Inelastic relaxation of electrons in superconductor-normal metal point-contact junctions

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An analysis is made of the nonlinear conductivity of point-contact ScN junctions with a constriction (c) between superconducting (S) and normal (N) metals in the range of voltages much higher than the superconducting energy gap. Nonlinearities of the junction current-voltage characteristics observed at these voltages are mainly due to the contribution of inelastic electron-phonon scattering to the current. It is postulated that the size of the constriction is considerably less than the following quantities: the mean free path of electrons in the case of elastic impurity scattering l_i, the inelastic electron-phonon interaction length l_ε, or the superconducting correlation length ξ_0. It is shown that the nature of phonon singularities of the conductivity and of the second derivative d^2I/dV^2 depends strongly on the reflection coefficient R of electrons incident on the NS interface. A strong temperature dependence of the second derivative is found below the critical temperature of the superconductor when the coefficient R is sufficiently large and also when the Éliashberg electron-phonon interaction function $g(\omega)$ has peaks of width comparable with the energy gap.

1. INTRODUCTION

The interest in the study of inelastic electron-phonon scattering in normal and superconducting metals by point-contact spectroscopy has recently increased.[1] The current-voltage characteristics of a point-contact junction and its derivatives at voltages corresponding to maxima and other singularities of the photon density of states $F(\omega)$ can be used to determine the frequency dependence of the electron-phonon interaction (EPI) function $g(\omega) = \alpha^2F(\omega)$, where α^2 is the square of a matrix element of the EPI averaged over the directions of the initial and final electron momenta.

The relationship between d^2I/dV^2 and $g(\omega)$ was obtained in Ref. 2 for the ballistic motion of electrons, which is optimal in the case of point-contact measurements.[1] This ballistic regime is realized when the size d of a microconstriction is small compared with the elastic and inelastic electron mean free paths l_i and l_ε. An increase in the precision of the method requires lowering of the temperature to reduce the influence of the thermal broadening of the Fermi distribution of electrons at the banks of a junction. The superconducting transition can occur in some of the materials. In some cases the superconductivity in the contact region may be destroyed by a magnetic field, but in the case of superconductors with high critical fields we can investigate the EPI in the superconducting state using a point-contact junction.

Information on the electron and phonon excitations in superconductors can also be deduced from the tunnel measurements[3–5] using superconductor-insulator-superconductor (or normal metal) [SIS (or SIN)] junctions. A nonlinear tunnel current-voltage characteristic typical of superconductors is due to the existence of a gap in the electron spectrum and also to the energy dependence of the complex energy parameter $\Delta(\varepsilon)$, introduced by Éliashberg,[6] which determines the tunnel density of states[3,4]

$$N_T(\varepsilon)=N(0)\,\mathrm{Re}[|\varepsilon|/(\varepsilon^2-\Delta^2)^{1/2}],$$

where $N(0)$ is the density of states on the Fermi surface of a normal metal. The value of $N_T(\varepsilon)$ can be found from the voltage dependence of the tunnel conductance dI/dV and then the EPI function can be reconstructed from the integral Éliashberg equations.[6–8]

In contrast to tunneling accompanied by conservation of the electron energy, in the case of point-contact junctions with direct conduction under voltages of the order of typical phonon frequencies the nonlinearities of the current-voltage characteristics are related mainly to the inelastic EPI in the junction region.

We shall consider the contribution of inelastic relaxation of nonequilibrium electron excitations to the nonlinear current-voltage characteristics of ScN junctions with a constriction (c) between superconducting (S) and normal (N) metals. This was done earlier[9] in an approximation which ignores the difference between the electron parameters of the metals in contact and the possibility of existence of an abrupt (nonsemiclassical) potential barrier at the interface.

An analysis given in Ref. 9 yielded a relationship between d^2I/dV^2 and the point-contact EPI function $g_{pc}(\omega)$ qualitatively similar to the case of a conventional ScN junction, but differing from the Éliashberg function $g(\omega)$ by the presence of a factor $K(\mathbf{p},\mathbf{p}_1)$, which in the case of averaging over the directions of the momenta introduces restrictions (due to the geometry of the point-contact junction) on the electron momenta before and after the emission of a phonon of frequency $\omega_{\mathbf{p}_1-\mathbf{p}}$. When one of the banks becomes superconducting, the familiar expression for the second derivative of the current-voltage characteristic[1,2]

$$d^2I/dV^2=-4\pi e^3\Omega_{eff}N(0)\langle K\rangle\int_0^\infty S_T\left(\frac{\omega-eV}{T}\right)g_{pc}(\omega)\,d\omega/T, \tag{1}$$

where Ω_{eff} is the effective generation volume,

$$S_T(x)=\frac{d^2}{dx^2}\left(\frac{x}{e^x-1}\right)$$

is the thermal broadening function, and $\langle K\rangle$ is the value of

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the K factor averaged over the directions of the momenta, has to be modified by replacing S_T with the broadening function of the ScN junction.[9] At low temperatures characterized by $T \ll \Delta$, this broadening function is

$$S(x)=\frac{2[x-(x^2-1)^{1/2}]^2}{(x^2-1)^{1/2}}\theta(x-1), \quad x=\frac{(\omega-eV)}{\Delta}. \qquad (2)$$

The above expression is valid at high voltages $eV \gg \Delta$ such that the inelastic scattering of electrons is the prime factor that governs the nonlinear $I(V)$ dependence.

In the superconducting state when temperature is lowered the thermal broadening of the phonon nonlinearities of the current-voltage characteristic changes to broadening associated with the finite size of the energy gap Δ. The maximum of the function $S_T(\omega)$ corresponding to the voltage $eV=\omega$ shifts at temperatures $T<\Delta$ to $\omega = eV+\Delta$ and the profile of the broadening curve becomes asymmetric.

The effects associated with the change in the broadening function had been observed already[10] for Sn-Cu point-contact junctions. Peaks of the second derivative of the current-voltage characteristic, corresponding to maxima of the EPI function, were found to shift because of a change from a conventional to an ScN contact in the direction of lower voltages by a value of the order of $\Delta_{\rm Sn}$. The observed shift of the phonon peaks is opposite to the shift, known from tunnel spectroscopy,[3-5] of singularities of the phonon density of states when a singularity $F(\omega)$ at a frequency ω_0 is manifested in the second derivative of the current-voltage characteristic at a voltage $eV=\omega_0+\Delta$. This is due to superposition of the singularity $F(\omega)$ on a square-root singularity of the tunnel density of states $N_T(\varepsilon)$ (Ref. 11).

The results given above for an ScN point-contact junction are valid if

$$d \ll l_i,\ \xi_0, \qquad (3)$$

where $\xi_0 \sim v_{FS}/T_c$ is the superconducting coherence length and v_{FS} is the Fermi velocity in the investigated superconductor. It is also assumed that the phonon structure of the current-voltage characteristic appears at voltages much higher than the energy gap $eV \gg \Delta$. Assuming that $eV \approx \Delta$, the nonlinear conductance of an ScN point-contact junction considered ignoring inelastic electron relaxation was calculated in Ref. 12 in the "pure" junction limit of Eq. (3) and in Ref. 13 in the "dirty" limit $l_i \ll d$, whereas the calculation in Ref. 14 was made allowing for the finite tunnel transparency of the interface. In all these cases a linear dependence $I(V)$ with an excess current $I_{\rm exc}$ was obtained for the $eV \gg \Delta$ case:

$$I(V)=V/R_N+I_{\rm exc}. \qquad (4)$$

Here, R_N is the normal resistance of the point-contact junction, the excess current $I_{\rm exc}$ corresponds to $eV \gg T$, and Δ is proportional to Δ/eR_N.

We calculated the nonlinear current-voltage characteristic of a point-contact ScN junction allowing for inelastic EPI processes in the junction region without restricting the value of the electron reflection coefficient, which depends on the relationship between the Fermi velocities and the parameters of the potential barrier at the junction. Nonlinear current-voltage characteristics of a normal junction between two different metals were calculated by Shekhter and Kulik.[15] The K factor and the EPI function of the heterojunction obtained by them contain a transparency coefficient D and the temperature dependence of the second derivative of the current-voltage characteristic is independent of this coefficient, but is governed by the thermal broadening function in Eq. (1).

We shall show that in the case of an ScN point-contact junction, depending on the value of D, i.e., on the degree of "tunneling," the form of the phonon singularities of the current-voltage characteristic and the influence of temperature on this characteristic may differ greatly from the cases of a normal heterojunction[15] or an ScN junction with a perfectly transparent $(D=1)$ interface.[9]

2. EQUATION FOR SEMICLASSICAL GREEN FUNCTIONS AND CALCULATIONS OF THE INELASTIC COMPONENT OF THE CURRENT

In view of the smallness of the ratio d/l_ε, the probability of phonon emission by a nonequilibrium electron experiencing the influence of a strong electric field at the junction is low. In the normal case this makes it possible to solve the Boltzmann transport equation for the electron distribution function using perturbation theory.[2] The nonequilibrium distribution considered in the zeroth approximation, dependent on the potential difference V and anisotropic in respect of the momentum, is substituted into the electron-phonon collision integral; this gives rise to corrections of the order of d/l_ε to the distribution function to the current, and these corrections are associated with the generation of phonons by nonequilibrium electrons. In the case of a superconductor, because of the inequality given by Eq. (3), we cannot use the kinetic equation for the quasiparticle distribution function, such as that obtained by Aronov and Gurevich.[16] We have to solve the matrix equations for a semiclassical Green function, which are integrated with respect to the electron energy $\varepsilon_{\mathbf{p}}$ (Refs. 17 and 18) and which describe not only the change in the energy distribution of electrons in the junction region, but also the change in the energy spectrum compared with that of a homogeneous superconductor. Allowance for inelastic processes of nonequilibrium electron relaxation gives rise to a correction of the order of d/l_ε to a Green function and the task of finding it is not trivial,[19] because of the matrix nature of the equations.

We now proceed to calculate the inelastic contribution to the Green function, which makes it possible to find the inelastic current component that depends nonlinearly on the voltage.

The point-contact model is similar to that considered earlier,[14,15] i.e., we shall assume that a superconductor and a normal metal are separated by a thin impermeable barrier with a small aperture (Fig. 1). Within this aperture the transparency coefficient of the barrier is D ($R=1-D$ is the electron reflection coefficient). The barrier width 2δ is considerably larger than the size of the aperture d, the potential of the barrier layer vanishes if $|z|>\delta$, and the z axis is perpendicular to the SN interface. The component of the electron momentum parallel to the interface is conserved. The coefficient D generally depends on the coordinate $\boldsymbol{\rho}$ in the plane of the interface and on the direction of the electron momentum. A normal metal is assumed to occupy the half-space $z<0$ and a superconductor occupies $z>0$. All the quantities referring to the left or right banks of the junction

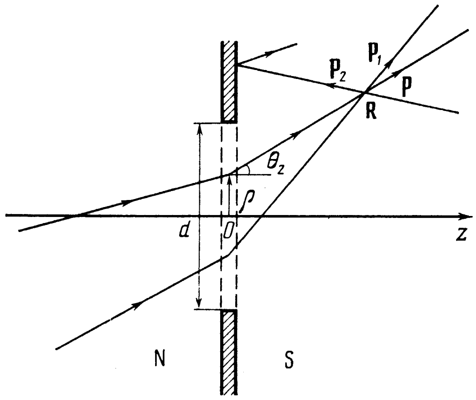

FIG. 1. Model of a junction and classification of electron paths. At a point **R**, located on a path the direction of which is governed by the momentum **p**, the momentum $\mathbf{p}_1$ corresponds to a transit path, whereas $\mathbf{p}_2$ corresponds to a nontransit path.

will be labeled with the index j which is 1 if $z<0$ and 2 if $z>0$.

We shall now write down the equation for a semiclassical Green function in the operator form[12,14]:

$$\mathbf{v}_{Fj}\partial\check{G}/\partial\mathbf{R}+[\check{H},\check{G}]_-=0. \tag{5}$$

The function $\check{G}(t,t';\mathbf{R},\mathbf{p})$, dependent on a "slow" semiclassical coordinate **R** and on the direction of the momentum p on the Fermi surface, satisfies Eq. (5) if $|z|>\delta$. The Fermi velocity vectors $\mathbf{v}_{Fj}=\partial\varepsilon_j/\partial\mathbf{p}$ are governed by the energy spectra $\varepsilon_j(\mathbf{p})$, which are assumed to be isotropic $v_{Fj}=p_{Fj}/m_j$, where p_{Fj} and m_j are the Fermi momenta and effective masses, respectively.

The matrix $\check{G}$ includes retarded, advanced, and Keldysh functions[20]

$$\check{G}=\begin{bmatrix}\hat{G}^R & \hat{G}^K\\ 0 & \hat{G}^A\end{bmatrix} \tag{6}$$

and these functions are themselves two-row matrices.[21] Their definition adopted here is identical with that used in Ref. 18. The operator $\check{H}$ in Eq. (5) is

$$\check{H}(t,t';\mathbf{R},\mathbf{p})=\left[\check{\tau}_3\frac{\partial}{\partial t}+ie\Phi(\mathbf{R},t)\right]\delta(t-t')+i\check{\Sigma}+\langle\check{G}\rangle/2\tau_j. \tag{7}$$

Here $\check{\tau}_3=\check{1}\otimes\hat{\tau}_3$, where $\check{1}$ is a unit 2×2 Keldysh matrix; $\hat{\tau}_3$ is a Pauli matrix; Φ is the potential of the electric field which appears due to the flow of the current across the junction interface; $\tau_j=l_i/v_{Fj}$ is the mean free time of electrons scattered by impurities; $\langle\ldots\rangle$ denotes averaging over the directions of the momentum. The self-energy operator $\check{\Sigma}$ can be represented as a matrix, by analogy with Eq. (6), and it describes superconducting pairing of electrons and renormalization of the spectrum due to the EPI,[6,7] as well as the inelastic electron-phonon scattering.[17,18] In calculation of the commutator in Eq. (5) and in the subsequent expressions the matrix product includes integration with respect to the time variables.

We shall simplify the calculations by ignoring the EPI in the normal metal. In experiments (see, for example, Ref. 10) it is usual to employ metals with a weak EPI, such as copper, silver, etc. Their contribution to the point-contact spectrum can be allowed for in accordance with Ref. 15. The inequalities $v_{F1}=v_{FN}>v_{F2}=v_{FS}$ and $p_{F1}>p_{F2}$ are also usually obeyed. The second equality has the effect that the total internal reflection is possible only in the case of those electrons which are incident on the interface from the normal metal side.

The voltage V is applied to the normal bank of the junction and the superconductor potential are assumed to be zero. Electrons are injected from the normal metal into the superconductor, where they relax emitting phonons. We shall now calculate the contribution made to the current by these inelastic processes.

We shall write down a Green function in the form

$$\check{G}=\check{G}_0+\check{g}, \tag{8}$$

where $\check{G}_0$ is a function considered in the zeroth approximation with respect to $d/\check{l}_\varepsilon$ and d/ξ (where $\xi^{-1}=\xi_0^{-1}+l_i^{-1}$). The function $\check{G}_0$ obeys Eq. (5) where the operator $\check{H}$ contains not the exact Green function $\check{G}$, but its limiting values $\check{G}_{1,2}$ at the banks of the junction.[12,14] The operator $\check{H}_0$ is then a discontinuous function and $z=0$. The value of $\check{G}_1$ can be obtained from the equilibrium normal quasiclassical Green function by a phase transformation described in Ref. 22 and $\check{G}_2$ is identical with the equilibrium superconductor function[17,18]:

$$\check{G}_2(\varepsilon)=\begin{bmatrix}\hat{g}^R(\varepsilon) & [\hat{g}^R(\varepsilon)-\hat{g}^A(\varepsilon)]\,\mathrm{th}(\varepsilon/2T)\\ 0 & \hat{g}^A(\varepsilon)\end{bmatrix}. \tag{9}$$

Here, $\hat{g}^{R,A}(\varepsilon)=g_\varepsilon^{R,A}\hat{\tau}_3+f_\varepsilon^{R,A}i\hat{\tau}_2$, where $g_\varepsilon^{R,A}=\varepsilon/\xi^{R,A}(\varepsilon)=(\varepsilon/\Delta)f_\varepsilon^{R,A}$. In the BCS theory the quantity Δ is a constant which in this case can be regarded as real and we then have $\xi^{R,A}(\varepsilon)=[(\varepsilon\pm i0)^2-\Delta^2]^{1/2}$. In the electron-phonon model of the superconductivity[6,7] the quantity Δ is regarded as a complex function of ε and $\xi^{R,A}(\varepsilon)$ is obtained by analytic continuation of the quantity $[\varepsilon^2-\Delta^2(\varepsilon)]^{1/2}$ from the upper and lower half-planes, respectively, to the real values of ε, and the choice of the branch of the square root is determined by the condition $\mathrm{Im}(\varepsilon^2-\Delta^2)^{1/2}>0$.

The first-order correction $\check{g}(t,t';\mathbf{R},\mathbf{p})$ is described by[19]

$$\eta_\mathbf{p}\partial\check{g}/\partial s+[\check{H}_0,\check{g}]_-+[\delta\check{H},\check{G}_0]_-=0, \tag{10}$$

where s is the time of motion along a path the direction of which is described by the vector $\mathbf{v}_{Fj}$; $\eta_\mathbf{p}\partial/\partial s=\mathbf{v}_{Fj}\partial/\partial\mathbf{R}$; $\eta_\mathbf{p}=\mathrm{sign}\,p_z$. The paths traversing the barrier are refracted (Fig. 1). The operator $\delta\check{H}$ is the difference between the exact operator (7) and its discontinuous approximation $\check{H}_0$, obtained by replacing $\check{G}$ with $\theta(-z)\check{G}_1+\theta(z)\check{G}_2$ and replacing a smooth potential $\Phi(\mathbf{R})$ with a step $\theta(-z)V$. The contribution of the last term in Eq. (7) can be ignored because it gives rise to voltage-independent corrections of the order of d/l_i to the normal resistance and to the excess current[19] in Eq. (4). The spatial variation of the potential $\Phi(\mathbf{R})$ can be found from the electrical neutrality condition when the potential does not occur in Eq. (10) because of the absence of a time dependence.

The part $\delta\check{H}$ needed to calculate the inelastic current is

$$\delta\check{H}(t,t';\mathbf{R},\mathbf{p})=i\delta\check{\Sigma}(t,t';\mathbf{R},\mathbf{p})\theta(z). \tag{11}$$

It is known that in the adiabatic approximation (see, for example, §21 in Ref. 21) the self-energy electron-phonon operator is a linear functional of an electron Green function. We can find $\delta\check{\Sigma}$ by substituting in the familiar expressions[18] the difference defined by $\check{G}_0(\mathbf{R},\mathbf{p}) - \check{G}_2$, so that at a given point R in the course of integration with respect to $\mathbf{p}$ we have to allow only for "transit" paths (Fig. 1) for which $\check{G}_0$ differs from $\check{G}_2$. The corresponding phase volume decreases away from the aperture as $(d/R)^2$.

A comparison of the second and third terms on the left-hand side of Eq. (10) shows that the ratio of these terms is of the order $R^2/d\lambda$, where $\lambda = \min\{\zeta, l_\varepsilon\}$. If $R \ll (d\lambda)^{1/2}$, we drop the second term and this gives the following solution:

$$\check{g}(s) = \check{g}(+) - i\eta_{\mathbf{p}} \int_0^s ds [\delta\check{\Sigma}, \check{G}_0]_-, \tag{12}$$

where the value of $\check{g}$ is taken directly to the right of the barrier, $\check{g}(+) = \check{g}(z=\delta)$, governs the inelastic component of the current. If $R \gg (d\lambda)^{1/2}$, the last term of Eq. (10) is small and $\check{g}$ satisfies a homogeneous equation the solution of which varies along the path in a distance of the order of $\lambda \gg d$. The asymptotic expression given by Eq. (12) corresponding to $s \gg d/v_{F2}$ and the limit of the solution of the homogeneous equation corresponding to $s \to 0$ are identical at distances $R \sim (\lambda d)^{1/2}$, so that far from the junction the solution can be represented by

$$\check{g}(s) = \exp(-\eta_{\mathbf{p}} \check{H}_0 s) \check{g}_2 \exp(\eta_{\mathbf{p}} \check{H}_0 s), \tag{13}$$

where

$$\check{g}_2 = \check{g}(+) - i\eta_{\mathbf{p}} \int_0^\infty ds [\delta\check{\Sigma}, \check{G}_0]_-. \tag{14}$$

In view of the above-mentioned restriction on the phase volume, the integral of Eq. (14) converges after a time $\sim d/v_{F2}$, i.e., after a time of the order of the transit time of an electron crossing the constriction. A characteristic length of the change in the zeroth-approximation solution is λ, so that we can ignore the dependence of the Green function in Eq. (14) on s and replace this function with its value $\check{G}_0(+)$ at the right-hand bank of the barrier. At high values of s the solution given by Eq. (13) should decrease so that the following boundary conditions apply to $\check{g}_2$ (Refs. 12 and 19):

$$\check{g}_2 = \eta_{\mathbf{p}} \check{G}_2 \check{g}_2 = -\eta_{\mathbf{p}} \check{g}_2 \check{G}_2. \tag{15}$$

Using Eq. (15) and similar relationships that follow from the requirements that $\check{g}$ falls to the left of the aperture in the limit $z \to -\infty$, we obtain the following system of equations for the Green function of Eq. (8) at the barrier banks, $\check{G}(\pm) = \check{G}(z = \pm\delta)$:

$$\begin{aligned}\check{G}(+) &= \check{G}_2 + \eta_{\mathbf{p}} \check{G}_2 \check{G}(+) - \eta_{\mathbf{p}} \check{1} + \check{K} \\ &= \check{G}_2 - \eta_{\mathbf{p}} \check{G}(+) \check{G}_2 + \eta_{\mathbf{p}} \check{1} + \check{K}',\end{aligned} \tag{16}$$

$$\check{G}(-) = \check{G}_1 - \eta_{\mathbf{p}} \check{G}_1 \check{G}(-) + \eta_{\mathbf{p}} \check{1} = \check{G}_1 + \eta_{\mathbf{p}} \check{G}(-) \check{G}_1 - \eta_{\mathbf{p}} \check{1}, \tag{17}$$

$$\check{K} = \eta_{\mathbf{p}} \check{J} - \check{G}_2 \check{J}, \quad \check{K}' = \eta_{\mathbf{p}} \check{J} + \check{J} \check{G}_2,$$

and

$$\check{J} = i \int_0^\infty ds [\delta\check{\Sigma}, \check{G}_0(+)]_- \tag{18}$$

is the electron-phonon collision operator.

The boundary conditions for semiclassical Green functions derived in Ref. 14 relate the part of $\check{G}_a$ odd in respect of the momentum and continuous at the junction interface to the even components $\check{G}_c(\pm)$ on both sides of the barrier. In Eqs. (16) and (17), we can substitute

$$\check{G}(\pm) = \check{G}_c(\pm) + \eta_{\mathbf{p}} \check{G}_a. \tag{19}$$

Moreover, the following equation is valid[14]

$$\begin{aligned}&[\check{1} - \check{G}_a^2 - D(\check{G}_c^+)^2] \check{G}_a = D\check{G}_c^- \check{G}_c^+, \\ &\check{G}_c^\pm = {}^1/_2[\check{G}_c(+) \pm \check{G}_c(-)].\end{aligned} \tag{20}$$

Equations (16)–(20) allow us to find (in the first order in respect of the electron-phonon collisions) a component of the function $\check{g}$ which is odd in respect of $\mathbf{p}$. It follows from Eqs. (16) and (17) that $\check{G}_c^\pm$ can be expressed in terms of $\check{G}_a$:

$$\check{G}_c^\pm = \check{G}_\pm + \check{G}_\mp \check{G}_a + {}^1/_2 \check{K}_c, \tag{21}$$

where

$$\check{G}_\pm = {}^1/_2(\check{G}_2 \pm \check{G}_1).$$

Substituting Eq. (21) into Eq. (20), we obtain an expression for $\check{G}_a$ where the Green function in all the terms containing the electron-phonon collision operators should be taken in the zeroth approximation. This derivation is based on the anticommutation relationships

$$\{\check{G}_a, \check{G}_\pm\}_+ = \{\check{g}_a, \check{G}_\pm\}_+ = {}^1/_2 \{\check{J}_c, \check{G}_2\}_+, \tag{22}$$

which follows from the definition of $\check{J}$ of Eq. (18) and from the normalization condition[18]

$$\check{G}^2 = \check{1}, \tag{23}$$

which is satisfied with the required precision, as can easily be shown.

After some simplifications the equation for $\check{G}_a$ becomes

$$\begin{aligned}\check{G}_a[\check{1} - D\check{G}_-^2] - D\check{G}_- \check{G}_+ = \frac{i}{2} D \int_0^\infty ds \{\check{G}_- \check{M} - \check{L}\check{G}_- \\ + (\check{\Sigma}_c - \check{G}_2 \check{\Sigma}_c \check{G}_2 + [\check{\Sigma}_a, \check{G}_2]_-) \check{G}_c^+\},\end{aligned} \tag{24}$$

where

$$\begin{aligned}\check{M} &= (\check{1} - \check{G}_a)(\check{\Sigma}_c - \check{G}_2 \check{\Sigma}_c \check{G}_2 + [\check{\Sigma}_a, \check{G}_2]_-), \\ \check{L} &= \check{G}_2 \check{\Sigma}_c \check{G}_c(+) - \check{G}_c(+) \check{\Sigma}_c \check{G}_2 + \check{G}_2 \check{\Sigma}_a \check{G}_a - \check{G}_a \check{\Sigma}_a \check{G}_2 \\ &\quad - \{\check{G}_c(+) - \check{G}_2, \check{\Sigma}_a\}_+ - \{\check{G}_a, \check{\Sigma}_c\}_+,\end{aligned}$$

and the Green functions on the right-hand side of Eq. (24) are taken in the zeroth approximation (the index "0" is omitted here and later) and are given by[14,22]:

$$\begin{aligned}\check{G}_a &= D\check{G}_- \check{G}_+ / (\check{1} - D\check{G}_-^2), \\ \check{G}_c(+) &= (\check{G}_+ + R\check{G}_-)/(\check{1} - D\check{G}_-^2), \\ \check{G}_c^+ &= \check{G}_+/(\check{1} - D\check{G}_-^2).\end{aligned} \tag{25}$$

We shall also introduce the notation

$$\delta\check{\Sigma} = \check{\Sigma}_c + \eta_{\mathbf{p}} \check{\Sigma}_a.$$

Transformation of the right-hand side of Eq. (24) gives

$$\check{g}_a = \frac{i}{2} \int_0^\infty ds (\check{X}_1 + \check{X}_2), \tag{26}$$

where

$$\check{X}_1=[\check{\Sigma}_c,\check{G}_c(+)]_- - \check{G}_a\check{\Sigma}_c\check{G}_c(+) - \check{G}_c(+)\check{\Sigma}_c\check{G}_a + \check{\mathcal{G}}_c(+)\check{\Sigma}_c\check{\mathcal{G}}_a(+) - \check{\mathcal{G}}_a(+)\check{\Sigma}_c\check{\mathcal{G}}_c(+), \tag{27}$$

$$\check{X}_2=\check{\Sigma}_a+[\check{\Sigma}_a,\check{G}_a]_- - \check{G}_c(+)\check{\Sigma}_a\check{G}_c(+) - \check{G}_a\check{\Sigma}_a\check{G}_a + \check{\mathcal{G}}_c(+)\check{\Sigma}_a\check{\mathcal{G}}_c(+) - \check{\mathcal{G}}_a(+)\check{\Sigma}_a\check{\mathcal{G}}_a(+). \tag{28}$$

These expressions contain a function $\check{\mathcal{G}}(+)$ introduced in Ref. 14 and obtained when a semiclassical description is used for systems with fast spatial variations of the potential and abrupt interfaces between regions with different electronic properties. The even and odd (in respect of the momentum) parts of this function $\check{\mathcal{G}}_{c,a}(+)$ are given by

$$\check{\mathcal{G}}_c(+)=-R^{1/2}\check{G}_2/(\check{1}-D\check{G}_-^{2}),$$
$$\check{\mathcal{G}}_a(+)=-R^{1/2}/(\check{1}-D\check{G}_-^{2}). \tag{29}$$

The expansions of Eqs. (25) and (29) in respect of the Pauli matrices are given in the Appendix.

The electron-phonon operators $\check{\Sigma}_a$ and $\check{\Sigma}_c$ are expressed in terms of Green functions integrated with respect to the energy[18]:

$$\hat{\Sigma}_c^{R,A}(\varepsilon,\mathbf{p})=\int d\varepsilon_1\int\frac{d\Omega_{\mathbf{p}_1}}{4\pi}w_{\mathbf{pp}_1}(\varepsilon_1-\varepsilon)\left\{\mathrm{cth}\left(\frac{\varepsilon_1-\varepsilon}{2T}\right)\times[\hat{G}_c^{R,A}(+;\varepsilon_1)-\hat{G}_2^{R,A}(\varepsilon_1)]\mp{}^1\!/_2[\hat{G}_c^{K}(+;\varepsilon_1)-\hat{G}_2^{K}(\varepsilon_1)]\right\}, \tag{30}$$

$$\hat{\Sigma}_c^{K}(\varepsilon,\mathbf{p})=\int d\varepsilon_1\int\frac{d\Omega_{\mathbf{p}_1}}{4\pi}w_{\mathbf{pp}_1}(\varepsilon_1-\varepsilon)\left\{\mathrm{cth}\left(\frac{\varepsilon_1-\varepsilon}{2T}\right)\times[\hat{G}_c^{K}(+;\varepsilon_1)-\hat{G}_2^{K}(\varepsilon_1)]-[\hat{G}_c^{R}(+)-\hat{G}_c^{A}(+)](\varepsilon_1)+[\hat{G}_2^{R}-\hat{G}_2^{A}](\varepsilon_1)\right\}, \tag{31}$$

where

$$w_{\mathbf{pp}_1}(\varepsilon_1-\varepsilon)={}^1\!/_4N_2(0)\sum_\lambda|g^\lambda_{\mathbf{pp}_1}|^2\theta(\mathbf{p}_1;\mathbf{p},\boldsymbol{\rho},s)\times[D^R-D^A]_{\mathbf{p}_1-\mathbf{p}}(\varepsilon_1-\varepsilon). \tag{32}$$

The phonon Green functions D^R and D^A are described by

$$D_{\mathbf{q}}^{R}(\omega)=[D_{\mathbf{q}}^{A}(\omega)]^*=\omega_\lambda^2(\mathbf{q})/[(\omega+i0)^2-\omega_\lambda^2(\mathbf{q})],$$

where $\omega^\lambda(\mathbf{q})$ is the dispersion law of phonons with a polarization λ; $g^\lambda_{\mathbf{pp}_1}$ is a matrix element of the EPI; $N_2(0)$ is the density of states on the Fermi surface in the investigated superconductor.

The step function is $\theta(\mathbf{p}_1;\mathbf{p},\boldsymbol{\rho},s)=1$ if the vector $\mathbf{p}_1$ corresponds to a transit path passing through a point $\mathbf{R}$; otherwise it vanishes (Fig. 1). The point $\mathbf{R}$ lies on a path passing through the aperture in the junction and the direction of this path in the superconductor is described by the vector $\mathbf{p}$; $\boldsymbol{\rho}$ is the coordinate of the point of intersection of the path with the $z=0$ plane; s is the time of motion from the junction plane to $\mathbf{R}$.

The operator $\check{\Sigma}_a$ satisfies expressions analogous to Eqs. (30) and (31), where $\check{G}_c(+)-\check{G}_2$ is replaced with $\check{G}_a$ and the functions $w_{\mathbf{pp}_1}$ are replaced with $\overline{w}_{\mathbf{pp}_1}$, the latter differing from Eq. (32) by the factor $\eta_{\mathbf{p}}\ \eta_{\mathbf{p}_1}$. All the zeroth-approximation Green functions in the above expressions are given explicitly in the Appendix.

The relationships given by Eqs. (25)-(32) allow us to calculate the nonlinear (in respect of the voltage) current component associated with the inelastic relaxation processes.

3. NONLINEAR CURRENT-VOLTAGE CHARACTERISTIC OF AN ScN POINT-CONTACT JUNCTION AT HIGH VOLTAGES

In this section we shall use the results of Sec. 2 to calculate the current-voltage characteristic of a point-contact junction between a normal metal and a superconductor subjected to voltages $eV\gg\Delta$ when the dependence $I(V)$ is linear and contains the excess current of Eq. (4) if we ignore the electron-phonon collisions. The inelastic component of the current $I_{\mathrm{ph}}(V)$ can be expressed in terms of the Keldysh function $\check{g}_a^K$:

$$I_{ph}(V)=-{}^1\!/_4eN_2(0)\int d^2\rho\int d\varepsilon\,\mathrm{Tr}\,\hat{\tau}_3\langle|v_z|\hat{g}_a^{K}(\varepsilon;\boldsymbol{\rho},s=0)\rangle, \tag{33}$$

where the integration with respect to $\boldsymbol{\rho}$ is limited to the area of the aperture in the junction.

Further calculations reduce to finding the coefficients in front of the matrix $\hat{\tau}_3$ in off-diagonal Keldysh components of the quantities $\check{X}_{1,2}$, which are defined by Eqs. (27) and (28). The expression for the current simplifies when $eV\gg\Delta$. The inelastic contribution to the current, which is a nonlinear function of V, can be written in the form

$$I_{ph}(V)=-eN_2(0)\Omega_{eff}\int_0^\infty d\omega[\tau_1^{-1}(\omega,V)+\tau_2^{-1}(\omega,V)]. \tag{34}$$

The above expression contains the inelastic relaxation times of electrons $\tau_{1,2}$ dependent on the voltage across the junction and associated with the processes that are accompanied by the emission of a phonon frequency ω:

$$\tau_1^{-1}(\omega,V)=2\pi\int d\varepsilon\left\langle w_1(\omega;\mathbf{p},\mathbf{p}_1)DD_1\left\{A(\varepsilon)+\frac{2(\varepsilon^2-\Delta^2)^{1/2}[|\varepsilon|+(\varepsilon^2-\Delta^2)^{1/2}]\theta(|\varepsilon|-\Delta)}{[D|\varepsilon|+(1+R)(\varepsilon^2-\Delta^2)^{1/2}]^2}[1-n(\varepsilon)]\right\}\right\rangle\times n(\varepsilon+\omega-eV),$$

$$\tau_2^{-1}(\omega,V)=2\pi\int d\varepsilon\left\langle w_2(\omega;\mathbf{p},\mathbf{p}_1)RDD_1\left\{A(\varepsilon)-\frac{2(\varepsilon^2-\Delta^2)^{1/2}[|\varepsilon|-(\varepsilon^2-\Delta^2)^{1/2}]\theta(|\varepsilon|-\Delta)}{[D|\varepsilon|+(1+R)(\varepsilon^2-\Delta^2)^{1/2}]^2}[1-n(\varepsilon)]\right\}\right\rangle\times n(\varepsilon+\omega-eV), \tag{36}$$

where

$$A(\varepsilon)=\Delta^2\{\theta(\Delta-|\varepsilon|)[(1+R)^2\Delta^2-4R\varepsilon^2]^{-1}+\theta(|\varepsilon|-\Delta)[D|\varepsilon|+(1+R)(\varepsilon^2-\Delta^2)^{1/2}]^2\},$$

and $n(\varepsilon)=[\exp(\varepsilon/T)+1]^{-1}$ is the Fermi distribution function. The effective phonon generation volume for a circular aperture with a diameter d in Eq. (34) is $\Omega_{eff}=d^3/3$. The EPI functions $w_{1,2}$ are

$$w_{1,2}(\omega; \mathbf{p}, \mathbf{p}_1) = {}^1/_2 N_2(0)\theta(\mp p_z p_{1z}) K(\mathbf{p}, \mathbf{p}_1)$$

$$\times \sum_\lambda \omega^\lambda_{\mathbf{p}_1-\mathbf{p}} |g^\lambda_{\mathbf{p}\mathbf{p}_1}|^2 \delta(\omega - \omega^\lambda_{\mathbf{p}_1-\mathbf{p}}), \qquad (37)$$

where the geometric K factor is[2]

$$K(\mathbf{p}, \mathbf{p}_1) = |p_z||p_{1z}|/|\mathbf{p}p_{1z} - \mathbf{p}_1 p_z|.$$

The transparency coefficient D and also the reflection coefficient R generally depend on the direction of the momentum. In Eqs. (35) and (36) we have $D = D(\mathbf{p})$ and $D_1 = D(\mathbf{p}_1)$, and the angular brackets denote averaging over the directions of $\mathbf{p}$ and $\mathbf{p}_1$. The actual form of the dependence on the momentum and on the barrier parameters is governed by the potential barrier model which is selected.[14,15]

It follows from Eq. (37) that the relaxation times τ_1 and τ_2 describe the processes of electron scattering accompanied by the emission of a phonon when the z component of the electron momentum may or may not change its sign.

The nonlinear part of the conductivity associated with phonon generation in the junction region can be described by a sum of two terms:

$$\sigma_{ph}(V) = \sigma_1(V) + \sigma_2(V). \qquad (38)$$

In the case of a junction in the form of a circular aperture of diameter d the conductivities $\sigma_{1,2}(V)$ are given by

$$\sigma_{1,2}(V) = -\frac{16d}{3v_{FS}R_N}\int_0^\infty d\omega \langle W_{1,2}(\omega; \mathbf{p}, \mathbf{p}_1)$$

$$\times F_{1,2}[(eV-\omega)/T; \Delta/T]\rangle, \qquad (39)$$

where

$$F_{1,2}[(eV-\omega)/T; \Delta/T]$$

$$= \int_{-\infty}^{+\infty} \frac{d\varepsilon}{4T} \mathrm{ch}^{-2}\left(\frac{\varepsilon+\omega-eV}{2T}\right)\left\{\frac{\theta(\Delta-|\varepsilon|)\Delta^2}{[(1+R)^2\Delta^2 - 4R\varepsilon^2]}\right.$$

$$\left. + \frac{[|\varepsilon| \pm (\varepsilon^2-\Delta^2)^{1/2}\,\mathrm{th}(\varepsilon/2T)][|\varepsilon| \pm (\varepsilon^2-\Delta^2)^{1/2}]}{[D|\varepsilon| + (1+R)(\varepsilon^2-\Delta^2)^{1/2}]^2}\theta(|\varepsilon|-\Delta)\right\}. \qquad (40)$$

The normal resistance of the junction is given by the usual expression

$$R_N^{-1} = 2\langle \alpha D(\alpha)\rangle / R_0, \quad \alpha = \cos\theta_2,$$

where θ_2 is the angle between the path of the electron to the right of the junction, inside the superconductor, and normal to the interface; R_0 is the resistance of a pure metallic constriction with a cross-sectional area S equal to $R_0 = 4\pi^2/e^2 p_{F2}^2 S$.

The EPI function W_1 is

$$W_1(\omega; \mathbf{p}, \mathbf{p}_1) = DD_1 w_1(\omega; \mathbf{p}, \mathbf{p}_1)/2\langle \alpha D(\alpha)\rangle$$

and averaging over the directions of the momenta makes this function identical with the point-contact function of a normal heterojunction,[15] whereas W_2 is given by

$$W_2(\omega; \mathbf{p}, \mathbf{p}_1) = RDD_1 w_2(\omega; \mathbf{p}, \mathbf{p}_1)/2\langle \alpha D(\alpha)\rangle.$$

If we ignore the angular dependences of the coefficients R and D, we find that averaging yields

$$\langle W_1\rangle = DG_1(\omega), \quad \langle W_2\rangle = RDG_2(\omega),$$

where the point-contact functions $G_{1,2}(\omega)$ have the usual form,[1,2] but contain different K factors:

$$K_{1,2}(\mathbf{p}, \mathbf{p}_1) = \theta(\mp p_z p_{1z}) K(\mathbf{p}, \mathbf{p}_1).$$

Let us assume that the EPI matrix element depends only on the modulus of the transferred momentum $q = |\mathbf{p} - \mathbf{p}_1|$, so that in the case of a spherical Fermi surface we average the K factors over the directions of $\mathbf{p}$ and $\mathbf{p}_1$ for a fixed scattering angle θ. This gives the familiar[23] "normal" factor

$$K_1(\theta) = {}^1/_8(1 - \theta\,\mathrm{ctg}\,\theta), \qquad (41)$$

corresponding to backscattering accompanied by a change in the sign of p_z, as well as the factor

$$K_2(\theta) = {}^1/_8[1 + (\pi - \theta)\,\mathrm{ctg}\,\theta], \qquad (42)$$

which determines the contribution of the forward scattering to the inelastic current.

When two normal metals are in contact and also when $D = 1$, the inelastic current and the conductivity are described by the first terms of Eqs. (34) and (38). If the transparency of the junction interface is finite and the temperature of the junction is $T < T_c$, an additional contribution σ_2 to the conductivity appears and, as demonstrated by Eq. (42), the role of the scattering processes accompanied by a small transfer of the momentum increases. The increase in $K_2(\theta)$ may have the effect that both terms in Eq. (38) become important even if the reflection coefficient R is small.

Figure 2 shows graphs of the functions $F_{1,2}$ obtained for various values of R. Near the voltages $eV = \omega \pm \Delta$ there are peaks associated with the gap singularities of the probabilities of the Andreev reflection and single-particle processes in an SN junction with a finite reflection coefficient.[24]

In numerical calculations we shall replace the EPI functions $G_{1,2}(\omega)$ with $\frac{1}{4}g_{pc}(\omega)$ and the factor $\frac{1}{4}$ then appears as a result of averaging of the factors $K_2(\theta)$ over the scattering angles. The point-contact function $g_{pc}(\omega)$ will be selected in the form of a Lorentzian curve with an average frequency ω_0 and a half-width Γ. The derivatives of the current-voltage characteristics with respect to the voltage and their temperature dependences are affected strongly by the value of R, and also by the relationship between the width Γ of a phonon peak and the gap Δ. Figure 3 shows the nonlinear correction to the resistivity of a contact $\rho(V)$, which corresponds to a narrow phonon peak such that $\gamma = \Gamma/\Delta = 0.2$. The gap singularities $F_{1,2}$ (Fig. 2) appear in the form of two maxima of $\rho(V)$ at $eV = \omega_0 \pm \Delta$, which disappear when the temperature of the junction is increased to T_c.

Values $\gamma \gtrsim 1$ are more realistic. The resistivity $\rho(V)$ is then a smooth function and the spectral singularities appear in the second derivative d^2I/dV^2 of the current-voltage characteristic. When the width of a phonon peak is large compared with Δ, the change in the point-contact spectrum as a result of lowering of T is relatively small and depends weakly on the reflection coefficient R (Fig. 4). If γ decreases but R remains sufficiently large, the temperature dependence of d^2I/dV^2 becomes very strong (Fig. 5). A Lorentzian phonon peak [deduced allowing for the thermal broadening—see Eq. (1)] is converted into an asymmetric curve as a

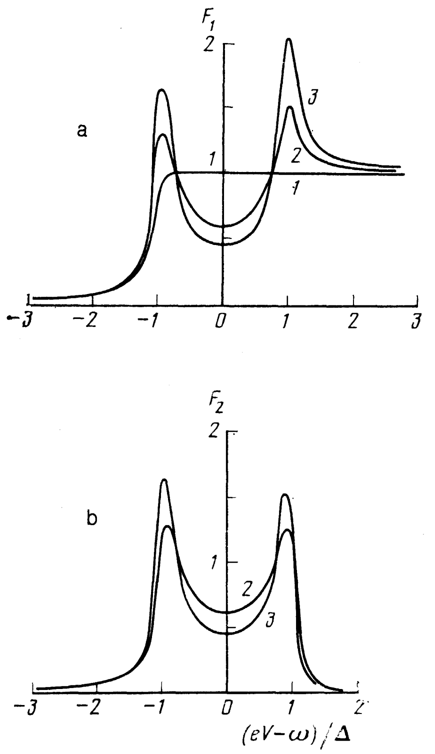

FIG. 2. Functions F_1 (a) and F_2 (b) plotted for $T/T_c = 0.1$ and different values of the reflection coefficient R: 1) 0; 2) 0.3; 3) 0.5.

result of the transition to the superconducting state and the derivative d^2I/dV^2 becomes an alternating function of the voltage.

The derivatives of the current-voltage characteristics plotted in Figs. 3–5 are given in relative units. The order of magnitude of the change in the differential resistivity due to the inelastic electron-phonon scattering is

$$\rho(V)/R_N \sim \lambda_{ph} D(d\omega_0/v_{FS}),$$

where $\lambda_{\rm ph}$ is the dimensionless EPI constant; in the selected model the phonon frequency ω_0 is equal to the average frequency of a Lorentzian peak.

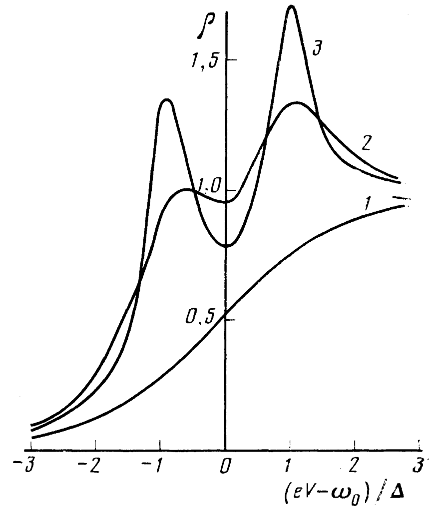

FIG. 3. Differential resistivity $\rho(V)$ at voltages corresponding to frequencies at which the point-contact function $g_{pc}(\omega)$ has a Lorentzian profile with an average frequency ω_0 and a half-width Γ. The reflection coefficient is $R = 0.5$ and we also have $\gamma = \Gamma/\Delta = 0.2$; the reduced temperature T/T_c is 1, 0.5, and 0.1 for curves 1, 2, and 3, respectively.

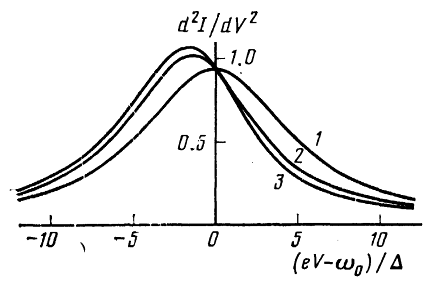

FIG. 4. Second derivative of the current-voltage characteristic with respect to the voltage for a Lorentzian function $g_{pc}(\omega)$; $R = 0.5$, $\gamma = 5$, and reduced temperature T/T_c amounting to 1, 0.8, and 0.5 for curves 1, 2, and 3, respectively.

For typical values of $d \sim 10^{-6}$ cm, $v_{FS} \sim 10^8$ cm/s, $D \sim 0.5$, $\omega_0 \sim 10^{13}$ s^{-1}, and $\lambda_{\rm ph} \sim 0.5$, we obtain $\rho/R_N \sim 2.5 \cdot 10^{-2}$.

In the case of superconductors with a strong EPI a nonlinear structure of the current-voltage characteristic of a point-contact junction can also be related to the energy dependence of the gap $\Delta(\varepsilon)$. At high voltages the "tail" of the gap singularity of the current-voltage characteristic makes a relative contribution of the order of $(\Delta/V)^2$ to the differential resistivity. Hence, it follows that the effects considered here may determine, for a given value of V, the nonlinear resistivity of the junction if

$$d \gg (v_{FS}/VD\lambda_{ph})(\Delta/V)^2. \tag{43}$$

This imposes restrictions on the transparency coefficient D, which must not be too small. If $D \ll 1$, we go back to the familiar elastic tunnel spectroscopy method.[3-5]

4. CONCLUSIONS

A theory of the nonlinear conductivity of ScN point-contact junctions given in Refs. 12–14 and 24 accounts for the dependence $I(V)$ at voltages $eV \sim \Delta$. When V is increased to values corresponding to typical phonon energies, a strong influence of the nonequilibrium effects and of heating in the junction region may result in major changes in the resistivity associated with partial or complete suppression of the superconductivity near the junction.

We considered a situation in which the voltage depend-

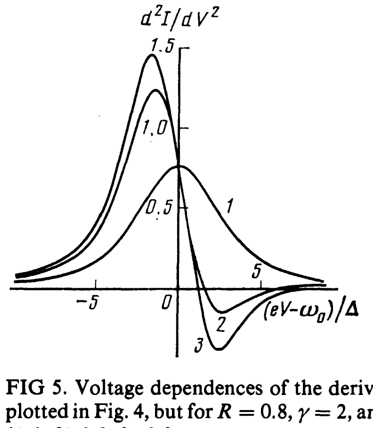

FIG 5. Voltage dependences of the derivative d^2I/dV^2 similar to those plotted in Fig. 4, but for $R = 0.8$, $\gamma = 2$, and the following values of T/T_c: 1) 1; 2) 0.8; 3) 0.3.

ence of the conductivity appears because of the contribution made to the current by "hot" electrons injected into a superconductor and relaxing by phonon emission. Phonons leave the region adjoining the constriction without heating this region significantly. The feasibility of realizing this regime experimentally in ScN point-contact junctions is supported by the observation that the excess current remains practically constant when V is increased,[25] i.e., the order parameter Δ for the banks is not suppressed on increase in the Joule dissipated power $R_N I^2$. The local state of the superconductor in the vicinity of a junction of size d nevertheless differs strongly from the homogeneous equilibrium state far from the junction and this is the reason for the complex voltage dependences of the probabilities of inelastic processes.

It is clear from our results that the phonon singularities of the current-voltage characteristic are affected by the finite transparency of the NS junction and/or the difference between the electronic characteristics of the normal and superconducting banks of the junction. Such effects are always present in ScN-type heterojunctions. In some cases[10] they are manifested weakly, but in other experiments[25-27] they influence considerably the point-contact spectra. For example, in the case of Ta-Cu contacts it is found[25] that the downward shift of d^2I/dV^2 along the ordinate and the appearance of negative values, in qualitative agreement with the above calculations, are correlated with the profiles of the gap singularities which indicate the reflection of electrons at the junction. Phonon singularities of the type shown in Fig. 5 have also been observed for Zn-Re point-contact junctions[26] below the superconducting temperature T_c of rhenium (1.7 K).

Technetium-silver junctions[27] exhibit a differential resistivity peak at a voltage slightly higher than the value corresponding to the maximum of the EPI function. In this case the mean free path of electrons is clearly small, the inequality of Eq. (3) is disobeyed, and the model proposed here is unacceptable.

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APPENDIX

The semiclassical Green functions described by Eq. (25) have the following Keldysh components:

$$\hat{G}_c^{R,A}(+)=\{[(1+R)g_\varepsilon^{R,A}\pm D]\hat{\tau}_3+(1+R)f_\varepsilon^{R,A}i\hat{\tau}_2\}/D^{R,A}(\varepsilon), \tag{A.1}$$

where $D^{R,A}(\varepsilon)=1+R\pm Dg_\varepsilon^{R,A}$ and the quantities $g_\varepsilon^{R,A}, f_\varepsilon^{R,A}$ are defined in the text above. The off-diagonal Keldysh function can be described by an expansion in terms of matrices $\hat{\tau}_{0,1,3}$ and $i\hat{\tau}_2$ with the following coefficients:

$$A_0(\varepsilon)=-\frac{D\alpha(\varepsilon)}{2D_\varepsilon^R D_\varepsilon^A}[(1+R)(1-g_\varepsilon^R g_\varepsilon^A+f_\varepsilon^R f_\varepsilon^A)+D(g_\varepsilon^R-g_\varepsilon^A)],$$

$$A_1(\varepsilon)=\frac{D\alpha(\varepsilon)}{2D_\varepsilon^R D_\varepsilon^A}D(f_\varepsilon^R+f_\varepsilon^A),$$

$$A_2(\varepsilon)=\frac{D\beta(\varepsilon)}{2D_\varepsilon^R D_\varepsilon^A}[D(f_\varepsilon^R-f_\varepsilon^A)-(1+R)(g_\varepsilon^R f_\varepsilon^A+g_\varepsilon^A f_\varepsilon^R)] + 4R\,\mathrm{th}(\varepsilon/2T)(f_\varepsilon^R-f_\varepsilon^A)/D_\varepsilon^R D_\varepsilon^A, \tag{A.2}$$

$$A_3(\varepsilon)=\frac{D\beta(\varepsilon)}{2D_\varepsilon^R D_\varepsilon^A}[(1+R)(1-g_\varepsilon^R g_\varepsilon^A-f_\varepsilon^R f_\varepsilon^A)+D(g_\varepsilon^R-g_\varepsilon^A)] + \frac{(1+R)\,\mathrm{th}(\varepsilon/2T)}{D_\varepsilon^R D_\varepsilon^A}[D(1-g_\varepsilon^R g_\varepsilon^A+f_\varepsilon^R f_\varepsilon^A)+(1+R)(g_\varepsilon^R-g_\varepsilon^A)].$$

The following notation is used above:

$$\alpha(\varepsilon)=\mathrm{th}\left(\frac{\varepsilon+eV}{2T}\right)-\mathrm{th}\left(\frac{\varepsilon-eV}{2T}\right),$$

$$\beta(\varepsilon)=\mathrm{th}\left(\frac{\varepsilon+eV}{2T}\right)+\mathrm{th}\left(\frac{\varepsilon-eV}{2T}\right).$$

The components of the Green function, which is odd in respect of the momentum, can be described similarly:

$$\hat{G}_a^{R,A}(\varepsilon)=\mp Df_\varepsilon^{R,A}\hat{\tau}_1/D_\varepsilon^{R,A}, \tag{A.3}$$

$$B_0(\varepsilon)=\frac{D(\beta(\varepsilon)-2\,\mathrm{th}(\varepsilon/2T))}{2D_\varepsilon^R D_\varepsilon^A}[D(1-g_\varepsilon^R g_\varepsilon^A+f_\varepsilon^R f_\varepsilon^A) + (1+R)(g_\varepsilon^R-g_\varepsilon^A)],$$

$$B_1(\varepsilon)=-D(1+R)\beta(\varepsilon)(f_\varepsilon^R+f_\varepsilon^A)/2D_\varepsilon^R D_\varepsilon^A,$$

$$B_2(\varepsilon)=-\frac{D\alpha(\varepsilon)}{2D_\varepsilon^R D_\varepsilon^A}[(1+R)(f_\varepsilon^R-f_\varepsilon^A)-D(g_\varepsilon^R f_\varepsilon^A+g_\varepsilon^A f_\varepsilon^R)], \tag{A.4}$$

$$B_3(\varepsilon)=-\frac{D\alpha(\varepsilon)}{2D_\varepsilon^R D_\varepsilon^A}[D(1-g_\varepsilon^R g_\varepsilon^A-f_\varepsilon^R f_\varepsilon^A) + (1+R)(g_\varepsilon^R-g_\varepsilon^A)].$$

In the case of the function $\mathscr{G}_{c,a}(+)$ defined by Eq. (29) we can use the following representations:

$$\mathscr{G}_c^{R,A}(+)=-\frac{2R^{1/2}}{D_\varepsilon^{R,A}}[g_\varepsilon^{R,A}\hat{\tau}_3+f_\varepsilon^{R,A}i\hat{\tau}_2], \tag{A.5}$$

$$\mathscr{G}_a^{R,A}(+)=-2R^{1/2}/D^{R,A}(\varepsilon); \tag{A.6}$$

the coefficients $C_n(\varepsilon)$ and $E_n(\varepsilon)$ in the expansions of the Keldysh functions are given by

$$C_0(\varepsilon)=-\frac{DR^{1/2}\alpha(\varepsilon)}{D_\varepsilon^R D_\varepsilon^A}[1+g_\varepsilon^R g_\varepsilon^A-f_\varepsilon^R f_\varepsilon^A],\qquad C_1(\varepsilon)=0,$$

$$C_2(\varepsilon)=-\frac{R^{1/2}}{D_\varepsilon^R D_\varepsilon^A}\Big[-D\beta(\varepsilon)(g_\varepsilon^R f_\varepsilon^A+g_\varepsilon^A f_\varepsilon^R) +2(1+R)\,\mathrm{th}\left(\frac{\varepsilon}{2T}\right)(f_\varepsilon^R-f_\varepsilon^A)\Big], \tag{A.7}$$

$$C_3(\varepsilon)=-\frac{R^{1/2}}{D_\varepsilon^R D_\varepsilon^A}\Big[-D\beta(\varepsilon)(1+g_\varepsilon^R g_\varepsilon^A+f_\varepsilon^R f_\varepsilon^A)+2D\,\mathrm{th}\left(\frac{\varepsilon}{2T}\right) \times(1-g_\varepsilon^R g_\varepsilon^A+f_\varepsilon^R f_\varepsilon^A)+2(1+R)\,\mathrm{th}\left(\frac{\varepsilon}{2T}\right)(g_\varepsilon^R-g_\varepsilon^A)\Big],$$

$$E_0(\varepsilon)=DR^{1/2}\beta(\varepsilon)(g_\varepsilon^R+g_\varepsilon^A)/D_\varepsilon^R D_\varepsilon^A,$$

$$E_1(\varepsilon)=-DR^{1/2}[\beta(\varepsilon)-2\,\mathrm{th}(\varepsilon/2T)](f_\varepsilon^R-f_\varepsilon^A)/D_\varepsilon^R D_\varepsilon^A,$$

$$E_2(\varepsilon)=-DR^{1/2}\alpha(\varepsilon)(f_\varepsilon^R+f_\varepsilon^A)/D_\varepsilon^R D_\varepsilon^A, \tag{A.8}$$

$$E_3(\varepsilon)=-DR^{1/2}\alpha(\varepsilon)(g_\varepsilon^R+g_\varepsilon^A)/D_\varepsilon^R D_\varepsilon^A.$$

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