Transformation of the statistical properties of light by a parametric system

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The plane-wave approximation is used to consider one-dimensional propagation of an electromagnetic field in a parametric system. The field is in an arbitrary state which can be squeezed or have a sub-Poisson photon statistics. It is shown that spontaneous parametric scattering plays a fundamental role in determining the light statistics at exit from the system. Conditions under which light with sub-Poisson statistics can be amplified without appreciably affecting its quantum properties are pointed out. When the system is in the absorption state, this ensures that the field has quantum properties throughout the spectral band in which observations are performed. Some of the states of the parametric system can be used, along with heterodyne detection, to detect squeezed states of the field.

Parametric systems are probably the most promising for producing quantum electromagnetic fields. Several specific systems based on this principle have now been proposed. Despite the large number of publications devoted to such systems (see, for example, the reviews in Ref. 1), several questions require further discussion. They include the role of spontaneous parametric scattering (SPS),[2] the effect of the dispersive properties of the medium, nonplane electromagnetic wavefronts, and so on. The interaction of the quantum field with media is of great interest at present. One of our purposes in this paper is to elucidate the possibility of amplification of light by a parametric system, without destroying its discrete properties. It is well known that this cannot be done with an ordinary resonance amplifier.[3]

Spontaneous parametric scattering, i.e., spontaneous decay of pump quanta into signal and idler wave quanta, plays a fundamental part in the correct description of the noise exhibited by light as it interacts with a parametric system. It is widely believed that a parametric system is a noiseless system (see, for example, Ref. 4). On the other hand, this is completely consistent with reality because the intrinsic noise level of the parametric system that is due to spontaneous parametric scattering is much lower than, for example, the intrinsic noise level of a resonance amplifier that is due to spontaneous emission. Under working conditions, SPS generates not only the intrinsic noise of the parametric system, but also beats with the coherently amplified light. This interference component is appreciable and has a significant effect on the light statistics. It destroys the quantum properties of light when the latter passes through a resonance amplifier.[3] In our previous paper,[5] we ignored this component and were therefore led to the incorrect conclusion that the limiting value of the sub-Poisson distribution could be reached.

In the great majority of published papers, the evolution of the statistical properties of light is described exclusively in the time language (see, for example, Ref. 6). This approach gives rise to two difficulties. The first is that the original field can be specified by one-time correlations alone. It is therefore impossible to specify the complete spectral and statistical state of light on the front boundary of the medium. The second difficulty lies in the formal replacement of the time variable t with the space variable z/c when an attempt is made to take the spatial aspect into account. This procedure (like any other intuitive procedure) is undoubtedly correct for many problems, but it can also lead to inadequate predictions of the statistical properties of light. The correct space-time problem must be formulated to avoid this. This was done in Ref. 5 for the case of the parametric system, using a specially adapted formalism of the transport equations for the density matrix of the electromagnetic field.[7]

In some respects, the problem that we shall consider is similar to that discussed in Ref. 8, which was concerned with the transformation of laser generation statistics when a parametric cell was turned on inside the cavity resonator. However, several questions are excluded when we consider the self-consistent resonator problem. Thus, in the case of the resonator, we cannot vary the different amplification and absorption states of the parametric system. It will be clear from our analysis that, by suitably choosing the transformation conditions, we can identify a range of possibilities of the parametric system that are of interest for different problems in statistical quantum optics.

FORMULATION OF THE PROBLEM AND THE OBSERVED SIGNAL

We shall consider the experimental setup illustrated schematically in the figure. Light in the form of the signal wave (SW) with characteristic frequency ω_0, produced by the source S, passes through the parametric system and the filter F with transmission bandwidth $\Delta\omega$. It is intercepted by the detector D whose output photocurrent is subjected to spectral analysis. The output characteristic of this experiment is the quantity[6] ($\hbar = c = e = 1$)

$$i_\omega^{(2)} = \frac{q}{\omega_0} \int_S dS \langle E^+(\mathbf{r}, t) E(\mathbf{r}, t) \rangle + \left(\frac{q}{\omega_0}\right)^2 \int_S dS_1\, dS_2 \times \int_{-\infty}^{+\infty} d\tau\, e^{i\omega\tau} \langle E^+(\mathbf{r}_1, t) E^+(\mathbf{r}_2, t+\tau) E(\mathbf{r}_2, t+\tau) E(\mathbf{r}_1, t) \rangle, \qquad (1)$$

where S is the surface area of the photocathode, q is the quantum efficiency of the photocathode, and E^+, E are the electromagnetic field operators in the Heisenberg representation. The first term in (1) is the photodetection shot noise, whereas the second term is conventionally referred to as the excess noise. In the case of the quantum field, the excess

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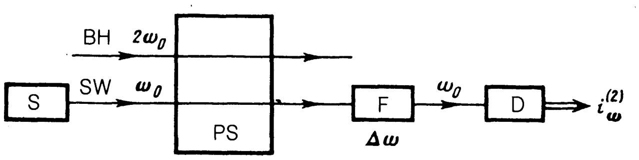

FIG. 1.

noise can be negative, so that only the entire expression given by (1) (and not each term separately) has a physical significance.

From now on, we confine our attention to the plane-wave approximation in which the waves propagate along the z axis, and take the positive-frequency operator E in the form

$$E(z,t)=\sum_{k\sim k_0}\left(\frac{\omega_k}{2L_0S}\right)^{1/2}a_k(t)\,e^{ikz}\approx\left(\frac{\omega_0}{2lS}\right)^{1/2}C(z,t)\,e^{ik_0z},$$

$$\tilde C(z,t)=\sum_{k\sim k_0}\left(\frac{l}{L_0}\right)^{1/2}a_k(t)\,e^{i(k-k_0)z} \tag{2}$$

where the sum is evaluated over wave numbers k in the interval $[k_0-\pi/l,\ k_0+\pi/l]$, $k_0=2\pi\omega_0$. The result is a wave packet with typical linear dimension $l=\Delta\omega^{-1}$. The parameter L_0 is the length of the auxiliary volume of quantization, which is eventually allowed to tend to infinity. The operators a_k and a_k^{+} satisfy the commutation relations $[a_k, a_k{'}^{+}]=\delta_{k,k'}$.

Equation (1) can now be rewritten in the form

$$i_\omega^{(2)}=A\left\{\frac{n(z)}{l}+\frac{2q}{l^2}\operatorname{Re}\int_0^\infty d\tau\,e^{i\omega\tau}g(z,\tau)\right\}. \tag{3}$$

The constant A is not significant and we shall therefore put $A=1$. The parameters $g(z,\tau)$ and $n(z)$ can be expressed in terms of the operators $\tilde C$ and $\tilde C^{+}$, as follows:

$$g(z,\tau)=\langle\tilde C^+(z,t)\tilde C^+(z,t+\tau)\tilde C(z,t+\tau)\tilde C(z,t)\rangle,$$

$$n(z)=\langle\tilde C^+(z)\tilde C(z)\rangle. \tag{4}$$

The operators $\tilde C^{+}$ and $\tilde C$ can be interpreted as the photon creation and annihilation operators at the point c because they obey the commutation rule $[\tilde C(z,t), \tilde C^{+}(z,t)]=1$. We shall find it convenient to use the following commutation relations for free fields:

$$[\tilde C(z,t),\ \tilde C^+(z,t+\tau)]=e^{i\omega_0\tau}l\delta_l(\tau),$$

$$l\delta_l(\tau)=\sum_{k\sim k_0}\frac{l}{L_0}e^{i\omega_k\tau}\Big|_{L_0\to\infty}=\frac{l\sin(\tau/2l)}{\tau/2}. \tag{5}$$

PHYSICAL MODEL AND BASIC EQUATION

The following equation can be written for one of the possible variants of parametric interaction:

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right)C(z,t)=-2i\lambda C^+(z,t),\qquad \tilde C(z,t)=C(z,t)\,e^{-i\omega_0t}. \tag{6}$$

A derivation of this equation is given in Ref. 5 and involves the assumption that the medium consists of fixed atoms that undergo a special $a\leftrightarrow b$ transition of frequency $2\omega_0$ (we recall that ω_0 is the signal-wave frequency). The medium interacts with the pump wave which is in resonance with the $a\leftrightarrow b$ transition. We thus have a degenerate parametric system, in which the idler wave coincides with the signal wave. Moreover, this is a resonant parametric system because the frequency of the pump wave and twice the frequency of the signal wave are equal to the $a\leftrightarrow b$ transition frequency. The right-hand side of (6) is written in the lowest order of perturbation theory in the interaction between the signal wave and the medium. Another important point is that we have neglected variations in the operators $C(z,t)$ over lengths of order l. This is equivalent to the assumption that the continuous space scale has been replaced by a discrete scale with a characteristic parameter l. This means that we have to consider the photon creation and annihilation in the corresponding cell and not at a point. Equation (6) must then be looked upon as written on a "coarse" spatial scale.

The coefficient λ in (6) represents the effectiveness of the interaction between the signal wave and the parametric system, and is given by

$$\lambda={}^1/{}_2 i\varkappa A_p(z,t)\alpha_p(z,t),$$

where α_p is the dimensionless complex amplitude of the pump wave and A_s is the saturated coefficient of amplification (absorption) of the medium for the pump wave. The quantity $\varkappa=f_{ab}/g_{ab}$ is the ratio of two-photon to one-photon interactions, which has the following explicit form:[5]

$$g_{ab}=-i\left(\frac{\omega_0}{2lS}\right)^{1/2}d_{ab}e^{ik_0z},$$

$$f_{ab}=\frac{1}{2}\sum_n g_{an}g_{nb}\left[\frac{1}{\omega_{nb}-\omega}-\frac{1}{\omega_{an}-\omega}\right].$$

It is also assumed for the medium that the widths of the levels that combine with the field are formed largely as a result of decays to other levels. These widths are assumed to be equal and to coincide with the transverse relaxation constant. An incoherent system capable of exciting the working levels is assumed to be present in the medium, so that, in the absence of external electromagnetic fields, the levels have their stationary populations.

Strictly speaking, the presence of the filter in front of the detector (see figure) does not free us from the necessity of taking into account in (6) not only the one selected quasimode, but also all the others, since the parametric interaction process is nonlinear, and the selected mode may be affected by the others. However, this coupling is determined exclusively by the phase matching conditions: the coupled waves are those with wave vectors k_1 and k_2 such that $k_p=k_1+k_2$, where k_p is the wave vector of the pump wave. Since the mode with wave vector $k_0=k_p/2$ is selected at the photocathode, it can only interact with itself,

The choice of the magnitude of $\Delta\omega$ is dictated by two conditions. First, this width must be much greater than the characteristic spectral width of the incident light. Second, we have ignored the dispersion relations typical for parametric systems, so that $\Delta\omega$ must be much smaller than the corresponding spectral scale.

LIGHT AMPLIFICATION COEFFICIENT OF THE PARAMETRIC SYSTEM

If we assume that the pump wave is nonfluctuating and monochromatic ($\lambda = \text{const}(t)$), the solution of (6) at entry to the parametric system is

$$C(z,\ t)=C(0,\ t-z)\,\text{ch}\,\mu+C^+(0,\ t-z)\,\text{sh}\,\mu e^{i\Phi_0}. \qquad (7)$$

The quantity

$$\mu=\int_0^z 2|\lambda(z')|\,dz'=\pm\varkappa(|\alpha_p(z)|-|\alpha_p(0)|)>0$$

determines the efficiency of parametric transformation and is uniquely related, as we shall see, to the efficiency of pump-wave transformation. The phase

$$\Phi_0=\arg\lambda=\arg(\alpha_p+\varkappa)+\pi/2$$

is the pump-wave phase, except for an unimportant additive component.

The operators $C(0,t)$ and $C^+(0,t)$ at entry to the medium satisfy the commutation relations given by (5) for free fields. Any average can now be constructed with the aid of (7). In particular, the signal-wave power liberated at the photocathode is

$$P(z)=K(z)P(0)+P_{sp}(z). \qquad (8)$$

The quantity

$$K(z)=\text{ch}\,2\mu+\text{sh}\,2\mu\cos\Phi, \qquad (9)$$

which we shall call the amplification coefficient of the parametric system, determines the coherent transformation (amplification or absorption) of the initial light by means of $P(0)$. The second term in (8), namely, $P_{sp}=\omega_0\Delta\omega\text{sh}\mu$, is the SPS power liberated at the photocathode within the bandwith $\Delta\omega$. The expression given by (8) takes into account the fact that $P(z)$ is related to the number $n(z)$ of photons per cell: $P(z)=\omega_0 l^{-1}n(z)$.

The phase $\Phi=\Phi_0-2\varphi$ is equal to the difference between the pump-wave phase and twice the phase of the signal wave φ. Its choice determines the different working states of the parametric system. When $\Phi=0$, the parametric system is an amplifier with amplification coefficient $K=e^{2\mu}$. We then speak of null amplification. Conversely, when $\Phi=\pi$, we have absorption (π-absorption) with amplification coefficient $K=e^{-2\mu}$. We are interested in a particular absorption state for which $\Phi=\pi+\varepsilon$, and the magnitude of ε is much less than unity but, at the same time, still quite large: $e^{-2\mu}\ll\varepsilon\ll 1$ for $\mu\gg 1$. We then have

$$K(z)\approx{}^1/_4\varepsilon^2 e^{2\mu}.$$

The absorption state for which $e^{-4\mu}\ll\varepsilon^2\ll e^{-2\mu}$ will be referred to as ε-absorption.

The transparent state of the parametric system, which arises for $\cos\Phi=-\text{sh}\mu/\text{ch}\mu$ is also interesting. It is readily verified that we then have $K=1$.

REPRESENTATION OF $g(z,\tau)$ IN TERMS OF THE PARAMETERS OF THE SIGNAL-WAVE SOURCE

Several types of correlator arise at entry to the parametric system when (7) is substituted into the fourth-order average given by (4). In addition to the normally ordered average $\langle C^+C^+CC\rangle$, we acquire, for example the average $\langle CCC^+C^+\rangle$ with the antinormal ordering. There are also terms that are sometimes referred to as anomalous, namely, $\langle CC^+CC\rangle$, $\langle CCCC\rangle$, etc. (see Appendix). These averages refer to the front boundary of the medium, and the operators in them commute in accordance with the rules defined by (5).

We shall suppose that a single-wave laser is the source of the initial radiation, which can be ordinary classical radiation with Poisson or super-Poisson photon statistics, or can have discrete properties such as sub-Poisson statistics or a squeezed state. Existing theories of such systems (see, for example, Refs. 8 and 9) then enable us to construct all the averages. However, we then encounter the problem of how to relate the required correlators at entry to the parametric system with the cavity averages for the laser. The correspondence principle provides us with a recipe for doing this: an average with normal and chronological ordering of the operators

$$\langle E^+(t_1)\ldots E^+(t_n)E(t_m')\ldots E(t_1')\rangle,\quad t_1<\ldots<t_n,\ t_m'>\ldots>t_1'$$

on one side of a semitransparent mirror will retain its form on the other side, and can be expressed in terms of the original average, just as in classical electrodynamics in which the field E transforms in TE on passing through the mirror, where T is the transmission coefficient. This can also be stated in the form of a theorem (I. V. Sokolov and M. I. Kolobov). Thus, all the correlators on the front boundary of the medium in the expression for $g(z,\tau)$ must be suitably ordered, so that they can be expressed in the simplest way in terms of the source averages (which are also given in the Appendix). The result is the following expression:

$$l^{-2}g(z,\ \tau)=C^2g_2(\tau)+Cg_1\delta_l(\tau)+g_0\delta_l^2(\tau), \qquad (10)$$

where

$$g_0=n_{sp}^2+n_{sp}\xi_{sp},\quad n_{sp}=\text{sh}^2\mu,\quad \xi_{sp}=\text{ch}^2\mu,$$
$$g_1=4Knn_{sp}+(K-1)n,$$
$$g_2(\tau)=Kn[K\xi\exp(-\Gamma_a\tau)+4n\overline{\psi^2}\exp(-\Gamma_\varphi\tau)\text{sh}^2 2\mu\sin\Phi]$$

in which n is the stationary number of photons in the cavity, ξ is the statistical parameter characterizing the mean square fluctuations in the number of photons in the cavity, $\langle n^2\rangle-\langle n\rangle^2=\langle n\rangle(1+\xi)$, $\overline{\psi^2}$ is the mean square fluctuation in the phase of generated radiation, Γ_a and Γ_φ are the spectral widths of the photon and phase fluctuations, and C is the cavity width (rate of escape of the field from the cavity). Terms independent of τ are discarded in (10).

ROLE OF SPS IN THE EVOLUTION OF THE STATISTICAL PROPERTIES OF LIGHT

The quantity g_0 in (10) is related by its origin to SPS and determines the intrinsic noise of the parametric system in the bandwidth $\Delta\omega$ in which observations are performed. As already noted, this component plays a relatively unimportant role. The quantity g_1 is due to beats between the coherent component of light and SPS. The term g_2 determines the intrinsic noise of the coherently amplified (absorbed) light.

We assume that there is no SPS, i.e., $g_0=g_1=0$. We then have

$$l^{-2}g(z,\tau)=C^2KnK\xi\exp(-\Gamma_a\tau)$$

(for simplicity, we take $\overline{\psi^2}=0$). However, this means that the parameter ξ transforms into $K\xi$ at exit from the parametric system, and becomes negative if $\xi<0$. On the other hand, because of its significance, it cannot be less than -1. It can be verified that, by including the interference term g_1, we can remove this type of difficulty. We may therefore conclude that SPS must be taken into account not only to achieve the correct quantitative estimates for the effect, but also from the fundamental point of view.

Let us now substitute (10) into the original formula given by (3), so that the photocurrent spectrum becomes

$$i_\omega^{(2)}=\bar{i}+qi_{sp}^{(2)}+qi_{coh}^{(2)}+qi_{int}^{(2)}, \tag{11}$$

where the photocurrent shot noise

$$\bar{i}=P(z)=KP(0)+P_{sp}$$

is independent of frequency and is proportional to the power released at the photocathode. The SPS noise (the intrinsic noise of the parametric system)

$$i_{sp}^{(2)}=P_{sp}(\mathrm{sh}^2\mu+\mathrm{ch}^2\mu)$$

is also independent of frequency within the bandwidth $\Delta\omega$ in which the observations are made. The noise of the coherently amplified (absorbed) light

$$i_{coh}^{(2)}=KP(0)\left[2K\xi\frac{C\Gamma_a}{\Gamma_a^2+\omega^2}+\frac{8n\overline{\psi^2}}{K}\frac{C\Gamma_\varphi}{\Gamma_\varphi^2+\omega^2}4\,\mathrm{ch}^2\mu\,\mathrm{sh}^2\mu\,\sin^2\Phi\right]$$

has characteristic resonance frequencies due to their presence in the original light noise. The interference noise

$$i_{int}^{(2)}=KP(0)\cdot 4\mathrm{sh}^2\mu+P(0)(K-1)$$

is also independent of frequency within the band-width $\Delta\omega$.

It is readily verified that, unless SPS is taken into account, the quantity $i_\omega^{(2)}$, known to be positive definite may, in fact, become negative.

Finally, one important point: the presence of the second term in the interference noise indicates the statistical dependence of SPS and the coherently amplified field.

AMPLIFICATION OF LIGHT WITH SUB-POISSON PHOTON STATISTICS

We shall now show that, in the null amplification state, the parametric system can produce a substantial increase in the light power without fundamentally affecting its quantum properties (in particular, the sub-Poisson statistics). As already noted, this cannot be done with an ordinary resonant amplifier.

We shall take the laser described in Ref. 10 as the source of initial radiation. For this system

$$\xi=I^{-1}-\frac{1}{2}\frac{\gamma_b}{\gamma_a+\gamma_b},$$
$$\overline{\psi^2}=0, \Gamma_a=CI(1+I)^{-1},$$

where I is the dimensionless generated power and γ_a and γ_b are the widths of the upper and lower working levels of the laser system. The limiting values $\xi=-\frac{1}{2}, \Gamma_a=C$ are obtained as $\gamma_a\to 0$ (since $I\to\infty$).

We must now write down (11) for the null amplification state ($\Phi=0, K=e^{2\mu}$), substituting $q=1$ and demanding that $\gamma_a\ll\gamma_b$:

$$i_\omega^{(2)}=2P_{sp}\,\mathrm{ch}^2\mu+KP(0)\left[\delta\frac{C^2}{C^2+\omega^2}+K\frac{\omega^2}{C^2+\omega^2}\right],$$
$$\delta=K(1+\tilde{I})\gamma_a/\gamma_b\tilde{I},\quad \tilde{I}=I\gamma_a/\gamma_b. \tag{12}$$

Since $\omega\ll C$, the noise level remains substantially below the shot level, provided

$$P_{sp}(z)\ll P(0),\quad \delta\ll 1 \tag{13}$$

(we are considering only effective processes for which $\mu\gg 1$). If the first condition can be satisfied easily, the second is not always satisfied by far because the ratio γ_a/γ_b is small and is readily compensated by factors such as the amplification coefficient K or the quantity $\tilde{I}^{-1}$, which, in contrast to I, can be small.

Conversely, when $\omega>C$, the noise level becomes much higher than the shot level (within the bandwidth $\Delta\omega$) and amounts to $\sim K^2P(0)$.

The ratio of the noise level for $\omega<C$ to the shot level is of the order of ($\tilde{I}>1$) $K\gamma_a/\gamma_b$. The parametric system is thus seen to destroy the quantum properties of light. Nevertheless, they remain well defined provided the conditions given by (13) are satisfied.

SUPPRESSION OF SHOT NOISE IN THE OBSERVATION BANDWIDTH $\Delta\omega$

Let us now consider the π-absorption by the parametric system, for which $K=e^{-2\mu}$ and (12) remains valid. When $\omega>C$, the noise level is determined by the quantity $2P_{sp}\mathrm{ch}^2\mu+K^2P(0)$, and its ratio to the shot component for $P_{sp}\ll P(0)$ is $K\ll 1$, i.e., definitely small if there is effective absorption. The quantum effect is observed in a wide band $\Delta\omega$, and is essentially independent of the statistical properties of the initial laser radiation.

For frequencies $\omega<C$, that are characteristic for the excess noise of laser radiation, the reduction in the noise level relative to the shot level continues in the case of π-absorption until the quantity $KP(0)\delta$ falls to $2P_{sp}\mathrm{ch}^2\mu$, and then begins to increase. An analogous increases will, of course, occur for $\omega>C$, as well, but later, depending on how small the ratio γ_a/γ_b is.

TRANSPARENT PARAMETRIC SYSTEM—A DETECTOR OF SQUEEZED STATES

We now turn to (11) or, more precisely, to the term $i_{coh}^{(2)}$. It contains two resonances at zero frequency, of which the first is proportional to ξ and is due to fluctuations in the energy of the initial light, and the second occurs only when the source of light (in our case, a laser) contains some stabilizing factors that force the field phase into oscillations about its mean: $\varphi-\bar{\varphi}\ll 1$. The first resonance carriers information about, for example, the sub-Poisson statistics, and the second about the squeezed state (for negative values of ξ and $\overline{\psi^2}$). When the parametric system is absent, and light reaches the photocathode directly, it is readily seen that, by putting $\mu=0$ in (11), we retain the information about the sub-Poisson distribution, but information about squeezing is lost.

Thus, if the initial state is squeezed, it does not directly manifest itself. One way of detecting the squeezed state is to perform heterodyne reception whereby this squeezed state is transformed into a state with sub-Poisson statistics, which can then be detected directly. Another method is also implicit in our analysis. A transparent parametric system (i.e., one with $K = 1$), which can be established by observing the condition $\cos\Phi = -\operatorname{sh}\mu/\operatorname{ch}\mu$, is inserted into the path of the squeezed light. The photocurrent spectrum (11) then assumes the following explicit form ($q = 1$):

$$i_\omega^{(2)} = P(0)\left[1 + 2\xi\frac{C\Gamma_a}{\Gamma_a^2+\omega^2} + 4\operatorname{sh}^2\mu\left(1 + 8n\overline{\psi^2}\frac{C\Gamma_\varphi}{\Gamma_\varphi^2+\omega^2}\right)\right].$$

We thus see that the transparent parametric system can serve as a means of detecting a squeezed state. To estimate the situation quantitatively, we must take a specific source of light. Suppose that this is a laser with a parametric cell, or a resonant parametric generator.[8,11] These sources can generate squeezed light for which $8n\overline{\psi^2} = -1$, $\Gamma_a \ll \Gamma_\varphi \approx C$. Consequently, when $\Gamma_a < \omega < C$, noise remains at the shot level, whereas, for $\omega > C$, it rapidly grows for $\mu \gg 1$, remaining constant in frequency within the bandwidth $\Delta\omega$. The presence of the valley in the spectrum up to the shot level is wholly due to the squeezed state of the initial electromagnetic field.

SUPPRESSION OF SHOT NOISE DURING ABSORPTION OF SQUEEZED LIGHT BY THE PARAMETRIC SYSTEM

The transparent parametric system will in no way enable us to use squeezed light for practical purposes, e.g., to increase the precision of optical measurements. As already noted, this essentially distinguishes this type of detection from heterodyne detection, in which the sub-Poisson distribution is produced. However, the parametric system can be used for these purposes. Consider ε-absorption. It occurs for $\Phi = \pi + \varepsilon$ when $e^{-4\mu} \ll \varepsilon^2 \ll e^{-2\mu}$. Let $\varepsilon^2 = e^{-3\mu}$, so that $K = e^{-\mu}/4$. As a result, the expression for $i_\omega^{(2)}$ becomes

$$i_\omega^{(2)} = KP(0)\left[\frac{1}{2}\xi e^{-\mu}\frac{C\Gamma_a}{\Gamma_a^2+\omega^2} + e^{2\mu}\left(1 + 8n\overline{\psi^2}\frac{C\Gamma_\varphi}{\Gamma_\varphi^2+\omega^2}\right)\right].$$

Suppose now that the source produces squeezed light: $8n\overline{\psi^2} \approx -1$, $\Gamma_a \ll \Gamma_\varphi \approx C$. It is clear that, for frequencies in the range $\Gamma_a < \omega < C$, the noise level will be approximately zero, so that the shot component of noise will almost completely cancel out. On the other hand, for frequencies $\omega > C$, the noise will substantially exceed the shot level because of the presence of the factor $e^{2\mu}$.

Hence ε-absorption by the parametric system can be used for precision optical measurements along with the method of heterodyne detection. In some respects, the parametric method is simpler because, for example, it is not subject to power limitations.

APPENDIX

After substitution of (7) in (4), the expression for $g(z,\tau)$ becomes

$$\begin{aligned} g(z,\tau) = {} & \operatorname{ch}^4\mu\langle C_1^+C_2^+C_2C_1\rangle + \operatorname{sh}^4\mu\langle C_1C_2C_2^+C_1^+\rangle \\ & + \operatorname{sh}^2\mu\operatorname{ch}^2\mu[\langle C_1C_2^+C_2C_1^+\rangle + \langle C_1^+C_2C_2^+C_1\rangle \\ & + (\langle C_1C_2C_2C_1\rangle e^{-2i\Phi_0} + \langle C_1C_2^+C_2^+C_1\rangle + \text{к.c.})] \\ & + \operatorname{sh}\mu\operatorname{ch}^3\mu[\langle C_1^+C_2C_2C_1\rangle e^{-i\Phi_0} + \langle C_1C_2^+C_2C_1\rangle e^{-i\Phi_0} + \text{h.c.}] \\ & + \operatorname{sh}^3\mu\operatorname{ch}\mu[\langle C_1C_2C_2^+C_1\rangle e^{-i\Phi_0} + \langle C_1C_2C_2C_1^+\rangle e^{-i\Phi_0} + \text{h.c.}], \end{aligned}$$

where the subscripts 1 and 2 on the operators refer to times t and $t + \tau$, respectively. The relationship between the correlators at entry to the parametric system and the cavity averages of the source is given by

$$\begin{aligned} &\langle C_1^+C_2C_2C_1\rangle = (Cl)^2\langle a_1^+a_2^+a_2a_1\rangle, \\ &\langle C_1C_2C_2^+C_1^+\rangle = (Cl)^2\langle a_1^+a_2^+a_2a_1\rangle + l^2\delta_l(\tau) \\ &\quad + (Cl)l\delta_l(\tau)[\langle a_1^+a_2\rangle + \langle a_2^+a_1\rangle], \\ &\langle C_1C_2^+C_2C_1^+\rangle = \langle C_1C_2C_2^+C_1^+\rangle - (Cl)\langle a_1^+a_1\rangle - 1, \\ &\langle C_1^+C_2C_2^+C_1\rangle = (Cl)^2\langle a_1^+a_2^+a_2a_1\rangle + (Cl)\langle a_1^+a_1\rangle, \\ &\langle C_1C_2C_2C_1\rangle = (Cl)^2\langle a_1a_2a_2a_1\rangle, \\ &\langle C_1C_2^+C_2^+C_1\rangle = (Cl)^2\langle a_2^+a_2^+a_1a_1\rangle + 2(Cl)l\delta_l(\tau)\langle a_2^+a_1\rangle, \\ &\langle C_1^+C_2C_2C_1\rangle = (Cl)^2\langle a_1^+a_2a_2a_1\rangle, \\ &\langle C_1C_2^+C_2C_1\rangle = (Cl)^2\langle a_2^+a_2a_1a_1\rangle + (Cl)l\delta_l(\tau)\langle a_2a_1\rangle, \\ &\langle C_1C_2C_2^+C_1\rangle = \langle C_1C_2^+C_2C_1\rangle + Cl\langle a_1a_1\rangle, \\ &\langle C_1C_2C_2C_1^+\rangle = (Cl)^2\langle a_1^+a_2a_2a_1\rangle + 2(Cl)l\delta_l(\tau)\langle a_2a_1\rangle. \end{aligned}$$

where C is the cavity width.

It follows from the results reported in Refs. 8, 10, and 11 that the internal cavity correlators are given by

$$\langle a_1^+a_2^+a_2a_1\rangle = n^2 + n\xi e^{-\Gamma_a\tau},$$

$$\langle a_2a_2a_1a_1\rangle = [n^2 + n\xi e^{-\Gamma_a\tau} - 4n\overline{\psi^2}(1 + e^{-\Gamma_\varphi\tau})]e^{4i\overline{\varphi}},$$

$$\langle a_2^+a_2^+a_1a_1\rangle = n^2 + n\xi e^{-\Gamma_a\tau} - 4n\overline{\psi^2}(1 - e^{-\Gamma_\varphi\tau}),$$

$$\langle a_1^+a_2a_2a_1\rangle = [n^2 + 2n\overline{\psi^2} + n\xi e^{-\Gamma_a\tau}]e^{2i\overline{\varphi}},$$

$$\langle a_2^+a_2a_1a_1\rangle = \langle a_1^+a_2a_2a_1\rangle,$$

$$\langle a_1^+a_2\rangle = n + {}^1/_4\xi(1 - e^{-\Gamma_a\tau}) - n\overline{\psi^2}(1 - e^{-\Gamma_\varphi\tau}),$$

$$\langle a_2a_1\rangle = [n - {}^1/_4\xi(1 - e^{-\Gamma_a\tau}) - n\overline{\psi^2}(1 + e^{-\Gamma_\varphi\tau})]e^{2i\overline{\varphi}}.$$

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