Static emf in metals in the cyclotron resonance region

I. E. Aronov and V. L. Fal'ko

Scientific-Research Institute of Radiophysics and Electronics, Academy of Sciences of the Ukrainian SSR, Kharkov

(Submitted 3 July 1988)
Zh. Eksp. Teor. Fiz. **95**, 301–310 (January 1989)

The existence of a cyclotron parametric instability is predicted for metals when a standing finite-amplitude cyclotron wave is excited. This instability results in strong inhomogeneous heating of electrons so that a static electric field $\mathbf{E}_0(x)$ is created in a metal. A study is made of the dynamics and kinetics of conduction electrons in the range of cyclotron resonances in a cyclotron resonance field and in an induced field $\mathbf{E}_0(x)$. The distribution of electrons and their effective temperature are found and an emf created by them is calculated. It is shown that a static emf is in the form of a resonance when considered as a function of a constant magnetic field $\mathbf{H}_0$. The line profile of a static resonance emf is determined and the conditions for experimental observation of this effect are identified.

INTRODUCTION

A high density of conduction electrons in a metal usually makes it impossible to establish a significant nonequilibrium in the electron system so that the deviation of the average energy of electrons from the Fermi value would exceed considerably the phonon temperature T. This is due to the fact that an electric field in a metal is always weak. In the static case the electric field intensity is limited by the Joule heating of carriers, whereas an alternating electric field is weak because of the skin effect. We can therefore assume[1] that the main source of nonlinear effects in metals is the change in the electron paths under the influence of a magnetic field of a wave. The majority effects of this kind—current states and spontaneous oscillations,[1–3] self-trapping under helicon resonance conditions,[4] and appearance of a current sheet traveling into a metal[5]—are observed at low frequencies which satisfy the inequalities $\omega \ll \Omega_0$, $v_F/l (\Omega_0 = eH_0/mc$ is the cyclotron frequency, H_0 is the intensity of a static magnetic field, l is the mean free path, and v_F is the Fermi velocity). The magnetic field H_1 of a wave can then be regarded as quasistatic compared with the characteristic time scale of the motion of an electron and the frequency ω simply determines the spatial distribution of the field H_1. The energy field of a wave has no significant influence on the electron paths because it is weak compared with the magnetic field H_1 (Refs. 1–5) and can be allowed for by perturbation theory.

A nonlinear cyclotron resonance is observed[7,8] at high frequencies when the frequency of a wave ω is comparable with the cyclotron frequency Ω_0. At high frequencies the dynamics of electrons depends strongly not only on the amplitude and phase of an alternating magnetic field, but also on an electric field along an electron path. This is due to the characteristic features that appear in an electron system under cyclotron parametric resonance (CPR) conditions[9,10] when

$$\Omega_0 \approx s\omega/2 \quad (s=1,\ 2). \tag{1}$$

Near cyclotron resonances a metal becomes transparent to electromagnetic radiation because the weakly damped oscillations of an electron–hole plasma in a metal are excited.[10,11] Cyclotron waves are excited in a magnetic field $\mathbf{H}_0$ parallel to the surface of a sample and they travel at right-angles to the vector $\mathbf{H}_0 (\mathbf{k} \perp \mathbf{H}_0)$, where $\mathbf{k}$ is the wave vector. Depending on the orientation of the vectors representing the electric field of the wave $\mathbf{E}$ and the field $\mathbf{H}_0$, we can distinguish ordinary ($\mathbf{E} \| \mathbf{H}_0$) and extraordinary ($\mathbf{E} \perp \mathbf{H}_0$) cyclotron modes. In metals we can expect propagation of both short-wavelength ($kR \gg 1$) and long-wavelength ($kR \ll 1$) cyclotron waves (R is the Larmor radius). Long-wavelength cyclotron waves are observed in a number of metals[10,11] and in bismuth[12–15] near electron and hole cyclotron resonances when standing waves are excited and $k \approx \pi/L$, where L is the thickness of a sample.

In recent experimental investigations of nonlinear effects in metals near a cyclotron resonance excitation of cyclotron waves of finite amplitude was accomplished.[6,7,14,15] When standing long-wavelength cyclotron waves are excited in a metal, the conditions for a CPR can be established[8,9,16] and then a strong overheating of the electron gas can be expected. When a CPR occurs in the electron system, modulation of the frequency of collective cyclotron motion of electrons gives rise to a parametric instability similar to a parametric instability of mechanical vibrations.[17,18] This instability can be suppressed by nonlinear processes. In the case of metals and semiconductors these nonlinear processes are the energy dependence of the electron relaxation frequencies and their nonlinear dynamics because of the nonquadratic dispersion law. The electron distribution function in the case of a CPR becomes nonequilibrium and isotropic, and it is determined by the average energy measured from the Fermi level ε_F, which exceeds greatly the phonon temperature T. In the field of a standing cyclotron wave under CPR conditions the electron distribution function is nonequilibrium and it depends on the coordinates. This gives rise to gradients of the average energy density of carriers and, as a consequence, creates a static electric field and a static emf, which exhibits a resonance as a function of a constant magnetic field H_0 (Refs. 16 and 19). It should be pointed out that in contrast to lf effects,[1] in the case of a CPR even the electric field of a cyclotron wave in a metal has a considerable influence on the nonlinear dynamics of electrons.

We shall predict and investigate a cyclotron parametric instability excited in metals by a standing cyclotron wave of finite amplitude. This instability is responsible for a strong inhomogeneous resonant heating of conduction electrons,

 0038-5646/89/010170-06$04.00

which rectifies an alternating hf signal, i.e., creates a static electric field (zeroth harmonic) and a resonant static emf in a metal.

1. FORMULATION OF THE PROBLEM. DYNAMICS OF ELECTRONS

We shall consider a metal plate of thickness L subjected to a static and homogeneous magnetic field $\mathbf{H}_0 \| z$ oriented parallel to the surface of the plate. The x axis is directed along the inward normal to the surface. An external electromagnetic wave of frequency ω excites a standing cyclotron wave inside the metal plate. The components of the cyclotron wave field are described by the following expressions (valid in the case of an extraordinary wave)[10,11]:

$$\begin{aligned} H_z&=H_1 e^{-x/\delta}\cos(kx+\alpha)\cos\omega t, \quad \operatorname{ctg}\alpha=k\delta,\\ E_y&=H_1(\beta\omega/ck)e^{-x/\delta}\sin kx\sin\omega t,\\ E_x&=-H_1(\beta\Omega_0/ck)e^{-x/\delta}\sin kx\cos\omega t,\\ k&=\pi/L, \quad \beta=[1+(k\delta)^{-2}]^{-1/2}, \end{aligned} \tag{2}$$

Here, H_1 the amplitude of the magnetic field of a cyclotron wave:

$$H_1 \ll H_0, \tag{3}$$

where δ is the wave attenuation depth. The cyclotron wave field is described by Eq. (2) in the long-wavelength limit when

$$kR \ll 1, \quad \Omega_0, \ \omega \gg \nu_i, \tag{4}$$

where ν_i is the relaxation frequency of the electron momentum. The spectrum and attenuation of long-wavelength cyclotron waves are well known.[10,11,13,14] The waves in question exist if

$$k\delta \sim \omega/\nu_i \gg 1. \tag{5}$$

We shall assume that long-wavelength cyclotron waves propagate in a metal with a quadratic dispersion law

$$\varepsilon=(p_x^2+p_y^2)/2m+p_z^2/2m_1 \tag{6}$$

(for example, in the case of holes in bismuth we have $m=0.064m_0$ and $m_1=0.703m_0$, where m_0 is the mass of a free electron).

As pointed out in the Introduction, the nonlinearity due to the influence of a finite-amplitude cyclotron wave field on the electron gas, creates a static field $\mathbf{E}_0(x) \| x$. Later we shall find this field self-consistently from the Poisson equation. Therefore, the dynamics of conduction electrons is governed by a constant and magnetic field $\mathbf{H}_0$, the cyclotron wave field of Eq. (2), and the static electric field $\mathbf{E}_0(x)$. The equation of motion of electrons in these fields considered in the approximation of Eq. (3) is

$$\begin{aligned} \ddot{x}+\Omega_0^2(x-x_0)=&-\Omega_1\Omega_0 e^{-x/\delta}\cos\omega t[(2\beta/k)\sin kx\\ &+(x-x_0)\cos(kx+\alpha)]+(e/m)E_0(x). \end{aligned} \tag{7}$$

Here, $\Omega_1=eH_1/mc$; $x_0=\mathscr{P}_y/m\Omega_0$ is the coordinate of the center of the electron orbit; $\mathscr{P}_y$ is the integral of motion;

$$p_x/m=v_x=\dot{x}, \quad p_y/m=v_y=\dot{y}=-\Omega_0(x-x_0), \quad p_z=\text{const}. \tag{8}$$

Obviously the induced field $\mathbf{E}_0(x)$ is a smooth function of the variable x and it does not exceed the amplitude of the cyclotron wave field. In other words, we shall assume that

$$|E_0(x)| \ll H_1(\Omega_0/ck). \tag{9}$$

When the conditions described by Eqs. (3) and (9) are satisfied, the right-hand side of Eq. (7) represents a small perturbation due to the cyclotron wave field and due to the static induced field $\mathbf{E}_0(x)$. However, this small perturbation may have a considerable influence on the electron paths because near the resonances of Eq. (1) the solution has secular terms and the usual perturbation theory is invalid. Therefore, we can solve Eq. (7) by going over to a rotating reference system[18]

$$x=x_0+R_\perp\cos(s\omega t/2+\theta) \quad (s=1,\ 2), \tag{10}$$

where the variables $R_\perp$ and θ are slow functions of time. We shall give the equations of motion for these variables in the scale of the "slow" time near the first and second cyclotron resonances ($s=1,2$). In the case of weak spatial dispersion [Eq. (4)] we find after averaging—subject to Eq. (10)—over the "fast" time that the principal parametric resonance in the $s=1$ case is described by

$$\begin{aligned} \dot{\xi}&=\varkappa_1(\xi_0)\xi\sin 2\theta,\\ \dot{\theta}&=\bar{\Delta}+\varkappa_1(\xi_0)\cos 2\theta; \end{aligned} \tag{11}$$

if $s=2$, then

$$\begin{aligned} \dot{\xi}&=\varkappa_2(\xi_0)\sin\theta,\\ \dot{\theta}&=\Delta_2+\varkappa_2(\xi_0)\xi^{-1}\cos\theta. \end{aligned} \tag{12}$$

The following notation is used above:

$$\begin{aligned} &\xi=kR_\perp, \quad \xi_0=kx_0, \quad \Delta_s=\Omega_0-s\omega/2,\\ &\bar{\Delta}=\Delta_1+(e/2m\Omega_0)E_0'(x),\\ &\varkappa_1(\xi_0)=(\Omega_1/4)e^{-\xi_0/k\delta}[2\beta\cos\xi_0+\cos(\xi_0+\alpha)-(2\beta/k\delta)\sin\xi_0],\\ &\varkappa_2(\xi_0)=\Omega_1\beta e^{-\xi_0/k\delta}\sin\xi_0. \end{aligned} \tag{13}$$

The prime denotes the derivative with respect to the argument; Δ_s is the detuning from a resonance and the quantities $\varkappa_1(\xi_0)$ and $\varkappa_2(\xi_0)$ determine the rate of energy pumping of the electron system under the CPR conditions in the cases when $s=1$ and $s=2$, respectively.

The equations of motion (11) and (12) can be integrated exactly because integrals of motion exist for them. In the case of the system (11) when $s=1$, we obtain

$$Q_1=\xi^2[\bar{\Delta}+\varkappa_1(\xi_0)\cos 2\theta]. \tag{14}$$

For the system (12) when $s=2$, we find that

$$Q_2=\Delta_2\xi^2+2\varkappa_2(\xi_0)\xi\cos\theta. \tag{15}$$

It must be stressed that the integrals of motion Q_1 and Q_2 represent generalized Hamiltonians of slow canonically conjugate variables ξ and θ. The equations of motion (11) and (12) can be written down in the canonical (Hamiltonian) form:

$$\dot{\xi}=-\partial Q_s/\partial\theta, \quad \dot{\theta}=\partial Q_s/\partial\xi \quad (s=1,2). \tag{16}$$

The quantities defined by Eqs. (14) and (15) determine the phase paths of electrons in the $(R_\perp,\theta)$ plane. At a resonance characterized by $s=1$ [Eq. (11)] the point $R_\perp=0$ is absolutely unstable and the phase paths diverge from this

point. The resonance condition for a cyclotron parametric instability is of the form

$$\varkappa_1^2(\xi_0)>(\tilde{\Delta})^2. \tag{17}$$

Usually the parametric instability is characterized by the threshold condition set by dissipative processes in the system.[8,9,17,18] Therefore, this condition should be found from the kinetic equation which contains a real collision integral for bulk scatterers.[9] It should be pointed out that in the region of the resonance described by Eq. (17) the electron energy rises exponentially with time and the growth rate is $\lambda=[\varkappa_1^2-(\tilde{\Delta})^2]^{1/2}$, which is exactly why weak friction cannot limit the instability[17] so that a threshold should exist. The condition (17) may not be met at various points in a sample, since the pump parameter $\varkappa_1$ is a function of the coordinate of the center x_0 of the electron orbit. This has the effect that a sample may have regions where the inequality (17) is obeyed and electrons have a high energy because of the instability, as well as regions where the condition (17) is disobeyed and there is no instability. Consequently, an inhomogeneously heated electron gas is established and the temperature of this gas exceeds the phonon temperature T and is a function of the coordinates.

If $s=2$ [Eqs. (12) and (15)], the width of the instability region decreases[17,18] compared with the principal resonance ($s=1$) by a factor $(H_0/H_1)\gg 1$, so that the inhomogeneous heating effect is manifested less strongly. Moreover, if $s=1$, there is a shift of the resonance position because of the induced field $E_0(x)$ of Eq. (13) and if $s=2$ there is no such shift.

2. KINETIC EQUATION. SOLUTION METHOD

The purpose of the analysis of the mechanical CPR problem in the preceding section was to provide a qualitative description of the cyclotron parametric instability. However, a consistent allowance for the relaxation of the electron energy and momentum cannot be made in mechanics. A full description of a CPR requires a kinetic approach from which we can deduce the CPR threshold and the distribution function of conduction electrons. The kinetic equation for a metal with a quadratic dispersion law (6) is of the form

$$\frac{\partial F}{\partial t}+v_x\frac{\partial F}{\partial x}+[\Omega(t,x)p_x+eE_x(t,x)+eE_0(x)]\frac{\partial F}{\partial p_x} -[\Omega(t,x)p_y-eE_y(t,x)]\frac{\partial F}{\partial p_y}=\hat{I}\{F\}. \tag{18}$$

Here,

$$\Omega(t,x)=\Omega_0+\Omega_1\exp(-x/\delta)\cos(kx+\alpha)\cos\omega t$$

is the cyclotron frequency modulated by the magnetic field of a cyclotron wave [Eq. (2)]. The collision integral $\hat{I}\{F\}$ describes relaxation of the electron energy and momentum[20,21]:

$$\hat{I}\{F\}=\varepsilon^{-1/2}\frac{\partial}{\partial\varepsilon}\left\{T\varepsilon^{1/2}\nu_e(\varepsilon)\left[\frac{\partial F}{\partial\varepsilon}+\frac{1}{T}F(1-F)\right]\right\} +\nu_i(\varepsilon)\left[\frac{1}{4\pi}\int d\Omega_{\mathbf{p}}F-F\right]. \tag{19}$$

Here, ε is the electron energy. The first term describes the process of electron energy relaxation involving a small amount of energy transferred in each electron–phonon scattering event. The phonon distribution function is assumed to be isotropic and the phonon temperature is T; $\nu_e(\varepsilon)$ is the electron energy relaxation frequency. The second term in Eq. (19) describes elastic scattering and $\nu_i(\varepsilon)$ is the electron momentum relaxation frequency.

This part of the collision integral describes electron momentum relaxation as a result of electron–impurity and electron–phonon scattering. Estimates indicate that in this case the electron–electron scattering is ineffective for a Fermi electron gas[20] and the relaxation to equilibrium is due to the electron–phonon interaction.

The relaxation process is usually quasielastic[20,21] and the following inequality is obeyed:

$$\nu_e\ll\nu_i\,. \tag{20}$$

The kinetic equation (18) can be solved by adopting the rotating reference system of Eq. (10) and the slow variables ξ and θ of Eq. (13). Since we are interested in a static induced field $E_0(x)$, it is sufficient to find just the static part of the distribution function $F_0(\xi,\theta,x_0)$. We can easily see that the field-dependent part of the kinetic equation is governed by the corresponding Hamiltonians expressed in terms of these variables [Eqs. (14) and (15)]. The hf part of the distribution function makes no contribution to the equation for the static part of $F_0(\xi,\theta)$. This is due to separation [Eq. (3)] of the fast (of frequency Ω_0) and slow (with an increment Ω_1) motion. We shall use a spherical coordinate system ε, Φ, and θ defined by the following relationships:

$$R_\perp=\left(\frac{2\varepsilon}{m\Omega_0^2}\right)^{1/2}\sin\Phi,\quad p_z=\left(\frac{2\varepsilon}{m\Omega_0^2}\right)^{1/2}\cos\Phi, \quad x=x_0+\left(\frac{2\varepsilon}{m\Omega_0^2}\right)^{1/2}\sin\Phi\cos\left(\frac{\omega}{n}t+\theta\right), \tag{21}$$

where Φ and θ are the polar and azimuthal angles in the $\mathbf{p}$ space [the expressions for p_x and p_y are given in Eq. (8)]. In terms of these variables the kinetic equation for the static part of the distribution function $F_0(\varepsilon,\theta,\Phi)$ becomes

$$2\varkappa_1(x_0)\varepsilon\sin^2\Phi\sin 2\theta\frac{\partial F_0}{\partial\varepsilon}+\frac{1}{2}\varkappa_1(x_0)\sin 2\Phi \times\sin 2\theta\frac{\partial F_0}{\partial\Phi}+[\Delta+\varkappa_1(x_0)\cos 2\theta]\frac{\partial F_0}{\partial\theta}=\hat{I}\{F_0\},\quad s=1. \tag{22}$$

The corresponding equation for $s=2$ is

$$\varkappa_2\sin\theta\sin\Phi(\varepsilon_0\varepsilon)^{1/2}\frac{\partial F_0}{\partial\varepsilon}+\frac{1}{2}\varkappa_2\sin\theta\cos\Phi\left(\frac{\varepsilon_0}{\varepsilon}\right)^{1/2}\frac{\partial F_0}{\partial\Phi} +\left[\Delta_2+\frac{1}{2}\varkappa_2\left(\frac{\varepsilon_0}{\varepsilon}\right)^{1/2}\frac{\cos\theta}{\sin\Phi}\right]\frac{\partial F_0}{\partial\theta}=\hat{I}\{F_0\}, \tag{23}$$
$$\varepsilon_0=2m\Omega_0^2/k^2.$$

The distribution function $F_0(\varepsilon,\theta,\Phi)$ can be used to determine the electron density $n(x)$, the average energy of electrons $\bar{\varepsilon}(x)$, and the static current $\mathbf{j}(x)$:

$$n(x)=\int d\tau_{\mathbf{p}}F_0,\quad \bar{\varepsilon}(x)=\int d\tau_{\mathbf{p}}\varepsilon(\mathbf{p})F_0, \quad \mathbf{j}(x)=e\int d\tau_{\mathbf{p}}\mathbf{v}F_0. \tag{24}$$

We shall assume that pumping is sufficiently weak so that

$$\Omega_1 \ll \nu_i \ll \Omega_0. \tag{25}$$

In this case the function $F_0(\varepsilon,\theta,\Phi)$ is isotropic in the principal approximation in terms of the parameter $(\Omega_1/\nu_i) \ll 1$. It can be found by the familiar method of expansion of the solution of the kinetic equation in terms of spherical harmonics (see, for example, Ref. 19). The isotropic part of the distribution function is of the Fermi type:

$$F_0 = \left[\exp\left[\frac{\varepsilon-\mu(x)}{T_{eff}(x)}\right]+1\right]^{-1}, \tag{26}$$

where $\mu(x)$ is the chemical potential of electrons and $T_{\text{eff}}(x)$ is the effective temperature of electrons under CPR conditions. The coordinate x occurs in Eqs. (22) and (23) as a parameter, because $kR \ll 1$, and the function $\mu(x)$ can be found from the transport equation and from the condition of normalization of the number of particles.

The expression given in Eq. (26) applies to the specific case of a slight heating of the electron system when the following conditions are obeyed:

$$T/\varepsilon_F \ll \Omega_1^2/\nu_3\nu_i \ll 1. \tag{27}$$

The left-hand side inequality in Eq. (27) is the threshold of the amplitude of a cyclotron wave needed to induce the parametric instability, whereas the right-hand inequality shows that overheating of the electron system is small compared with the Fermi energy ε_F.

The effective electron temperature $T_{\text{eff}}(x)$ should be found from

$$T_{eff}(x) = \begin{cases} T + \frac{2}{5}\mu(x)\varkappa_1^2(x)/\nu_3\left(\frac{T_{eff}}{\Theta_D}\right)\nu_i\left(\frac{T_{eff}}{\Theta_D}\right)\left(1+\frac{4\Delta_1^2}{\nu_i^2}\right), \\ T + \frac{2}{3}m\Omega_0^2\varkappa_2^2(x)/k^2\nu_3\left(\frac{T_{eff}}{\Theta_D}\right)\nu_i\left(\frac{T_{eff}}{\Theta_D}\right)\left(1+\frac{\Delta_2^2}{\nu_i^2}\right), \end{cases} \tag{28}$$

where Θ_D is the Debye temperature.

We have ignored here the term $(eE_0'/2m\Omega_0)$ in the detuning of a resonance in the case when $s = 1$ [Eq. (13)] on the assumption that this term shifts slightly the position of a resonance singularity of the static induced field of $E_0(x)$ and the resonance static emf. (Estimates of the shift of the resonance position and the criterion of validity of the above approximation will be given below.)

The chemical potential of $\mu(x)$ can be found from the transport equation. This can be done by multiplying the kinetic equation (18) in turn by ep_x/m and ep_y/m and integrating this equation with respect to the momenta. The result is a system of equations relating the currents j_x and j_y to the electron density $n(x)$ and their average energy $\bar{\varepsilon}(x)$ [Eq. (24)]. If we use the time averages of these equations and assume that the total current in a sample is zero (when the contacts are open):

$$\langle j_x\rangle_t = \langle j_y\rangle_t = 0, \tag{29}$$

we obtain the transport equation (representing the law of conservation of the total electron momentum) in the form

$$\frac{\partial\bar{\varepsilon}}{\partial x} - \frac{3}{2}eE_0(x)n(x) = 0. \tag{30}$$

Moreover, in order to determine $\mu(x)$, we have to use the normalization of the electron density:

$$n_0 = L^{-1}\int_0^L dx\, n(x) = \langle n(x)\rangle. \tag{31}$$

Here, n_0 is the total density of electrons in a metal.

The chemical potential is

$$\mu(x) = \mu_0 - e\varphi(x) - (\pi^2/4)T_{eff}^2(x)/\varepsilon_F. \tag{32}$$

The quantity μ_0 is found from Eqs. (30) and (31):

$$\mu_0 = \varepsilon_F + e\langle\varphi(x)\rangle + (\pi^2/6)\langle T_{eff}^2(x)\rangle/\varepsilon_F. \tag{33}$$

Here,

$$\varphi(x) = -\int_0^x dx'\, E_0(x')$$

is the potential of the induced electrostatic field.

3. STATIC ELECTRIC FIELD. APPEARANCE OF AN EMF

The induced electric field $E_0(x)$ or the potential $\varphi(x)$ can be found from the Poisson equation:

$$\varphi'' = \lambda^{-2}\{\varphi - \langle\varphi\rangle + (\pi^2/6e\varepsilon_F)\,[T_{eff}^2(x) - \langle T_{eff}^2(x)\rangle]\} \tag{34}$$

subject to the boundary conditions $E_0(0) = E_0(L) = 0$; $\lambda = (\varepsilon\varepsilon_F/6\pi e^2 n_0)^{1/2}$ is the Debye screening radius; ε is the static permittivity. Since the Debye radius λ is the smallest parameter with the dimensions of length in our problem (for example, in the case of bismuth we have $\lambda \approx 4.5\times10^{-6}$ cm and for sodium we obtain $\lambda \approx 1\times10^{-8}$ cm), Eq. (34) is solved in the quasineutral approximation: $\lambda \to 0$. In this approximation the potential φ is

$$\varphi = \langle\varphi\rangle - (\pi^2/6e\varepsilon_F)\,[T_{eff}^2(x) - \langle T_{eff}^2(x)\rangle]. \tag{35}$$

The static emf is the difference between the potentials at the points where a measuring instrument is connected. Let us assume that the coordinates of these points are $x = 0$ and $x = L_1 \leqslant L$, which leads to

$$\mathscr{E} = \int_0^{L_1} dx\, E_0(x) = \varphi(0) - \varphi(L_1). \tag{36}$$

Therefore, the emf is governed by the effective electron temperature at the points $x = 0$ and $x = L_1$:[1)]

$$\mathscr{E}_s = \frac{\pi^2}{6e\varepsilon_F}[T_{eff}^2(L_1) - T_{eff}^2(0)] \qquad (s=1,2). \tag{37}$$

We shall give the result for the principal CPR ($s = 1$). We shall consider two limiting cases.

1) First, we shall assume that the heating of the electron system is so small that the electron relaxation frequencies ν_e and ν_i can be regarded as independent of T_{eff}. Then, T_{eff} is given by Eq. (28) and the value of $\mathscr{E}_1$ is

$$\mathscr{E}_1 = c_1 \frac{\varepsilon_F}{e}\left[\frac{\Omega_1^2}{\nu_e\nu_i}\frac{1}{1+4(\Delta_1/\nu_i)^2}\right]\Psi_1(L_1,\delta). \quad (38)$$

The quantity $c_1 = [\pi(2\beta+\cos\alpha)^2/40]^2/6$ is a numerical coefficient. The function $\Psi_1(L_1,\delta)$ is governed by the geometry of connection of measuring instruments:

$$\Psi_1 = 1-\exp(-LL_1/\delta)\,[2\beta\cos(\pi L_1/L) + \cos(\pi L_1/L+\alpha) - (2\beta/k\delta)\sin(\pi L_1/L)]^4/(2\beta+\cos\alpha)^4. \quad (39)$$

If contact is made with both faces of a plate, i.e., if $L_1 = L$, then $\Psi_1 = 1 - e^{-4L/\delta}$. If the measuring contacts are located on the $x = 0$ face of the plate and in the middle of the plate $L_1 = L/2$, then

$$\Psi_1 = 1-e^{-2L/\delta}/(3k\delta)^4 \approx 1.$$

We can see that the static emf of Eq. (38) is in the form of a resonance when considered as a function of the constant magnetic field H_0 and it rises rapidly on increase in the pump amplitude $\mathscr{E}_1 \propto H_1^4$.

2) At high pumping rates the effective temperature $T_{\text{eff}}(x)$ increases and, therefore, it should be found from Eq. (28) where we have to allow for the dependences of ν_e and ν_i on T_{eff}. In general,[20,21] the values of ν_e and ν_i for a Fermi gas can be approximated by

$$\nu_e = \nu_{e1} + \nu_{e0}\,(T_{eff}/\Theta_D)^a, \quad \nu_i = \nu_{i1} + \nu_{i0}(T_{eff}/\Theta_D)^b, \quad (40)$$

where $a > 0$ and $b > 0$, and both depend on the overheating of the electron system[20,21]; ν_{e1}, ν_{e0}, ν_{i1}, and ν_{i0} are all constants. The result given by Eq. (38) corresponds to the case when $\nu_e = \nu_{e1}$ and $\nu_i = \nu_{i1}$, and it is independent of T_{eff}. We are considering here the case when the second terms in Eq. (40) are much larger than the first. The effective temperature $T_{\text{eff}}(x)$ is described by

$$T_{eff}(x) = \frac{T}{a+b+1} + \Theta_D\left[\frac{\varepsilon_F}{\Theta_D}\frac{2}{5}\frac{\varkappa_1^2(x)}{\nu_{e0}\nu_{i0}}\right]^{1/(a+b+1)}, \quad (41)$$

so that the resonance emf is given by

$$\mathscr{E}_1 = c_2\frac{\varepsilon_F}{e}\left(\frac{\Theta_D}{\varepsilon_F}\right)^{2(a+b)/(a+b+1)} \times\left[\frac{\Omega_1^2}{\nu_{e0}\nu_{i0}[1+4(\Delta_1/\nu_i)^2]^2}\right]^{2/(a+b+1)}\Psi_2(L_1,\delta). \quad (42)$$

The quantity $c_2 = (\pi^2/6)[(2\beta+\cos\alpha)^2/40]^{2/(a+b+1)}$ is a numerical coefficient. The function Ψ_2 in Eq. (42) is governed, like Ψ_1 in Eq. (38), by the geometry of the measuring contacts:

$$\Psi_2(L_1,\delta) = 1-\exp[-4L_1/\delta(a+b+1)]\,(2\beta+\cos\alpha)^{-4/(a+b+1)} \times[2\beta\cos(\pi L_1/L)+\cos(\pi L_1/L+\alpha) - (2\beta/k\delta)\sin(\pi L_1/L)]^{4/(a+b+1)}. \quad (43)$$

If $L_1 = L$, then

$$\Psi_2 = 1-\exp[-4L/\delta(a+b+1)];$$

whereas for $L_1 = L/2$, we have

$$\Psi_2 = 1-\exp[-2L/\delta(a+b+1)]/(3k\delta)^{4/(a+b+1)} \sim 1.$$

We can see that the emf of Eq. (42) is, as in the case described by Eq. (38), in the form of a resonance when considered as a function of H_0 and it increases on increase in the pump amplitude: $\mathscr{E}_1 \propto H_1^{4/(a+b+1)}$. However, this increase is much slower than that described by Eq. (38), because a and b are subject to the inequalities $4 < (a+b+1) \geqslant 9$ (Ref. 21). The width of a resonance profile increases because the resonance factor $[1+4(\Delta_1/\nu_i)^2]^{-1}$ occurs in Eq. (42) as a term with a smaller power exponent than in Eq. (38). This is due to the fact that an increase in $T_{\text{eff}}(x)$ of the Fermi gas causes a strong rise of the relaxation frequencies ν_e and ν_i of Eq. (40), so that the effective pump parameter $\Omega_1^2/\nu_e\nu_i$ decreases. These changes in the line profile an intensity on increase in the pump amplitude H_1 can be used to determine the parameters a and b of Eq. (40).

It should be pointed out that in the case of a resonance ($s = 1$) the appearance of an induced electric field $E_0(x)$ shifts the position of the resonance of Eq. (13), which is due to the dynamics of electrons in the field of a cyclotron wave and in the static field $E_0(x)$. This shift should be manifested also in the surface impedance near a cyclotron resonance ($2\Omega_0 \approx \omega$) and in the line representing the static resonance emf $\mathscr{E}_1(H_0)$. The shift of the center of the resonance line is of the order of $0.1\Omega_0(kR)^2(\Omega_1^2/\nu_e\nu_i)^2 \ll \Omega_0$ in the case of a slight overheating of the electron gas when it follows from Eq. (40) that $\nu_e \approx \nu_{e1}$ and $\nu_i \approx \nu_{i1}$ are independent of T_{eff}. This shift decreases at high values of T_{eff}.

In the case of the resonance described by $s = 1$ [Eq. (1)] the emf $\mathscr{E}_2$ is still given by Eq. (37). We can see that when contacts are connected to both $x = 0$ and $x = L$ faces, the effective temperature $T_{\text{eff}}(x)$ vanishes at these points [Eq. (28)] and, consequently, the emf also vanishes. It follows that the field should be found from Eq. (34) and we then have $\mathscr{E}_2 \propto (\lambda/R)^4\mathscr{E}_1$, which is considerably less than in the case when $s = 1$ [Eqs. (38) and (42)]. However, if the measuring contacts are located at the points $x = 0$ and $L_1 = L/2$, the temperature difference and the emf do not vanish. This case corresponds to a strong overheating of electrons, since if we assume that ν_e and ν_i are constant, then

$$T_{eff}(L/2) \sim \varepsilon_F(\Omega_1^2/\nu_3\nu_i)/(kR)^2,$$

so that T_{eff} should be found from Eq. (28) allowing for the dependences of the relaxation frequencies ν_e and ν_i on T_{eff} [Eq. (40)]:

$$T_{eff}(x) = \frac{T}{a+b+1} + \Theta_D\left\{\frac{2}{3}\frac{\varkappa_2^2(x)}{\nu_{e0}\nu_{i0}[1+\Delta_2/\nu_i]^2}\frac{m\Omega_0^2}{k^2\Theta_D}\right\}^{1/(a+b+1)} \quad (44)$$

The static resonance emf is then described by

$$\mathscr{E}_2 = c_3\frac{\varepsilon_F}{e}\left(\frac{\Theta_D}{\varepsilon_F}\right)^{2(a+b)/(a+b+1)}(kR)^{-4/(a+b+1)} \times\left\{\frac{\Omega_1^2}{\nu_{e0}\nu_{i0}[1+(\Delta_2/\nu_i)^2]}\right\}^{2/(a+b+1)} e^{-2L/\delta}. \quad (45)$$

Here, $c_3 = (\pi^2/6)(\beta/3)^{2/(a+b+1)}$. The dependence on the pump parameter and the resonance singularity are the same as in Eq. (42). The amplitude of the effect is now $(kR)^{-4/(a+b+1)} > 1$ times greater.

The effect under discussion represents generation of the zeroth harmonic as a result of inhomogeneous resonant heating of electrons in the field of a standing finite-amplitude cyclotron wave. This heating shapes the profile of the electron temperature $T_{\text{eff}}(x)$ in a metal. It follows that the emf that appears in a sample is analogous to the thermoelectric emf when the electron temperature $T_{\text{eff}}(x)$ is governed by nonlinear dynamics in the case of a CPR and scattering processes.

The static resonance emf should be detectable at low temperatures in a sample of bismuth ($L \sim 1$ mm) with the mean free path of electrons $l \sim 0.1$–1 mm in a magnetic field $H_0 \sim 1$ kOe at frequencies $\omega \sim 10^{10}$–10^{11} s^{-1}. The pump amplitude should be $H_1 > 0.1$ Oe. If contacts are then attached at the points $x = 0$ and $x = L/2$, the static resonance emf should be $\mathscr{E} \sim 0.1$ mV. In the case of a metal (such as potassium or copper) estimates indicate that under the same conditions we should have $\mathscr{E} \sim 10^{-2}$ mV.

[1)]This expression is valid whenever the temperature profile $T_{\text{eff}}(x)$ is given for a metal and $T_{\text{eff}} \ll \varepsilon_F$ is obeyed.

[1]V. T. Dolgopolov, Usp. Fiz. Nauk **130**, 241 (1980) [Sov. Phys. Usp. **23**, 134 (1980)].
[2]N. M. Makarov and V. A. Yampol'skiĭ, Zh. Eksp. Teor. Fiz. **85**, 614 (1983) [Sov. Phys. JETP **58**, 357 (1983)].
[3]I. F. Voloshin, S. V. Kravchenko, and L. M. Fisher, Zh. Eksp. Teor. Fiz. **92**, 1050 (1987) [Sov. Phys. JETP **65**, 596 (1987)].
[4]V. I. Bozhko and E. P. Vol'skiĭ, Zh. Eksp. Teor. Fiz. **72**, 257 (1977) [Sov. Phys. JETP **45**, 135 (1977)].
[5]V. T. Dolgopolov and P. N. Chuprov, Solid State Commun. **48**, 165 (1983).
[6]V. F. Gantmakher, G. I. Leviev, and M. R. Trunin, Zh. Eksp. Teor. Fiz. **82**, 1607 (1982) [Sov. Phys. JETP **55**, 931 (1982)].
[7]V. F. Gantmakher, G. I. Leviev, and M. R. Trunin, Zh. Eksp. Teor. Fiz. **85**, 1489 (1983) [Sov. Phys. JETP **58**, 863 (1983)].
[8]I. E. Aronov, E. A. Kaner, and A. A. Slutskin, Solid State Commun. **38**, 245 (1981).
[9]I. E. Aronov and É. A. Kaner, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 223 (1984) [JETP Lett. **40**, 992 (1984)].
[10]E. A. Kaner and V. G. Skobov, Adv. Phys. **17**, 605 (1968).
[11]P. M. Platzman and P. A. Wolff, *Waves and Interactions in Solid State Plasma, Suppl. 13 to Solid State Phys.*, Academic Press, New York (1973).
[12]V. S. Édel'man, Pis'ma Zh. Eksp. Teor. Fiz. **9**, 302 (1969) [JETP Lett. **9**, 177 (1969)].
[13]V. P. Naberezhnykh, D. É. Zherebchevskiĭ, and V. L. Mel'nik, Zh. Eksp. Teor. Fiz. **63**, 169 (1972) [Sov. Phys. JETP **36**, 89 (1973)].
[14]M. R. Trunin, Zh. Eksp. Teor. Fiz. **88**, 1834 (1985) [Sov. Phys. JETP **61**, 1087 (1985)].
[15]M. R. Trunin and V. S. Édel'man, Zh. Eksp. Teor. Fiz. **92**, 988 (1987) [Sov. Phys. JETP **65**, 560 (1987)].
[16]I. E. Aronov and V. L. Fal'ko, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 516 (1988) [JETP Lett. **47**, 603 (1988)].
[17]L. D. Landau and E. M. Lifshitz, *Mechanics, 2nd ed.*, Pergamon Press, Oxford (1969).
[18]N. N. Bogolyubov and Yu. A. Mitropol'skii, *Asymptotic Methods in the Theory of Nonlinear Oscillations*, Gordon and Breach, New York (1964).
[19]I. E. Aronov, O. N. Baranetz, and E. A. Kaner, Solid State Commun. **58**, 355 (1986).
[20]F. G. Bass and Yu. G. Gurevich, *Hot Electrons and Strong Electromagnetic Waves in Semiconductor and Gas-Discharge Plasmas* [in Russian], Nauka, Moscow (1975), Chap. 1.
[21]V. F. Gantmakher and I. B. Levinson, *Scattering of Carriers in Metals and Semiconductors* [in Russian], Nauka, Moscow (1984), Chaps. 4 and 6.

Translated by A. Tybulewicz