Thermoelectric coefficients of a 2D electron gas in a quantizing magnetic field

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(Submitted 24 November 1987)
Zh. Eksp. Teor. Fiz. **94**, 216–224 (October 1988)

A theory is proposed of the thermoelectric coefficients of a 2D electron gas in a quantizing magnetic field. This theory explains the nonmonotonic temperature dependence and the order of magnitude of the transverse thermoelectric effect observed for a GaAs–$Al_x Ga_{1-x}As$ heterostructure.

INTRODUCTION

The temperature and field dependence of the transverse thermoelectric coefficient[1] of a 2D electron gas in a GaAs–$Al_x Ga_{1-x}As$ heterostructure subject to quantizing magnetic fields exhibits a number of special features which have not yet been explained. Firstly, a nonmonotonic temperature dependence of the peak value of the thermoelectric coefficient is in conflict with the existing theories of thermomagnetic effects in an inversion layer,[3-9] according to which the peak value of the electron thermoelectric coefficient of a 2D electron gas is independent of temperature:

$$\alpha_p=-\frac{k}{|e|}\frac{\ln 2}{\nu}=-\frac{0{,}06}{\nu}\left[\frac{\mathrm{mV}}{\mathrm{K}}\right], \tag{1}$$

where $\nu=n+\frac{1}{2}$ in the absence of spin splitting and $\nu=2n+1\pm\frac{1}{2}$ if there is spin splitting of the Landau level with index n; k is the Boltzmann constant.

A second special feature is the anomalously high value of α_p found experimentally ($\alpha_p \approx 0.12$ mV/K),[1] which is two orders of magnitude higher than α_p predicted by Eq. (1) for the case when $\nu=3/2$. Allowance for the broadening of the Landau levels in Ref. 10 simply increases monotonically the peak value of the transverse thermoelectric coefficient with temperature.[2] We therefore have to admit that the current theories of thermomagnetic effects occurring in inversion layers subjected to quantizing magnetic fields are unsatisfactory. As pointed out in Ref. 1, the observed anomalies in the behavior of α_p can be explained if we allow for the deviation of phonons from local equilibrium (due to entrainment of electrons by phonons). It should be noted that the drag thermoelectric coefficient of a 2D electron gas in zero magnetic field has already been considered theoretically.[11,12]

We calculate the transverse thermoelectric coefficient of a quasi-two-dimensional electron gas due to nonequilibrium of phonons (frictional thermoelectric coefficient) and study its temperature and field dependences in quantizing magnetic fields in the case of a GaAs–$Al_x Ga_{1-x}As$ heterostructure. We assume that the electron gas fills a layer on a xy plane and the effective thickness of this layer differs from zero and is equal to $\langle z\rangle$ (quasi-two-dimensional electron gas). A magnetic field is applied at right-angles to this layer. The phonon system is essentially three-dimensional.

BASIC EQUATIONS

We can calculate the surface density of the charge flux if we know the equations of motion of the one-particle electron and phonon density matrices. If the amplitude of the scattering of electrons by phonons is considered in the Born approximation, the kinetic equations for one-particle density matrices can be found from the generalized transport equation[13,14]

$$\frac{d}{dt}\langle P_k\rangle^t=\frac{1}{i\hbar}\langle[P_k,\mathscr{H}]\rangle^t, \tag{2}$$

where P_k are the occupation numbers of one-particle states and the Hamiltonian of the electron–phonon system $\mathscr{H}$ is

$$\mathscr{H}=\mathscr{H}_e+\mathscr{H}_p+\mathscr{H}_{ep}, \tag{3}$$

$$\mathscr{H}_e=\sum_\nu \varepsilon_\nu a_\nu^+a_\nu,\quad \mathscr{H}_p=\sum_\mathbf{q}\hbar\omega_\mathbf{q}b_\mathbf{q}^+b_\mathbf{q},$$
$$\mathscr{H}_{ep}=\sum_{\nu'\nu}U_{\nu'\nu}a_{\nu'}^+a_\nu, \tag{4}$$

$$U_{\nu'\nu}=\sum_\mathbf{q}(U_{\nu'\nu}^\mathbf{q}b_\mathbf{q}+U_{\nu'\nu}^{-\mathbf{q}}b_\mathbf{q}^+),\quad U_{\nu'\nu}^\mathbf{q}=C_\mathbf{q}e_{\nu'\nu}^{i\mathbf{q}\mathbf{x}},\quad U_{\nu'\nu}^{-\mathbf{q}}=(U_{\nu\nu'}^\mathbf{q})^*, \tag{5}$$

and $C_\mathbf{q}$ is a Fourier component of the energy of the interaction of electrons with phonons. In the case of homopolar acoustic phonons, we have[15,16]

$$|C_\mathbf{q}|^2=\hbar E_0^2sq/2C_L. \tag{6}$$

Here, E_0 is the deformation potential constant, $C_L=\rho s^2$ is the elastic constant, ρ is the density of the investigated crystal, and s is the velocity of sound.

We assume that the scattering of electrons by the deformation potential predominates over the scattering by piezoelectric vibrations.[16] We define one-particle matrices[13]

$$f_{\nu'\nu}=\langle a_{\nu'}^+a_\nu\rangle,\quad N_{\mathbf{q}\mathbf{q}'}=\langle b_\mathbf{q}^+b_{\mathbf{q}'}\rangle \tag{7}$$

and the correlation matrices

$$h_{\nu'\nu}(\mathbf{q})=\langle a_{\nu'}^+a_\nu b_\mathbf{q}^+\rangle,\quad h_{\nu'\nu}(\mathbf{q})=\langle a_{\nu'}^+a_\nu b_\mathbf{q}\rangle. \tag{8}$$

The averaging in Eqs. (7) and (8) is carried out over a Gibbs ensemble.

Using Eqs. (2), (7), and (8), we obtain

$$\left(i\hbar\frac{\partial}{\partial t}+\varepsilon_1-\varepsilon_2\right)f_{12}=\sum_{3\mathbf{q}}\{U_{23}^\mathbf{q}h_{13}^*(\mathbf{q})+U_{23}^{-\mathbf{q}}h_{13}(\mathbf{q})-U_{31}^\mathbf{q}h_{32}^*(\mathbf{q})-U_{31}^{-\mathbf{q}}h_{32}(\mathbf{q})\}, \tag{9}$$

$$\left(i\hbar\frac{\partial}{\partial t}+\hbar\omega_\mathbf{q}-\hbar\omega_{\mathbf{q}'}\right)N_{\mathbf{q}\mathbf{q}'}=\sum_{1,2}\{U_{12}^{-\mathbf{q}'}h_{12}(\mathbf{q})-U_{12}^\mathbf{q}h_{12}^*(\mathbf{q}')\}. \tag{10}$$

We can similarly write down equations for the matrices (8) which contain derivatives of h with respect to time. In the case of slow processes the time derivatives of h can be ignored if we consider the adiabatic parameter $\varepsilon\to 0$:

$$i\hbar\frac{\partial}{\partial t}h_{12}(\mathbf{q})\to i\varepsilon h_{12}(\mathbf{q}). \tag{11}$$

We then obtain

$$h_{12}(\mathbf{q})=(\varepsilon_1-\varepsilon_2+\hbar\omega_{\mathbf{q}}+i\varepsilon)^{-1}\sum_{34\mathbf{q}'}U_{34}{}^{\mathbf{q}'}[(\delta_{32}f_{14}-\delta_{14}f_{32})N_{\mathbf{q}\mathbf{q}'}-\delta_{qq'}(f_{34}f_{12}+f_{32}(\delta_{41}-f_{14}))], \tag{12}$$

$$h_{12}{}^{*}(\mathbf{q})=(\varepsilon_1-\varepsilon_2-\hbar\omega_{\mathbf{q}}+i\varepsilon)^{-1}\cdot\sum_{34\mathbf{q}'}U_{34}{}^{-\mathbf{q}'}[(\delta_{32}f_{14}-\delta_{41}f_{32})(\delta_{\mathbf{q}\mathbf{q}'}+N_{\mathbf{q}'\mathbf{q}})+\delta_{\mathbf{q}'\mathbf{q}}(f_{34}f_{12}+f_{32}(\delta_{41}-f_{14}))]. \tag{13}$$

Substituting Eqs. (12) and (13) into Eqs. (9) and (10), we obtain a system of kinetic equations for the one-particle electron and phonon density matrices.

We now consider the case of a quantizing magnetic field applied at right-angles to the surface of an inversion layer. We describe the one-particle states of a quasi-two-dimensional electron by a wave function that allows for electrical quantization along the z direction and for Landau quantization in the xy plane[17]:

$$\begin{gathered}|v\rangle=|nk_x\rangle=\xi_0(z)\exp(ik_xx)\Phi_n((y-y_0)/\alpha),\\ \Phi_n(x)=(2^n n!\pi^{1/2}\alpha)^{-1/2}\exp(-x^2/2)H_n(x).\end{gathered} \tag{14}$$

Here, $y_0=\alpha^2k_x$ is the projection of the center of a Larmor orbit along the y axis, Φ_n is the eigenfunction of a harmonic oscillator, $H_n(x)$ is a Hermitian polynomial, α is the magnetic length, $\xi_0(z)$ is the envelope of the wave function of an electron in the inversion layer at the electrical quantum limit[18] given by

$$\xi_0(z)=(b^3/2)^{1/2}ze^{-bz/2}, \tag{15}$$

where the quantity b that minimizes the total energy is[19]

$$b=\left[\frac{12me^2}{\hbar^2\varkappa}\left(N_d+\frac{11}{32}N_s\right)\right]^{1/3}, \tag{16}$$

N_s and N_d are the electron densities in inversion and depletion layers, and $\varkappa$ is the permittivity. The effective thickness $\langle z\rangle$ of an inversion layer is approximately equal to the average separation of the inversion layer electrons from the interface[19]:

$$\langle z\rangle\simeq\frac{1}{N_s}\sum_i N_i\int_0^\infty z\xi_i^2(z)\,dz \tag{17}$$

[N_i is the density of electrons in the ith electrical subband and $\xi_i(z)$ is the corresponding normalized envelope of the wave function]. In the electrical quantum limit, when only the lowest electrical subband ($i=0$) is filled with electrons, we have

$$\langle z\rangle=3/b\propto N_s^{-1/3}. \tag{18}$$

The matrix elements of the electron–phonon interaction which occur in Eq. (5) can be represented, allowing for the explicit form of the wave functions of Eq. (14), in the form[17]

$$\langle n'k_x'|e^{i\mathbf{qr}}|nk_x\rangle=\frac{b^3}{(b-iq_z)^3}\delta(p_x+\hbar q_x-p_x')I^{\mathbf{q}}_{n'n}, \tag{19}$$

where

$$\begin{gathered}I^{\mathbf{q}}_{n'n}=\exp\left(-\frac{iq_xq_y\alpha^2}{2}\right)e^{iq_yy_0}\frac{n!}{(n!n'!)^{1/2}}L_{n_m}^{|n'-n|}(u)\,u^{|n'-n|/2}e^{-u/2},\\ u=(\alpha q_\perp)^2/2,\qquad q_\perp{}^2=q_x{}^2+q_y{}^2,\qquad n_m=\min(n,n'),\\ L_n{}^r(u)=\frac{1}{n!}e^uu^{-r}\frac{d^n}{du^n}e^{-u}u^{r+n}.\end{gathered} \tag{20}$$

The effects of the drag on electrons by phonons can be investigated by considering two-dimensional spatial inhomogeneities distributed in a plane perpendicular to the magnetic field. Such inhomogeneities are described by off-diagonal elements of the density matrix only with respect of the quantum number k_x, i.e., f_{nk_x,nk'_x}. Calculation of the charge flux in such systems can be carried out conveniently using the Wigner representation of the density matrix[13]:

$$f_{nk_xx}=\sum_{k_x'}f_{nk_x,\,nk_x'}\exp[i(k_x'-k_x)x], \tag{21}$$

$$N_{\mathbf{q}}(x,y)=\sum_{\mathbf{q}'}N_{\mathbf{q}\mathbf{q}'}\delta_{q_zq'_z}\exp[i(q_x-q_x')x+i(q_y-q_y')y], \tag{22}$$

where the particle number density is given by the expression

$$n(x,y)=\sum_{nk_x}f_{nk_xx}\Phi_n{}^2((y-y_0)/\alpha). \tag{23}$$

If the density is slowly varying, we have

$$\Phi_n{}^2((y-y_0)/\alpha)\approx\delta(y-y_0). \tag{24}$$

We then have

$$n(x,y)=\frac{1}{2\pi\alpha^2}\sum_n f_{nyx}. \tag{25}$$

Substituting Eqs. (12) and (13) into Eqs. (9) and (10) subject to Eqs. (21), (22), and (25), we obtain

$$\begin{aligned}\frac{\partial}{\partial t}n(x,y)=\frac{1}{\alpha^2\hbar}\sum_{n'n\mathbf{q}}A_{\mathbf{q}}G_{n'n}\{[(f_{n'y'x'}-f_{nyx})N_{\mathbf{q}}(x,y)\\ +f_{n'y'x'}(1-f_{nyx})]\delta(\varepsilon_n-\varepsilon_{n'}+\hbar\omega_{\mathbf{q}})-[(f_{nyx}-f_{n'y''x''})N_{\mathbf{q}}(x',y')\\ +f_{nyx}(1-f_{n'y''x''})]\delta(\varepsilon_{n'}-\varepsilon_n+\hbar\omega_{\mathbf{q}})\},\end{aligned} \tag{26}$$

$$\begin{aligned}&\left(\frac{\partial}{\partial t}+\frac{\partial\omega_{\mathbf{q}}}{\partial q_x}\frac{\partial}{\partial x}+\frac{\partial\omega_{\mathbf{q}}}{\partial q_y}\frac{\partial}{\partial y}\right)N_{\mathbf{q}}(x,y)\\ &=\frac{1}{\alpha^2\hbar}\sum_{nn'}A_{\mathbf{q}}G_{n'n}[(f_{n'y'x'}-f_{nyx})N_{\mathbf{q}}(x,y)\\ &\quad+f_{n'y'x'}(1-f_{nyx})]\delta(\varepsilon_n-\varepsilon_{n'}+\hbar\omega_{\mathbf{q}})\\ &\quad+[N_{\mathbf{q}}{}^0(x,y)-N_{\mathbf{q}}(x,y)]\omega_{pp}.\end{aligned} \tag{27}$$

Here,

$$\begin{gathered}y'=y+\alpha^2q_x,\quad x'=x-\alpha^2q_y,\quad y''=y-\alpha^2q_x,\quad x''=x+\alpha^2q_y,\\ A_{\mathbf{q}}=(1+q_z{}^2/b^2)^{-3}\hbar E_0{}^2sq/2C_L,\quad G_{n'n}=|I_{n'n}|^2;\end{gathered} \tag{28}$$

the last term on the right-hand side of Eq. (27) allows for nonelectron relaxation of phonons, i.e., for the relaxation of phonons by interaction with other phonons, boundaries of a sample, defects, etc.; $\omega_{pp}(\mathbf{q})$ is the effective relaxation frequency of phonons in a thermostat (nonelectron relaxation frequency of phonons); and $N^0_{\mathbf{q}}$ is a local-equilibrium phonon distribution function:

$$N_{\mathbf{q}}^0(x,y)=\{\exp[\hbar\omega_{\mathbf{q}}/kT(x,y)]-1\}^{-1}, \tag{29}$$

where $T(x,y)$ is the thermostat temperature.

We assume, as usual, that the distribution function of phonons differs little from the local-equilibrium function of Eq. (29). We shall therefore assume that

$$N_{\mathbf{q}}(x,y)=N_{\mathbf{q}}^0(x,y)+g(x,y), \qquad |g|\ll N_{\mathbf{q}}^0. \tag{30}$$

Substituting Eq. (30) into Eq. (27), we find the equation for the determination of $g(x,y)$, which contains the electron distribution function f_{nyx}. We can determine $g(x,y)$ to first order $(\omega_0\tau)^{-1}$ by assuming that f_{nyx} is the local-equilibrium electron distribution function:

$$f_{nyx}=\{\exp[(\varepsilon_n-\xi(x,y))/kT(x,y)]+1\}^{-1}, \tag{31}$$

where in the case of a standard parabolic energy band in the electrical quantum limit we have

$$\varepsilon_n=\varepsilon_0+\hbar\omega_0(n+{}^1/_2) \tag{32}$$

(ε_0 is the energy of the lowest electrical subband, $\omega_0 = eH/mc$ is the cyclotron frequency, and m is the effective mass of an electron).

Substituting next Eq. (31) into the equation for the function $g(x,y)$, we find that to first order in the gradients of T and ξ, we obtain

$$\begin{aligned} f_{n'y'x'}=f_{n'}-(1-f_{n'})f_{n'}\{\alpha^2q_x[-(kT_0)^{-1}\nabla_y\xi &+(\varepsilon_{n'}-\xi_0)\nabla_y(kT)^{-1}] \\ -\alpha^2q_y[-(kT_0)^{-1}\nabla_x\xi&+(\varepsilon_n-\xi_0)\nabla_x(kT)^{-1}]\}, \end{aligned} \tag{33}$$

$$\begin{aligned} \frac{\partial\omega_{\mathbf{q}}}{\partial q_\alpha}\frac{\partial}{\partial x_\alpha}N_{\mathbf{q}}^0(x,y)=\Big\{&(f_{n'}-f_n)g(x,y)-\frac{f_n-f_{n'}}{2[\mathrm{ch}[\hbar\omega_{\mathbf{q}}/kT_0)-1]} \\ &\cdot\{\alpha^2q_x[-(kT_0)^{-1}\nabla_y\xi+(\varepsilon_{n'}-\xi_0)\nabla_y(kT)^{-1}] \\ &-\alpha^2q_y[-(kT_0)^{-1}\nabla_x\xi \\ &+(\varepsilon_{n'}-\xi_0)\nabla_x(kT)^{-1}]\}\Big\}\delta(\varepsilon_n-\varepsilon_{n'}+\hbar\omega_{\mathbf{q}}), \\ g(x,y)=\{\hbar s^2(q_\alpha\nabla_\alpha)(kT)^{-1}&-[\alpha^2q_x(-(kT_0)^{-1}\nabla_y\xi\omega_{ep} \\ &+\omega_{ep}'\nabla_y(kT)^{-1})-\alpha^2q_y(-(kT_0)^{-1}\nabla_x\xi\omega_{ep}+ \\ &+\omega_{ep}'\nabla_x(kT)^{-1})]\}\{2[\mathrm{ch}(\hbar\omega_{\mathbf{q}}/kT)-1](\omega_{ep}+\omega_{\mathbf{pp}})\}^{-1}. \end{aligned} \tag{34}$$

Here, ω_{ep} is the relaxation frequency of phonons interacting with electrons:

$$\omega_{ep}=\frac{1}{\alpha^2\hbar\langle z\rangle}\sum_{n'n}A_{\mathbf{q}}G_{n'n}[f(\varepsilon_n)-f(\varepsilon_n+\hbar\omega_{\mathbf{q}})]\delta(\varepsilon_n-\varepsilon_{n'}+\hbar\omega_{\mathbf{q}}), \tag{35}$$

$$\begin{aligned}\omega_{ep}'=\frac{1}{\alpha^2\hbar\langle z\rangle}\sum_{n'n}&A_{\mathbf{q}}G_{n'n}[f(\varepsilon_n)-f(\varepsilon_n+\hbar\omega_{\mathbf{q}})] \\ &\cdot\delta(\varepsilon_n+\hbar\omega_{\mathbf{q}}-\xi_0)\delta(\varepsilon_n-\varepsilon_{n'}+\hbar\omega_{\mathbf{q}}),\end{aligned} \tag{36}$$

$$f_n=f(\varepsilon_n)=\{\exp[(\varepsilon_n-\xi_0)/kT_0]+1\}^{-1}, \tag{37}$$

f_n is the equilibrium electron distribution function, and ξ_0 and T_0 are the average values of the chemical potential and temperature.

We can find the conduction current using the equation for the conservation of the charge $en(x,y)$:

$$\frac{\partial}{\partial t}en(x,y)=-\mathrm{div}\,\mathbf{j}=-\nabla_xj_x-\nabla_yj_y. \tag{38}$$

Substituting Eqs. (30) and (31) into Eq. (26) and using the expansions

$$\begin{aligned} f_{n'y'x'}=f_{n'}+f_{n'}(1-f_{n'})\{(\varepsilon_n-\xi_0)/(kT_0)^2[\nabla_ykT\alpha^2q_x \\ -\nabla_xkT\alpha^2q_y+{}^1/_2(\nabla_y^2kT\alpha^4q_x^2-2\nabla_y\nabla_xkT\alpha^4q_xq_y+\nabla_x^2kT\alpha^4q_y^2)] \\ +(kT_0)^{-1}[\nabla_y\xi\alpha^2q_x-\nabla_x\xi\alpha^2q_y+{}^1/_2(\nabla_y^2\xi\alpha^4q_x^2 \\ -2\nabla_y\nabla_x\xi\alpha^4q_xq_y+\nabla_x^2\xi\alpha^4q_y^2)]\}, \end{aligned} \tag{39}$$

$$\begin{aligned} N_{\mathbf{q}}(x'',y'')=N_0+&\frac{\hbar\omega_{\mathbf{q}}}{2(kT)^2[\mathrm{ch}(\hbar\omega_{\mathbf{q}}/kT_0)-1]} \\ &\cdot[-\nabla_ykT\alpha^2q_x+\nabla_xkT\alpha^2q_y \\ +{}^1/_2(\nabla_y^2kT\alpha^4q_x^2&-2\nabla_x\nabla_ykT\alpha^4q_xq_y+\nabla_x^2kT\alpha^4q_y^2)]+g(x,y) \\ &-\nabla_yg(x,y)\alpha^2q_x+\nabla_xg(x,y)\alpha^2q_y, \end{aligned} \tag{40}$$

where

$$N_0=[\exp(\hbar\omega_{\mathbf{q}}/kT_0)-1]^{-1}, \tag{41}$$

we find to first order in $\nabla_\alpha T$ and $\nabla_\alpha\xi$ the following expression for the conduction current which allows for the drag on the electrons by phonons:

$$\begin{aligned} j_{x,y}=-\sum_{\mathbf{q}}&\frac{e\alpha^4q_x^2\langle z\rangle}{2[\mathrm{ch}(\hbar\omega_{\mathbf{q}}/kT_0)-1](\omega_{ep}+\omega_{\mathbf{pp}})} \\ &\cdot\Big[\omega_{ep}'\omega_{\mathbf{pp}}(kT)^{-2}\nabla_{x,y}kT \\ &+\omega_{ep}\omega_{\mathbf{pp}}(kT)^{-1}\nabla_{x,y}\xi\mp\omega_{ep}\frac{\hbar s^2}{(\alpha kT)^2}\nabla_ykT\Big], \end{aligned} \tag{42}$$

where the minus sign applies to j_x and the plus sign applies to j_y. The matrix form of Eq. (42) is

$$j_i=-\sigma_{ik}e^{-1}\nabla_k\xi-\beta_{ik}\nabla_kT, \tag{43}$$

where the tensors of the transport coefficients are

$$\begin{aligned} \sigma_{xx}^{(p)}=\sigma_{yy}^{(p)}&=\frac{1}{kT}\sum_{\mathbf{q}}\frac{e^2\alpha^4q_x^2\omega_{ep}\omega_{\mathbf{pp}}\langle z\rangle}{2[\mathrm{ch}(\hbar\omega_{\mathbf{q}}/kT)-1](\omega_{ep}+\omega_{\mathbf{pp}})}, \\ \beta_{xx}^{(p)}=\beta_{yy}^{(p)}&=\frac{1}{kT^2}\sum_{\mathbf{q}}\frac{e\alpha^4q_x^2\omega_{ep}'\omega_{\mathbf{pp}}\langle z\rangle}{2[\mathrm{ch}(\hbar\omega_{\mathbf{q}}/kT)-1](\omega_{ep}+\omega_{\mathbf{pp}})}, \\ \beta_{xy}^{(p)}=-\beta_{yx}^{(p)}&=-\sum_{\mathbf{q}}\frac{e\alpha^4q_x^2\omega_{ep}(\hbar s^2/\alpha^2kT^2)\langle z\rangle}{2[\mathrm{ch}(\hbar\omega_{\mathbf{q}}/kT)-1](\omega_{ep}+\omega_{\mathbf{pp}})}. \end{aligned} \tag{44}$$

Equations (42) and (43) determine the dissipative (collisional) charge flux allowing for the scattering of electrons and phonons.

The transport coefficients for a nondissipative electron flux can be described by the following expressions[3]

$$\sigma_{xy}=\sigma_{yx}=\frac{ecN_s}{H}=-\frac{e^2}{2\pi\hbar}\sum_n f(\varepsilon_n)=-\frac{e^2\nu}{2\pi\hbar}, \tag{45}$$

$$\beta_{xy}=-\frac{ke}{2\pi\hbar}\sum_n\left(\ln 2+\ln\mathrm{ch}\frac{x_n}{2}-\frac{x_n}{2}\mathrm{th}\frac{x_n}{2}\right), \tag{46}$$

where

$$x_n=(\varepsilon_n-\xi)/kT.$$

The expression for the isothermal transverse thermoelectric coefficient is[13]

$$\alpha_\perp=\alpha_{xx}=\alpha_{yy}=(\sigma_{xx}\beta_{xx}+\sigma_{xy}\beta_{xy})/(\sigma_{xx}^2+\sigma_{xy}^2)\approx\beta_{xy}/\sigma_{xy} \tag{47}$$

and it follows from Eqs. (44)–(46) that it consists of two components: the electron component

$$\alpha^e=-\frac{k}{|e|}\sum_n[\ln 2+\ln\mathrm{ch}(x_n/2)-(x_n/2)\,\mathrm{th}(x_n/2)]\Big[\sum_n(e^{x_n}+1)^{-1}\Big]^{-1}, \tag{48}$$

which is entirely due to nonequilibrium electrons, and the phonon component

$$\alpha^p=\beta_{xy}^{(p)}/\sigma_{xy}, \tag{49}$$

which is due to deviation of phonons from the equilibrium distribution because of electron–phonon collisions:

$$\alpha_\perp=\alpha^e+\alpha^p. \tag{50}$$

At high temperatures the frequency ω_{pp} of collisions of phonons with the "thermostat" is considerably greater than the frequency of their collisions with electrons ω_{ep} ($\omega_{pp}\gg\omega_{ep}$) and in Eq. (44) we can go to the limit $\omega_{pp}\to\infty$. Consequently, we have $\beta_{xy}^{(0)}\to 0$ and the drag thermoelectric coefficient approaches $\alpha^{(p)}\to 0$ (i.e., there is no drag on the electrons by phonons). It then follows from Eq. (44) that

$$\sigma_{xx}^{(p)}=\sigma_{yy}^{(p)}=\frac{\alpha^4e^2\langle z\rangle}{2kT}\sum_{\mathbf{q}}\frac{q_x^2\omega_{ep}}{\mathrm{ch}(\hbar\omega_{\mathbf{q}}/kT)-1}.$$

In the opposite case of sufficiently low temperatures the nonelectron frequency ω_{pp} of phonon relaxation (relaxation of phonons due to interaction with the "thermostat" is less than the frequency ω_{ep} of phonon relaxation due to interaction of electrons ($\omega_{ep}>\omega_{pp}$). In the limiting case when $\omega_{pp}\to 0$ we find from Eq. (44) that $\sigma_{\alpha\alpha}^{(p)}\to 0$, $\beta_{\alpha\alpha}^{(p)}\to 0$,

$$\alpha^{(p)}=\frac{c^{(p)}}{3en},\quad c^{(p)}=\frac{1}{2kT^2}\sum_{\mathbf{q}}\frac{(\hbar\omega_{\mathbf{q}})^2}{\mathrm{ch}(\hbar\omega_{\mathbf{q}}/kT)-1} \tag{49'}$$

($c^{(p)}$ is the phonon or lattice specific heat, $n=N_s\langle z\rangle$ is the bulk electron density).

Equation (49′) is identical with the expression for $\alpha^{(p)}$ obtained in Ref. 20 using a phenomenological theory and assuming an overall drift of a system of electrons and phonons at a shared velocity (total entrainment of phonons).

According to Refs. 13, 21, and 22, the damping rate of longitudinal long-wavelength sound is

$$\omega_{pp}(\mathbf{q})=\frac{(kT)^4q}{\hbar^3C_LS^2}+\frac{s}{L} \tag{51}$$

(L represents the size of a sample). The first term in Eq. (51) describes the absorption of long-wavelength sound by short-wavelength thermal phonons; the second term corresponds to the relaxation of phonons at the boundaries of a sample, which is important at low temperatures.

The presence of scatterers broadens the Landau levels. There are many ways of allowing for such broadening. The simplest is the smearing of the δ function.[17] Introducing smearing of the Landau levels, we obtain the following expression for the frequency of collisions of phonons with electrons [Eq. (34)]:

$$\omega_{ep}=\frac{E_0^2sT_D}{\pi\langle z\rangle\alpha^2C_LkT^2}\frac{q[\exp(\hbar\omega_{\mathbf{q}}/kT)-1]}{(1+q_z^2/b^2)^3}\cdot\sum_{n'n}\frac{G_{n'n}f(\varepsilon_n+\hbar\omega_{\mathbf{q}})[1-f(\varepsilon_n)]}{(T_D/T)^2+[(\varepsilon_n-\varepsilon_{n'}+\hbar\omega_{\mathbf{q}})/kT]^2}, \tag{52}$$

where T_D is the Dingle temperature governing the Landau level broadening.

If the applied magnetic field is sufficiently strong, we can ignore transitions between different Landau levels and assume that $G_{n'n}=G_n\delta_{n'n}$. Then, the drag thermoelectric power of three-dimensional phonons is described by

$$\alpha^p=-\frac{k}{|e|}\frac{\langle z\rangle b\Theta^2}{4\pi T^2\sum_n(e^{x_n}+1)^{-1}}\cdot\int_0^\infty dV\int_0^\infty du\frac{u[1-(1+\omega_{ep}/\omega_{pp})^{-1}]}{\mathrm{ch}[\Theta(u+\alpha^2b^2V^2/2)^{1/2}/T]-1}, \tag{53}$$

where $\omega_{ep}/\omega_{pp}\equiv\Omega$ and it follows from Eqs. (52) and (51) that

$$\Omega=\frac{2^{1/2}(E_0/k)^2T_DL}{\pi\Theta^2\langle z\rangle C_L\alpha^3/k}\Big[\frac{4T^4(u+\alpha^2b^2V^2/2)^{1/2}}{\Theta^3C_L\alpha^4/kL}+1\Big]^{-1}\cdot\frac{(u+\alpha^2b^2V^2/2)^{1/2}}{(1+V^2)^3}\frac{\{\exp[\Theta(u+\alpha^2b^2V^2/2)^{1/2}/T]-1\}}{(T_D/\Theta)^2+u+\alpha^2b^2V^2/2}\cdot\sum_n\frac{G_n\exp(x_n)}{\{\exp[x_n+\Theta(u+\alpha^2b^2V^2/2)^{1/2}/T]+1\}[\exp(x_n)+1]}, \tag{54}$$

where $\Theta\equiv 2^{1/2}\hbar s/k\alpha$. The order of magnitude of the momentum of transverse motion is $\hbar/\alpha$. Therefore, an electron traveling across a magnetic field can interact only with phonons whose momentum is $\hbar q_\perp\lesssim\hbar/\alpha$. The law of conservation of momentum imposes rigid restrictions only on the phonon momentum component $q_\perp$ parallel to the surface, whereas in the case of q_z it is found that phonons characterized by $|q|\lesssim 1/\langle z\rangle$ participate equally in the scattering process. Since $\langle z\rangle\propto N_s^{-1/3}$, it follows that $q_z\ll q_\perp$ at low densities of the $2D$ gas, so that $q\approx q_\perp$. If we assume that the Fermi

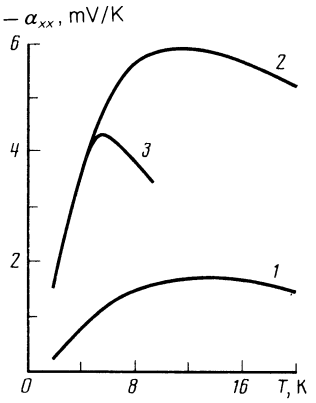

FIG. 1. Temperature dependence of the transverse thermoelectric power α_{xx}: 1) $\langle z\rangle=10^{-5}$ cm; 2) $\langle z\rangle=5\times10^{-5}$ cm; 3) experimental curve taken from Ref. 1; $N_s=6.82\times10^{11}$ cm^{-2}, Fermi temperature $T_F=297$ K, $H=18.5$ T, $\nu=3/2$.

level coincides with the $n = 0$ Landau level and derive α^p for the case of weak drag of electrons by phonons, when $\Omega < 1$ and $(1+\Omega)^{-1} \propto 1-\Omega$, we find from Eqs. (53) and (54) that

$$\alpha^p = -\frac{k}{|e|}\frac{3\cdot 2^{1/2}}{32\pi}\frac{(E_0/k)^2 bLTT_D}{\Theta^3 C_L \alpha^3/k}\int_0^\infty dx\, x^4\, \mathrm{sh}^{-1} x$$

$$\cdot\frac{\exp[-(Tx)^2/\Theta^2]}{(T_D/T)^2+x^2}\left[\frac{4T^5x}{\Theta^4 C_L\alpha^4/kL}+1\right]^{-1}. \qquad (55)$$

We analyze the temperature dependence of the drag thermoelectric coefficient α^p allowing only for the phonon–phonon relaxation and we ignore completely the scattering of phonons on the boundaries of a sample. We consider magnetic fields and temperatures such that $T \gg \Theta$. It then follows from Eq. (55) that

$$\alpha^p = \begin{cases} -\dfrac{k}{|e|}\dfrac{3}{128(2\pi)^{1/2}}\dfrac{(E_0/k)^2\alpha b\Theta^2 T_D}{T^5}, & T_D < \Theta \\ -\dfrac{k}{|e|}\dfrac{3}{256(2\pi)^{1/2}}\dfrac{(E_0/k)^2\alpha b\Theta^4}{T^5 T_D}, & T_D \gg \Theta \end{cases}. \qquad (56)$$

Figure 1 shows the results of a numerical analysis carried out using Eq. (53) assuming parameters typical of a GaAs–$Al_xGa_{1-x}As$ heterostructure[16]: $m = 0.065 m_0$ (m_0 is the mass of a free electron), $L = 0.08$ cm, $s = 3.9\times 10^5$ cm/s, $E_0 \approx 13.5$ eV, $C_L = 1.397\times 10^{12}$ erg/cm^3, $\varkappa = 11.5$, $N_d \approx 5\times 10^{10}$ cm^{-2}, $T_D = 3$ K, and $H = 18.5$ T.

It follows from our calculations that allowance for the drag effects makes it possible to account for the nonmonotonic temperature dependence and for the order of magnitude of the transverse thermoelectric force observed experimentally.[1]

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Translated by A. Tybulewicz