With what topology could the universe be created?

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The topological and geometrical properties of a self-created universe with cosmological constant are considered in the framework of Hawking's quantum cosmology. The probabilities for creation of the universe with different topologies (including a torus, sphere, hyperbolic space) are calculated; for an inflationary universe these topologies are found to be equally probable. The probability of a quantum change of the topology during the evolution of the universe is calculated for a concrete model.

1. INTRODUCTION

The possibility of a nontrivial topology of the universe became particularly acute at the beginning of the sixties after Wheeler's work on geometrodynamics, including the idea of a foamlike structure of space–time.[1] The recent development of ideas about the quantum creation of the universe (see, for example, Refs. 2–4) have put this question into a somewhat different form, namely, that of the topology with which the universe can be created, and with what probability.[5] However, as was already noted by Zel'dovich and Starobinskiĭ, difficulties arise already in the very formulation of this problem, in particular, "it is not clear what is the meaning of the probability of creation of a closed universe and how this probability is to be normalized."

The framework of quantum cosmology developed by Hawking and his collaborators[6–8] appears very promising for discussing the topology of a created universe. This approach to the determination and interpretation of the wave function of the universe has already been used to consider many very important problems relating to the cosmological constant, initial perturbations, the inflationary stage, the CPT theorem, etc.[9–13] In the framework of the Euclidean formulation of the path integral given by the authors of these studies it has been possible not only to define the concept of probability but also to find a natural condition of normalization of the wave function. Another important advantage (particularly in the context of the present paper and emphasized by Hartle and Hawking[7]) is the possibility of direct calculations of 3-geometries with nontrivial topology. In Ref. 7 Hartle and Hawking also discussed topological problems of the creation of the universe, including the properties of a 4-manifold whose edge is a given topologically trivial 3-manifold. These questions were considered in more detail by Mkrtchyan,[14,15] who for a number of special cases succeeded in obtaining restrictions on the properties of a created universe with matter. In this connection it should be noted that in accordance with cobordism theory and two closed 3-manifolds are cobordant (Rokhlin's theorem), i.e., there do not exist restrictions on the topology of a trivial 3-manifold.[16])

In this paper, using the framework of the approach of Hawking and his collaborators, we attempt to investigate both the topological and geometrical properties of a created universe. In the semiclassical approximation we estimate the probabilities for creation of a universe with different topologies in superspace. We shall see that the study of this problem requires the finding, for a given value of the cosmological constant Λ, of solutions of the Einstein equations for spaces without matter,

$$R_{ab}=\Lambda g_{ab}$$

with homogeneous isotropic metric.

Among these solutions there are both compact gravitational instantons[17,18] and complex solutions, which, as we shall show, contribute to the required wave function. We shall call the latter pseudoinstantons.

Having the necessary solutions, we then calculate the Euclidean gravitational action, which occurs in the path integral for the wave function. The method of steepest descent is used then to calculate the integrals of the wave functions for spaces with different topologies and thus determine their relative probabilities of creation. For spaces capable of undergoing an inflationary stage after creation these topologies are found to be equally probable.

Taking into account the possibility of a quantum change of the topology of the universe after creation, we calculate for a concrete example (with $\Lambda=0$) the probability amplitudes for some transitions. We find that the transition of a sphere into a torus is an event that is extremely improbable compared with the transition of a sphere into a sphere, i.e., without change of topology.

2. CANONICAL QUANTUM COSMOLOGY

In accordance with the quantum-geometrodynamic formalism, a certain quantum state of the universe is described by a wave function $\psi(h_{ij})$ that satisfies the Wheeler–DeWitt equation on superspace, i.e, the infinite-dimensional space of all Riemannian metrices h_{ij} (for a discussion of the properties of superspace, see Ref. 19). The square of the wave function determines the probability of creation of the universe on the 3-manifold S metric h_{ij} (in the absence of matter).

Hawking and his collaborators assume that the quantum state of the real universe is determined by a wave function of the form

$$\psi(h_{ij},S)=\int_C d[g_{ab}]\exp(-I[g_{ab}]), \qquad (1)$$

where the intergration is over all 4-dimensional compact manifolds M with Euclidean metric g_{ab} that induces the metric h_{ij} on the boundary $\partial M=S$.

The Euclidean quantum action has the form

$$I[g_{ab}]=-\frac{1}{16\pi l^2}\int d^4x\, g^{1/2}(R-2\Lambda)-\frac{1}{8\pi l^2}\int_{\partial M} d^3x\, h^{1/2}K, \quad (2)$$

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where

$$g=\det g_{ab},\quad h=\det h_{ij},$$

and K is the trace of the second fundamental form of the embedding of S in M.

If near S the metric g_{ab} can be represented in the form

$$ds^2=N^2dt^2+h_{ij}dx^idx^j, \tag{3}$$

then the second fundamental form K_{ij} is

$$K_{ij}=\frac{1}{2N}\frac{\partial h_{ij}}{\partial t}.$$

In what follows we shall consider isotropic and homogeneous closed (compact without boundaries) cosmological models with Λ term and without matter. In this case the metric on the 3-manifold S, i.e., for $t=$ const, depends on the single parameter a:

$$h_{ij}(x,\ t)=\sigma^2a^2(t)\tilde{h}_{ij}(x), \tag{4}$$

where

$$\sigma^2=\frac{4\pi l^2}{3}\left[\int\limits_S d^3x\,\tilde{h}^{1/2}\right]^{-1},\quad \tilde{h}=\det\tilde{h}_{ij}.$$

The curvature for the induced metric $\tilde{h}_{ij}$ is[20]

$${}^3R_{ijkl}=k(\tilde{h}_{ik}\tilde{h}_{jl}-\tilde{h}_{il}\tilde{h}_{jk}) \tag{5}$$

for $k=+1$, when S is the 3-sphere S^3 or the 3-sphere factorized with respect to a discrete group (S topology); for $k=0$,[2)] when S is the 3-torus $T^3=S^1\times S^1\times S^1$ or another flat space (T topology); and for $k=-1$, when S is the 3-hyperbolic space H^3 factorized with respect to a discrete group (H topology).

The space of the metrics (3)–(5) determines a minisuperspace. For the metric (5) the action (2) has the form

$$I_k[a]=\frac{1}{2}\int dt\left\{\frac{N}{a}\right\}\left\{-\left(\frac{a\dot{a}}{N}\right)^2-ka^2+\lambda a^4\right\}, \tag{6}$$

where

$$\dot{a}=da/dt,\quad \lambda=\Lambda\sigma^3/3\equiv H^2.$$

In the integral

$$\psi_k(a_0)=\int d[a]\exp(-I_k[a]) \tag{7}$$

for the wave function the integration is over all $a(t)$ that take the value a_0 on S.

Using these expressions, we can estimate in the semiclassical approximation the probability of creation of a universe with $k=0,\pm1$. But it is first of all necessary to determine the solutions of the Einstein equations that can contribute to the wave function.

3. ISOTROPIC PSEUDOINSTANTONS

To estimate the wave function $\psi_K(a_0)$, it would appear at the first glance that one can proceed as follows. For given value of Λ we find compact gravitational instantons with metric g_{ab} in the form (3). It is known, for example, that for $\Lambda>0$ an instanton solution with metric (3) is S^4 with radius $(\Lambda/3)^{-1/2}$ (Ref. 18). Further, if it is found that the metric (3)–(5) cannot be ascribed to the instanton that is found then one could expect that the corresponding wave function must be zero in the semiclassical approximation.

In fact this procedure is not always correct, since when we are calculating the integral in (7) we must take into account not only real, i.e., Euclidean, but also complex solutions, (cf. the calculations of integrals by the method of steepest descent, when all complex saddle points are taken into account). Although the physical meaning of such solutions is obscure, they nevertheless contribute to the wave function.

The saddle points for (7) are found from the Einstein equation written down for the metric (3),

$$(\dot{a}/N)^2+H^2a^2=k, \tag{8}$$

subject to the condition that there exist t_1 and t_2 such that

$$a(t_1)=0,\quad t_1<t_2,\quad a(t_2)=a_0. \tag{9}$$

The results of the calculations can be represented in the following form:

for $\Lambda=0$

$k=0$	$k=+1$	$k=-1$
$a(t)=\text{const}$	$a(t)=\pm\zeta(t)$	$a(t)=\pm i\zeta(t)$
R^4[3)]	$\zeta(t)=\zeta(0)+\int\limits_0^t d\tau N(t)$	$H^3\times R$
$R^4/\Gamma\equiv T^4$	$R^4,\ R^4/\Gamma$	$H^3/\Gamma\times R$
$N=\begin{cases}1\\-i\end{cases}$	$N=1$	$N=-i$
$d\zeta^2+d\Omega_0^2$ $-d\zeta^2+d\Omega_0^2$	$d\zeta^2+\zeta^2d\Omega_1^2=$ $=d\vartheta^2+d\Omega_0^2$	$-d\zeta^2+\zeta^2d\Omega_1^2$ $=d\zeta^2+d\Omega_0^2$

(Γ is some discrete group);

for $\Lambda>0$

$k=0$	$k=+1$	$k=-1$
$a(t)=\frac{\text{const}}{H}e^{\pm H\zeta(t)}$	$a(t)=\pm\frac{1}{H}\sin\{H(\zeta(t))\}$	$a(t)=\pm\frac{i}{H}\sin\{H(\zeta(t))\}$
"Incomplete" de Sitter	S^4 $R\times S^3$	$R_+\times R^3$ Noncompact with singularity
$N=-i$	$N=\begin{cases}1\\-i\end{cases}$	$N=-i$
$-d\zeta^2+\frac{e^{\pm2H\zeta}}{H^2}d\Omega_0^2$	$\frac{1}{H^2}[d\vartheta^2+\sin^2(\vartheta)\,d\Omega_1^2]$	$-d\tau^2+\frac{\text{ch}\,H\tau}{H^2}d\Omega_1^2$;

for $\Lambda < 0$

$k=0$	$k=+1$	$k=-1$
$a=\frac{c}{H}e^{\pm H\tau}$	$a=\frac{1}{H}\,\mathrm{sh}\,H\tau$	$a=\begin{cases}\frac{1}{H}\sin H\tau\\ \frac{1}{H}\,\mathrm{ch}\,H\tau\end{cases}$
$N=1$	$N=1$	$N=\begin{cases}-i\\ 1\end{cases}$
R^4	$R\times S^3$	$S\times R^3$
		$R\times H^3/\Gamma$
$d\tau^2+e^{\pm 2H\tau}d\Omega_0{}^2$	$d\tau^2+\frac{1}{H^2}\mathrm{sh}^2(H\tau)d\Omega_1{}^2$	$-d\tau^2+\frac{1}{H}\sin^2(H\tau)d\Omega^2_{-1}$
		$-d\tau^2+\frac{1}{H^2}\mathrm{ch}^2(H\tau)d\Omega^2_{-1}$

The topologies given above are incomplete and correspond, strictly speaking, only to real solutions.

We discuss in more detail the topological and geometrical properties of some solutions, namely, for $\Lambda>0, k=+1$

$$a(t)=\pm\frac{1}{H}\sin\{H(\xi(t))\}.$$

When $N(t)=1$ and $\xi(0)=0$ we have for the $+$ sign

$$a(t)=\frac{1}{H}\sin(Ht),$$

and this is the Euclidean instanton solution S^4.

But when $N(t)=-i, \xi(0)=\pi/2$, we have

$$a(t)=\frac{1}{H}\mathrm{ch}(Ht),$$

the de Sitter solution, which does not, however, satisfy the condition (9).

Very interesting is the solution with

$$N(t)=\begin{cases}1, & 0\leqslant t\leqslant\pi/2H-\varepsilon\\ -i, & \pi/2H+\varepsilon<t\end{cases},$$

where $0<\varepsilon\leqslant 1$ and the function $N(t)$ is defined continuously in the interval[4)]

$$t\in[\pi/2H-\varepsilon,\ \pi/2H+\varepsilon].$$

This solution actually describes creation of the universe from "nothing," making a transition from the Euclidean hemisphere to the de Sitter stage of expansion (for $Ha_0>1$ it in fact makes the main contribution to $\psi_{+1}(a_0)$).

It is interesting to note that there are some exotic solutions among the ones that we have obtained. Thus, for $N(t)=-1$, $\xi(0)=0$ $(k=+1)$ we have the same 4-sphere but with proper time

$$d\tau=N(t)dt=-dt,$$

i.e., with opposite direction with respect to the coordinate time. This means that a test particle for $N=1$ is an antiparticle for $N=-1$. In this connection we should recall the fundamental question, recently discussed by Hawking[13] and Page,[21] of the arrow of time in cosmology.

Analogous solutions also exist for other values of k and $\Lambda=H^2$.

4. WAVE FUNCTIONS FOR DIFFERENT TOPOLOGIES

We now turn to the calculation of the wave functions for the solutions given above. We first of all calculate the action with the boundary conditions

$$a(t_1)=0,\quad a(t_2)=a_0. \tag{10}$$

We consider the action for $H^2>0$. Rewriting (6) in the form

$$I_k[a]=\frac{1}{2}\int_{t_1}^{t_2}dt\,Na\Big[-\Big(\frac{\dot a}{N}\Big)^2-k+H^2a^2\Big] \tag{11}$$

and using (8), we obtain

$$I_k[a]=\frac{1}{2}\int_{t_1}^{t_2}dt\,Na\Big[-2\Big(\frac{\dot a}{N}\Big)^2\Big]=-\int_0^{a_0}da\,a\Big(\frac{\dot a}{N}\Big)$$
$$=-\int_0^{a_0}da\,a(k-H^2a^2)^{1/2}=\frac{1}{2H^2}\int_k^{k-H^2a_0^2}dz\,z^{1/2}. \tag{12}$$

It can be seen from (10) and (11) that for $k=0$ the action $I_0[a]$ takes two values,

$$I_0{}^{\pm}[a]=\pm i(Ha_0{}^3/3),$$

from which it follows that the wave function is

$$\psi_0(a_0)\approx e^{-I_0{}^+(a)}+e^{-I_0{}^-(a)}\approx\cos\Big[\frac{Ha_0{}^3}{3}\Big].$$

To calculate $\psi_k(a_0)$ for $k\neq 0$ we shall proceed from the representation (for more details, see [7])

$$\psi_k(a_0)\approx\frac{1}{2\pi i}\int_C dp\exp\Big(\frac{H}{3}pa_0{}^3\Big)\phi_k(p),$$

where

$$p=\frac{1}{NH}\frac{\dot a}{a},$$

and C is a contour in the complex plane of p parallel to the imaginary coordinate axis and to the right of all singularities of the function $\phi_k(p)$:

$$\phi_k(p)=\int d[a]\exp(-\mathcal{I}_k[a]),$$
$$\mathcal{I}_k[a]=Hpa_0^3/3+I_k[a_0]. \tag{13}$$

We note a difference relating to the wave function $\psi_k(a)$ and to the wave function in the momentum representation, $\phi_k(p)$. In the first case there is a description of creation from "nothing," and a varies from $a=0$ to $a=a_0$ for $k=\pm 1$, while in the second case the momentum of the universe arrives at the value p having being created "from different states": from $p_0=\pm\infty$ for $k=1$ and from $p_0=\pm i\infty$ for $k=-1$.

From (13) we can obtain

$$\phi_k^{\pm}(p)=\int d[a]\exp(-\mathcal{I}^{\pm}[a]),$$

where

$$\mathcal{I}_k^{\pm}[a]=\pm\frac{k^{1/2}}{3H^2}+\frac{k^{1/2}p}{3H^2(1+p^2)^{1/2}},\quad k^{1/2}=\begin{cases}1, & k=1\\ i, & k=-1\end{cases}$$

and the symbols $\pm$ correspond to the sign of p_0. Then the wave function can be rewritten in the form

$$\psi_k(a_0)\approx\frac{1}{2\pi i}\int_c dp\exp\left(\frac{H}{3}pa_0^3\right)\{\phi_k^+(p)+\phi_k^-(p)\}.$$

For each of the integrals in this expression we obtain, after integrating by the method of steepest descent (cf. Ref. 7),

$$\frac{1}{2\pi i}\int_c dp\exp\left[\frac{H}{3}pa_0^3-\frac{p}{3H^2(1+p^2)^{1/2}}\right]$$
$$\approx\begin{cases}\exp\left[-\dfrac{(1-H^2a_0^2)^{3/2}}{3H^2}\right], & Ha_0<1\\ \cos\left[\dfrac{(H^2a_0^2-1)^{3/2}}{3H^2}-\dfrac{\pi}{4}\right], & Ha_0>1\end{cases}$$

$$\frac{1}{2\pi i}\int_c dp\exp\left[\frac{H}{3}pa_0^3-\frac{ip}{3H^2(1+p^2)^{1/2}}\right]$$
$$\approx\cos\left[\frac{(H^2a_0^2+1)^{3/2}}{3H^2}+\frac{\pi}{4}\right],$$

from which we obtain for $\psi_{+1}(a_0)$

$$\psi_{+1}(a_0)\approx\exp\left(\frac{1}{3H^2}\right)\exp\left[-\frac{(1-H^2a_0^2)^{3/2}}{3H^2}\right]$$
$$+\exp\left(-\frac{1}{3H^2}\right)\exp\left[-\frac{(1-H^2a_0^2)^{3/2}}{3H^2}\right]$$
$$\approx\exp\left(\frac{1}{3H^2}\right)\exp\left[-\frac{(1-H^2a_0^2)^{3/2}}{3H^2}\right],$$
$$Ha_0<1.$$

For $Ha_0>1$ we obtain similarly

$$\psi_{+1}(a_0)\approx\exp\left(\frac{1}{3H^2}\right)\cos\left[\frac{(H^2a_0^2-1)^{3/2}}{3H^2}-\frac{\pi}{4}\right].$$

It is now clear why for $\Lambda=H^2>0$ and $k=1$ the probability of creation of a universe with a_0 such that $Ha_0>1$ is not zero despite the fact that it is not possible to embed S^3 of radius a_0 in S^4 with radius H^{-1}. The probability is nonzero because there exists a complex solution—a pseudoinstanton.

Finally, we have

$$\psi_0(a_0)\approx\cos[Ha_0^3/3],$$

$$\psi_{+1}(a_0)\approx\begin{cases}\exp\left(\dfrac{1}{3H^2}\right)\exp\left[-\dfrac{(1-H^2a_0^2)^{3/2}}{3H^2}\right], & Ha_0<1\\ \exp\left(\dfrac{1}{3H^2}\right)\cos\left[\dfrac{(H^2a_0^2-1)^{3/2}}{3H^2}-\dfrac{\pi}{4}\right], & Ha_0>1\end{cases} \tag{14}$$

$$\psi_{-1}(a_0)\approx\cos\left(\frac{1}{3H^2}\right)\cos\left[\frac{(H^2a_0^2+1)^{3/2}}{3H^2}+\frac{\pi}{4}\right].$$

For $Ha_0\gg 1$ the wave functions take the form

$$\psi_0(a_0)\approx\cos\left(\frac{Ha_0^3}{3}\right),$$
$$\psi_{+1}(a_0)\approx\exp\left(\frac{1}{3H^2}\right)\cos\left(\frac{Ha_0^3}{3}\right), \tag{15}$$
$$\psi_{-1}(a_0)\approx\cos\left(\frac{1}{3H^2}\right)\cos\left(\frac{Ha_0^3}{3}\right),$$

from which we find the probability ratios

$$|\psi_{-1}|^2:|\psi_0|^2:|\psi_{+1}|^2\approx\cos\frac{1}{3H^2}:1:\exp\left(\frac{1}{3H^2}\right)$$

or

$$|\psi_{-1}|^2\leqslant|\psi_0|^2<|\psi_{+1}|^2, \tag{16}$$

i.e., in this case the probability of creation of a sphere is greatest. This inequality is not changed for $Ha_0\ll 1$, $H^2\gg 1$.

One can also estimate the probability of creation of an inflationary universe.[5)] As is well known,[4] one of the necessary conditions for inflation is a large value of a massive scalar field: $m^2\varphi^2\gg 1$. Since during this stage the field evolves slowly, $\varphi/\varphi\ll H$, and, therefore, $m^2\varphi^2$ plays the role of H^2, we can obtain from (15) for $H^2\gg 1$

$$|\psi_{-1}|^2:|\psi_0|^2:|\psi_{+1}|^2\approx 1:1:1,$$

i.e., the creation of inflationary universes with the S, T, and H topologies is equally probable.

5. PROBABILITY OF A CHANGE IN THE TOPOLOGY OF THE UNIVERSE

Thus, we have determined the probability of quantum creation of a universe with different topologies from "nothing," i.e., transition from the state $a=0$ to a_0. Can we now draw unambiguous conclusions about the topology of the present universe? This is obviously not possible, since the topology of the universe could have changed during evolution. The classical theory prohibits transitions with a change of the topology,[22] but in the quantum theory there are no such limitations.

Here, considering the example of a "toy" model, we find the probability of a quantum change of topology, for which it is necesary to extend the mini-superspace representation used above.

We consider the case when $H^2=0$. Then in the semiclassical approximation the main contribution to the wave function (1) will be made by the Euclidean 4-torus $S^1\times S^1\times S^1\times S^1=T^4$ with metric

$$ds^2=8\pi l^2L^2[d\vartheta_0^2+d\vartheta_1^2+d\vartheta_2^2+d\vartheta_3^2],\quad 0\leqslant\vartheta_i<1,$$

where L is a dimensionless constant greater than unity. Into this torus we can embed any 3-sphere of radius $R < L/2$ [in units of $(8\pi)^{1/2}l$], near which the metric has the form

$$ds^2=8\pi l^2L^2\{dR^2+R^2[d\alpha_1^2+\sin^2\alpha_1(d\alpha_2^2+\sin^2\alpha_2 d\alpha_3^2)]\},$$
$$R=\text{const}<L/2.$$

Into the same 4-torus we can embed the 3-torus with metric

$$ds^2=8\pi l^2L^2(d\chi^2+\chi^2d\varphi^2+d\vartheta_2^2+d\vartheta_3^2),\quad \chi=1,\quad 0\leqslant\varphi\leqslant1.$$

Since we consider spaces without matter and for $\Lambda = 0$, only the last term (T^4 instanton for $\Lambda = 0$) will contribute to the action (2):

$$I=-\frac{L^2}{8\pi l^2}\int\limits_{\partial M} d^3\vartheta\, h^{1/2}K,$$

where ∂M is, with allowance for the orientation of the torus and sphere,

$$\partial M=T^3-S^3.$$

Then, using the relation

$$\int\limits_S d^3x\, h^{1/2}K=\partial_n\int\limits_S d^3x\, h^{1/2},$$

where n is the unit normal vector to S, we can calculate the required action

$$I=-\frac{L^2}{8\pi l^2}\int\limits_{T^3} d^3\vartheta\, h^{1/2}K+\frac{L^2}{8\pi l^2}\int\limits_{S^3} d^3\vartheta\, h^{1/2}K$$
$$=-L^2\partial_\chi\Big[\chi\int\limits_{T^3} d\varphi\, d\vartheta_2\, d\vartheta_3\Big]$$
$$+L^2\partial_{+R}\Big[R^3\int\limits_{S^3} d\alpha_1\, d\alpha_2\, d\alpha_3 \sin^2\alpha_2\sin\alpha_3\Big]=-(1-6\pi^2R^2)L^2.$$

Therefore, the amplitude for transition of the sphere of radius R_0 into a torus is

$$\psi(S^3\to T^3)\approx e^{-I}=\exp[(1-6\pi^2R_0^2)L^2],$$

and, as is readily seen, the amplitude for transition of the sphere of radius R_1 is

$$\psi(S^3\to S^3)\approx\exp[6\pi^2(-R_0^2+R_1^2)L^2].$$

Hence, for the ratio of the probabilities of these transitions we have

$$|\psi(S^3\to T^3)|^2:|\psi(S^3\to S^3)|^2\approx\exp[2(1-6\pi^2R_1^2)L^2].$$

If the radius of the sphere is chosen in such a way that the volume of the 3-sphere is equal to the volume of the 3-torus,

$$2\pi^2R_1^3=L^3,$$

then we find

$$|\psi(S^3\to T^3)|^2:|\psi(S^3\to S^3)|^2\approx\exp[2-6L^2(2\pi^2)^{1/3}]L^2\approx e^{-16L^4},\qquad(17)$$

i.e., a change of the topology, of the sphere into a torus, is practically impossible in the considered problem.

There is another possibility for transition of a sphere into a torus, namely, annihilation of the initial sphere and creation of a new torus or sphere (in a 4-torus). It is readily seen that the result (17) remains valid, since it does not depend on the radius of the initial sphere.

6. CONCLUSIONS

In this paper we have considered the topological and geometrical aspects of quantum creation of the universe. In our view, it would be valuable to restrict the general formulation of the problem of the geometry of the 3-manifold[7] in such a way as to permit some detailed mathematical study. This circumstance forced us to consider the problem of the origin of a homogeneous isotropic universe without matter but with cosmological constant (in particular, the presence of this term makes it possible to interpret some conclusions in the context of models of an inflationary universe).

The basis of our investigation has been Hawking's approach to determination of the wave function of the universe by a Euclidean integral over compact metrics. We have found that not only gravitational instantons, i.e., real compact solutions of the Einstein equations, but also complex solutions—pseudoinstantons—contribute to the wave function. Among the pseudoinstantons there are solutions with interesting properties that describe a continuous transition from the Euclidean hemisphere to a stage of exponential expansion with reversed direction of the proper time relative to the coordinate time, i.e., with transition of particles into antiparticles.

Calculation of the functional integrals by a method of steepest descent analogous to the procedure used by Hartle and Hawking made it possible to find in the semiclassical approximation the probabilities of creation of a universe (transitions from the state $a = 0$ to the state $a = a_0$) with the T, S, and H topologies ($k = 0$, $k = +1$, $k = -1$, respectively). The results of the calculations have shown that for $Ha \gg 1$ the probability of creation of the S topology is greatest, while in the case of creation of an inflationary universe ($H^2 \gg 1$) these topologies are equally probable.

In the final part of the paper we have considered a toy model and determined the probability of transitions with a change of topology, a possibility that is, as is well known, permitted by quantum theory. We have shown that for $\Lambda = 0$ transition of a 3-sphere into a 3-torus in a 4-dimensional torus is strongly suppressed compared with the transition of a sphere into a sphere (with different radius).

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1) As is well known, the geometry defined on the 3-manifold can be smoothly extended to a 4-manifold (if the condition of paracompactness is satisfied).

2) For $k = 0$ we require fulfillment of the condition $\int_S d^3x\bar{h}^{1/2} = 1$.

3) Here noncompact solutions are also given.

4) One can show that such functions exist.

5) Without allowance for quantum creation of matter.

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