solid hydrogen and neon, by internal and surface vibrations of the lattice. It is shown that, in contrast to electrons traveling above the surface of liquid helium, the main reason for the scattering is not the bending of the surface but modulation of the bulk permittivity. Calculations are reported of the energy relaxation time for electrons at a lower transverse quantization level. The process of relaxation is dominated by bulk vibrations. Scattering by these vibrations is not quasielastic, in contrast to the scattering by Rayleigh vibrations.

INTRODUCTION

Electrons on the surface of liquid helium (levitating electrons) have been the subject of experimental investigations for some time (for reviews, see Refs. 1 and 2). Similar experiments with electrons on the surface of solid hydrogen, deuterium, and neon have appeared since the early eighties.[3–12]

The present paper reports calculations of the scattering of such electrons by crystal lattice vibrations. It must be stressed at the outset that the mechanism of the scattering of electrons in the case of a solid "substrate" is very different from the mechanism of the scattering in the case of a liquid "substrate."

The main source of the scattering perturbations originating from a liquid "substrate" are distortions of the image forces (which confine an electron to the vicinity of the surface) due to the bending of the surface of the liquid by capillary waves (this is known as the electron–ripplon interaction). Capillary waves are much "softer" than acoustic waves: if the surface tension is estimated using the expression $\alpha \propto Ms^2/a_0^2$, where M is the mass of an atom, s is the velocity of sound, and a_0 is the interatomic distance, it is found that the frequency of a long ($qa_0 \ll 1$) capillary wave is $\omega_{\text{cap}} \propto sq(qa_0)^{1/2}$, which is much less than the frequency of an acoustic wave $\omega_{\text{ac}} \propto sq$. Soft waves are easier to excite and they scatter more strongly (because of the higher density of states and higher Planck occupation numbers). This is why in the case of scattering of electrons above liquid helium we can ignore internal acoustic vibrations, i.e., we can ignore the compressibility.

However, if the "substrate" is solid, then the surface (Rayleigh) and internal waves have frequencies of the same order of magnitude. This means that we cannot ignore the compressibility and have to allow for internal vibrations. If the compressibility is finite, there is another reason for distortion of the image forces, which is modulation of the permittivity in the bulk of a crystal. This scattering mechanism is stronger than that due to the bending of the surface of a liquid and it includes contributions not only of surface but also of internal vibrations.

In the majority of the reported experiments on electrons on surfaces of cryogenic crystals attention has been concentrated on the effects associated with the elastic scattering of electrons. In these experiments the scattering by lattice vibrations does not appear because of the strong background of the scattering by defects of the crystal surface and by atoms or molecules of the vapor above the surface. However, in the case of the phenomena associated with the heating of electrons and also in the case of weak localization effects the inelastic scattering is important and in this case the scattering by lattice vibrations should predominate. Attempts to detect weak localization of electrons on the surface of solid hydrogen and to determine the dephasing time have already been reported.[11]

1. SCATTERING PERTURBATION

We shall consider the simplest model of the potential of image forces with an untruncated "Coulomb" attraction and an infinitely strong repulsion:

$$U(z)=\begin{cases} -Qe^2/z, & z>0 \\ +\infty, & z<0 \end{cases}. \tag{1}$$

Here, $Q=(\epsilon-1)/4(\epsilon+1)$, where ϵ is the permittivity of the crystal and z is the axis directed along the normal to the surface of the crystal which occupies the half-space $z<0$. The value of Q (and of the other parameters which will be required later) of solid hydrogen and neon are listed in Table I. The parameters of deuterium are not given because they differ from the parameters of hydrogen by amounts smaller than the error inherent in the adopted model. For comparison, Table I lists also the parameters of liquid helium.

Perturbation of the potential U by bending of the surface in the case of a constant value ϵ of is given in Ref. 13. It can be represented in the form

$$\delta U^s(\mathbf{r})=-\frac{Qe^2}{\pi}\int_{z'=0} d^2R'\,\frac{u(\mathbf{R}')-u(\mathbf{R})}{|\mathbf{r}-\mathbf{r}'|^4}. \tag{2}$$

The vector $\mathbf{R}$ is the projection of $\mathbf{r}$ onto the (x, y) plane and $u(\mathbf{R})$ is the normal displacement u_z of the point $\mathbf{R}$ of the surface of a crystal; integration is carried out over the points $\mathbf{r}'$ on the surface of a crystal ($z'=0$).

Perturbation of the potential U because of the weak spatial modulation of ϵ for a constant plane boundary is (see Appendix 1)

$$\delta U^b(\mathbf{r})=\frac{e^2}{2\pi}\,\frac{1}{(\epsilon+1)^2}\int_{z'<0} d^3r'\,\frac{\delta\epsilon(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^4}. \tag{3}$$

The permittivity modulation $\delta\epsilon(\mathbf{r})$ appears because of modulation of the crystal density ρ as a result of vibrations. Using

0038-5646/89/020395-07$04.00

TABLE I. Parameters describing scattering of electrons by phonons above the surface of cryogenic crystals and liquid helium

	H_2	Ne	^{4}He		H_2	Ne	^{4}He
T_{mp}, K	14	25	—	c/s	0,58	0,56	—
Q	$3{,}1\cdot 10^{-2}$	$2{,}4\cdot 10^{-2}$	$6{,}9\cdot 10^{-3}$	ms^2, K	0,5	0,17	0,005
γ^{-1}, Å	17	22	77	ε_γ, K	17	8	0,4
E_1, K	150	91	7,5	ε_γ', K	5	2	—
ρ, g/cm^3	0,08	1,51	0,15	F_0, kV/cm	150	71	1
s, 10^5 cm/s	1,9	1,13	0,2	$\tilde{\tau}$, ms	0,02	1,2	50

the Clausius–Mossoti relationship, we obtain

$$\delta\epsilon=(\epsilon-1)(\delta\rho/\rho)=-(\epsilon-1)\,\mathrm{div}\,\mathbf{u}, \tag{4}$$

where $\mathbf{u}$ is the displacement vector. Assuming that $\epsilon\approx 1$, we can write down

$$\delta U^b(\mathbf{r})=-\frac{Qe^2}{\pi}\int\limits_{z'<0} d^3r'\,\frac{\mathrm{div}\,\mathbf{u}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^4}. \tag{5}$$

In the presence of a surface force F there is also a perturbation[13]

$$\delta U^F(\mathbf{r})=e\mathbf{F}\mathbf{u}(\mathbf{R}). \tag{6}$$

2. SCATTERING PROBABILITY

We shall calculate the scattering probability using methods developed in Ref. 14. The wave function of an electron is

$$\frac{1}{L}e^{i\mathbf{k}\mathbf{R}}\,\psi_n(z), \tag{7}$$

where L^2 is the normalization area; $\mathbf{k}$ is the wave vector of free motion in the (x,y) plane, and $\psi_n(z)$ is the eigenfunction in the potential (1).

We shall average the scattering perturbation $\delta U(\mathbf{r})$ over the motion along the z axis, i.e., we shall calculate

$$\delta\bar{U}_{nn'}(\mathbf{R})=\int dz\,\delta U(\mathbf{r})\rho_{nn'}(z),\quad \rho_{nn'}(z)=\psi_n(z)\psi_{n'}(z). \tag{8}$$

We have allowed here for the fact that the functions ψ_n are real; δU is the sum of the perturbations δU^s, δU^b, and δU^F. The displacement $\mathbf{u}$, which occurs in δU, should be regarded as the phonon field operator in the Heisenberg representation $\mathbf{u}(\mathbf{r},t)$. We shall construct a correlation function

$$\langle\delta\bar{U}_{nn'}(\mathbf{R}_1,t_1)\,\delta\bar{U}_{nn'}(\mathbf{R}_2,t_2)\rangle$$
$$=\int\frac{d\omega}{2\pi}\int\frac{d^2q}{(2\pi)^2}\exp[-i\omega(t_1-t_2)+i\mathbf{q}(\mathbf{R}_1-\mathbf{R}_2)]\langle|\delta\bar{U}_{nn'}|^2\rangle_{\omega\mathbf{q}}. \tag{9}$$

We shall use its Fourier component to express the scattering probability:

$$W_{n\mathbf{k}\to n'\mathbf{k}'}=\frac{1}{\hbar^2L^2}\langle|\delta\bar{U}_{nn'}|^2\rangle_{\omega\mathbf{q}}, \tag{10}$$

where the transferred energy and momentum are

$$\hbar\omega=\varepsilon_{n\mathbf{k}}-\varepsilon_{n'\mathbf{k}'},\quad \mathbf{q}=\mathbf{k}'-\mathbf{k}. \tag{11}$$

The correlation function (9) contains diagonal terms of the form $\langle\delta\bar{U}^s\delta\bar{U}^s\rangle$ as well as cross terms $\langle\delta\bar{U}^s\delta\bar{U}^b\rangle$, i.e., the different scattering mechanisms interfere. We shall consider only the situations when one scattering mechanism predominates. It is then sufficient to know the diagonal correlation functions. A comparison of these functions makes it possible to determine which mechanism predominates.

We shall use the representation

$$\frac{1}{r^4}=\frac{1}{(R^2+z^2)^2}=\int\frac{d^2q}{(2\pi)^2}\,e^{i\mathbf{q}\mathbf{R}}\,\frac{\pi}{z^2}\,\mathcal{K}(qz),$$
$$\mathcal{K}(x)=xK_1(x), \tag{12}$$

where K_1 is the modified Bessel function of the second kind. We note that

$$\mathcal{K}(x)=1+\frac{1}{2}x^2\ln\frac{x}{2}+\ldots,\quad x\to 0. \tag{13}$$

A direct calculation gives the following results. In the case of the perturbation (2), because of the bending of the surface of a crystal, we have

$$\langle|\delta\bar{U}^s_{nn'}|^2\rangle_{\omega\mathbf{q}}=Q^2e^4\langle uu\rangle_{\omega\mathbf{q}}\left\{\int\limits_0^\infty\frac{dz}{z^2}[1-\mathcal{K}(qz)]\rho_{nn'}(z)\right\}^2. \tag{14}$$

This includes the Fourier component of the correlation function of normal displacements of points on the surface. In the case of the perturbation (5), which is due to modulation of the bulk permittivity, we have

$$\langle|\delta\bar{U}^b_{nn'}|^2\rangle_{\omega\mathbf{q}}=Q^2e^4\iint\limits_0^\infty dz_1\,dz_2\iint\limits_{-\infty}^0 dz_1'\,dz_2'\,\rho_{nn'}(z_1)\rho_{nn'}(z_2)$$
$$\times\langle\mathrm{div}\,\mathbf{u}(z_1')\,\mathrm{div}\,\mathbf{u}(z_2')\rangle_{\omega\mathbf{q}}\frac{1}{|z_1-z_1'|^2}\frac{1}{|z_2-z_2'|^2}$$
$$\times\mathcal{K}(q|z_1-z_1'|)\mathcal{K}(q|z_2-z_2'|). \tag{15}$$

In this case we have a correlation function of dilatations

$$\langle\mathrm{div}\,\mathbf{u}(\mathbf{r}_1,t_1)\,\mathrm{div}\,\mathbf{u}(\mathbf{r}_2,t_2)\rangle, \tag{16}$$

which can be used to calculate the Fourier component in terms of $\mathbf{R}_1-\mathbf{R}_2$ and t_1-t_2. Finally, for the perturbation (6), which appears because of a pressing-down field, we have

$$\langle|\delta\bar{U}^F_{nn'}|^2\rangle=\delta_{nn'}(eF)^2\langle uu\rangle_{\omega\mathbf{q}}. \tag{17}$$

The required correlation functions of displacements and dilatations can be found using the familiar relationship between the equilibrium correlation functions and the retarded Green's functions of the phonon field[15]:

$$\langle u_\alpha(\mathbf{r}_1)u_\beta(\mathbf{r}_2)\rangle_\omega=-2\hbar(N_\omega+1)\,\mathrm{Im}\,G_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2|\omega+i0),$$
$$N_\omega=(e^{\hbar\omega/T}-1)^{-1}. \tag{18}$$

Here, G is the displacement along an axis α at the point $\mathbf{r}_1$ when the density of the force directed along an axis β is

$$-\delta(\mathbf{r}-\mathbf{r}_2)e^{-i\omega t}.$$

The function G was calculated in Ref. 16 for an elastic isotropic half-space.

We separate the Bose factors by writing down

$$\langle \mathrm{div}\,\mathbf{u}(z_1)\,\mathrm{div}\,\mathbf{u}(z_2)\rangle_{\omega\mathbf{q}}=2\hbar(N_\omega+1)\,\mathrm{sign}\,\omega K(|\omega|,q|z_1,z_2),$$
$$\langle uu\rangle_{\omega\mathbf{q}}=2\hbar(N_\omega+1)\,\mathrm{sign}\,\omega L(|\omega|,q). \tag{19}$$

We introduce the notation

$$a=(\omega^2/s^2-q^2)^{1/2},\quad b=(\omega^2/c^2-q^2)^{1/2},$$
$$\alpha=(q^2-\omega^2/s^2)^{1/2},\quad \beta=(q^2-\omega^2/c^2)^{1/2}, \tag{20}$$

where s and c are the velocities of longitudinal and transverse sound. Simple but fairly cumbersome operations yield the following results in the case when $\omega>0$. If $q<\omega/s$, then

$$K=\frac{\omega^2}{\rho s^4}\frac{1}{2a}[\cos a(z_1-z_2)+\mathscr{R}\cos a(z_1+z_2)],$$
$$L=\frac{\pi\omega^2}{\rho c^4}\frac{a}{(b^2-q^2)^2+4abq^2},$$
$$\mathscr{R}=[4abq^2-(b^2-q^2)^2]/[4abq^2+(b^2-q^2)^2]; \tag{21}$$

if $\omega/s<q<\omega/c$, then

$$K=\frac{\omega^2}{\rho s^4}\frac{1}{2\alpha}\frac{8\alpha bq^2(b^2-q^2)^2}{16\alpha^2b^2q^4+(b^2-q^2)^4}e^{-\alpha(z_1+z_2)},$$
$$L=\frac{\pi\omega^2}{\rho c^4}\frac{4\alpha^2bq^2}{16\alpha^2b^2q^4+(b^2-q^2)^4}; \tag{22}$$

if $q>\omega/c$, then

$$K=\frac{\omega^2}{\rho s^4}4\beta q^2e^{-\alpha(z_1+z_2)}\pi\delta[4\alpha\beta q^2-(q^2+\beta^2)^2],$$
$$L=\frac{\pi\omega^2}{\rho c^4}\alpha\pi\delta[4\alpha\beta q^2-(q^2+\beta^2)^2]. \tag{23}$$

Equations (21)–(23) are written down for the case when the elastic half-space occupies $z>0$. When they are substituted in Eqs. (14), (15), and (17), the signs of z_1 and z_2 must be reversed.

The delta function occurring in the last two expressions determines the dispersion law of Rayleigh waves:

$$\delta[\ldots]=\frac{c}{q^3}\frac{1}{|f'(\xi_0)|}\delta(\omega-vq),$$
$$f(\xi)=(2-\xi^2)^2-4(1-\xi^2)^{1/2}\left(1-\frac{c^2}{s^2}\xi^2\right)^{1/2},$$
$$v=c\xi_0,\quad f(\xi_0)=0. \tag{24}$$

Here, v is the velocity of Rayleigh waves.

3. SCATTERING AT A LOWER LEVEL

The expressions given in Sec. 2 allow us to calculate, in principle, any scattering characteristics. In order to obtain explicit expressions, we shall consider the scattering at a lower level of transverse motion ($n=n'=1$). At this level we have

$$\psi_1(z)=2\gamma^{3/2}ze^{-\gamma z},\quad \gamma^{-1}=a_B/Q,\quad a_B=\hbar^2/me^2. \tag{25}$$

The energy of an electron at the level $n=1$ is $-E_1+\varepsilon_{\mathbf{k}}$, where the binding energy is

$$E_1=\hbar^2\gamma^2/2m={}^1/_2Q^2R,\quad R=me^4/\hbar^2, \tag{26}$$

whereas $\varepsilon_{\mathbf{k}}=\hbar^2k^2/2m$ is the kinetic energy of motion in the (x,y) plane.

For reasons given in the Introduction we shall calculate the relaxation time $\tilde{\tau}(\varepsilon)$, which is found from the relationship

$$\frac{\varepsilon-\varepsilon_T}{\tilde{\tau}(\varepsilon)}=P(\varepsilon)\equiv\sum_{\mathbf{k}'}W_{\mathbf{k}\to\mathbf{k}'}(\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}'}). \tag{27}$$

Here, ε_T is that value of ε at which the power of the energy losses $P(\varepsilon)$ changes its sign.

Integration with respect to d_2k' in Eq. (27) reduces to integration with respect to $d\omega dq$ (see Appendix 2). It is clear from Eqs. (21)–(24) that K and L depend on ω and q very differently in the range of internal vibrations ($q>\omega/c$) and in the range of surface (Rayleigh) vibrations ($q<\omega/c$). Therefore, we have to calculate separately the contributions made to $P(\varepsilon)$ by bulk and Rayleigh waves: $P=P^B+P^R$.

We shall see later that P^B is dominated by those transitions which are characterized by

$$q\sim k,\quad \hbar\omega\sim\varepsilon, \tag{28}$$

whereas in the case of P^R the important transitions are those characterized by

$$q\sim k,\quad \hbar\omega=\hbar vq\sim\hbar sq\sim(\varepsilon ms^2)^{1/2}\ll\varepsilon. \tag{29}$$

In other words, the scattering by bulk vibrations is inelastic and that by Rayleigh vibrations is quasielastic.

In the calculation of $\tilde{\tau}(\varepsilon)$, we shall confine ourselves to the energy range

$$ms^2\ll\varepsilon\ll E_1. \tag{30}$$

It is clear from Table I that in the case of thermal vibrations ($\varepsilon\sim T$) below the melting point T_{mp} this range covers all the important temperatures with the exception of the very lowest.

We shall calculate the Fourier components of the correlation functions (14), (15), and (17) for internal waves on the basis of the assumptions represented by Eqs. (28) and (30). We note that these assumptions lead to

$$sq/\omega\sim(ms^2/\varepsilon)^{1/2}\ll1. \tag{31}$$

We shall first consider the simpler Fourier components given by Eqs. (14) and (17). The inequality of Eq. (31) allows us to calculate the correlation function of displacements in Eq. (19) in the case when $q=0$, which gives

$$\langle uu\rangle^B_{\omega\mathbf{q}}=(N_\omega+1)\frac{2\pi\hbar}{\omega\rho s}. \tag{32}$$

We can then find directly the correlation function associated with the surface force field:

$$\langle|\delta\bar{U}^F|^2\rangle^B_{\omega\mathbf{q}}=(N_\omega+1)\frac{2\pi\hbar}{\omega\rho s}(eF)^2. \tag{33}$$

In calculation of the integral in Eq. (14) we find that because of the factor $\rho_{11}=\psi_1^2(z)$, we have $z\lesssim\gamma^{-1}$ and, therefore $qz\sim kz\ll1$ follows from Eq. (30). Using the expansion of Eq. (13), we obtain a correlation function due to bending of the surface:

$$\langle|\delta\bar{U}^s|^2\rangle^B_{\omega\mathbf{q}}=Q^2e^4(N_\omega+1)\frac{2\pi\hbar}{\omega\rho s}q^4|\mathscr{L}_q|^2,$$

$$\mathscr{L}_q=-\frac{1}{2}\int_0^\infty dz|\psi_1(z)|^2\ln\frac{qz}{2}. \tag{34}$$

When the correlation function of dilatations in Eq. (19) is expanded in terms of q, we need to retain only two terms of the expansion (this will be demonstrated later), so that

$$\langle\operatorname{div}\mathbf{u}(z_1)\operatorname{div}\mathbf{u}(z_2)\rangle^B_{\omega\mathbf{q}}=(N_\omega+1)\frac{2\hbar\omega}{\rho s^3}$$
$$\times\left[\sin\frac{\omega}{s}z_1\sin\frac{\omega}{s}z_2+4\left(\frac{c}{s}\right)^3\left(\frac{sq}{\omega}\right)^2\cos\frac{\omega}{s}z_1\cos\frac{\omega}{s}z_2\right]. \tag{35}$$

We now consider the integral in Eq. (15). In this integral we have $z\lesssim\gamma^{-1}$ for the same reason as that in the case of Eq. (14). Oscillations of the correlation function of Eq. (35) give $z'\lesssim(\omega/s)^{-1}$. Therefore, according to Eq. (31), we find that $qz'\lesssim sq/\omega\ll1$. Hence, $q|z-z'|\ll1$ and we can assume that $\mathscr{K}=1$. We then obtain the following expression for the correlation function associated with the bulk modulation of the permittivity:

$$\langle|\delta\bar{U}^b|^2\rangle^B_{\omega\mathbf{q}}=Q^2e^4(N_\omega+1)\frac{2\hbar\omega}{\rho s^3}\gamma^2$$
$$\times\left[S^2(t)+4\left(\frac{c}{s}\right)^3\left(\frac{sq}{\omega}\right)^2C^2(t)\right]. \tag{36}$$

Here, $t=\omega/2\gamma s$ and we have

$$C(t)+iS(t)=\int_0^\infty dx\,W(x)e^{itx}, \tag{37}$$

where

$$W(x)=\int_0^\infty dy\frac{y^2e^{-y}}{(x+y)^2}=\begin{cases}1+2x\ln x+\dots, & x\to0\\ 2x^{-2}+\dots, & x\to\infty\end{cases}. \tag{38}$$

At low values of t, we find that

$$C(t)\to1,\quad S(t)=t\ln(a/t)\to0,\quad a\sim1. \tag{39}$$

This is why we have to retain in Eq. (35) the second term which originates from the second term of the expansion of the correlation function of the dilatations of Eq. (19) in terms of q. At high values of t, we obtain

$$S(t)=t^{-1},\quad C(t)\propto t^{-2}\ln t. \tag{40}$$

In this case the second term in Eq. (36) is always small.

In calculating the Fourier components of the correlation functions (14), (15), and (17) for Rayleigh waves we can make the same approximations with respect to $\mathscr{K}$ as in the case of internal waves. We then find that for the mechanism associated with bulk modulation of ϵ, we obtain

$$\langle|\delta\bar{U}^b|^2\rangle^R_{\omega\mathbf{q}}=Q^2e^4(N_\omega+1)\operatorname{sign}\omega\,\delta(|\omega|-vq)$$
$$\times\frac{\xi_0^2}{|f'(\xi_0)|}\left(1-\frac{v^2}{c^2}\right)^{1/2}\frac{8\pi\hbar}{\rho s}q^2\gamma^2; \tag{41}$$

in the case of the mechanism associated with bending of the surface, we find that

$$\langle|\delta\bar{U}^s|^2\rangle^R_{\omega\mathbf{q}}=Q^2e^4(N_\omega+1)\operatorname{sign}\omega\,\delta(|\omega|-vq)$$
$$\times\frac{\xi_0^2}{|f'(\xi_0)|}\left(1-\frac{v^2}{s^2}\right)^{1/2}\frac{\pi^2\hbar}{2\rho c}q^4|\mathscr{L}_q|^2; \tag{42}$$

for the mechanism associated with the surface force field, we have

$$\langle|\delta\bar{U}^F|^2\rangle^R_{\omega\mathbf{q}}=(eF)^2(N_\omega+1)\operatorname{sign}\omega\,\delta(|\omega|-vq)$$
$$\times\frac{\xi_0^2}{|f'(\xi_0)|}\left(1-\frac{v^2}{s^2}\right)^{1/2}\frac{2\pi^2\hbar}{\rho c}. \tag{43}$$

4. COMPARATIVE ROLE OF DIFFERENT SCATTERING MECHANISMS

We shall first estimate the comparative effectiveness of the scattering mechanisms associated with the bending of the surface and with the bulk modulation of the permittivity in the case of bulk waves. In our estimates we shall assume that logarithmic factors are of order unity, which leads to

$$\frac{\langle|\delta\bar{U}^b|^2\rangle^B}{\langle|\delta\bar{U}^s|^2\rangle^B}\propto\begin{cases}(\omega/sq)^4+(\gamma/q)^2, & t\ll1\\ (\gamma/q)^4, & t\gg1\end{cases}. \tag{44}$$

Using the estimates given by Eq. (28) we find that the mechanism for modulation of ϵ dominates in the case when $\varepsilon\ll E_1$ because $k\ll\gamma$. A comparison of Eqs. (41) and (42) readily shows that this mechanism predominates also in the case of surface waves and this is again due to the inequality $\gamma/q\gg1$.

The physical reason for the mechanism of modulation of ϵ being stronger than the mechanism associated with bending of the surface in the $q\ll\gamma$ case is as follows. A displacement of the surface alters the spectrum of an electron in the field of image forces only if the surface is bent. Hence, it follows that δU^s is proportional to the second derivative of u_z $(z=0)$ with respect to x and y. Therefore, for a wave with a momentum q the amplitude of the potential $\delta U^s_{\mathbf{q}}$ is proportional to q^2 at low values of q, as demonstrated by Eq. (34). Bulk modulation of ϵ may distort the spectrum in the field of image forces also when $\delta\epsilon$ is independent of x and y. Therefore, for a wave with a momentum q the amplitude δU^b_q is finite in the limit $q\to0$, as demonstrated in Eq. (36).

We shall now turn to the scattering mechanism associated with the surface force. In the case of internal vibrations we obtain the following estimates from Eqs. (36) and (33):

$$\frac{\langle|\delta\overline{U}^{b}|^{2}\rangle^{B}}{\langle|\delta\overline{U}^{F}|^{2}\rangle^{B}}\sim\begin{cases}\left(\dfrac{F_{0}}{F}\right)^{2}\left[\left(\dfrac{\omega}{\gamma s}\right)^{2}+\left(\dfrac{q}{\gamma}\right)^{2}\right], & t\ll1\\ \left(\dfrac{F_{0}}{F}\right)^{2}, & t\gg1\end{cases}, \tag{45}$$

where

$$F_{0}=Qe\gamma^{2}=Q^{3}(e/a_{B}^{2}). \tag{46}$$

The role of the surface force is greatest at low energies. Assuming that $\varepsilon\sim ms^2$, we find that the minimum value of the ratio (45) in the range of energies defined by Eq. (30) is $(F_0/F)^2(ms^2/E_1)$. It readily follows from a comparison of Eqs. (41) and (43) that the same estimate applies also in the case of surface waves. Therefore, the surface force can become significant only at temperatures $T\lesssim 1\,K$ in fields $F\gtrsim 10$ kV/cm. Therefore, in the case of cryogenic crystals the role of the surface force as the scattering mechanism is considerably less than in the case of liquid helium. This is due to the face that the characteristic field F_0 is two orders of magnitude less for helium.

Obviously, the distortion of the image forces is described by Eqs. (2) and (3) only if we adopt the model of the potential described by Eq. (1). For example, if the barrier at the $z = 0$ boundary is not infinite, we can expect a perturbation due to the fact that the bending of the surface or the bulk deformation near the surface does not alter the barrier height. We shall ignore such effects for two reasons. Firstly, the potential (1) gives a spectrum of levels which in the case of hydrogen differs by no more than 25% from that found by observation.[10] Secondly, inclusion of these effects in scattering would give rise to new unknown parameters (such as the derivative of the height of the barrier with respect to the surface curvature).

5. ENERGY RELAXATION TIME

In this section we calculate the energy relaxation time $\tilde{\tau}(\varepsilon)$ for the scattering within the $n = 1$ lower level of transverse quantization at energies in the range $ms^2\ll\varepsilon\ll E_1$. We allow only for the dominant scattering mechanism due to the permittivity modulation. The different behavior of the Fourier component of Eq. (36) at high and low values of t gives rise to a characteristic energy

$$\varepsilon_{\gamma}=2\hbar\gamma s=2(2ms^{2}E_{1})^{1/2}. \tag{47}$$

It should be noted that in the case of hydrogen the thermal energies $\varepsilon\sim T$ below the melting point T_{mp} lie lower than ε_γ, but in the case of neon they are higher than ε_γ.

We shall consider first the range $\varepsilon\ll\varepsilon_\gamma$. Using Eqs. (36) and (39), we find the contribution of internal modes:

$$P^{B}(\varepsilon)=\frac{\varepsilon}{\bar{\tau}}\frac{\varepsilon}{ms^{2}}\left\{4\left(\frac{c}{s}\right)^{3}\frac{2ms^{2}}{\varepsilon_{\gamma}}\left[2\Phi_{0}\left(\frac{\varepsilon}{T}\right)-\Phi_{1}\left(\frac{\varepsilon}{T}\right)\right]+\left(\frac{\varepsilon}{\varepsilon_{\gamma}}\right)^{3}|\mathscr{L}_{\varepsilon}|^{2}\,\Phi_{4}\left(\frac{\varepsilon}{T}\right)\right\}. \tag{48}$$

We have introduced here a characteristic scattering time

$$\frac{1}{\bar{\tau}}=Q^{5}\frac{R}{\hbar}\frac{m}{2\rho a_{B}^{3}}=\frac{4}{\pi}\frac{E_{1}}{\hbar}\frac{m}{\rho\gamma^{-3}} \tag{49}$$

and functions

$$\Phi_{k}(\xi)=\int_{-\infty}^{1}dx\,\frac{x^{k}}{1-e^{-\xi x}}. \tag{50}$$

In the integration with respect to ω the logarithm occurring in Eq. (39) is taken outside the integral at $\hbar\omega = \varepsilon$, so that

$$\mathscr{L}_{\varepsilon}=\ln(a\varepsilon_{\gamma}/\varepsilon). \tag{51}$$

If $\varepsilon\gg\varepsilon_\gamma$, we similarly obtain

$$P^{B}(\varepsilon)=\frac{\varepsilon}{\bar{\tau}}\frac{\varepsilon_{\gamma}}{ms^{2}}\Phi_{0}\left(\frac{\varepsilon}{T}\right). \tag{52}$$

If $\varepsilon\gg T$, we can assume that $T = 0$ and use

$$\Phi_{k}(\infty)=(k+1)^{-1}. \tag{53}$$

This gives

$$\left.\frac{1}{\bar{\tau}(\varepsilon)}\right|_{T=0}^{B}=\begin{cases}\dfrac{1}{\bar{\tau}}\left\{12\left(\dfrac{c}{s}\right)^{3}\dfrac{\varepsilon}{\varepsilon_{\gamma}}+\dfrac{2}{5}|\mathscr{L}_{\varepsilon}|^{2}\dfrac{\varepsilon^{4}}{2ms^{2}\varepsilon_{\gamma}^{3}}\right\}, & \varepsilon\ll\varepsilon_{\gamma}\\ \dfrac{1}{\bar{\tau}}\dfrac{\varepsilon_{\gamma}}{ms^{2}} & \varepsilon\gg\varepsilon_{\gamma}\end{cases} \tag{54}$$

The simpler expression for $\tilde{\tau}(\varepsilon)$ at $T = 0$ can be used to obtain order-of-magnitude estimates also when $\varepsilon\sim T$.

Since the scattering by bulk vibrations is not quasielastic nor of the low-angle type [see Eq. (25)], the momentum relaxation time $\tau_1(\varepsilon)$ for the scattering by these vibrations is of the same order of magnitude as the energy relaxation time $\tilde{\tau}(\varepsilon)$ found from Eq. (54).

We shall now consider Rayleigh vibrations. In this case there is no characteristic energy ε_γ. In estimating the role of Rayleigh vibrations we shall calculate the momentum relaxation time $\tau_1(\varepsilon)$ for scattering by these vibrations. Since the scattering is quasielastic, it follows that

$$\frac{1}{\tau_{1}(\varepsilon)}=\sum_{\mathbf{k}'}W_{\mathbf{k}\to\mathbf{k}'}(1-\cos\chi), \tag{55}$$

where χ is the angle between **k** and **k**′. Using Eq. (41), we find that at $T = 0$ the required expression is

$$\left.\frac{1}{\tau_{1}(\varepsilon)}\right|_{T=0}^{R}=\frac{1}{\bar{\tau}}\frac{\varepsilon}{\varepsilon_{\gamma}}12\left(\frac{c}{s}\right)^{3}\frac{2\pi\xi_{0}^{2}}{|f'(\xi_{0})|}(1-\xi_{0}^{2})^{1/2}. \tag{56}$$

Comparing Eqs. (54) and (56), we can see that in the energy range

$$ms^{2}\lesssim\varepsilon\lesssim(2ms^{2}\varepsilon_{\gamma}^{2})^{1/3}\equiv\varepsilon_{\gamma}' \tag{57}$$

the contribution of Rayleigh vibrations to the process of momentum relaxation is the same as that of bulk vibrations. If $\varepsilon\ll\varepsilon_\gamma'$, the contribution of Rayleigh vibrations to τ_1 is small. In the case of energy relaxation, it follows that due to quasielasticity of the scattering by Rayleigh vibrations throughout the investigated range of energies given by Eq. (30) the dominant contribution comes from internal vibrations.

In the calculation of $P^B(\varepsilon)$ during the stage of integration with respect to $d\omega dq$ it is clear that the main contribution is indeed made by the range defined by Eq. (28), which justifies the above assumption. It is instructive also to know the surface layer z' which when deformed determines the scattering potential δU^b. If $\varepsilon \ll \varepsilon_\gamma$, the integral in Eq. (37) includes contributions from $x \sim 1$, i.e., $z' \sim \gamma^{-1}$. However, if $\varepsilon \gg \varepsilon_\gamma$, then $x \sim t^{-1}$, i.e., $z' \sim s/\omega \ll \gamma^{-1}$. In the case of scattering by Rayleigh waves the assumption represented by Eq. (29) is justified. We then always have $z \sim \gamma^{-1}$.

We shall now consider the behavior and the orders of magnitude of the energy relaxation time given by Eq. (54). The two terms in the first row of Eq. (54) become comparable when $\varepsilon \sim \varepsilon'_\gamma \ll \varepsilon_\gamma$. Therefore, as ε is increased from ms^2 to ε_γ, the quantity $\tilde{\tau}(\varepsilon)$ first falls proportionally to ε^{-1} and then proportionally to ε^{-4}. If $\varepsilon \gg \varepsilon_\gamma$ the time $\tilde{\tau}(\varepsilon)$ reaches saturation. Some idea of the orders of magnitude is given by the values of $\tilde{\tau}(\varepsilon)$ obtained for $\varepsilon = 1$ K and $\varepsilon = T_{\text{mp}}$. In the case of hydrogen the values are 150 and 10 ns, whereas for neon they are 4 and 0.13 ms. It is clear from Eq. (49) that the much stronger scattering by hydrogen is associataed with the lower density of this crystal, i.e., with a larger amplitude of zero-point vibrations.

The time $\tau_1 = 10$ ns corresponds to a mobility $\mu = 2\times10^7$ cm$^2\cdot$V$^{-1}\cdot$s^{-1}. This mobility is somewhat less than that obtained in Ref. 4, where only a weaker mechanism associated with bending of the surface is allowed for.

The energy relaxation time for the scattering by lattice vibrations should be compared with the corresponding time for the scattering by molecules or atoms in a vapor. In the case of the latter scattering mechanism (see Ref. 2), we have

$$\tau_1 = \frac{8}{3\pi}\frac{m\gamma^{-1}}{\hbar\sigma N}, \tag{58}$$

where N is the vapor density and σ is the cross section for the scattering of an electron by a vapor molecule. The energy relaxation time for the scattering in a vapor is

$$\tilde{\tau} = (M/2m)\tau_1. \tag{59}$$

The experiments reported in Ref. 12 give the scattering cross section for hydrogen as $\sigma = 50$ Å^2. If we assume that $N = 2\times10^{19}$ cm^{-2} (which in the case of hydrogen corresponds to the saturated vapor density at $T = 13$ K), we find that $\tilde{\tau}_{\text{vap}} = 5$ ns. If $\varepsilon = 13$ K, Eq. (54) gives $\tilde{\tau}_{\text{phon}} = 4$ ns for the scattering by lattice vibrations (phonons). Since T decreases the density N falls exponentially, the above estimate means that in the case of thermal electrons above hydrogen the process of energy relaxation by scattering on vapor molecules is important only near the melting point.

The author is grateful to E. I. Rashba, V. B. Shikin, V. S. Édel'man, and V. M. Édel'shtein for discussing the various aspects of this paper.

APPENDIX 1

We shall consider the distribution of the charge $\rho(r)$ in a medium with a permittivity $\epsilon(r)$. The potential $\varphi(r)$ created by this charge satisfies the following equation:

$$\operatorname{div}(\epsilon\nabla\varphi) = -4\pi\rho. \tag{60}$$

The energy of the field is

$$\mathscr{E} = \frac{1}{2}\int d^3r\,\rho\varphi. \tag{61}$$

We shall assume that for a fixed charge ρ the change in the permittivity is $\epsilon \to +\delta\epsilon$. The resultant change in the potential $\delta\varphi$ is easily found by varying Eq. (60):

$$\delta\varphi(\mathbf{r}) = \frac{1}{4\pi}\int d^3r'\,G(\mathbf{r},\mathbf{r}')\operatorname{div}'[\delta\epsilon(\mathbf{r}')\nabla'\varphi(\mathbf{r}')], \tag{62}$$

where the Green's function G satisfies

$$\operatorname{div}[\epsilon(\mathbf{r})\nabla G(\mathbf{r},\mathbf{r}')] = -4\pi\delta(\mathbf{r}-\mathbf{r}'). \tag{63}$$

In the case of a point charge,

$$\rho(\mathbf{r}) = e\delta(\mathbf{r}-\mathbf{r}_0), \tag{64}$$

we have

$$\varphi(\mathbf{r}) = eG(\mathbf{r},\mathbf{r}_0). \tag{65}$$

Then, the change in the energy deduced from Eq. (61) is

$$\delta\mathscr{E} = {}^1/_2 e\delta\varphi(\mathbf{r}_0). \tag{66}$$

Substituting Eq. (65) into Eq. (62) and integrating by parts, we obtain

$$\delta\mathscr{E} = \frac{e^2}{8\pi}\int d^3r'\,\delta\epsilon(\mathbf{r}')\nabla' G(\mathbf{r}_0,\mathbf{r}')\nabla' G(\mathbf{r}',\mathbf{r}_0). \tag{67}$$

If the unperturbed problem corresponds to the half-space characterized by $\epsilon = \text{const}$, then G is readily found by the mapping method, which gives Eq. (3).

APPENDIX 2

If in the course of integration with respect to d^2k' the integrand depends only on the moduli of the momenta $\mathbf{k}$ and $\mathbf{k}'$, and on the angle χ between them, then

$$\int d^2k' \ldots = \frac{2m\varepsilon}{\hbar^2}\int_{-\infty}^{1} dx \int_{t_1(x)}^{t_2(x)} \frac{dt}{R(t,x)} \ldots, \tag{68}$$

where

$$x = \omega/\varepsilon, \quad t = (q/k)^2, \quad t_{1,2}(x) = [1 \mp (1-x)^{1/2}]^2,$$
$$R(t,x) = \{[t - t_1(x)][t_2(x) - 1]\}^{1/2}. \tag{69}$$

The range of integration $(-\infty, 0)$ with respect to dx corresponds to $k' > k$, and the range $(0, 1)$ corresponds to $k' < k$. It is useful to bear in mind that

$$\int\frac{dt}{R} = \pi, \quad \int\frac{dt\,t}{R} = \pi(2-x), \quad \int\frac{dt\,t^2}{R} = \pi(6-6x+x^2).$$

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Translated by A. Tybulewicz