The process $n \to n\gamma$ in the field of a circularly polarized plane wave

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We obtain an invariant solution of the generalized Dirac equation in the field of a circularly polarized plane wave for an uncharged particle possessing an anomalous magnetic moment. We determine the probability and intensity of photon emission by a neutron in such a field. It consists of four contributions, stemming from wave capture at four frequencies, three of them nonmultiples of the fundamental. To first order in the energy of the wave, the results agree with those previously obtained by the author for this special case. Thus, in conjunction with previous work, this completely solves the problem of the emission by a neutron in the standard set of plane wave fields (linear and circularly polarized waves and crossed constant fields).

Consideration of the electromagnetic interaction of particles with the plane wave field $A = A(\varphi)$, $\varphi = kx$ is an important element of quantum theory, since on the one hand it is a generally applicable approach in a constant crossed field $A = a\varphi$ at ultrarelativistic particle energies,[1] and on the other, it makes it possible in certain cases to calculate probabilities for various processes in terms of the wave amplitude. Besides constant crossed fields, other examples of simple configurations of plane wave fields include those with linear

$$A=a \sin \varphi \tag{1a}$$

or circular

$$A=a_1 \cos \varphi+a_2 \sin \varphi, \quad a_1^2=a_2^2=a^2, \quad (a_1a_2)=0 \tag{1b}$$

polarization.

In the presence of charged-particle interaction, the Dirac equation reduces to the form[2]

$$\Phi'=B'\Phi, \tag{2}$$

where B' is a matrix that combines A and A', and the derivatives are taken with respect to the phase. If the commutator of B and B' vanishes, the solution of Eq. (2) will take the form

$$\Phi=\exp(B)\Phi_0. \tag{3}$$

This is true for any potential involving charged-particle interaction, and the corresponding Volkov solution is universally applicable.[1,2]

In the generalized Dirac equation, which describes the interaction of an uncharged particle that possesses an anomalous magnetic moment with the field of a plane wave, the form of B is such that $[B,B'] = 0$ only for potentials of the form $A = af(\varphi)$, and the approach to obtaining a solution then remains unaltered. For example, in two previous papers[3,4] we have derived a relativistically invariant solution for this case, and more specifically have calculated the probability and intensity of photon emission by a neutron in the field of a linearly polarized wave and in a constant crossed field.[4] In the present paper, we obtain an invariant solution in a circularly polarized wave ($[B,B'] \neq 0$), as well as the probability and intensity of photon emission by a neutron.

The neutron wave function in a plane wave field takes the form

$$\Psi = \Phi \frac{u}{(2p_0)^{1/2}} e^{-i(px)}, \tag{3}$$

where the matrix Φ satisfies Eq. (2), and[3]

$$B=\frac{\mu}{2(kp)}(\hat{k}\hat{A}\hat{p}+\hat{p}\hat{k}\hat{A}). \tag{4}$$

Here $u(p)$ is a Dirac spinor, and μ is the anomalous magnetic moment of the neutron. For the potential (1b), we have

$$B^2=B'^2=-z^2, \quad z=(-\mu^2a^2)^{1/2}, \tag{5a}$$

$$(BB')^2=-z^4, \quad B''=-B, \tag{5b}$$

$$BB'=z^2N, \tag{5c}$$

$$N=\frac{1}{-a^2}\left(\hat{a}_1\hat{a}_2-\frac{pa_1}{kp}\hat{k}\hat{a}_2+\frac{pa_2}{kp}\hat{k}\hat{a}_1\right), \tag{5d}$$

with N independent of φ. Φ should thus be expressible as a linear combination of three linearly independent matrices specific to the problem, namely B, B', and BB', and the unit matrix, i.e.,

$$\Phi=f_1(\varphi)+f_2(\varphi)B+f_3(\varphi)B'+f_4(\varphi)BB', \tag{6}$$

where the f_i are phase-dependent scalar coefficients. Substituting Φ into Eq. (2), equating factors of identical matrices on both sides of the resulting equation, and making use of (5a)–(5d), we obtain the system of differential equations

$$f_1'+z^2f_3=0, \quad f_2'-f_3-z^2f_4=0,$$
$$f_1-f_2-f_3'=0, \quad f_2+f_4'=0, \tag{7}$$

which has the solution (up to an overall constant factor)

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -g\dfrac{\delta_g}{z^2} \\ -i\xi\dfrac{\delta_g}{z^2} \\ -i\dfrac{g\xi}{z^2} \end{Bmatrix} e^{i\xi\delta_g\varphi}, \tag{8}$$

where

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$$\delta_g = [(1+4z^2)^{1/2}+g]/2; \quad \xi,\ g=\pm 1. \tag{8a}$$

If we substitute these values into (6), we obtain

$$\Phi(\xi, g) = \left\{1 - \frac{g\delta_g}{2(kp)z(-a^2)^{1/2}}[\hat{k}\hat{a}_{\xi g}\hat{p} + \hat{p}\hat{k}\hat{a}_{\xi g}]e^{-i\xi g\varphi} - i\xi g N\right\} e^{i\xi\delta_g\varphi},$$

$$a_{\xi g} = a_1 + i\xi g a_2. \tag{9}$$

When there is no field ($z\to 0$), we ought to have $\Phi = 1$. This is satisfied uniquely (with normalization $\overline{\Phi}\Phi = I$) by the combination

$$\Phi = [\Phi(1, -1) + \Phi(-1, -1)]/2\varepsilon^{1/2},$$

and after some rearrangement, we finally obtain

$$\varepsilon^{1/2}\Phi = \cos\varphi_- + \frac{\delta_-}{2(kp)z(-a^2)^{1/2}}[\hat{k}\hat{A}_+\hat{p} + \hat{p}\hat{k}\hat{A}_+] - N\sin\varphi_-, \tag{10}$$

where

$$A_+ = a_1\cos\varphi_+ + a_2\sin\varphi_+, \quad \varepsilon = 1+\delta_-^2/z^2, \quad \varphi_\pm = \delta_\pm\varphi,$$
$$\delta_\pm = [(1+4z^2)^{1/2}\pm 1]/2, \tag{10a}$$

and N is defined by Eq. (5d). The validity of the solution (10) can, with (3) taken into account, be checked by direct substitution into the generalized Dirac equation [Eq. (1) of Ref. 3]. It can be seen that in the process $n\to n\gamma$, the present solution leads to the capture from the wave (or release to the wave, in other processes) of four frequencies

$$k_1 = k, \quad k_1' = k(1+4z^2)^{1/2}, \quad k_2 = 2k\delta_+, \quad k_2' = 2k\delta_-, \tag{11}$$

where three of these are nonmultiples of the fundamental. The latter circumstance is quite surprising, and was first pointed out by Ternov *et al.*[5] In a low-intensity wave ($z\to 0$) we have

$$k_1' \approx k_1 = k, \quad k_2 \approx 2k, \quad k_2' \approx 0,$$

which can be interpreted as one- and two-photon capture.

On an elementary level, one can discern the reason for the appearance of anharmonic frequencies, which is formally a result of the harmonic time dependence of the matrix (10) with argument $\delta_\pm \omega t$, (ω is the wave frequency), in the complicated way in which the spin moves in a circularly polarized wave ("precession" relative to the magnetic field of the wave, and "rotation" relative to **k** at frequency ω), which leads to the mixing of the two kinds of corresponding quantized transitions in the process $n\to n\gamma$. A related problem is solved by Landau and Lifshitz[6] for the nonrelativistic theory: they find the wave function of a neutral spin-1/2 particle with an anomalous magnetic moment that is moving in a constant-amplitude magnetic field. The latter is inclined at an angle θ to the z-axis and rotates about it at anglar velocity ω. When $\theta = \pi/2$, the time dependence of the wave function is identical to our result expressed in appropriate notation.

The term $-i\mu\sigma_{\alpha\beta}F^{\alpha\beta}/2$ in the "Dirac equation" is the relativistic generalization of the usual Pauli spin operator[3] for the interaction of a magnetic moment $\hat{\mu}$ with an external field ($\hat{\mu}\cdot\mathbf{H}$). The basic prerequisite for its applicability in the present instance is that the interaction energy be small, and that the wave frequency be low compared with the energy difference between the ground and first excited states of the quark system responsible for the neutron's anomalous moment. The information at our disposal inclines us to believe that this condition is satisfied by a large margin in all cases of present interest.

2. The matrix element of the process $n\to n\gamma$ in a circularly polarized wave consists of four noninterfering contributions which describe the capture of the aforementioned frequencies, and by analogy with Ref. 4, we find the following expressions for the corresponding emission probabilities per unit time for unpolarized particles (p and p' are the initial and final neutron momenta, $\varkappa$ is the photon momentum):

$$W_1 = w_0\left(\frac{\delta_-}{z}\right)^2 \mathcal{F}_1\left\{\frac{1}{4}\mathrm{Sp}[(\hat{p}'+m)(-M'\sigma^{\mu\nu}\tilde{N} + \tilde{N}'^{*}\sigma^{\mu\nu}M)(\hat{p}+m)(\tilde{N}\sigma^{\alpha\beta}M'^{*} - M^{*}\sigma^{\alpha\beta}\tilde{N}'^{*})]g_{\mu\alpha}\varkappa_\nu\varkappa_\beta\right\}, \tag{12a}$$

$$W_1' = w_0\left(\frac{\delta_-}{z}\right)^2 \mathcal{F}_1'\left\{\frac{1}{4}\mathrm{Sp}[(\hat{p}'+m)(-M'\sigma^{\mu\nu}\tilde{N}^{*} + \tilde{N}'\sigma^{\mu\nu}M)(\hat{p}+m)(\tilde{N}^{*}\sigma^{\alpha\beta}M'^{*} - M^{*}\sigma^{\alpha\beta}\tilde{N}')]g_{\mu\alpha}\varkappa_\nu\varkappa_\beta\right\}, \tag{12b}$$

$$W_2 = w_0\left(\frac{\delta_-}{z}\right)^4 \mathcal{F}_2\{{}^1/_4\,\mathrm{Sp}[(\hat{p}'+m) \times M'\sigma^{\mu\nu}M(\hat{p}+m)M^{*}\sigma^{\alpha\beta}M'^{*}]g_{\mu\alpha}\varkappa_\nu\varkappa_\beta\}, \tag{12c}$$

$$W_2' = w_0\mathcal{F}_2'\{{}^1/_4\mathrm{Sp}[(\hat{p}'+m)\tilde{N}'\sigma^{\mu\nu}\tilde{N}^{*}(\hat{p}+m)\tilde{N}^{*}\sigma^{\alpha\beta}\tilde{N}']g_{\mu\alpha}\varkappa_\nu\varkappa_\beta\}, \tag{12d}$$

where we use the notation

$$\tilde{N} = 1 + iN, \quad M = \frac{1}{2(kp)(-a^2)^{1/2}}(\hat{k}\hat{\tilde{a}}\hat{p} + \hat{p}\hat{k}\hat{\tilde{a}}),$$

$$\tilde{a} = a_1 + ia_2,$$

$$w_0 = \frac{\mu^2 m^4}{4p_0\varepsilon^2}, \tag{13a}$$

$$\mathcal{F}_1[G] = \frac{1}{4\pi m^4}\int\frac{d^3\varkappa}{2\varkappa_0}\int\frac{d^3p'}{2p_0'}\delta(p+k_1-p'-\varkappa)G, \tag{13b}$$

and the operators $\mathcal{F}_2$, $\mathcal{F}_2'$, $\mathcal{F}_1'$ are obtained from (13b) by appropriately transforming the captured momentum to k_2, k_2', and k_1'. Furthermore,

$$M' = M(p\to p'), \quad \tilde{N}' = \tilde{N}(p\to p'),$$

and in M^* and $\tilde{N}^*$ we put $a_2 \to -a_2$.

Applying the conservation laws to the expressions in curly brackets (12) leads to the following results:

$$\{(12a)\}=32(k_1p)(k_1p')\left[2+m^2\left(\frac{p}{k_1p}-\frac{p'}{k_1p'}\right)^2\right],$$

$$\{(12b)\}=64m^2[(k_1'p)-(k_1'p')]\left[2+m^2\left(\frac{1}{k_1'p}-\frac{1}{k_1'p'}\right)\right],$$

$$\{(12c)\}=16(k_2p)(k_2p')\left[2+m^2\left(\frac{p}{k_1p}-\frac{p'}{k_1p'}\right)^2\right],$$

$$\{(12d)\}=16(k_2'p)(k_2'p')\left[2+m^2\left(\frac{p}{k_2'p}-\frac{p'}{k_2'p'}\right)^2\right],$$

where we have used the identity[1]

$$\frac{1}{a^2}\left[\left(\frac{a_1p}{kp}-\frac{a_1p'}{kp'}\right)^2+\left(\frac{a_2p}{kp}-\frac{a_2p'}{kp'}\right)^2\right]=\left(\frac{p}{kp}-\frac{p'}{kp'}\right)^2.$$

The invariant integral relations in Ref. 4 may be used to calculate the phase-space integrals. As a result, we obtain

$$W_1=w_0\left(\frac{\delta_-}{z}\right)^2 g(u_1),\qquad u_1=\frac{2(k_1p)}{m^2},\tag{14a}$$

$$W_1'=w_0\left(\frac{\delta_-}{z}\right)^2 \tilde{g}(u_1'),\qquad u_1'=\frac{2(k_1'p)}{m^2},\tag{14b}$$

$$W_2={}^1/_2w_0\left(\frac{\delta_-}{z}\right)^4 g(u_2),\qquad u_2=\frac{2(k_2p)}{m^2},\tag{14c}$$

$$W_2'={}^1/_2w_0 g(u_2'),\qquad u_2'=\frac{2(k_2'p)}{m^2},\tag{14d}$$

where

$$g(u)=\frac{u(u+2)}{(1+u)^2}(u^2-2u-2)+4\ln(1+u),\tag{15a}$$

$$\tilde{g}(u)=\frac{4u(u+2)}{(1+u)}-8\ln(1+u),\tag{15b}$$

and in the nonrelativistic limit, we have

$$g(u)\approx\tilde{g}(u)\approx{}^4/_3u^3.$$

The functions $g(u)$ and $\tilde{g}(u)$ are plotted in Fig. 1. For $z\ll1$, the quantity W_1+W_1', which is proportional to z^2, can be interpreted as the probability of one-photon capture, and to the same order, the result is the same as that obtained in Ref. 4. Note that we then have $W_2\sim z^4$, $W_2'\sim z^6$.

3. The emitted "intensity" of the four photon momenta in a completely circularly polarized wave can be obtained by adding a factor $\varkappa_\sigma$ to the curly brackets in (12). Making use of the integral relations from Ref. 4 once again, we obtain four expressions for the emitted intensity, corresponding to the four types of capture:

$$I_\sigma^{(1)}=w_0\left(\frac{\delta_-}{z}\right)^2[f(u_1)k_{1\sigma}+h(u_1)p_\sigma],\tag{16a}$$

$$I_\sigma^{(1)\prime}=w_0\left(\frac{\delta_-}{z}\right)^2[\tilde{f}(u_1')k_{1\sigma}'+\tilde{h}(u_1')p_\sigma],\tag{16b}$$

$$I_\sigma^{(2)}={}^1/_2w_0\left(\frac{\delta_-}{z}\right)^4[f(u_2)k_{2\sigma}+h(u_2)p_\sigma],\tag{16c}$$

$$I_\sigma^{(2)\prime}={}^1/_2w_0[f(u_2')k_{2\sigma}'+h(u_2')p_\sigma],\tag{16d}$$

where

$$f(u)=\frac{2}{3(1+u)^3}(u^5+u^4+3u^3+22u^2+30u+12)-\frac{8}{u}\ln(1+u),\tag{17a}$$

$$h(u)=\frac{u}{3(1+u)^3}(u^4-u^3-22u^2-30u-12)+4\ln(1+u),\tag{17b}$$

$$\tilde{f}(u)=\frac{4}{3(1+u)^2}(u^3-4u^2-18u-12)+\frac{16}{u}\ln(1+u),\tag{17c}$$

$$\tilde{h}(u)=\frac{4u}{3(1+u)^2}(2u^2+9u+6)-8\ln(1+u),\tag{17d}$$

and in the nonrelativistic limit,

$$f(u)\approx{}^{16}/_{15}u^4,\ h(u)\approx{}^2/_3u^4,\ \tilde{f}(u)\approx{}^8/_{15}u^4,\ \tilde{h}(u)\approx{}^2/_3u^4.$$

The functions $f(u)$ and $h(u)$ are plotted in Fig. 2 and $\tilde{f}(u)$ and $\tilde{h}(u)$ are plotted in Fig. 3. For $z\ll1$, the quantity $I_\sigma^{(1)}+I_\sigma^{(1)\prime}$ is proportional to z^2, and we can interpret it to be the radiation intensity in one-photon capture, in agreement with the result obtained to the same order in Ref. 4.

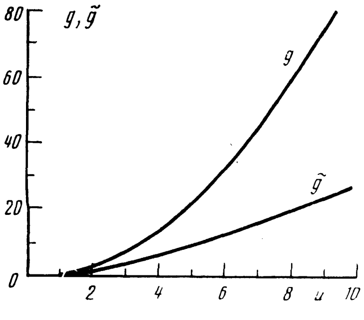

FIG. 1.

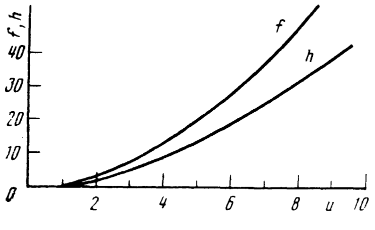

FIG. 2.

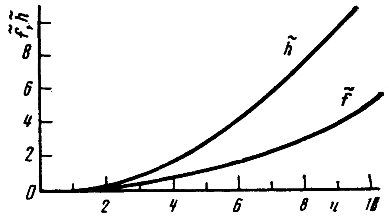

FIG. 3.

4. For neutrons, the largest value of z obtainable with high-power lasers is $\sim 10^{-3}$, so that while the capture effect at anharmonic frequencies in a circularly polarized wave is not very large, it is nonetheless observable. Experimentally, this would typically require being able to distinguish between the contributions due to W_1 and W_1' (or $I_\sigma^{(1)}$ and $I_\sigma^{(1)\prime}$). The latter would go a long way towards elucidating the applicability of allowing for the anomalous magnetic moment of uncharged particles using the generalized Dirac equation.

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