Production of massless particles in collisions of strings at high energy

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We obtain explicit formulas (generalizing the Low-Gribov expressions) for the production amplitude of massless particles (gluons and gravitons) with low transverse momenta in the scattering of lower-mass excitations of open and closed strings. In the limit $\alpha' \to 0$ we reproduce the results of calculation of effective vertices for inelastic scattering amplitudes in multi-Regge kinematics in the Yang-Mills theory and in gravitation.

1. INTRODUCTION

At the present time the superstring represents a real candidate for "pre-matter," determining the observed spectrum of elementary particles.[1] In this model in the critical dimension ($D = 26$ for the bosonic string and $D = 10$ for its supersymmetric generalization) the massless particles are united in multiplets, which include the gluon (for the open string) and the graviton (for the closed string).[1] The heterotic string[2] and its generalizations[3] make possible the construction of a model, whose low-energy limit is Einstein gravitation and various versions of grand unification theories. Moreover, the string nature of elementary particles manifests itself only at Planck energies $s^{1/2} \sim (\alpha')^{-1/2}$, where α' is the slope of the Regge trajectories for string excitations.

At the same time it was shown in calculations of scattering amplitudes of various particles in the Yang-Mills theory[4,5] and in gravitation[6] that the gluon and graviton lie on Regge trajectories, with the one-loop trajectory of the graviton in extended ($N = 4$) supergravity being linear (accurate to within the standard infrared divergence). Therefore the possibility is not excluded, that the string may arise as a solution (possibly outside the framework of perturbation theory) of the equations of a local field theory of the supergravity type. To clarify this possibility it is necessary to study in more detail the connection between the predictions of field theory and the string model.[1)] In this paper we derive from known expressions for many-particle scattering amplitudes of string excitations in the tree approximation the results of Refs. 5 and 6 for multi-Regge processes in the Yang-Mills theory and in gravity. In addition we obtain simple formulas for production amplitudes of massless particles with low transverse momentum, generalizing Gribov's[8] results on photon bremsstrahlung at high energies.

2. MULTI-REGGE PROCESSES IN THE OPEN STRING SECTOR

In this section we discuss production of massless particles—gluons—in the scattering of the lowest mass states of the open string—tachyons. The limitation to tachyonic states is due to the fact that in the multi-Regge kinematics the scattering amplitudes factor, so that the results for the interactions of other string excitations differ only by irrelevant factors.[9]

We consider first the elastic scattering amplitude of tachyons in the open string, corresponding to the Veneziano formula in the Koba-Nielsen variables:[1]

$$A(s,t,u) \sim \int \prod_{i=1}^{4} dx_i \prod_{i<j} |x_i - x_j|^{-2\alpha' k_i k_j}, \tag{1}$$

where k_1, k_2, k_3, and k_4 are momenta of the external particles on the mass shell (see Fig. 1):

$$k^2 = (k^0)^2 - \mathbf{k}^2 = -1/\alpha', \quad \alpha(s) = 1 + \alpha' t, \tag{2}$$

α' is the slope of the Regge trajectories $\alpha(s)$. In Eq. (1) the proportionality factor cancels the infinite phase-space volume, connected with integration over the parameters of the Moebius group:

$$x_i \to (ax_i + b)/(cx_i + d), \qquad ad - bc = 1, \tag{3}$$

with respect to which the integrand in (1) is invariant, and also contains beside the square of the Yang-Mills coupling constant the Chan-Paton color factors[1]:

$$C_{i_1 i_2 \ldots i_r} = {}^1/_2 \operatorname{tr}(\lambda_{i_1} \lambda_{i_2} \ldots \lambda_{i_r}), \qquad \operatorname{tr} \lambda_i \lambda_j = 2\delta_{ij}. \tag{4}$$

Here λ_i are the generators of the $U(n)$ group in the quark representation. We note that the factors (4) are different for the three quark diagrams of Fig. 1, corresponding to the three regions of integration over x in the invariant amplitude (1):

$$\begin{gathered} A_{i_1 i_2 i_3 i_4} = g^2 \int_{-\infty}^{+\infty} dx \, |x|^{-\alpha(s)-1} |1-x|^{-\alpha(t)-1} \\ \times [C^s_{i_1 i_2 i_3 i_4} \theta(x) \theta(1-x) + C^s_{i_1 i_3 i_2 i_4} \theta(x-1) + C_{i_2 i_1 i_3 i_4} \theta(-x)], \\ C^s_{i_1 i_2 \ldots i_r} \equiv {}^1/_2 (C_{i_1 i_2 \ldots i_r} + C_{i_r i_{r-1} \ldots i_1}), \end{gathered} \tag{5}$$

where we have restored the absolute normalization of the amplitude. The anharmonic ratio

$$x = x_{12} x_{34} / x_{13} x_{24}, \qquad x_{ij} \equiv x_i - x_j, \tag{6}$$

arises as the integration variable upon taking into account the invariance of (3) for the following choice of "calibration" of the parameters

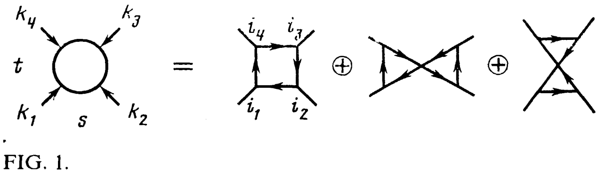

FIG. 1.

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$$x_1=0,\quad x_2=x,\quad x_3=1,\quad x_4=\infty. \tag{7}$$

In the Regge asymptotics

$$s\gg 1/\alpha',\quad t\sim 1/\alpha' \tag{8}$$

in expression (5) the following region of integration is important:

$$|1-x|\sim 1/\alpha' s\to 0, \tag{9}$$

to which the first two terms contribute

$$A_{i_1i_2i_3i_4}=g^2[(-\alpha' s)^{\alpha(t)}C^{*}_{i_1i_2i_3i_4}+(\alpha' s)^{\alpha(t)}C^{*}_{i_1i_3i_2i_4}]\Gamma(-\alpha(t)), \tag{10}$$

where $\Gamma(x)$ is the gamma function.

Equation (10) corresponds to Regge behavior of the scattering amplitude A with a trajectory $\alpha^{\pm}=\alpha(t)$ which is degenerate in signature:

$$A_{i_1i_2i_3i_4}=A^{+}_{i_1i_2i_3i_4}+A^{-}_{i_1i_2i_3i_4},$$
$$A^{+}_{i_1i_2i_3i_4}=g^2\xi^{+}(t)(\alpha' s)^{\alpha(t)}C^{+}_{i_1i_2i_3i_4}, \tag{11}$$
$$A^{-}_{i_1i_2i_3i_4}=g^2\xi^{-}(t)(\alpha' s)^{\alpha(t)}C^{-}_{i_1i_2i_3i_4},$$

where the signature ($\xi^{\pm}$) and color ($C^{\pm}$) factors equal

$$\xi^{\pm}(t)=\{\exp[-i\pi\alpha(t)]\pm 1\}\Gamma(-\alpha(t)),$$
$$C^{\pm}_{i_1i_2i_3i_4}={}^1/_2(C^{*}_{i_1i_2i_3i_4}\pm C^{*}_{i_1i_3i_2i_4}). \tag{12}$$

The contributions A^{+} and A^{-} in the case $i_k\neq 0$ ($k=1,2,3,4$) correspond respectively to the singlet-octet (d-coupling) and octet (f-coupling) in color intermediate states in the t-channel, because for the $C_{i_1i_2\ldots i_k}$ (4) we have the factorization relation[1]

$$C_{i_1i_2\ldots i_k}=\sum_j C_{i_1\ldots i_{r-1}j}C_{ji_r\ldots i_k}, \tag{13}$$

which gives

$$C^{+}_{i_1i_2i_3i_4}=\sum_j C^{+}_{i_1i_4j}C^{+}_{ji_3i_2},$$
$$C^{-}_{i_1i_2i_3i_4}=\sum_{j\neq 0}T_{i_1i_4j}T_{ji_3i_2},\qquad T_{ijk}={}^1/_2(C_{jik}-C_{ijk}), \tag{14}$$

where T_{ijk} coincides with the generator T_k of the $SU(n)$ group in the adjoint representation.

For small t the main contribution to (11) comes from the state with negative signature, corresponding to gluon exchange:

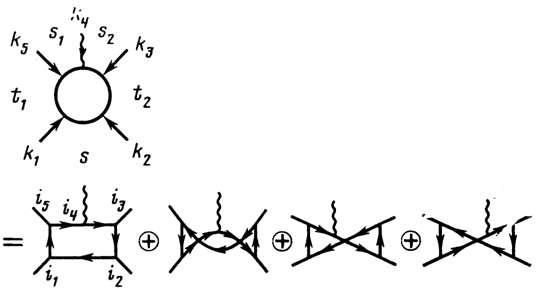

FIG. 2.

$$A_{i_1i_2i_3i_4}|_{\alpha' t\to 0}=2g^2T_{ji_4i_1}\frac{s}{t}T_{ji_3i_2}. \tag{15}$$

We consider now the emission of a particle (gluon) with polarization vector $e_\mu(-k_4)$ in tachyon scattering (see Fig. 2). In place of Eq. (1) we have[1]

$$A(s,s_1,s_2,t_1,t_2)\sim\int\prod_{i=1}^{5}dx_i\prod_{i<j}|x_i-x_j|^{-2\alpha' k_ik_j}e_\mu(-k_4)a_\mu(x_4), \tag{16}$$

where the "vertex" $a_\mu(x_i)$ equals

$$a(x_4)=\sum_{i\neq 4}\frac{k_i}{x_i-x_4}. \tag{17}$$

It is not hard to verify the invariance of (16) with respect to a gauge change of the gluon polarization vector $e_\mu(-k_4)$:

$$e\to e+ck_4, \tag{18}$$

where we make use of the identity

$$k_4a(x_4)=-\frac{\partial}{\partial x_4}\sum_{i<j}k_ik_j\ln|x_i-x_j|. \tag{19}$$

It is convenient to choose the following as independent invariants in the amplitude (16) (see Fig. 2):

$$s=2k_1k_2,\quad s_1=-2k_1k_4,\quad s_2=-2k_2k_4,$$
$$t_1=q_1^2=2k_1k_5+2m^2,\qquad t_2=q_2^2=2k_2k_3+2m^2, \tag{20}$$

where it is assumed that the external particles lie on the mass shell:

$$k_i^2|_{i\neq 4}=m^2=-1/\alpha',\qquad k_4^2=0. \tag{21}$$

Then in the multi-Regge kinematics we have

$$s\gg 1/\alpha',\quad s_1\gg 1/\alpha',\quad s_2\gg 1/\alpha',\quad s_1s_2=s\mathbf{k}_\perp^2,$$
$$q_1^2\sim q_2^2\sim \mathbf{k}_\perp^2\sim 1/\alpha',\qquad k\equiv -k_4=q_1-q_2. \tag{22}$$

The remaining scalar products k_ik_j can be expressed linearly in terms of the quantities (20):

$$2k_1k_3=-s+s_1-t_1,\qquad 2k_2k_5=-s+s_2-t_2,$$
$$2k_3k_4=s_2+t_1-t_2,\qquad 2k_4k_5=s_1+t_2-t_1,\qquad 2k_3k_5=s-s_1-s_2. \tag{23}$$

Using (20) and (23) to combine factors in (16) involving the same powers we arrive at

$$A(s,s_1,s_2,t_1,t_2)\sim\int\prod_{i=1}^{5}dx_i|x_{15}x_{23}|^{2\alpha' m^2}e_\mu a_\mu(x_i)$$
$$\times\left|\frac{x_{12}x_{35}}{x_{13}x_{25}}\right|^{-\alpha' s}\left|\frac{x_{13}x_{45}}{x_{14}x_{35}}\right|^{-\alpha' s_1}\left|\frac{x_{25}x_{34}}{x_{24}x_{35}}\right|^{-\alpha' s_2}$$
$$\times\left|\frac{x_{15}x_{34}}{x_{13}x_{45}}\right|^{-\alpha' t_1}\left|\frac{x_{23}x_{45}}{x_{25}x_{34}}\right|^{-\alpha' t_2} \tag{24}$$

This expression is invariant on the mass shell (21) with respect to the transformations (3), if one notes that as a consequence of $(k_i,e)=0$ the following relation is valid

$$(e, a(x_i))|_{x_i\to -1/x_i} = x_i^2(e, a(x_i)). \qquad (25)$$

This invariance permits the reduction of the number of integration variables to two by choosing the following parametrization [compare (7)]:

$$x_1=0, \quad x_2=1, \quad x_3=x, \quad x_4=y, \quad x_5=\infty. \qquad (26)$$

Further, the main contribution to the kinematics (22) comes from the integration region in (24), where [compare (9)]

$$\begin{aligned} \frac{x_{13}x_{45}}{x_{14}x_{35}} - 1 &= \frac{x-y}{y} \equiv \varepsilon_1 \sim \frac{1}{\alpha' s_1} \to 0, \\ \frac{x_{25}x_{34}}{x_{24}x_{35}} - 1 &= \frac{x-1}{1-y} \equiv \varepsilon_2 \sim \frac{1}{\alpha' s_2} \to 0. \end{aligned} \qquad (27)$$

The following approximate expressions result from relations (27) for the x and y in (26):

$$x \approx 1+\varepsilon_1\varepsilon_2, \qquad y \approx 1-\varepsilon_1. \qquad (28)$$

Upon passing in (24) to the integral variables ε_1 and ε_2 using (26) and (28) we obtain for the amplitude A in the region (22) (cf. Ref. 9)

$$\begin{aligned} A(s, s_1, s_2, t_1, t_2) \sim \int \frac{d\varepsilon_1}{\varepsilon_1^2}\frac{d\varepsilon_2}{\varepsilon_2^2} |\varepsilon_1|^{-\alpha' t_1} |\varepsilon_2|^{-\alpha' t_2} \\ \times e_\mu b_\mu(\varepsilon) \exp[-\varepsilon_1\alpha' s_1 - \varepsilon_2\alpha' s_2 + \varepsilon_1\varepsilon_2\alpha' s], \end{aligned} \qquad (29)$$

where the vector b_μ equals

$$b(\varepsilon) = \varepsilon_1 a(x_i) \approx -q_2 - k_1\varepsilon_1 + k_2\varepsilon_2, \qquad q_2 = -k_2 - k_3. \qquad (30)$$

The integrals in (29) should be divided into four contributions, corresponding to positive or negative values of the variables ε_1 and ε_2. For each of these contributions one should choose definite signs of the invariants s_1, s_2, and s, for which the corresponding integrals converge. Then it is necessary to analytically continue the resultant expressions into the physical region of the s-channel (or another possible channel), where s, s_1, and s_2 are positive (compare Ref. 9). After taking into account that each of the indicated contributions is accompanied by its color factor (4) we obtain corresponding to the quark diagrams of Fig. 2 [compare (10)] the expression

$$\begin{aligned} &A(s, s_1, s_2, t_1, t_2) \\ &= \alpha' g^3 J(\Lambda, t_1, t_2)\,[(-\alpha' s_1)^{\alpha(t_1)}(-\alpha' s_2)^{\alpha(t_2)} C^a_{i_1i_2i_3i_4i_5} \\ &- (\alpha' s_1)^{\alpha(t_1)}(\alpha' s_2)^{\alpha(t_2)} C^a_{i_1i_4i_2i_3i_5} + (-\alpha' s_1)^{\alpha(t_1)}(\alpha' s_2)^{\alpha(t_2)} C^a_{i_1i_3i_2i_4i_5} \\ &- (\alpha' s_1)^{\alpha(t_1)}(-\alpha' s_2)^{\alpha(t_2)} C_{i_1i_4i_3i_2i_5}], \\ &C^a_{i_1i_2i_3i_4i_5} = {}^1/_2 (C_{i_1i_2i_3i_4i_5} - C_{i_5i_4i_3i_2i_1}), \\ &\Lambda = -s/\alpha' s_1 s_2 = -1/\alpha' \mathbf{k}_\perp^2, \end{aligned} \qquad (31)$$

where the function $J(\Lambda, t_1, t_2)$ is defined for $\Lambda > 0$ by the formula

$$\begin{aligned} J(\Lambda, t_1, t_2) = \int_0^\infty dx_1\, x_1^{-2-\alpha' t_1} \int_0^\infty dx_2\, x_2^{-2-\alpha' t_2}\, e_\mu b_\mu(x) \\ \times \exp[-x_1 - x_2 - x_1 x_2 \Lambda], \\ b(x) = -q_2 - k_1\frac{x_1}{\alpha' s_1} + k_2\frac{x_2}{\alpha' s_2}. \end{aligned} \qquad (32)$$

For $\Lambda\to\infty$ the main contribution in the integrals (32) comes from the regions $x_2\sim 1$, $x_1 \sim 1/\Lambda$, and $x_1\sim 1$, $x_2\sim 1/\Lambda$, so that $J(\Lambda, t_1, t_2)$ can be represented in the form

$$J(\Lambda, t_1, t_2) = \Lambda^{\alpha(t_1)} J_2(\Lambda, t_1, t_2) + \Lambda^{\alpha(t_2)} J_1(\Lambda, t_1, t_2), \qquad (33)$$

where the functions J_1 and J_2 are analytic for large Λ, being given in that limit by the following gauge-invariant expressions:

$$\begin{aligned} J_1(\Lambda, t_1, t_2)|_{\Lambda\to\infty} &= \Gamma(-\alpha(t_2))\Gamma(\alpha(t_2)-\alpha(t_1)) B^1_\mu e_\mu, \\ J_2(\Lambda, t_1, t_2)|_{\Lambda\to\infty} &= \Gamma(-\alpha(t_1))\Gamma(\alpha(t_1)-\alpha(t_2)) B^2_\mu e_\mu, \\ B^1 = k_1\frac{t_1-t_2}{s_1} - \frac{1}{2}(q_1+q_2), &\qquad B^2 = k_2\frac{t_1-t_2}{s_2} - \frac{1}{2}(q_1+q_2). \end{aligned} \qquad (34)$$

We note that in view of (22) $\Lambda = -1/\alpha' \mathbf{k}_\perp^2$, so that (34) holds in the limit $\mathbf{k}_\perp^2 \to 0$, with q_1^2 and q_2^2 not coinciding (which is impossible in the physical region of the s-channel, where $q_1^2 - q_2^2 \approx 2qk_\perp$).

Substituting (33) into (31) we obtain the amplitude A with correct analyticity properties in the multi-Regge kinematics (compare Ref. 9):

$$\begin{aligned} &A(s, s_1, s_2, t_1, t_2) = \alpha' g^3 \\ &\times\{C^a_{i_1i_2i_3i_4i_5}[(-\alpha' s)^{\alpha(t_1)}(-\alpha' s_2)^{\alpha(t_2)-\alpha(t_1)} J_2 \\ &+ (-\alpha' s)^{\alpha(t_2)}(-\alpha' s_1)^{\alpha(t_1)-\alpha(t_2)} J_1] \\ &+ C^a_{i_5i_2i_3i_4i_1}[(-\alpha' s)^{\alpha(t_1)}(\alpha' s_2)^{\alpha(t_2)-\alpha(t_1)} J_2 \\ &+ (-\alpha' s)^{\alpha(t_2)}(\alpha' s_1)^{\alpha(t_1)-\alpha(t_2)} J_1] \\ &+ C^a_{i_1i_3i_2i_4i_5}[(\alpha' s)^{\alpha(t_1)}(\alpha' s_2)^{\alpha(t_2)-\alpha(t_1)} J_2 \\ &+ (\alpha' s)^{\alpha(t_2)}(-\alpha' s_1)^{\alpha(t_1)-\alpha(t_2)} J_1] \\ &+ C^a_{i_5i_3i_2i_4i_1}[(\alpha' s)^{\alpha(t_1)}(-\alpha' s_2)^{\alpha(t_2)-\alpha(t_1)} J_2 \\ &+ (\alpha' s)^{\alpha(t_2)}(\alpha' s_1)^{\alpha(t_1)-\alpha(t_2)} J_1]\}. \end{aligned} \qquad (35)$$

The amplitude for the production of several additional particles may be studied analogously.[9]

3. MULTI-REGGE PROCESSES IN THE CLOSED STRING SECTOR

We consider now production of massless particles in the scattering of tachyons—the lowest-mass excitations for closed strings. We start again with the elastic amplitude—the Virasoro formula in the Koba-Nielsen variables:[1]

$$A(s, t, u) \sim \int \prod_{i=1}^{4} d^2 z_i \prod_{i<j} |z_i - z_j|^{-2\alpha_p' k_i k_j}, \qquad (36)$$

where $\alpha_p' = \alpha'/2$ is the slope of the Regge trajectories $\alpha_p(s)$ for closed strings. The integration in (36) is over the whole complex plane. Making use of the invariance of (36) under the transformations (3) with complex parameters one may bring (36) to the form [compare (5)]

$$\begin{aligned} A(s, t, u) = \frac{\varkappa^2}{\pi}(\alpha_p')^{-1}\int d^2z\, |z|^{-\alpha_p(s)-2}|1-z|^{-\alpha_p(t)-2}, \\ \alpha_p(s) = 2 + \alpha_p' s, \end{aligned} \qquad (37)$$

where $\varkappa$ is the three-graviton coupling constant.

In the asymptotics form (8) the principal integration region is given by

$$|z-1|\sim 1/\alpha_p's\to 0, \tag{38}$$

and (37) needs to be decomposed in two contributions, corresponding to $\mathrm{Re}(z-1)>0$ and $\mathrm{Re}(z-1)<0$. In these contributions one must assume respectively $s>0$ and $s<0$, and then continue analytically the resultant expressions:

$$A(s,t,u)|_{\alpha_p's\to\infty}=(\alpha_p')^{-1}\varkappa^2\Gamma(-\alpha_p(t))[(\alpha_p's)^{\alpha_p(t)}+(-\alpha_p's)^{\alpha_p(t)}]\int\limits_{-\pi/2}^{\pi/2}\frac{d\varphi}{\pi}(\cos\varphi)^{\alpha_p(t)}$$

$$=(\alpha_p')^{-1}\varkappa^2[(\alpha_p's)^{\alpha_p(t)}+(-\alpha_p's)^{\alpha_p(t)}]\times\frac{\Gamma(-\alpha_p(t))\Gamma((1+\alpha_p(t))/2)}{\Gamma(1/2)\Gamma(\alpha_p(t)/2+1)} \tag{39}$$

In this manner we obtain the Regge asymptotics, corresponding to the exchange of a graviton Regge pole with positive signature. For $\alpha_p'\to 0$ Eqs. (39) simplify:

$$A(s,t,u)\Big|_{\substack{s\to\infty\\ \alpha_p'\to 0}}=-\varkappa^2s^2/2t, \tag{40}$$

i.e., the parameter $\varkappa$ is connected with the universal gravitation constant G by the formula

$$\varkappa^2=8\pi G. \tag{41}$$

We go over now to the study of the amplitude for the production of a graviton and other massless particles in the collision of tachyons[1] [compare (16), (36)]:

$$A(s,s_1,s_2,t_1,t_2)\sim\int\prod_{i=1}^{5}d^2z_i\prod_{i<j}|z_i-z_j|^{-2\alpha_p'k_ik_j}e_{\mu\nu}(-k_4)a_{\mu\nu}(z_r,z_r^*), \tag{42}$$

where $e_{\mu\nu}(-k_4)$ is the polarization tensor of the created particle, and the tensor vertex $a_{\mu\nu}(z,z^*)$ is formed as a product of gluon vertices $a_\mu(x)$ [Eq. (17)]:

$$a_{\mu\nu}(z_r,z_r^*)=a_\mu(z_r)a_\nu(z_r^*). \tag{43}$$

Applying to Eq. (42) the same transformations as in the case of the open string in the previous section, we obtain for the inelastic amplitude in the multi-Regge kinematics [compare (29), (30), (37)]

$$A(s,s_1,s_2,t_1,t_2)=\frac{\varkappa^3}{\pi^2}\int\frac{d^2\varepsilon_1}{|\varepsilon_1|^4}\frac{d^2\varepsilon_2}{|\varepsilon_2|^4}|\varepsilon_1|^{-\alpha_p't_1}|\varepsilon_2|^{-\alpha_p't_2}$$
$$\times e_{\mu\nu}b_{\mu\nu}(\varepsilon,\varepsilon^*)\exp[-\alpha_p's_1\,\mathrm{Re}\,\varepsilon_1-\alpha_p's_2\,\mathrm{Re}\,\varepsilon_2+\alpha's\,\mathrm{Re}(\varepsilon_1\varepsilon_2)],$$
$$b_{\mu\nu}(\varepsilon,\varepsilon^*)=b_\mu(\varepsilon)b_\nu(\varepsilon^*), \tag{44}$$

where the vector $b_\mu(x)$ is given by Eq. (30).

Analogously to the case of the open string one may go over in Eq. (44) to new integration variables z_1, z_2 according to the rules

$$z_1=\alpha_p's_1\varepsilon_1,\quad z_2=\alpha_p's_2\varepsilon_2. \tag{45}$$

Then after extracting the factors

$$\prod_{i=1}^{2}(\alpha_p's_i)^{\alpha(t_i)},$$

we find that the remaining integral depends on the invariants s_1, s_2, and s only in the combination corresponding to the variable Λ_p [see (31)]. In the neighborhood of the point $\Lambda_p=\infty$ the main contribution comes from the integration region $z_1\sim(\Lambda_p)^{-1}, z_2\sim 1$ and $z_2\sim(\Lambda_p)^{-1}, z_1\sim 1$. In each of the two regions the integral should be decomposed into four contributions. For example, in the first region these contributions will correspond to different signs of $\mathrm{Re}z_2$ and $\Lambda_p\mathrm{Re}(z_1z_2)\equiv\tilde z_1$. Choosing for each contribution the sign of the invariants s_1,s_2 and s so that the corresponding integral converges and then continuing analytically the resultant expressions, we obtain for (44) the following representation [compare (35)]:

$$A(s,s_1,s_2,t_1,t_2)=\{[(-\alpha_p's)^{\alpha_p(t_1)}+(\alpha_p's)^{\alpha_p(t_1)}][(-\alpha_p's_2)^{\alpha_p(t_2)-\alpha_p(t_1)}+(\alpha_p's_2)^{\alpha_p(t_2)-\alpha_p(t_1)}]I_2(\Lambda_p,t_1,t_2)+[(-\alpha_p's)^{\alpha_p(t_2)}+(\alpha_p's)^{\alpha_p(t_2)}][(-\alpha_p's_1)^{\alpha_p(t_1)-\alpha_p(t_2)}+(\alpha_p's_1)^{\alpha_p(t_1)-\alpha_p(t_2)}]I_1(\Lambda_p,t_1,t_2)\}, \tag{46}$$

where

$$I_1(\Lambda_p,t_1,t_2)=\pi^{-2}\int d^2z_1|z_1|^{-2-\alpha_p'(t_1-t_2)}\exp(-\mathrm{Re}\,z_1)\times\int d^2\tilde z_2|\tilde z_2|^{-4-\alpha_p't_2}\exp\left[-\mathrm{Re}\,\tilde z_2-\Lambda_p^{-1}\mathrm{Re}\frac{\tilde z_2}{z_1}\right]\times e_{\mu\nu}(k)b_\mu(z)b_\nu(z^*), \tag{47}$$

$$\mathrm{Re}\,z_1>0,\qquad \mathrm{Re}\,\tilde z_2>0,$$

$$I_2(\Lambda_p,t_1,t_2)=\pi^{-2}\int d^2z_2|z_2|^{-2-\alpha_p'(t_2-t_1)}\exp(-\mathrm{Re}\,z_2)\times\int d^2\tilde z_1|\tilde z_1|^{-4-\alpha_p't_1}\exp\left[-\mathrm{Re}\,\tilde z_1-\Lambda_p^{-1}\mathrm{Re}\frac{\tilde z_1}{z_2}\right]\times e_{\mu\nu}(k)b_\mu(z)b_\nu(z^*),$$

$$\mathrm{Re}\,z_2>0,\qquad \mathrm{Re}\,\tilde z_1>0,\qquad \Lambda_p=-s/\alpha_p's_1s_2.$$

It is assumed here that in I_1 and I_2 only contributions analytic for $\Lambda_p\to\infty$ are isolated. In particular, as is easily verified by direct calculation, the main asymptotic contributions equal [compare (34)]

$$I_1(\Lambda_p,t_1,t_2)|_{\Lambda_p=\infty}=B_\mu^1B_\nu^1e_{\mu\nu}\frac{\Gamma(-\alpha_p(t_2))\Gamma(1/2(\alpha_p(t_2)+1))\Gamma(\alpha_p(t_2)-\alpha_p(t_1))\Gamma(1/2(\alpha_p(t_1)-\alpha_p(t_2)+1))}{\Gamma(1/2)\Gamma(1/2\alpha_p(t_2)+1)\Gamma(1/2)\Gamma(1/2(\alpha_p(t_1)-\alpha_p(t_2))+1)}$$

$$I_2(\Lambda_p,t_1,t_2)|_{\Lambda_p=\infty}=B_\mu^2B_\nu^2e_{\mu\nu}\frac{\Gamma(-\alpha_p(t_1))\Gamma(1/2(\alpha_p(t_1)+1))\Gamma(\alpha_p(t_1)-\alpha_p(t_2))\Gamma(1/2(\alpha_p(t_2)-\alpha_p(t_1))+1)}{\Gamma(1/2)\Gamma(1/2\alpha_p(t_1)+1)\Gamma(1/2)\Gamma(1/2(\alpha_p(t_2)-\alpha_p(t_1))+1)}, \tag{48}$$

where the vectors $B_\mu^{1,2}$ are given in Eqs. (34).

4. PRODUCTION OF PARTICLES WITH LOW TRANSVERSE MOMENTUM

As is known, Low's theorem[10] permits the determination of the amplitude for the production of soft photons through the amplitude of the main process and its derivatives. At high energies the region of applicability of classical expressions for the accompanying radiation is substantially widened, as was shown by Gribov.[8] Specifically, it is sufficient to view as small just the transverse component $k_\perp$ of the photon momentum:

$$|\mathbf{k}_\perp| \ll m_{\text{char}}, \tag{49}$$

where m_{char} is a characteristic transverse momentum for the main process: the momentum transfer q, the Regge scale $(\alpha')^{-1/2}$ or the mass of an exchanged particle. In Ref. 8 the leading asymptotics $\sim 1/k_\perp$ of the bremsstrahlung amplitude was found in the limit (49). It is of interest to calculate the correction $\sim (k_\perp)^0$ to this asymptotics, in analogy to the way Low found the correction $\sim\omega^0$ to the amplitude for the emission of photons of low frequency ω.[10] String theory with Regge behavior of the scattering amplitude is a good model of strong interactions, in which the question of universality of formulas for bremsstrahlung of masless particles (gluons, gravitons, etc.) can be studied.

We consider Eqs. (31) and (46) in the region (49). In this case $m_{\text{char}} \sim q \sim (\alpha')^{-1/2}$. It must be remembered that in the physical region of the process not only $\Lambda^{-1/2}$, but also $\alpha_p'(t_1 - t_2)$ tends to zero:

$$\Lambda^{-1/2} \sim \alpha_p'(t_1 - t_2) \to 0. \tag{50}$$

Expanding expression (34) in a Taylor series in $t_1 - t = t_{12}/2$ and $t_2 - t = -t_{12}/2$, where $t = (t_1 + t_2)/2$, we obtain

$$\begin{aligned}
J_1|_{k_\perp \to 0} &= \frac{\Gamma(-\alpha(t))}{\alpha'(t_2 - t_1)} B_\mu^1 e_\mu [1 + \alpha'(t_2 - t_1)\chi(t)], \\
J_2|_{k_\perp \to 0} &= \frac{\Gamma(-\alpha(t))}{\alpha'(t_1 - t_2)} B_\mu^2 e_\mu [1 + \alpha'(t_1 - t_2)\chi(t)], \qquad (51)\\
I_1|_{k_\perp \to 0} &= B_\mu^1 B_\nu^1 e_{\mu\nu} \frac{1}{\alpha_p'(t_2 - t_1)} \frac{\Gamma(-\alpha_p(t))\Gamma(\frac{1}{2}(\alpha_p(t)+1))}{\pi^{1/2}\Gamma(\frac{1}{2}\alpha_p(t)+1)} \\
&\quad \times [1 + \alpha_p'(t_2 - t_1)\Phi(t)], \\
I_2|_{k \to 0} &= B_\mu^2 B_\nu^2 e_{\mu\nu} \frac{1}{\alpha_p'(t_1 - t_2)} \frac{\Gamma(-\alpha_p(t))\Gamma(\frac{1}{2}(\alpha_p(t)+1))}{\pi^{1/2}\Gamma(\frac{1}{2}\alpha_p(t)+1)} \\
&\quad \times [1 + \alpha_p'(t_1 - t_2)\Phi(t)],
\end{aligned}$$

where we have introduced the functions χ and Φ given by

$$\begin{aligned}
\chi(t) &= -\tfrac{1}{2}\psi(-\alpha(t)) + \psi(1), \\
\Phi(t) &= -\tfrac{1}{2}\psi(-\alpha_p(t)) + \tfrac{1}{4}\psi(\tfrac{1}{2}(\alpha_p(t)+1)) \\
&\quad - \tfrac{1}{4}\psi(\tfrac{1}{2}\alpha_p(t)+1) + \tfrac{3}{2}\psi(1) - \tfrac{1}{2}\psi(\tfrac{1}{2}), \qquad (52)\\
\psi(x) &= \Gamma'(x)/\Gamma(x), \quad \psi(1) = -c_E, \quad \psi(\tfrac{1}{2}) = -c_E - 2\ln 2.
\end{aligned}$$

Substituting the functions J_1, J_2 into formula (35) and expanding the resultant expression in a series near the point $t_{1,2} = t$, we obtain for the amplitude for gluon bremsstrahlung the expression

$$\begin{aligned}
A(s, s_1, s_2, t_1, t_2)|_{k_\perp \to 0} &= g e_\mu \Big[2\Big(\frac{k_2}{s_2} - \frac{k_1}{s_1}\Big)_\mu \\
&+ (B_2 + B_1)_\mu \frac{\partial}{\partial t} \Big] g^2 \Gamma(-\alpha(t)) [\tfrac{1}{2}(C^a_{i_1 i_2 i_3 i_4 i_5} + C^a_{i_5 i_3 i_2 i_4 i_1})(-\alpha' s)^{\alpha(t)} \\
&+ \tfrac{1}{2}(C^a_{i_1 i_3 i_2 i_4 i_5} + C^a_{i_5 i_2 i_3 i_4 i_1})(\alpha' s)^{\alpha(t)}] \\
&+ \Gamma(-\alpha(t)) g^3 \{(B_2 e)[(-\ln(-\alpha' s_2) - c_E)((-\alpha' s)^{\alpha(t)} C^a_{i_1 i_2 i_3 i_4 i_5} \\
&+ (\alpha' s)^{\alpha(t)} C^a_{i_5 i_2 i_3 i_4 i_1}) + (-\ln(\alpha' s_2) - c_E)((-\alpha' s)^{\alpha(t)} C^a_{i_5 i_3 i_2 i_4 i_1} \\
&+ (\alpha' s)^{\alpha(t)} C^a_{i_1 i_3 i_2 i_4 i_5})] + (B_1 e)[(-\ln(-\alpha' s_1) - c_E)((-\alpha' s)^{\alpha(t)} \\
&\times C^a_{i_1 i_2 i_3 i_4 i_5} + (\alpha' s)^{\alpha(t)} C^a_{i_1 i_3 i_2 i_4 i_5}) + (-\ln(\alpha' s_1) \\
&- c_E)((-\alpha' s)^{\alpha(t)} C^a_{i_5 i_3 i_2 i_4 i_1} + (\alpha' s)^{\alpha(t)} C^a_{i_5 i_2 i_3 i_4 i_1})]\}. \qquad (53)
\end{aligned}$$

The first term in formula (53) corresponds to the non-Abelian generalization of Low's theorem.[10] Here the gluons are emitted from the external real particles, but the contribution of the color anomalous magnetic moment is absent, since the particles participating in the reaction are spinless. Further, as is easily verified, the first term is different from zero only when the signature and color spin in the channels t_1 and t_2 coincide, which is connected with the coherence phenomenon in the gluon emission by the initial (i) and scattered (i') particle (compare Ref. 11).

Equation (53) at the same time generalizes the Gribov relation[8] for the amplitude for the emission of a photon with low transverse momentum $k_\perp$, since it takes into account in addition to the leading contribution $\sim k_\perp^{-1}$ also correction terms $\sim (k_\perp)^0$. The second term in (53) represents the contribution of excited intermediate states in dispersion relations in the variables s_1 and s_2, which is absent in the Gribov approximation. The requirement of analyticity of the inelastic amplitude in the variables s_1, s_2, and s, corresponding to the representation (31), fixes this additional contribution accurate to within an additive constant $\Gamma'(1) = -c_E$.

We go now to the production of a graviton and other massless particles with low transverse momentum. Substituting I_1 and I_2 from (51) into Eq. (46) we obtain [compare (39), (53)]:

$$\begin{aligned}
A(s, s_1, s_2, t_1, t_2)|_{k_\perp \to 0} &= \varkappa \Big\{ \frac{2(B_\mu^1 B_\nu^1 - B_\mu^2 B_\nu^2) e_{\mu\nu}}{t_2 - t_1} + (B_\mu^1 B_\nu^1 + B_\mu^2 B_\nu^2) e_{\mu\nu} \frac{d}{dt} \\
&+ \alpha_p' B_\mu^2 B_\nu^2 e_{\mu\nu} [-\ln(-\alpha_p' s_2) - \ln(\alpha_p' s_2) + 2\ln 2 - 2c_E] \\
&+ \alpha_p' B_\mu^1 B_\nu^1 e_{\mu\nu} [-\ln(-\alpha_p' s_1) - \ln(\alpha_p' s_1) + 2\ln 2 - 2c_E] \Big\} \\
&\times \varkappa^2 [(-\alpha_p' s)^{\alpha_p(t)} + (\alpha_p' s)^{\alpha_p(t)}] \frac{\Gamma(-\alpha_p(t))\Gamma(\frac{1}{2}(\alpha_p(t)+1))}{\alpha_p' \Gamma(\frac{1}{2})\Gamma(\frac{1}{2}\alpha_p(t)+1)}.
\end{aligned} \tag{54}$$

As above, the first two terms in the round brackets correspond to the generalization of Low's theorem for the emission of soft massless particles—the graviton and the dilaton—while the last terms take into account the contribution of excited intermediate states in dispersion relations in s_1 and s_2, and are necessary in order that representation (46) be satisfied.

Equations (53) and (54) are valid in the tree approximation, where in the j-plane the t-channel partial waves $f_j(t)$ have only Regge poles. In calculating the contribution of higher loop corrections the trajectories of these Regge poles curve, their degeneracy in signature and color is lifted and $f_j(t)$ develop Mandelstam branch points.[1] Nevertheless it is not out of the question that also in this case the production amplitude for massless particles with low transverse momentum may be expressible in terms of the elastic amplitude and its derivatives with respect to t.

5. MULTI-REGGE PROCESSES IN THE YANG-MILLS THEORY AND IN GRAVITATION

We return to Eqs. (32), (33) and find J_1 and J_2 in the limit $\alpha' \to 0$:

$$J_2|_{\alpha'\to 0} = \int_0^\infty \frac{dx_2}{x_2^{1+\alpha'(t_2-t_1)}} \exp(-x_2) \int_0^\infty \frac{d\tilde{x}_1}{\tilde{x}_1^{2+\alpha' t_1}} \exp(-x_1)$$

$$\times b_\mu(x) e_\mu \exp\left[-\frac{\tilde{x}_1}{x_2} \Lambda^{-1} \right]_{\alpha'\to 0}$$

$$= (\alpha' t_1)^{-1} [\alpha'(t_1-t_2)]^{-1} \left[B_\mu{}^2 - \frac{1}{\Lambda\alpha'} \left(\frac{k_1}{s_1} - \frac{k_2}{s_2} \right)_\mu \right] e_\mu$$

$$= (\alpha')^{-2} t_1^{-1} (t_1-t_2)^{-1} \frac{1}{2} \left(-q_1^\perp - q_2^\perp + k_1 \frac{s_2}{s} - k_2 \left(\frac{s_1}{s} + 2\frac{t_2-t_1}{s_2} \right) \right)_\mu e_\mu,$$

$$J_1|_{\alpha'\to 0} = (\alpha')^{-2} t_2^{-1} (t_2-t_1)^{-1}\, {}^1/_2 \left(-q_1^\perp - q_2^\perp - k_2 \frac{s_1}{s} - k_1 \left(\frac{s_2}{s} + 2\frac{t_1-t_2}{s_1} \right) \right)_\mu e_\mu,$$

$$q_1 \approx q_1^\perp + \frac{s_2}{s} k_1, \qquad q_2 \approx q_2^\perp - \frac{s_1}{s} k_2. \tag{55}$$

In comparing Eqs. (34) and (55) it is seen that it is necessary in this case to take into consideration the correction $\sim \Lambda^{-1} (\alpha')^{-1}$ to the result of evaluation of $J_{1,2}$ for $\Lambda \to \infty$.

Substituting (55) into (35) and finding the limit of the resulting expression as $\alpha' \to 0$ we obtain [compare (15)]

$$A(s, s_1, s_2, t_1, t_2)|_{\alpha'\to 0} = 2sgT_{j_1 i_5 i_1} t_1^{-1} g T_{i_4 j_2 j_1} C_\mu(q_2, q_1) e_\mu t_2^{-1} g T_{j_2 i_3 i_2}, \tag{56}$$

where we made use of relations (13), (14), to express the color matrices in terms of the generators T of the gauge group in the adjoint representation, and the effective vertex $C_\mu(q_2, q_1)$ is given by the expression

$$C_\mu e_\mu = -2(\alpha')^2 t_1 t_2 (J_1 + J_2)$$
$$= (-q_1^\perp - q_2^\perp + k_1(s_2/s + 2t_1/s_1) - k_2(s_1/s + 2t_2/s_2))_\mu e_\mu. \tag{57}$$

Expressions (56) and (57) coincide with the results obtained previously in calculating amplitudes for inelastic processes in non-Abelian gauge theory.[4] We note that the vector C_μ satisfies the following relations:[4]

$$C_\mu (q_1 - q_2)_\mu = 0, \quad C_\mu e_\mu|_{q_1^\perp \to 0} = C_\mu e_\mu|_{q_2^\perp \to 0} = 0. \tag{58}$$

We pass now to the case of closed strings. Substituting into Eqs. (47) expressions (32) for the vectors $b_\mu(z)$ and $b_\nu(z^*)$:

$$b(z) = -\frac{1}{2}(q_1 + q_2) + k_2 \frac{z_2}{\alpha_p' s_2} - k_1 \frac{\tilde{z}_1}{\alpha_p' s_1 \Lambda_p z_2}, \tag{59}$$

and evaluating the corresponding integrals in the limit $\alpha_p' \to 0$, we obtain the functions I_1 and I_2 in the form [compare (48), (55)]

$$I_1|_{\alpha'_p\to 0} = \frac{{}^1/_4 e_{\mu\nu}}{(\alpha_p')^2 (t_1 - t_2) t_2} \left[B^1 - \frac{1}{\Lambda_p' \alpha_p'} \left(\frac{k_2}{s_2} - \frac{k_1}{s_1} \right) \right]_\mu \times \left[B^1 - \frac{1}{\Lambda_p \alpha_p'} \left(\frac{k_2}{s_2} - \frac{k_1}{s_1} \right) \right]_\nu, \tag{60}$$

$$I_2|_{\alpha'_p\to 0} = \frac{{}^1/_4 e_{\mu\nu}}{(\alpha_p')^2 (t_2 - t_1) t_1} \left[B^2 - \frac{1}{\Lambda_p \alpha_p'} \left(\frac{k_1}{s_1} - \frac{k_2}{s_2} \right) \right]_\mu \times \left[B^2 - \frac{1}{\Lambda_p \alpha_p'} \left(\frac{k_1}{s_1} - \frac{k_2}{s_2} \right) \right]_\nu,$$

where we used the following elementary formulas:

$$\int \frac{d^2 z}{|z|^2} |z|^{-\varepsilon} e^{-\mathrm{Re}\, z} \Big|_{\substack{\mathrm{Re}\, z>0 \\ \varepsilon\to 0}} = \frac{\pi}{\varepsilon}, \quad \int \frac{d^2 z}{|z|^2} \mathrm{Re}\, z e^{-\mathrm{Re}\, z} \Big|_{\mathrm{Re}\, z>0} = \pi,$$

$$\int d^2 z\, |z|^{-\varepsilon} e^{-\mathrm{Re}\, z} \Big|_{\substack{\mathrm{Re}\, z>0 \\ \varepsilon\to 0}} = -\pi\varepsilon, \quad \int \frac{d^2 z}{|z|^2} |z|^{-\varepsilon} \mathrm{Re}\, \frac{z}{z^*} e^{-\mathrm{Re}\, z} \Big|_{\substack{\mathrm{Re}\, z>0 \\ \varepsilon\to 0}} = -\frac{\pi}{2}. \tag{61}$$

Substituting (60) into (46) and letting α_p' go to zero we finally obtain

$$A(s, s_1, s_2, t_1, t_2)|_{\alpha_p'\to 0} = s^2 \varkappa \frac{1}{t_1} \varkappa C_{\mu\nu}(q_2, q_1) e_{\mu\nu} \frac{1}{t_2} \varkappa, \tag{62}$$

where the tensor $C_{\mu\nu}$ is given by the relation

$$C_{\mu\nu} = \frac{1}{2} C_\mu C_\nu - \frac{1}{2} N_\mu N_\nu, \qquad N = 2(t_1 t_2)^{1/2} \left(\frac{k_1}{s_1} - \frac{k_2}{s_2} \right). \tag{63}$$

Formulas (62) and (63) for the emission amplitude of high energy gravitons coincide with those that were previously obtained for Einstein gravity.[6] We note that the effective vertex $C_{\mu\nu}$ in (63) is not a product of gluon vertices C_μ and C_ν, as might have been expected from the factorization of the integrand (44) [compare (29)]. The presence of the additional term $-\frac{1}{2} N_\mu N_\nu$ in Eq. (63) is connected with the fact that the amplitude (62) must satisfy the Steinmann relations,[12] i.e., the simultaneous pole in the variables s_1 and s_2 in expression (63) should cancel.[6]

6. CONCLUSION

In this paper we studied at the tree-diagram level of the dual resonance model the bremsstrahlung of soft gluons and gravitons and obtained formulas generalizing the Low-Gribov results for the quantum electrodynamics of hadrons. Using the two-fold limit $s, s_1, s_2 \to \infty$ and $\alpha' \to 0$ permits in a most economical fashion reconstruction of results of calculations of tree amplitudes of multi-Regge processes in the Yang-Mills theory and in gravitation. One would like to generalize this approach to multi-loop diagrams. Since the main interest is in four-dimensional field theory the starting point should be a string model with critical dimension $D = 4$.[4]

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[1] In particular, as is well-known,[1] the "pomeron" which arises as a Regge pole in the one-loop diagram for the scattering amplitude for open strings, coincides with the Reggeon corresponding to the graviton. At the same time in the Yang-Mills theory the pomeron is the bound state of two reggeized gluons with intercept $j(0) > 1$.[7] The question arises whether a gauge group for the Yang-Mills model can be found (e.g., in a Grand Unified Theory) such that the pomeron intercept would equal

two [$j(0) = 2$], i.e., would coincide with the spin of a composite graviton. Such a theory would contain gravity and would be renormalizable.

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