Interference phenomena and radiation-force rectification

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It is shown that particles in interfering optical fields are acted upon by a radiation force that varies little over the light wavelength (rectification effect). In a monochromatic field the rectified force is of the order of the spontaneous light pressure and is mainly solenoidal. In a bichromatic field the rectified force is of the order of the stimulated light pressure and can be potential. The potential wells in tunable-laser fields can reach a depth 10 K.

1. INTRODUCTION

It is known[1–3] that a resonance light field can act effectively on the translational motion of atoms. An atom in a spatially homogeneous field of a traveling wave is acted upon by a spontaneous light-pressure field whose magnitude exceeds $\hbar \mathbf{k}\gamma/2$.[2] Here $\mathbf{k}$ is the resonance wave vector, γ is the width of the upper level, and the lower is taken to be the ground level.

Predominant in strong inhomogeneous fields is the gradient force due to stimulated transitions.[1] In a field given by $E(\mathbf{r})e^{-i\Delta t}$ (Δ is the detuning from resonance and d is the dipole moment of the transition) the potential of the gradient force of slow atoms is[3]

$$U(\mathbf{r})=\frac{1}{2}\hbar\Delta\ln\left(1+\frac{2|V(\mathbf{r})|^2}{\Delta^2+\gamma^2/4}\right),\qquad (1)$$
$$V(\mathbf{r})=dE(\mathbf{r})/\hbar.$$

In a standing-wave field $V(x) = V_0 \cos kx$ the potential oscillates in space with a period π/k and has potential wells with a depth on the order of $\hbar V_0$ with $V_0 \sim \Delta \gg \gamma$.

The hysteresis due to spontaneous relaxation produces also a friction force that acts on the atom and depends on the velocity.[3,6–9] In the field of an intense standing wave, at $\Delta < 0$, the atoms execute above-barrier motion, at fixed velocities (bunching in velocity space), while at $\Delta > 0$ the atoms are captured by the potential wells of the standing wave.[10] The velocity bunching effect and localization (channeling) of atoms in a standing wave have recently been observed in experiment.[11,12]

The oscillating character of the gradient wave, however, imposes certain restrictions on its use. We show in the present paper that in a bichromatic field of standing waves the atoms are acted upon by a force of order $\hbar kV$, with a sign that is constant over large ($\gg 1/k$) spatial scales. In other words, the radiation force is rectified in interfering fields.[1)] Note that rectification of a gradient force in a bichromatic field, with no account taken of spontaneous relaxation, was discussed in Refs. 4 and 5. This imposes very stringent resonance conditions on the external field (the Rabi resonance).

We consider here another limiting case ($\gamma t \gg 1$), in which the spontaneous relaxation plays an important role. In particular, the conditions for a generalized ("global") Rabi resonance[14,15] turn out to be much simpler. The spatial structure of the rectified radiation force can be either potential or solenoidal. This means that the atom can be localized in deep potential wells or can rotate.

2. INTERFERENCE PHENOMENA IN A MONOCHROMATIC FIELD

The force acting on an atom in a resonance light field is given by

$$\mathbf{f}=\hbar p\nabla V^* + \text{c.c.},\qquad V(\mathbf{r}t)=dE(\mathbf{r}t)/\hbar.\qquad (2)$$

The induced dipole moment $p(t)$ is determined from the Bloch optical equations for p and for the difference q of the working-level populations:

$$i\left(\frac{d}{dt}+\frac{\gamma}{2}\right)p=V(t)q,\qquad V(t)\equiv V(\mathbf{r}(t),t),\qquad (3)$$
$$\frac{dq}{dt}+\gamma(q+1)=2iV(t)p^*+\text{c.c.}\qquad (4)$$

It is assumed that the atom moves at constant velocity $\mathbf{r} = \mathbf{v}(t)$. We assume hereafter that the atoms move slowly ($kv \ll \gamma$) and represent the radiation force in the form

$$f(\mathbf{r},\mathbf{v})=\mathbf{f}_0(\mathbf{r})+\mathbf{f}_1(\mathbf{r},\mathbf{v}),\qquad (5)$$

where $\mathbf{f}_1$ is the friction force and is proportional to the velocity, while $\mathbf{f}_0$ is independent of the particle velocity. We are interested mainly in the $\mathbf{f}_0$ singularities due to the interference between the fields.

We begin the study of the interference phenomena with the case of a monochromatic external field of the form $V(\mathbf{r})e^{-i\Delta t}$. In the zeroth approximation in velocity we have for the force the expression

$$\mathbf{f}_0(\mathbf{r})=\frac{\hbar q(\mathbf{r})}{|\nu|^2}(\Delta\nabla I+\gamma\mathbf{J}(\mathbf{r})),\qquad q=-(1+2I/|\nu|^2)^{-1},$$
$$I=|V|^2,\qquad \mathbf{J}=\frac{i}{2}(V^*\nabla V-\text{c.c.}),\qquad \nu=\Delta+i\gamma/2.\qquad (6)$$

I is proportional to the field intensity and J has the meaning of the field-momentum flux. For example, in a traveling wave $V(\mathbf{r}) = V_0\exp(i\mathbf{k}\cdot\mathbf{r})$ we have $\nabla I = 0$ and $\mathbf{J} = \mathbf{k}I$. In a standing wave $V(\mathbf{r}) = V_0\cos(\mathbf{k}\cdot\mathbf{r})$ there is no momentum flux ($\mathbf{J} = 0$) and $\nabla I \neq 0$.

The first term in (6) is the gradient force and the second is the force of spontaneous light pressure. To simplify the calculation we expand the expression for the force in powers of the field, assuming that $I/|\nu|^2 \ll 1$:

$$\mathbf{f}_0=\mathbf{f}_0^{(2)}+\mathbf{f}_0^{(4)}+\dots.\qquad (7)$$

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Linear approximation with respect to field intensity

In second order in the field we have

$$\mathbf{f}_0^{(2)}(\mathbf{r}) = -\frac{\hbar}{|\nu|^2}[\Delta\nabla I(\mathbf{r}) + \gamma\mathbf{J}(\mathbf{r})]. \tag{8}$$

Using the wave equation for the field $V(\mathbf{r})$, we readily see that

$$\operatorname{div}\mathbf{J}(\mathbf{r}) = 0. \tag{9}$$

The condition (9) means absence of local potential minima of the spontaneous-light pressure force (the optical analog of the Earnshaw theorem[16]). It can be stated that the spontaneous light-pressure force has a solenoidal structure. We examine this question using as an example a field in the form of a superposition of several plane waves:

$$V(\mathbf{r}) = V_0\sum_j a_j\exp(i\varphi_j), \quad \varphi_j = k\mathbf{n}_j\mathbf{r} + \varphi_j^{(0)}. \tag{10}$$

The momentum flux in such a field is given by

$$\mathbf{J}(\mathbf{r}) = -kV_0^2\Big[\sum_j \mathbf{n}_j a_j^2 + \sum_{j>j_1} a_j a_{j_1}(\mathbf{n}_j+\mathbf{n}_{j_1})\cos(\varphi_j-\varphi_{j_1})\Big]. \tag{11}$$

The interference effects of interest to us are most pronounced if there is no average momentum flux, i.e., if

$$\sum_j \mathbf{n}_j a_j^2 = 0. \tag{12}$$

The coordinate-independent component of the spontaneous light-pressure force is then zero. The variable part of the force is proportional to the double sum in (11).

If the directions of the vectors $\mathbf{n}_j$ and $\mathbf{n}_{j_1}$ differ substantially, the corresponding force components oscillate rapidly in space, with a period $\sim 1/k$. If, however, the directions of $\mathbf{n}_j$ and $\mathbf{n}_{j_1}$ are close, Eq. (11) acquires a constant-sign term over distances much larger than $1/k$.

We calculate now the "rectified" radiation force, i.e., the light-pressure force component which a) varies slowly over the length of the light wave and b) has a maximum possible value at a given field strength.[2)] This value is of the order of $\hbar k\gamma$ for the spontaneous light-pressure force and of the order of $\hbar kV$ for the gradient force.

To separate the continuous (rectified) component, we average the force over a spatial scale much larger than $2\pi/k$. We use angle brackets to denote averaging. We obtain then from (8)

$$\langle\mathbf{f}_0^{(2)}(\mathbf{r})\rangle = -\frac{\hbar\gamma}{|\nu|^2}\langle\mathbf{J}(\mathbf{r})\rangle. \tag{13}$$

Obviously, the gradient force drops out in this averaging. The only terms left in $\langle\mathbf{J}\rangle$ are those with $|\mathbf{n}_j - \mathbf{n}_{j_1}| \ll 1$.

By way of a concrete example, we consider a two-dimensional field configuration in the form of a superposition of six fields with equal amplitudes ($a_j = 1$) and directions of the wave vectors $\mathbf{n}_j$ and $\mathbf{n}_j'$, where

$$\begin{gathered}\mathbf{n}_j = (\cos(2\pi j/3),\ \sin(2\pi j/3),\ 0), \quad j=1,2,3,\\ \mathbf{n}_j' = \mathbf{n}_j + [\boldsymbol{\alpha}\mathbf{n}_j], \quad \boldsymbol{\alpha} = (0,0,\alpha), \quad \alpha \ll 1.\end{gathered} \tag{14}$$

The angle between the vectors $\mathbf{n}_j$ is 120°. The system of vectors $\mathbf{n}_j'$ is "rigidly" rotated around the z axis by a small angle α (relative to the vectors $\mathbf{n}_j$ (see Fig. 1). Obviously, the condition (12) is satisfied in this case.

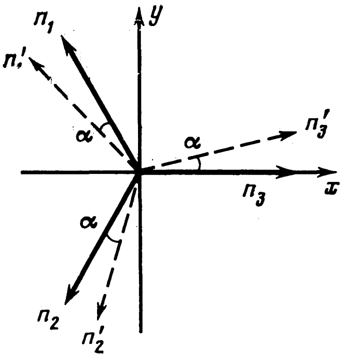

FIG. 1.

The averaged radiation force is purely solenoidal:

$$\begin{gathered}\langle\mathbf{f}_0^{(2)}(\mathbf{r})\rangle = \operatorname{rot}\mathbf{A}, \quad \mathbf{A} = (0,0,A),\\ A(\mathbf{r}) = A_0\sum_{j=1}^{3}\sin\psi_j, \quad A_0 = \frac{2\hbar\gamma|V_0|^2}{\alpha|\nu|^2},\\ \psi_j = k[\boldsymbol{\alpha}\mathbf{n}_j]\mathbf{r} + \varphi_j^{(0)\prime} - \varphi_j^{(0)}.\end{gathered} \tag{15}$$

The solenoidal force (15) has the scale $\hbar k\gamma$ of the light-pressure force and oscillates in space with a period $1/\alpha k \gg 1/k$.

The friction force is given by

$$\langle\mathbf{f}_1^{(2)}(\mathbf{r},\mathbf{v})\rangle = -\varkappa\mathbf{v}, \quad \varkappa = -6\gamma\Delta\hbar k^2|V_0|^2/|\nu|. \tag{16}$$

For simplicity, oscillating terms of the type $\cos(\varphi_j - \varphi_j')$ have been left out of (16), since they do not change qualitatively the picture of the motion.

We consider by way of illustration the motion of a particle near a node of the force (15), when $\alpha kr \ll 1$. Linearizing the rotational force with respect to the small displacement, we obtain the following equation of motion:

$$m\ddot{\mathbf{r}} = -\varkappa\dot{\mathbf{r}} + m\Gamma_0^2[\mathbf{e}_z\mathbf{r}], \quad \Gamma_0^2 = \frac{3\alpha\hbar k^2\gamma|V_0|^2}{m|\nu|^2}|C|, \tag{17}$$

where the constant C with $|C| < 1$ depends on the phases of the interfering fields. Putting $x + iy \sim \exp(-\Gamma t)$, we obtain from (17)

$$\Gamma = \frac{\varkappa}{2m}\{1 \pm [1 - 4i(\Gamma_0 m/\varkappa)^2]^{1/2}\}. \tag{18}$$

The character of the motion is determined by the magnitude of the parameter $m\Gamma_0/\varkappa$, if $|m\Gamma_0/\varkappa| < 1$, i.e., if

$$\alpha \ll \alpha_{cr} = \frac{3\gamma\Delta^2\hbar k^2|V_0|^2}{|\nu|^6|C|m} \sim \frac{\hbar k^2}{m\gamma} \ll 1, \tag{19}$$

the rotational force acts little on the particle motion, and the particle velocity attenuates exponentially at $\Delta < 0$. Note that for atoms with strong (allowed) transitions we have $\hbar k^2/m\gamma \sim 10^{-3}$. If $\alpha \gtrsim \alpha_{cr}$, the particle motion becomes unstable and the particle leaves the vicinity of the rotational-force node after a time $t \sim \Gamma_0^{-1}$.

It is thus possible to produce in a weak monochromatic field a rectified radiation force having a solenoidal structure. By varying the angle between the propogation directions of the interfering fields it is possible to vary the spatial period of the force $\langle \mathbf{f}_0^{(2)}\rangle$.

Quadratic approximation in intensity

The averaged radiation force is expressed in fourth order in the field amplitude in terms of the spatial correlator of the intensity and the momentum flux:

$$\langle \mathbf{f}_0^{(4)}(\mathbf{r})\rangle = \frac{2\hbar\gamma}{|\nu|^4}\langle I(\mathbf{r})\mathbf{J}(\mathbf{r})\rangle. \tag{20}$$

For a field in the form (14) we obtain thus

$$\langle \mathbf{f}_0^{(4)}(\mathbf{r})\rangle = \operatorname{rot}\mathbf{A} + \nabla U, \quad \mathbf{A} = (0, 0, A), \tag{21}$$

$$A = A_1\Big[\sum_{j=1}^{3}\Big(8\sin\psi_j - \frac{1}{2}\sin 2\psi_j\Big) + \sum_{j,j_1=1}^{3}\sin(\psi_j+\psi_{j_1})\Big],$$

$$A_1 = -\frac{8\hbar|V_0|^4\gamma}{|\nu|^4\alpha},$$

$$U = U_0\sum_{j=1}^{3}\sin(3^{1/2}\alpha k\mathbf{n}_j\mathbf{r}+\xi_j), \qquad U_0 = \frac{8\hbar\gamma}{3^{1/2}\alpha}\Big|\frac{V_0}{\nu}\Big|^4,$$

where ξ_j are constant phases, $\xi_3 = \varphi_1^{(0)} - \varphi_2^{(0)} + \varphi_2'^{(0)}$, and ξ_1 and ξ_2 are obtained by cyclic permutation of the indices.

We see that in the quadratic approximation in the intensity there appears, besides the small increment to the solenoidal force, and "admixture" of a potential field. The depths of the potential wells are quite large:

$$U_0 \sim \hbar\gamma/\alpha \gg \hbar\gamma \tag{22}$$

for $|V_0/\nu| \sim 1$. The period of the negative relief is of the order of $1/\alpha k$. In a monochromatic field the averaged (rectified) radiation force has thus a characteristic scale $\hbar k\gamma$, i.e., it is determined by the strength of the spontaneous light pressure.

The Earnshaw theorem is valid for the averaged force only in the lowest (linear) approximation in the field intensity. In the next (quadratic) approximation the force has a potential component. The solenoidal force, however, predominates.

A different force picture appears in a bichromatic inhomogeneous light field.

3. INTERFERENCE PHENOMENA IN A WEAK BICHROMATIC FIELD

Let the external field be a superposition of two monochromatic fields with detunings from resonance Δ_0 and Δ_1:

$$V(\mathbf{r}t) = V_0(\mathbf{r})e^{-i\Delta_0 t} + V_1(\mathbf{r})e^{-i\Delta_1 t}. \tag{23}$$

In the calculation of the force (2) we can leave out the terms that oscillate at frequencies that are multiples of the difference. Allowance for these terms leads to small oscillatory increments, of order $\lesssim \hbar k$, to the particle momentum. We seek a perturbation-theory solution of the Bloch equations in weak fields, $|V_l/\gamma| \ll 1$ with $l = 0$ and 1. In the lowest order in intensity we have for the force and for the averaged force the expressions

$$\mathbf{f}_0^{(2)}(\mathbf{r}) = -\hbar\sum_{l=0,1}[\Delta_l\nabla I_l + \gamma\mathbf{J}_l]/|\nu_l|^2,$$
$$\langle\mathbf{f}_0^{(2)}(\mathbf{r})\rangle = -\hbar\gamma\sum_{l=0,1}\langle\mathbf{J}_l(\mathbf{r})\rangle/|\nu_l|^2, \tag{24}$$

where I_l and $\mathbf{J}_l$ are the intensity and momentum flux in the lth mode of the field. In fourth order in the field we obtain the following expression for the averaged force:

$$\begin{aligned}\langle\mathbf{f}_0^{(4)}(\mathbf{r})\rangle = {}& 2\hbar\gamma\sum_{l=0,1}\langle I_l\mathbf{J}_l\rangle/|\nu_l|^4 \\ & + \frac{2\hbar}{|\nu_0|^2|\nu_1|^2}\{[(\Delta_1-\Delta_0)(1-\cos\chi) \\ & -\gamma\sin\chi]\langle I_0\nabla I_1\rangle + [\gamma(1+\cos\chi)+2\Delta_0\sin\chi]\langle I_0\mathbf{J}_1\rangle \\ & + [\gamma(1+\cos\chi)-2\Delta_1\sin\chi]\langle I_1\mathbf{J}_0\rangle\},\end{aligned} \tag{25}$$

$$\nu_l = |\nu_l|\exp(i\chi_l), \quad \chi = 2(\chi_1-\chi_0).$$

A new correlator $\langle I_0\nabla I_1\rangle$ of the intensity–intensity type appears in expression (25) for the force. We consider next the case of large detunings ($\Delta_l \gg \gamma$). In this limit Eq. (25) takes the simpler form

$$\langle\mathbf{f}_0^{(4)}(\mathbf{r})\rangle = 2\hbar\gamma\sum_{l=0,1}\langle I_l\mathbf{J}_l\rangle/|\nu_l|^4 + \frac{4\hbar\gamma}{\Delta_0^2\Delta_1^2}\Big[\frac{\Delta_0}{\Delta_1}\langle I_0\mathbf{J}_1\rangle + \frac{\Delta_1}{\Delta_0}\langle I_1\mathbf{J}_0\rangle\Big].$$

In a field of the form

$$V_0(\mathbf{r}) = V_0\sum_{j=1}^{3}\exp(i\varphi_j), \qquad V_1(\mathbf{r}) = V_1\sum_{j=1}^{3}\exp(i\varphi_j'),$$

where the vectors $\mathbf{n}_j$ and $\mathbf{n}_j'$ are defined by relations (14), there are no momentum fluxes in any of the modes: $\langle\mathbf{J}_l\rangle = \langle I_l\mathbf{J}_l\rangle = 0$. The rectified force is in this case potential:

$$\langle\mathbf{f}_0^{(2)}(\mathbf{r})\rangle = 0,$$
$$\langle\mathbf{f}_0^{(4)}(\mathbf{r})\rangle = \nabla U, \qquad U(r) = U_0\sum_{j=1}^{3}\sin(3^{1/2}\alpha k\mathbf{n}_j\mathbf{r}+\xi_j),$$
$$U_0 = \frac{2\hbar\gamma|V_0|^2|V_1|^2}{3^{1/2}\alpha\Delta_0\Delta_1}(1/\Delta_0^2+1/\Delta_1^2). \tag{27}$$

We arrive at the following conclusion: interference of the fields $V_0(\mathbf{r})$ and $V_1(\mathbf{r})$ produces in a bichromatic field a radiation force in which the principal role is played by the potential component. This circumstance can be most important for the solution of the problem of atom localization by light fields.

4. BICHROMATIC FIELD OF FINITE AMPLITUDE

We proceed now to consider the rectification of the radiation force in sufficiently strong fields, when stimulated transitions play an important role.

We regard the field $V_1e^{-i\Delta_1 t}$ in the superposition (23) as "high-frequency" and assume the following hierarchy of the frequencies of the problem:

$$\Delta_1 \gg |V_1| \gg |V_1|^2/\Delta_1, \quad \Delta_0,\ V_0,\ kv,\ \gamma. \tag{28}$$

The frequencies in the right-hand side of the inequality (28) are taken to be slow, and the relations between them can be quite arbitrary. Under these conditions the action of the high-frequency field reduces to formation of a spatially inhomogeneous Stark shift $\pm |V_1(\mathbf{r})|^2/\Delta_1$ of the atom levels. The question is: what is the light-pressure force if the atom is acted upon also by an inhomogeneous "low-frequency" field $V_0(\mathbf{r})$? Note that the inequalities in (28) constitute the condition of the generalized Rabi resonance,[14,15] which is not local in space, in contrast to the case considered in Refs. 4 and 5.

We derive first the basic equations for the slowly varying quantities, the atom density matrix and the light-pressure force. To this end we represent the dipole moment and the atom level-population difference as expansions in the high-frequency harmonics:

$$p=p_0e^{-i\Delta_0 t}+p_1e^{-i\Delta_1 t}+\dots,\quad q=q_0+q_1e^{-i\Delta_1 t}+q_1^*e^{i\Delta_1 t}+\dots.$$

The higher harmonics of this expansion can be omitted in the first approximation in the parameter $1/\Delta_1$. From the Bloch equations (3) and (4) we get

$$p_1=V_1q_0/\Delta_1,\quad q_1=-2p_0^*V_1/\Delta_1. \tag{29}$$

Let us substitute these quantities in the equations for the slow density-matrix components and omit for brevity the zero subscripts of p_0, q_0, and V_0. The initial equations for the slow quantities take then the form

$$\mathbf{f}=\hbar p^*\nabla V(\mathbf{r})+\hbar p\nabla V^*(\mathbf{r})+\frac{\hbar}{2}q\nabla\Delta(\mathbf{r}), \tag{2a}$$

$$i\frac{dp}{dt}+\nu(\mathbf{r})p=V(\mathbf{r})q,\quad \nu(\mathbf{r})=\Delta(\mathbf{r})+i\gamma/2, \tag{3a}$$

$$\frac{dq}{dt}+\gamma(q+1)=2ip^*V(\mathbf{r})+\text{c.c.},\quad \Delta(\mathbf{r})=\Delta_0+\frac{2|V_1(\mathbf{r})|^2}{\Delta_1}. \tag{4a}$$

These equations generalize the Bloch equations (3) and (4) and expression (2) for the force to include the case of an inhomogeneous Stark shift of the atomic levels. The effective detuning $\Delta(\mathbf{r})$ depends now on the coordinate, viz., it plays the role of a potential in expression (2a). In particular, for $V=0$ we have $U(\mathbf{r})=\hbar\Delta(\mathbf{r})/2$.

Slow atoms

We consider in greater detail the case of slow atoms ($kv\ll\gamma$). In the zeroth approximation in the velocity we have

$$\mathbf{f}_0(\mathbf{r})=\hbar q\left\{\frac{1}{|\nu(\mathbf{r})|^2}[\Delta(\mathbf{r})\nabla I+\gamma\mathbf{J}(\mathbf{r})]+\frac{1}{2}\nabla\Delta(\mathbf{r})\right\},$$
$$q(\mathbf{r})=-(1+2I(\mathbf{r})/|\nu(\mathbf{r})|^2)^{-1}. \tag{30}$$

This expression generalizes Eq. (6) to include the case of the inhomogeneous Stark shift due to the additional high-frequency field. It must be emphasized that in Eq. (30) the induced light pressure is no longer a gradient of an effective potential. In other words, the gradient force is rectified. The expression for $\mathbf{f}_1$

$$\mathbf{f}_1(\mathbf{r},\mathbf{v})=\frac{1}{\gamma}\mathbf{f}_0(r)\left[\frac{dq}{dt}+\frac{2V}{\nu^*}\frac{d}{dt}\left(\frac{V^*q}{\nu^*}\right)+\frac{2V^*}{\nu}\frac{d}{dt}\right.$$
$$\left.\times\left(\frac{Vq}{\nu}\right)\right]+\frac{\hbar i\nabla V^*}{\nu}\frac{d}{dt}\left(\frac{Vq}{\nu}\right)+\frac{\hbar i\nabla V}{\nu^*}\frac{d}{dt}\left(\frac{V^*q}{\nu^*}\right), \tag{31}$$

is where q is determined by Eq. (30) and $d/dt=\mathbf{v}\nabla$. Expressions (30) and (31) become considerably simpler in the limit of a weak or strong low-frequency field $V(\mathbf{r})$.

Weak field

Let $V\ll\gamma$ *and* $\Delta(\mathbf{r})\gg\gamma$. We have then

$$\mathbf{f}_0(\mathbf{r})=-\nabla U,\quad U(\mathbf{r})=\hbar\Delta(\mathbf{r})/2, \tag{32}$$

and the force $\mathbf{f}_1$ can be represented as a sum of the rotational and friction forces:

$$\mathbf{f}_1(\mathbf{r},\mathbf{v})=\mathbf{f}_{\text{rot}}+\mathbf{f}_{\text{fr}},$$

$$\mathbf{f}_{\text{rot}}=\frac{\hbar}{\Delta^3(\mathbf{r})}\left[2\mathbf{J}(\mathbf{r})\frac{d\Delta(\mathbf{r})}{dt}-2(\mathbf{v}\mathbf{J}(\mathbf{r}))\nabla\Delta(\mathbf{r})\right.$$
$$\left.+\Delta(\mathbf{r})\left(i\frac{dV}{dt}\nabla V^*-i\nabla V\frac{dV^*}{dt}\right)\right], \tag{33}$$
$$\mathbf{f}_{\text{fr}}=\frac{\hbar\gamma}{\Delta^3(\mathbf{r})}\left(\frac{dV}{dt}\nabla V^*+\text{c.c.}\right)$$
$$+\frac{1}{2}\hbar\gamma\left[\nabla I\frac{d\Delta^{-3}(\mathbf{r})}{dt}-\nabla\Delta(\mathbf{r})\frac{d}{dt}(I/\Delta^4(\mathbf{r}))\right].$$

It is easily seen that $\mathbf{v}\cdot\mathbf{f}_{\text{rot}}=0$, i.e., the rotational force performs no work on the particle.

In the limit considered, the high-frequency field produces thus an effective potential $\hbar\Delta(\mathbf{r})/2$ and the weak low-frequency field rotates and decelerates (or accelerates) the particles.

Strong field

Let now the low-frequency field be strong:

$$V(\mathbf{r})\sim\Delta(\mathbf{r})\gg\gamma,\ kv.$$

Using the quasistationary approximation to obtain the dipole moment, we reduce the system (3a) and (4a) to the form[13]

$$\frac{dA}{dt}=-\frac{\gamma}{2}(1+1/\varepsilon^2)A+\gamma/\varepsilon,\quad A=-\varepsilon q, \tag{34}$$

$$\mathbf{f}=-\frac{\hbar}{2}A\nabla(\Delta(\mathbf{r})\varepsilon(\mathbf{r})),\quad \varepsilon(\mathbf{r})=[1+|2V(\mathbf{r})/\Delta(\mathbf{r})|^2]^{1/2}. \tag{35}$$

The force is proportional to the population difference A of the quasienergy states and to the quasienergy gradient $\Delta(\mathbf{r})\varepsilon(\mathbf{r})/2$. The quantity A is determined by the function $\varepsilon(\mathbf{r})$ and is, generally speaking, not a function of the quasienergy. It is this which leads to the rectification—to the appearance of a constant-sign component of the force component over "macroscopic" spatial scales much larger than $1/k$. For slow atoms ($kv\ll\gamma$) we have in the zeroth approximation in the velocity

$$A=\frac{2\varepsilon}{1+\varepsilon^2},\quad \langle\mathbf{f}_0(\mathbf{r})\rangle=-\left\langle\frac{\hbar\varepsilon(r)}{1+\varepsilon^2(r)}\nabla(\Delta(\mathbf{r})\varepsilon(\mathbf{r}))\right\rangle. \tag{36}$$

The averaged radiation force (36) is determined by the rates of the induced processes. We shall call it the "rectified gradient" force. For weak saturation ($|V/\Delta_0|^2\ll 1$ and $\Delta_0\gg|V_1|^2/\Delta_1$) we have

$$\mathbf{f}_0(\mathbf{r})=2\hbar\left\langle\left|\frac{V(\mathbf{r})}{\Delta_0}\right|^4\nabla\frac{|V_1(\mathbf{r})|^2}{\Delta_1}\right\rangle. \tag{37}$$

We use now (37) to consider several simple examples. In the one-dimensional case of two standing wave, $V(x) = V_0 \cos kx$ and $V_1(x) = V_1 \cos[(k+\delta k)x + \varphi]$, $\delta k \ll k$, we get from (37)

$$\langle f_{0x}(x)\rangle = -\frac{dU}{dx}, \quad U(x) = U_0 \cos(2\delta kx + 2\varphi),$$

$$U_0 = -\frac{\hbar k}{\delta k}\left|\frac{V_0}{\Delta_0}\right|^4 \frac{|V_1|^2}{\Delta_1}. \qquad (38)$$

The averaged radiation force is of the order of the gradient force $\hbar kV$ and oscillates with a period $\pi/\delta k$ that can exceed by several orders the wavelength of the light. The potential-well depth $U_0 \sim k\hbar V/\delta k$ exceeds the usual depth $\hbar V$ by a factor $k/\delta k$.

In fields of tunable lasers the value of $\hbar V$ is 0.01 K. For $\delta k/k \sim 10^{-3}$ we have $U_0 \sim 10$ K.

Rectification of the radiation force produces thus in a bichromatic field superdeep potential wells.

Force rectification in the two-dimensional case

Let a two-dimensional field constitute the following superposition of traveling waves:

$$V(\mathbf{r}) = V_0[\exp(ik\mathbf{n}_0\mathbf{r}) + a\exp(ik\mathbf{n}_1\mathbf{r}) + a\exp(ik\mathbf{n}_2\mathbf{r})],$$
$$V_1(\mathbf{r}) = V_1[\exp(ik\mathbf{n}_0'\mathbf{r} + i\varphi_0)$$
$$+ b\exp(ik\mathbf{n}_1'\mathbf{r} + i\varphi_1) + b\exp(ik\mathbf{n}_2'\mathbf{r} + i\varphi_2)],$$

where the unit vectors $\mathbf{n}_i$ and $\mathbf{n}_i'$ $(i = 0, 1, 2)$ lie in the (x, y) plane. We align the vectors $\mathbf{n}_i$ with the bisectors of the first, second, and fourth quadrants of a Cartesian coordinate frame with unit vectors $\mathbf{e}_x$ and $\mathbf{e}_y$, i.e., we put $\mathbf{n}_0 = 2^{-1/2}(\mathbf{e}_x + \mathbf{e}_y)$, $\mathbf{n}_1 = 2^{-1/2}(\mathbf{e}_x - \mathbf{e}_y)$ and $\mathbf{n}_2 = -\mathbf{n}_1$. The vectors $\mathbf{n}_i' = \mathbf{n}_i + \delta\mathbf{n}_i$ differ little from the vectors $\mathbf{n}_i$, $|\delta\mathbf{n}_i| \ll 1$. Note that $(\delta\mathbf{n}_i \cdot \mathbf{n}) = 0$ (see Fig. 2).

We assume for simplicity that the coefficients a and b are small. The averaged force takes then the form

$$\langle \mathbf{f}_0(\mathbf{r})\rangle = f\{\sin[k(\delta\mathbf{n}_2 - \delta\mathbf{n}_0)\mathbf{r} + \varphi_2$$
$$-\varphi_0]\mathbf{e}_x + \sin[k(\delta\mathbf{n}_1 - \delta\mathbf{n}_0)\mathbf{r} + \varphi_1 - \varphi_0]\mathbf{e}_y\},$$
$$f = 8\cdot 2^{1/2}\hbar kab\left|\frac{V_0}{\Delta_0}\right|^4 \frac{|V_1|^2}{\Delta_1} \qquad (39)$$

We introduce small rotation angles α_i $(\alpha_i \ll 1)$ defined by the relations $\delta\mathbf{n}_1 = -\alpha_1\mathbf{n}_0$, $\delta\mathbf{n}_0 = \alpha_0\mathbf{n}_1$ and $\delta\mathbf{n}_2 = \alpha_2\mathbf{n}_0$. The vector detunings entering in the force (39) are then given by

$$\delta\mathbf{n}_2 - \delta\mathbf{n}_0 = (\alpha_2 - \alpha_0)\mathbf{e}_x/2^{1/2} + (\alpha_2 + \alpha_0)\mathbf{e}_y/2^{1/2},$$
$$\delta\mathbf{n}_1 - \delta\mathbf{n}_0 = -(\alpha_1 + \alpha_0)\mathbf{e}_x/2^{1/2} + (\alpha_0 - \alpha_1)\mathbf{e}_y/2^{1/2}.$$

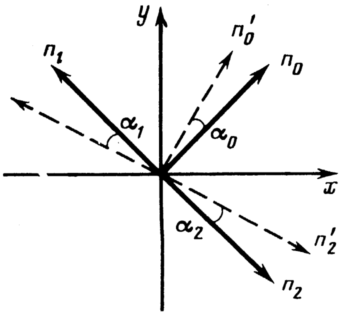

FIG. 2.

By varying the angles α_i we can control the vector detunings. We note the following two qualitatively different situations:

Potential field

If $\alpha_2 = -\alpha_0$ and $\alpha_1 = -\alpha_0$, a potential force field is produced, with a potential

$$U(\mathbf{r}) = \frac{f}{2^{1/2}\alpha k}[-\cos(-2^{1/2}\alpha_0 kx + \varphi_2 - \varphi_0)$$
$$+\cos(2^{1/2}\alpha_0 ky + \varphi_1 - \varphi_0)], \qquad (40)$$

The well depths are of the order of $f/k\alpha_0 \gg \hbar V_0$ and the period is $2^{1/2}\pi/\alpha_0 k$.

Solenoidal field

If $\alpha_1 = \alpha_0$ and $\alpha_2 = \alpha_0$, we have the case of a solenoidal field:

$$\langle \mathbf{f}_0(\mathbf{r})\rangle = \mathrm{rot}\,\mathbf{A}, \quad \mathbf{A} = (0, 0, A), \qquad (41)$$
$$A = -\frac{f}{2^{1/2}\alpha_0 k}[\cos(2^{1/2}\alpha_0 ky + \varphi_2 - \varphi_0) + \cos(-2^{1/2}\alpha_0 kx + \varphi_1 - \varphi_0)].$$

Such a force causes particle rotation. The characteristic frequency of the rotation can be determined from the linearized equation of motion of the particle near a node of the force (41):

$$m\ddot{\mathbf{r}} = -2^{1/2}f\frac{k\mathbf{v}}{\gamma} + 2^{1/2}f\alpha_0 k[\mathbf{e}_z\mathbf{r}], \qquad (42)$$

where f is defined in (39). We have used here as the friction force the retarded gradient force[10] for $V_0^2/\Delta_0 < V_1^2/\Delta_1$.

If the field is not very weak, namely,

$$f \gg f_c = 2^{1/2}\alpha_0 m\gamma^2/k, \qquad (43)$$

the rotation frequency $(x + iy \propto \exp(i\Omega t))$ is given by

$$\Omega = \Omega_0(1 - if_c/f), \quad \Omega_0 = \alpha_0\gamma. \qquad (44)$$

The particle rotates with a frequency Ω_0, and the rotation radius increases slowly if $f > 0$.

Thus, by varying the directions of the light rays, one can obtain either a potential or a solenoidal force field. In the general case, of course, the rectified radiation force (39) is some combination of a potential and a solenoidal field.

5. CONCLUSION

We have shown that field interference causes rectification of a radiation force. In a monochromatic field the rectified radiation force is of the order of the spontaneous light-pressure force, in which case the solenoidal component predominates. In a bichromatic field one can produce a rectified radiation force in which the potential component predominates. Superdeep potential wells are produced, $1/\alpha \sim 10^3$ times larger than the characteristic depth $\hbar V$. Such wells can be used for prolonged localization of atoms in a light field.

[1)] This was briefly reported in Ref. 13.

[2)] To avoid misunderstandings, we note that not all continuously varying gradient force is rectified. For example, in the field of a light-beam field $V_0(x,y)e^{ikz}$ having an amplitude $V_0(x,y)$ and varying over a length l $(kl \gg 1)$ there is no rectification, since the gradient force $\nabla|V_0(x,y)|^2\Delta|\nu|^2$ is small in respect to the parameter $1/kl$ compared with the maximum possible value $k|V_0|^2\Delta/|\nu|^2$.

[1]G. A. Askar'yan, Zh. Eksp. Teor. Fiz. **42**, 1567 (1962) [Sov. Phys. JETP **15**, 1088 (1962)].
[2]A. Ashkin, Phys. Rev. Lett. **24**, 156 (1970).
[3]A. P. Kazantsev, Zh. Eksp. Teor. Fiz. **66**, 1599 (1974) [Sov. Phys. JETP **39**, 784 (1974)].
[4]A. P. Kazantsev, Pis'ma Zh. Eksp. Teor. Fiz. **17**, 212 (1973) [JETP Lett. **17**, 150 (1973)].
[5]G. A. Askar'yan and V. A. Namiot, Zh. Eksp. Teor. Fiz. **69**, 1986 (1975) [Sov. Phys. JETP **42**, 1009 (1975)].
[6]J. P. Gordon and A. Ashkin, Phys. Rev. **21**, 1606 (1980).
[7]J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am. **2**, 1707 (1985).
[8]A. P. Kazantsev *et al.*, *ibid.* **2**, 1725 (1985).
[9]A. P. Kazantsev, D. O. Chudesnikov, and V. P. Yakovlev, Zh. Eksp. Teor. Fiz. **90**, 1619 (1986) [Sov. Phys. JETP **63**, 951 (1986).
[10]A. P. Kazantsev *et al.*, Phys. Rep. **129**, 1606 (1985).
[11]A. Aspect *et al.*, Phys. Rev. Lett. **57**, 1686 (1986).
[12]C. Salomon *et al.*, *ibid.* in press (1989).
[13]A. P. Kazantsev and I. V. Krasnov, Pis'ma Zh. Eksp. Teor. Fiz. **46**, 264 (1987) [JETP Lett. **46**, 332 (1987)].
[14]I. V. Krasnov, Zh. Eksp. Teor. Fiz. **89**, 420 (1985) [Sov. Phys. JETP **62**, 238 (1985)].
[15]I. V. Krasnov, *ibid*, **92**, 1265 (1987) [**65**, 708 (1987)].
[16]A. Ashkin and J. P. Gordon, Opt. Lett. **8**, 511 (1983).

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