Ponderomotive broadening in above-threshold ionization spectra

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The influence of a spatial-temporal inhomogeneity of a strong optical field on the electron dynamics and on the spectrum of above-threshold ionization is investigated. A modified two-step model of photoelectron ionization and detection is formulated and is based on the premise that in an optical field the electron oscillations follow the translational motion adiabatically. The peak widths and shifts produced in the above-threshold ionization spectrum by the spatial and temporal inhomogeneity of the light pulse are calculated. It is shown that the suppression of the low-energy channels is relative in character; in short pulses the suppressed peaks become superimposed in the low-energy part of the spectrum.

1. INTRODUCTION

A distinctive feature of above-threshold-ionization (ATI) experiments is that the ionized electron is found to be in a strong nonstationary field of the focused pulsed-laser radiation. Nonlinear scattering by this field (the Kapitza–Dirac effect[1,2]) alters the initial momentum and energy of the electron. In the classical description of the field, the photoelectron is scattered[3] by a ponderomotive potential ($\hbar = m = e = 1$)

$$U(x,\ t) = E^2(x,\ t)/4\omega^2, \tag{1}$$

where $E(x,t)$ is the amplitude of the field of frequency ω. This scattering must be taken into account for an adequate description of the experimental ATI data. The influence of the potential (1) on the electron motion was directly confirmed by an experiment[4] in which reflection of the electron from the light focus was observed.

The energy spectrum of the ATI ionization constitutes a succession of peaks separated by the photon energy. The spectrum structure depends relatively weakly on the type of atom, but is sensitive to the properties of the laser emission. In long pulses, the peak positions do not depend on the emission intensity. As the intensity increases, the low-energy part of the spectrum is suppressed and each peak is broadened. Decreasing the pulse duration shifts the peaks towards lower energies and changes their widths.

Starting with Ref. 5 (see also Refs. 6–8) the ATI theory makes use of a two-step model in which the elementary ionization act is calculated in a locally homogeneous field, while electron acceleration in the potential (1) is taken into account in the region away from the focus.

The ionization probability in a long pulse was calculated in Ref. 9 with wave functions that take accurate account of the ponderomotive acceleration. In the physical picture corresponding to these functions, the electron translational motion is determined by a ponderomotive potential, while the oscillations follow adiabatically the field variation due to the electron displacement in space. It has been found that the energy relations in both models are formally identical but differ in the physical meaning of the quantity $U = E^2/4\omega^2$. In the two-step model this quantity is regarded as the average electron-oscillation energy in the optical field in the treatment of the elementary act, and of the potential energy (1) during the stage of departure from the focus. In the approach of Ref. 9, U enters only as the potential energy of the electron translational motion. The models differ essentially in the mechanisms that suppress the channels. In the first case the suppression of the low-energy peaks is due to the increase of the ionization potential by an amount equal to the oscillation energy, while in the second case it is due to the inability of an electron with energy lower than that of the potential barrier to penetrate into the classically inaccessible region.[9]

Within the framework of the two-step model, the physical reason for the shifts and widths is that the decrease of the photoelectron kinetic energy as a result of the increase of the ionization potential[10] is not completely offset by its subsequent increase by the ponderomotive acceleration on leaving the focus. To our knowledge the latter mechanism has not been taken into account in any calculations of the shape of the peak. The results of numerical simulation[11] demonstrate the presence of shifts and widths, but cannot explain their dependences on the field parameters.

Mechanisms other than ponderomotive can produce shifts and widths, but will not be considered here. The width contribution due to the uncertainty relation $\Delta E \Delta t \sim 1$ in the elementary act is too small compared with the observed quantities, and the influence of the residual space charge[12] comes into play only at high densities of the atomic target.

We describe here a modified two-step model in which the elementary ionization act is regarded in the spirit of the adiabatic approach[9] extended to include the case of pulses of finite duration (Secs. 2 and 3). General expressions are obtained for the peak profile due to the spatial and temporal inhomogeneity of the field (Sec. 4). The peak shifts and widths are obtained for a Gaussian envelope of the field in the limiting cases of long (Sec. 5) and very short (Sec. 6) pulses. In Sec. 7 is discussed the constant-flight-velocity approximation for calculations of the spectrum shape. The results of Secs. 5–7 are compared in Sec. 8 with the experimental data of Refs. 13 and 14.

The use in Sec. 3 of the transition probability per unit time to describe the elementary act presupposes the absence of ionization saturation. Allowance for this factor in the evolution of the spectrum may be essential and requires a separate analysis.

We use a one-dimensional model in which the electron moves along the linear field polarization that coincides in direction with the field inhomogeneity.

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2. ELECTRON IN AN OPTICAL FIELD

A ponderomotive potential enters in the analysis of electron motion in a weakly inhomogeneous optical field $E(X,t)\cos\omega t$ through the use of the method of averaging over fast oscillations.[15] The coordinate X and the generalized momentum $\mathscr{P}$ of the electron are represented as sums of slow translational and rapid oscillatory terms:

$$X=x+\xi, \quad \mathscr{P}=p+p_\xi. \tag{2}$$

The vibrational and oscillatory motions can be effectively separated if the characteristic dimension R and time τ_{int} which characterize the translational motion are respectively large compared with the oscillation amplitude and with the optical period:

$$R\gg\xi, \quad \omega\tau_{int}\gg 1. \tag{3}$$

At such a substantial difference between motion scales, it is natural to assume that the oscillations are determined by the local value of the field amplitude at the center of the oscillation, and when the electron moves along the translational trajectory $x(t)$ the oscillations follow adiabatically the changing conditions:

$$\xi=-\frac{E(x(t),t)}{\omega^2}\cos\omega t. \tag{4}$$

We shall consider below the properties of electron motion in an optical field in the framework of the adiabatic picture (4). A quantum description of the motion, in the same approximation, is given for a stationary field in Ref. 9.

Application of a standard procedure[15] to the equations of motion that follow from the exact Hamiltonian

$$H_r=\frac{1}{2}\left(\mathscr{P}+\frac{1}{c}A\right)^2 \tag{5}$$

with a field vector potential

$$A=-\frac{cE(X,t)}{\omega}\sin\omega t,$$

leads to translational-motion equations corresponding to a Hamiltonian

$$H(p,x,t)=\frac{p^2}{2}+U(x,t). \tag{6}$$

The oscillation dynamics is described in the adiabatic picture by the equations

$$\dot{p}_\xi=0, \quad \dot{\xi}=\frac{E(x,t)}{\omega}\sin\omega t, \tag{7}$$

from which are omitted terms that add to the solutions only small corrections ($\sim\xi/R$, $\sim 1/\omega\tau_{int}$). In the integration of the fast equations (7) the field amplitude is assumed constant and the values averaged over the optical period are assumed to be zero:

$$\langle\xi\rangle=\langle p_\xi\rangle=0.$$

Equation (7) leads thus to (4) and to $p_\xi=0$, the latter meaning that in a weakly inhomogeneous field the generalized electron momentum is equal to the translational momentum.

The Hamiltonian (6) depends explicitly on the time, and the translational energy is strictly speaking not conserved. Since the potential energy in (6) has a temporal scale on the order of the pulse duration τ, one can speak only of approximate conservation of the translational energy during time intervals shorter than τ (as discussed in the present section below and in Secs. 3 and 5). An electron in an oscillating optical field can be ascribed an energy equal to $\langle H_r\rangle$. Since $\langle H_r\rangle=H(p,x,t)$ in the approximation considered, one can speak to equal accuracy of a "dressed"-electron energy equal to the translational energy. The fast-motion energy should be assumed to be zero. The reason is that the nonoscillating part of the exact Hamiltonian

$$\frac{1}{2c^2}\langle A^2\rangle=\left\langle\frac{\xi^2}{2}\right\rangle=U$$

is included, in the form of potential energy, in the slow-motion Hamiltonian.

Solution of the equations of motion with a Hamiltonian (6) and with a gradient force

$$f(x,\,t)=-\partial U(x,\,t)/\partial x, \tag{8}$$

together with the boundary conditions (t_0, x_0, p_0), yields the time dependences of the coordinate $x(t)$ and momentum $p(t)$ of the translational motion. In our units, the velocity is

$$v(t)\equiv\dot{x}(t)=p(t).$$

The asymptotic ($t\gg\tau_{int}$) value of the momentum p determines the kinetic energy of the free electron, no longer interacting with the optical field, recorded by the detector. The change of the kinetic energy can be expressed in the form of the work performed by the force (8) along the trajectory

$$\mathscr{A}(x_0,t_0)=\frac{p^2}{2}-\frac{p_0^2}{2}=\int_{t_0}^{\infty}f(x(t),t)\,v(t)\,dt. \tag{9}$$

The difficulties encountered in the solution of the slow-motion equations and in the determination of the work (9) are due to nonconservation of the time-dependent gradient force.

The character of the translational motion and the magnitude of the work (9) depend substantially on the relation between the time R/v of emergence of the electron from the focal region and the pulse duration τ (Ref. 16). The time of electron interaction with the optical field is equal to the smaller of the two:

$$\tau_{int}=\min\,(R/v,\,\tau). \tag{10}$$

If $R/v\tau\ll 1$, the long-pulse regime is realized: the electron traverses the focus so rapidly that no explicit dependence on the time manages to manifest itself in the field amplitude and in the force (8). The motion is close to conservative. In the intermediate case $R/v\tau\sim 1$ the light pulse is short and the time dependence cannot be neglected. The strong inequality $R/v\tau\gg 1$ (ultrashort pulses) means that the electron displacement during the time of the light pulse is small. In this limiting case the electron is moved by a force $f(x_0, t)$ that depends only on the time.

If it turns out in the latter case that the momentum increment $\Delta p\sim f\tau$ is short, and the conditions

$$\frac{v\tau}{R}\ll 1, \quad \Delta p\sim\frac{U}{R}\tau\ll p \tag{11}$$

are simultaneously met, the free electron momentum is equal to the initial value. Since the translational momentum is equal to the generalized momentum, the inequalities (11) determine the conditions under which the action of an optical field bounded in space and in time can be regarded as an adiabatic application of a spatially homogeneous field.

3. ELEMENTARY IONIZATION ACT

We describe ionization of an atom by means of the probability w_n, per unit time, of a transition from a bound state of energy ε_g to a continuous spectrum, accompanied by absorption of n photons. If an atom located at a point x_0 is in a bound state at an instant of time t_0, its ionization takes place in the same way as in a homogeneous monochromatic field of amplitude $E(x_0, t_0)$. At distances Δx and times Δt satisfying the conditions

$$R \gg \Delta x \gg a_B, \quad \tau_{ini} \gg \Delta t \gg \omega^{-1}, \tag{12}$$

the field amplitude has not yet changed, but the electron no longer interacts with the atom and has an energy determined by the conservation law

$$p^2(x_0, t_0)/2 + U(x_0, t_0) = n\omega + \varepsilon_g. \tag{13}$$

The excess of the energy of n photons over the ionization potential (the channel energy $\varepsilon_n \equiv n\omega + \varepsilon_g$) is transformed in the optical field into an electron energy equal to the translational energy. The role of the fast oscillations reduces to the appearance of the term $n\omega$ in (13). The left-hand side of (13) can be represented as the sum of the kinetic and potential energies only in the semiclassical description of the translational motion, while the constancy of the energy is due to the approximation conservation of the translational-motion energy over short time intervals. The quantities t_0, x_0, and $p(x_0, t_0)$ introduced above determine the initial conditions for the motion of an ionized electron in an optical field.

The central part of the focus is classically inaccessible to an electron having an energy lower than the height of the ponderomotive barrier. The probability, in the nth channel ($\varepsilon_n < U_0$) of ionizing an atom located between the turning points x_n of this channel is practically zero in view of the exponentially small overlap of the localized initial state and the final-state wave function in the ponderomotive potential (Fig. 1). For atoms in the region classically accessible to the

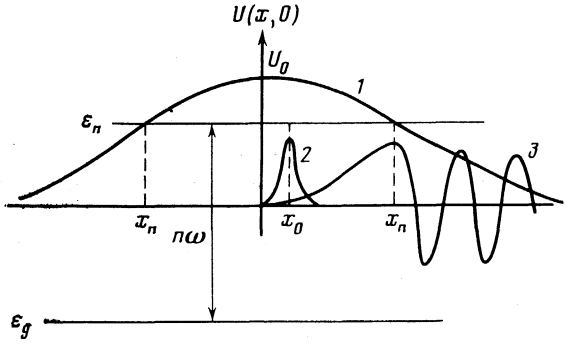

FIG. 1. Mechanism of suppression of below-barrier channel: 1—ponderomotive potential, 2—wave function of bound state ε_g of an atom located at the point x_0, 3—wave function of final state with energy ε_n; x_n is a turning point.

nth channel potential we shall assume a power-law probability with a large nonlinearity exponent $\tilde{n} \gg 1$, as is usually the case in the multiphoton limit.[17] In perturbation theory we have $\tilde{n} = n$, but no such equality obtains in the other models.[7,9,18] Ultimately we have

$$w_n(x_0, t_0) = \begin{cases} aU^{\tilde{n}}(x_0, t_0), & U(x_0, t_0) > \varepsilon_n \\ 0, & U(x_0, t_0) < \varepsilon_n \end{cases}. \tag{14}$$

A quantum-theoretical verification of the ionization-channel suppression mechanism (14) is given in Ref. 9 for a stationary beam ($R/v\tau \to 0$). The slow time variation of the barrier height leads to motion of the turning points, but does not alter qualitatively the damped behavior of the electron wave functions in the below-barrier region. The generalization of the result of Ref. 9 to include a nonstationary case is therefore justified.

4. PHOTOELECTRON SPECTRUM

The number of electrons produced when n photons are absorbed in a time dt_0 in a volume dx_0 by ionization of a gas of density c_a is

$$dN_n = c_a w_n(x_0, t_0)\, dx_0 dt_0. \tag{15}$$

As seen from (13), in the elementary act all the electrons produced have the same energy, equal to the channel energy. In the succeeding large-scale motion (Sec. 2) in a nonstationary potential the energy is not conserved and it follows from (8) and (13) that the electron energy recorded by the detector is

$$\varepsilon = \varepsilon_n - U(x_0, t_0) + \mathcal{A}(x_0, t_0), \tag{16}$$

and depends not only on the creation place and time, but also on the motion at later instants of time, since the last term of (16) is integral in character. The energy spread is due both to the inhomogeneity of the potential and to the difference between the work values. We shall show below that the spectrum is formed mainly in a narrow range of variables (x_0, t_0) in which it is convenient to represent the energy (16) in the form

$$\varepsilon = \varepsilon_n + \delta + \gamma(x_0, t_0), \tag{17}$$

with separation of the homogeneous (δ) and inhomogeneous [$\gamma(x_0, t_0)$] parts that determine the shift and broadening of the peak, respectively.

The distribution of the detected electrons in energy is obtained by transforming in (15) from the variables (x_0, t_0) to the variables (ε, y) with the aid of (16) or (17):

$$\frac{dN_n}{d\varepsilon} = c_a \int w_n(x_0, t_0) \frac{\partial(x_0, t_0)}{\partial(\varepsilon, y)} dy. \tag{18}$$

The choice of the variable y is arbitrary and is governed by convenience considerations; in particular, we can put $y = x_0$ or $y = t_0$.

If

$$\max\,(U(x_0, t_0) - \mathcal{A}(x_0, t_0)) > \omega,$$

the electrons produced in different space points and times (x_0, t_0) can reach the detector with equal energies by absorbing different numbers of photons. In this case it is necessary to sum the contributions of different channels (see Sec. 6).

Owing to the high nonlinearity of the probability, the main contribution to the integral in (18) is produced near points where maximum channel intensity is reached. For above-barrier channels, this intensity is equal to the maximum intensity of the light pulse and is reached at the point $x_0 = 0, t_0 = 0$. For a partially closed channel, the maximum channel intensity is reached near a turning point and is determined by the condition $U = \varepsilon_n$. It is independent of the maximum light-pulse intensity. It follows therefore that the number N_n of electrons in the partially closed channel does not depend (or depends weakly) on the maximum light-pulse intensity. Once the nth channel becomes below-barrier, the electron-production rate w_n ceases to increase, but in above-barrier channels the nonlinear increase of the probability continues until the light-pulse intensity reaches its maximum. The channel suppression thus has a relative character,[13] but owing to the high nonlinearity of the ionization the electron fraction in the suppressed channels is small.[9]

5. LONG PULSES

In the zeroth approximation in the parameter $R/v\tau \ll 1$ one can neglect the change of the force during the exit of the electron from the focus, and put $t = t_0$ by virtue of (8). The translational motion becomes conservative, and the work of the gradient force is exactly equal to the initial potential energy. The electrons resulting from an n-photon transition are thus detected, as follows from (16), with equal energies ε_n regardless of where and when they were produced. In this approximation, the spectrum has an infinitesimally thin peak at the energy ε_n, with an area determined by the total number of electrons in the given channel:

$$N_n = \int_{-\infty}^{\infty} c_a w_n(x_0, t_0)\, dx_0\, dt_0. \tag{19}$$

To determine the work performed, with account taken of the corrections for the parameter $R/v\tau \ll 1$, we expand the force (8) in powers $(t - t_0)$ of the explicit time, and to calculate the work (9) we use the $x(t)$ dependence obtained for the potential motion. Integration by parts yields

$$\mathscr{A}(x_0, t_0) = U(x_0, t_0) + \delta\mathscr{A}^{(1)}(x_0, t_0), \tag{20}$$

where

$$\delta\mathscr{A}^{(1)}(x_0, t_0) = \int_{x_0}^{\infty} dx \frac{\partial U(x, t_0)}{\partial t_0} \frac{1}{\{2(\varepsilon_n - U(x, t_0))\}^{1/2}}. \tag{21}$$

The sign of the nonstationary correction is determined by the sign of the derivative $\partial U/\partial t$ at the instant of the photoelectron creation. The electrons created on the leading front of the laser pulse are accelerated as they leave the focus more strongly than in potential motion, since the force is increased. On the contrary, electrons created on the trailing edge are detected with an energy lower than ε_n (Ref. 4). The largest number of electrons is created at the instant $t_0 = 0$ and at maximum intensity, but have $\delta\mathscr{A}^{(1)} \approx 0$. Thus, in a long pulse that is symmetric in time the electron peak is broadened symmetrically near the energy ε_n. The shift of the peak position is of second order in the small parameter $R/v\tau$.

Consider the case of a Gaussian pulse

$$U(x, t) = U_0 \exp(-x^2/R^2 - t^2/\tau^2) \tag{22}$$

and of fully open ionization channels: $\varepsilon_n > U_0$. Owing to the high nonlinearity of the probability (14), the bulk of the electrons is produced at

$$|x_0| \leqslant \frac{R}{\tilde{n}^{1/2}}, \quad |t_0| \leqslant \frac{\tau}{\tilde{n}^{1/2}}. \tag{23}$$

In this space–time region expression (21) can be easily calculated and leads to a detected-electron energy

$$\varepsilon = \varepsilon_n - \frac{t_0}{\tau} \frac{\pi^{1/2} R}{v_n \tau} U_0, \tag{24}$$

where $v_n = \{2(\varepsilon_n - U_0)\}^{1/2}$ is the initial velocity of the photoelectron in the region (23). If the barrier height U_0 is close to the channel energy ε_n, the interaction time is lengthened and the energy spread is increased. Expression (24) is not valid for $\varepsilon_n - U_0 \ll U_0$, when the exit time becomes logarithmically large.

Using relation (24) in (18) and integrating over the atom positions, we obtain the shape of the electron peak

$$\frac{dN_n(\varepsilon)}{d\varepsilon} = \frac{N_n}{(\pi\Delta_n^2)^{1/2}} \exp\left(-\frac{(\varepsilon - \varepsilon_n)^2}{\Delta_n^2}\right) \tag{25}$$

with a width equal to

$$\Gamma_n = 2\Delta_n = 2\left(\frac{\pi}{\tilde{n}}\right)^{1/2} \frac{R}{v_n \tau} U_0. \tag{26}$$

Expression (26) determines the dependence of the width on the laser-emission characteristics and on the degree of nonlinearity. In long pulses, the peak width is a small fraction of the height of the ponderomotive barrier, and the peaks do not coalesce ($\Gamma_n < \omega$) even in strong fields with $U_0/\omega > 1$, when a certain number of channels is suppressed.

6. ULTRASHORT PULSES

In the ultrashort-pulse regime, $v\tau/R \ll 1$ and the electron is practically stationary during the time of interaction with the light pulse. When calculating the work (9) we can put in the force $x(t) \approx x_0$ and represent the momentum as the sum of the initial momentum p_0 and the increment acquired by the action of the force $f(x_0, t)$. Equation (9) is then transformed into the obvious expression

$$\mathscr{A}(x_0, t_0) = \frac{(p_0 + \Delta p)^2}{2} - \frac{p_0^2}{2}, \tag{27}$$

where

$$\Delta p = \int_{t_0}^{\infty} f(x_0, t)\, dt. \tag{28}$$

The work (27) is a quantity of first order of smallness in $v\tau/R \ll 1$ with a velocity value $v = p_0 + \Delta p$.

We consider first channels with $\varepsilon_n > U_0$, which remain above-barrier during the entire laser pulse. The maximum probability is reached in them at $x_0 = 0$ and $t_0 = 0$, and it follows from (16) and (17) that

$$\delta = -U_0, \quad \gamma(x_0, t_0) = U_0 - U(x_0, t_0) + \mathscr{A}(x_0, t_0), \tag{29}$$

where $\mathscr{A}(x_0, t_0)$ is determined by (27) and (28). All the above-barrier peaks are shifted towards lower energies by the same amount. The lowest of them lands in the energy

region $0<\varepsilon<\omega$, where the threshold channel corresponding to absorption of a maximum number of electrons was located in the case of long pulses and weak fields.

We shall analyze the peak broadening for the Gaussian envelope (22). In the multiphoton limiting case, we can confine ourselves in expression (9) for the work in the significant region (23), to the term linear in Δp for all channels with the possible exception of the first below-barrier one. We have then

$$\mathscr{A}(x_0, t_0)=\alpha_n \frac{x_0}{R} U_0, \quad \alpha_n=\frac{\pi^{1/2} v_n \tau}{R}. \tag{30}$$

In this region $\mathscr{A}\sim\alpha_n U_0/\tilde{n}^{1/2}$, while the spread of the potential energy is $\delta U = U_0 - U(x_0, t_0)\sim U_0/\tilde{n}$; consequently the relative contribution of these two terms to the broadening is determined by the dimensionless parameter $\alpha_n \tilde{n}^{1/2}$. In the case

$$\alpha_n \tilde{n}^{1/4}\ll 1 \tag{31}$$

we can neglect the work in (29). This means that the observed quantity is the distribution in the initial kinetic energies, a distribution not distorted by the influence of the ponderomotive scattering (cf. Ref. 11). At the instant of creation the electron has an energy equal to the channel energy ε_n (Sec. 3). It moves subsequently in the optical field without energy conservation, since the external field is rapidly turned off whereas the momentum remains constant.

In Sec. 2 we formulated the conditions for approximate momentum conservation (11), in which it was implied that the motion is characterized by distances $\sim R$ and by a force $f\sim U_0/R$. When the strongly nonlinear ionization spectrum is calculated, the effective dimension with which the shift $v\tau$ is compared becomes $\sim R/\tilde{n}^{1/2}$, and the force becomes $f\sim U_0/(R\tilde{n}^{1/2})$. As a result, the conditions for observing an undistorted spectrum differ from the adiabatic-switching conditions: the first in (11) becomes more stringent, and the second less so.

It follows from (18) and (29), subject to the condition (31), that the distribution in energy takes in the below-barrier channel the form

$$\frac{dN_n}{d\varepsilon}=N_n \frac{\tilde{n}}{U_0}\left(1-\frac{\varepsilon-\tilde{\varepsilon}_n}{U_0}\right)^{\tilde{n}-1}. \tag{32}$$

This equation describes an asymmetric peak with a maximum near $\tilde{\varepsilon}_n=\varepsilon_n - U_0$ and smeared out on the high-energy side. Formally, the electron energies are contained in the interval $\tilde{\varepsilon}_n<\varepsilon<\varepsilon_n$, but since $\tilde{n}\gg 1$ the distribution decreases rapidly and has a width

$$\Gamma_n=U_0/\tilde{n}, \tag{33}$$

which is independent of the form of the light pulse. The peaks do not coalesce in the considered multiphoton limiting case ($\Gamma_n<\omega$).

A detailed analysis of relations (16)–(18) and (9), with account taken of terms of second order in $\alpha_n\ll 1$ and with arbitrary value of the parameter $\alpha_n\tilde{n}^{1/2}$, leads to a positive correction $\sim\alpha_n{}^2 U_0$ for the shift δ and to the following expression for the width:

$$\Gamma_n=\begin{cases}\left(\dfrac{1}{\tilde{n}}+\dfrac{\alpha_n}{\tilde{n}^{1/2}}+\dfrac{\alpha_n{}^2}{4}\right)U_0, & \alpha_n<\dfrac{2}{\tilde{n}^{1/2}}\\ \dfrac{2\alpha_n}{\tilde{n}^{1/2}}U_0, & \alpha_n>\dfrac{2}{\tilde{n}^{1/2}}\end{cases} \tag{34}$$

If $\alpha_n\tilde{n}^{1/2}\ll 1$, Eq. (34) goes over into (33). When α_n increases the contribution of the work to (29) becomes dominant and the peak becomes symmetric. Let the initial velocity be along the x axis. The electrons created on the right of the center of the focus ($x_0>0$) are accelerated, and those on the left ($x_0<0$) are decelerated by the gradient force.

A few remarks concerning the below-barrier channels. The electrons are created in them, with overwhelming probability, near the turning points (Sec. 2), and have therefore low initial kinetic energies ($<\omega$). Since the initial electron momentum changes insignificantly in very short light pulses, the contributions of all the partially closed channels are summed in the low-energy spectral region. The main contribution to this sum is made the uppermost below-barrier channel, and can be comparable with that of the lowest above-barrier channel.

7. CONSTANT-FLIGHT-VELOCITY APPROXIMATION

The work can be easily calculated for any value of the parameter $R/v\tau$ by assuming the electron velocity constant[14] and by putting in (9)

$$v(t)=v, \quad x(t)=x_0+v(t-t_0).$$

In the case of the Gaussian envelope (22), the answer is expressed in terms of the error integral. We present its limiting expression for the region $|x_0|\ll R$ and $|t_0|\ll\tau$. The homogeneous shift is equal to

$$\delta=-\frac{U_0}{1+(v\tau/R)^2}, \tag{35}$$

and the inhomogeneous part of the shift, with account taken of only the terms linear in x_0/R and t_0/τ, takes the form

$$\gamma(x_0, t_0)=-U_0\frac{\pi^{1/2} v\tau}{R}\left(1+\left(\frac{v\tau}{R}\right)^2\right)^{-3/2}\left(\frac{v\tau}{R}\frac{t_0}{\tau}-\frac{x_0}{R}\right) \tag{36}$$

and leads in accordance with (18) to a Gaussian peak of width

$$\Gamma_n=2\frac{U_0}{\tilde{n}^{1/2}}\frac{\pi^{1/2} v\tau/R}{1+(v\tau/R)^2}. \tag{37}$$

In the limiting cases of long and ultrashort pulses, expressions (35)–(37) differ only by the definition of the velocity from the corresponding results of Secs. 4 and 5 for above-barrier channels. Since terms quadratic in (x_0, t_0) have been left out of (36), the region of validity of Eqs. (36) and (37) on the short-pulse side is limited by the condition $v\tau/R>2\tilde{n}^{1/2}$ [see the discussion following Eq. (31)].

The constant-flight-velocity approximation is justified if the initial and final electron velocities are close and it is immaterial which one is used in the definition of v, i.e., if $\Delta p\ll p_0$. This condition is equivalent to the inequality $\varepsilon_n\gg U_0$ (it is satisfied by only high-lying above-barrier channel) in the case of long pulses, and to the weaker inequality $\alpha_n U_0/(\tilde{n}^{1/2}v^2)\ll 1$ in the case of short ones. Under long-pulse conditions, for above-barrier channels with energy on the order

TABLE I. Comparison of experimental and theoretical widths $\tilde{\Gamma}$ and shifts δ of the photoelectron peaks ($\tau_p = 50$ ps).

		$S=1$	$S=2$	$S=3$	$S=4$
$\tilde{\Gamma}$, eV	Exp.	0.60	0.47	0.49	0.43
	Theor.	0.67	0.30	0.23	0.20
δ, eV	Exp.	−0.41	−0.24	−0.19	−0.12
	Theor.	−0.33	−0.19	−0.14	−0.10

of the barrier height (these are precisely the channels in which the largest number of electrons land), and all the more for below-barrier channels (at any pulse duration), the initial and final velocities differ insignificantly and the constant-flight-velocity approximation can no longer be used for quantitative calculations. One can hope, however, for a suitably defined velocity v, that expressions (35) and (37) will yield reasonable interpolation equations. Comparison of (37) with (26) and with (34) shows that (37) can be used for all above-barrier channels by assuming the velocity v to be equal to the initial one.

8. CONCLUSION

Shifts and widths of electron peaks under conditions of negligible ionization saturation and space charge were measured in Refs. 14 and 15. (The influence of saturation on the above-threshold ionization spectrum may turn out to be important and calls for special consideration.) Xenon atoms were ionized by Nd-laser radiation ($\omega = 1.17$ eV) at a maximum intensity $7.5 \cdot 10^{12}$ W/cm^3 ($U_0 = 0.79$ eV). The focal radius in the experiment was $R = 14\,\mu$m (field-envelope radius $\Omega_0 = 2^{1/2}R = 20\,\mu$m), and the pulse durations were $\tau_p = 136$ ps and $\tau_p = 50$ ps. The effective pulse duration τ_p is connected with the Gaussian-envelope parameter τ by the relation $\tau_p = \pi^{1/2}\tau$.

Just as in Refs. 13 and 14, we assign to the channels integer numbers 0, 1, 2,..., i.e., we put $n = 11 + S$. The channel $S = 0$ has an energy $\varepsilon_0 = 0.6$ eV lower than U_0, and does not appear in the experiment. The initial electron velocity is $v_S = \{2(\varepsilon_0 + S\omega - U_0)\}^{1/2}$. The parameter $v_s\tau/R \gtrsim 1$ for all the observed peaks, with the exception of the case $S = 1$ and $\tau_p = 50$ ps.

Table I lists the values of the shift δ and of the full width at half maximum $\tilde{\Gamma}$, obtained from experiment and calculated using Eqs. (34), (26), and (35). (One-dimensional calculations can be used in the three-dimensional problem because of the sharp directivity of the angular distribution in the direction of the linear polarization.[19]) The table demonstrates the reasonable agreement between the theory and experiment.

In the ultrashort-pulse region, the widths obtained from Eq. (33) agree well with the experimental data of Ref. 20,vis., $\Gamma_{\text{theor}} \approx 0.20$ eV and $\Gamma_{\text{exp}} \approx 0.26$ eV for $U_0 = 1.6$ eV and $n = 8$ ($S = 1$).

Comparison with the experimental data leads to the conclusion that the broadening in above-barrier ionization spectra is due in the main to the space–time inhomogeneity of the optical field.

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