Experimental test of gravitation at small distances

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One possible deviation from a Newtonian gravitational potential that occurs in supergravity theories, as well as in recently hypothesized interactions that contribute to the gravitational interaction, is of the form $V(r) = -G_\infty (M/r)\{1+\alpha \exp(-r/r_0)\}$. We describe here an experiment to test the gravitational interaction between two spherical masses separated by a distance in the range 3.8–6.5 mm. Over this range we detect no dependence of the gravitational constant on the distance between the test masses. Limits are placed on possible values of the parameters α and r_0 emerging from our results.

INTRODUCTION

The possibility of a deviation from Newton's Law of Universal Gravitation has been the subject of numerous recent studies. One particular problem that has been posed concerns the dependence of the gravitational constant on the distance between interacting masses. This problem has aroused interest for a number of reasons. Of all the constants of physics, the gravitational constant is the least accurately known, and the values determined in different laboratories do not always agree to within their quoted systematic errors.[1] Possible modifications to the law of gravitation arise in certain versions of the scalar-tensor theory of gravity and supergravity,[2,3] and it has been claimed in a number of experimental papers[4,5] that a distance-dependence has been detected for the gravitational constant.

A variety of theoretical approaches lead to a gravitational potential due to a mass M at distance r given by

$$V(r)=-G_\infty Mr^{-1}\{1+\alpha \exp(-r/r_0)\}. \qquad (1)$$

Besides the usual Newtonian term, there is a term in the form of a Yukawa potential, which might be looked upon as an additional interaction due to the exchange of virtual particles with nonvanishing mass and a corresponding finite range r_0. The dimensionless constant α characterizes the strength of the latter interaction as compared with gravitation.

A series of laboratory experiments[6–9] conducted at distances between 2 and 10 cm, however, failed to reliably detect any distance-dependence of the gravitational constant. A recent hypothesis[10] suggests the existence of a new form of interaction having a range of the order of several hundred meters—the so-called "fifth force." This force contributes to the gravitational interaction between masses and is also described by the potential (1), with α being proportional to the ratio of the baryon numbers to the atomic masses of the interacting matter. Other suggestions have also been made relating α to the atomic structure of the matter constituting the interacting masses.[11] Experiments designed to detect a new interaction have thus far yielded contradictory results, and they neither confirm nor refute its hypothesized existence.[12–15]

The range of terrestrial distances for which gravitation has been tested experimentally thus extends from centimeters to kilometers. Distances less than 1 cm are also of interest, but progress in that realm entails overcoming certain problems. In this paper, we describe an experiment that tests Newton's Law of Gravitation at distances from 3.8 mm to 6.5 mm.

EXPERIMENTAL ARRANGEMENT AND MEASUREMENT TECHNIQUE

As the distance between masses (for example, spheres of radius R) decreases, the gravitational force between them decrease at least as R^4, and it becomes difficult to distinguish it from the background gravitational attraction of surrounding masses, as well as seismic noise. With this in mind, we opted for a Cavendish design for our experiment, but making use of a torsion balance operating under dynamic rather than quasistatic conditions, and taking advantage of the resonant enhancement of the oscillation amplitude of such a balance when a test mass mounted at one end of the balance arm was subjected to a periodically varying attractive gravitational force from a third, movable mass.

Figure 1 presents a block diagram of the arrangement used to measure the gravitational force between two spherical masses. Two spheres were affixed to the ends of the beam of the torsion balance, which was then suspended in a vacuum chamber (evacuated to a pressure of less than 10^{-5} Torr) by a quartz fiber 5 μm in diameter and 14 mm long. The fiber was aluminized to mitigate the effects of electrostatic charge. A test mass A ($m_A = 59.25 \pm 0.1$ mg) was prepared from platinum. The torsional oscillation period of the balance was $T = 42$ sec, and the relaxation time was $\tau^* = 2.5\cdot 10^3$ sec, the latter was governed by losses in the surface layer of the suspension fiber. The vacuum chamber was fitted with a cylindrical insert having a thin end-wall 0.25 mm thick. The balance was suspended and adjusted in such a way that test mass A was located adjacent to the center of the cylindrical insert, 0.2 mm from its surface. Another mass B was located outside the chamber but within the cylindrical insert, from whence it exerted a gravitational force on mass A. Mass B was a 706 $\pm$ 0.5 mg tungsten sphere attached to the end of a fine molybdenum needle. The external mass could be moved along a guide by a servo-controlled motor, and could be set at a series of specified distances from mass A.

Torsional oscillations of the balance were detected by a capacitive parametric displacement transducer whose high-frequency drive was provided by a stabilized quartz oscillator. The effective capacitance was that between the end wall of the cylindical insert and test mass A. This same sensor recorded pendulum oscillations of the balance arm perpen-

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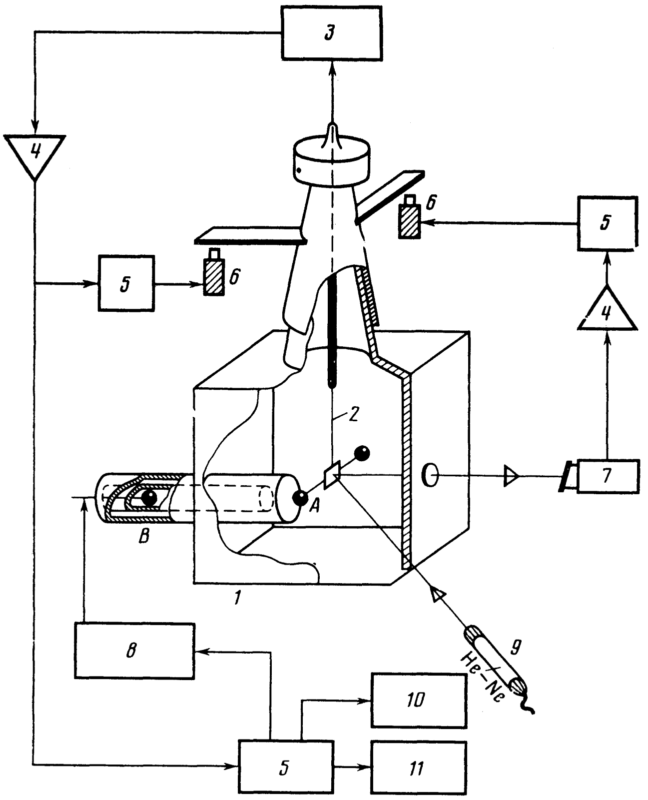

FIG. 1. Block diagram of the experimental arrangement for measuring the gravitational force between two spherical masses. 1) Vacuum chamber; 2) torsion balance; 3) capacitive displacement transducer; 4) amplifier; 5) filter; 6) electromagnet; 7) photodetector; 8) displacement control system for mass B; 9) laser; 10) strip-chart recorder; 11) voltmeter.

dicular to the plane formed by the arm and the suspension fiber, and the two types of oscillation were discriminated by appropriate filters.

To reduce the effective noise temperature of torsional oscillations of the balance induced by seismic effects, we instituted damping of the pendulum modes. Pendulum oscillations both in the plane of the balance arm and suspension fiber and perpendicular to it were detected by optical and capacitive sensors whose outputs were amplified and set to electromagnets with the appropriate phase. The latter then drove the entire vacuum chamber, including the balance suspension assembly, synchronously with seismic oscillations of the balance arm. The phase of the drive was so chosen as to quench spurious pendulum oscillations. In the process, the relaxation time of pendulum oscillations was reduced from 10^5 sec to several tenths of a second, significantly diminishing the coupling of pendulum-mode energy into the torsional mode, and lowering the effective noise temperature of the torsional oscillations to $3\cdot 10^3$ K (at night). This enabled us to achieve a sensitivity in measurements of the torque acting on the test mass A of $P_{\min} \approx 5\cdot 10^{-10}$ dyne·cm for an integration time of $\tau \approx 10^3$ sec.

The measurements were made as follows. Gauge blocks were used to set the closest position of mass B, at a distance r_1 from test mass A. The displacement control system, driven by the signal from a torsional oscillation sensor on the balance, was then turned on. Mass B was automatically moved from its closest position to its farthest (40 mm travel) synchronously with the oscillations of the balance. Depending on the phase of the motion of mass B, torsional oscillations would first be excited by the gravitational attraction between masses A and B, then damped. The amplified and filtered signal from the sensor output, which was proportional to the displacement of the test mass, was recorded by a strip-chart recorder, and for every oscillation period, its amplitude was measured by a digital voltmeter. A new closest position was then set up for mass B, corresponding to a distance r_2, and the measurement cycle was repeated.

The change in oscillation amplitude per period under the influence of a periodic torque that consists of a series of rectangular pulses of amplitude P_0, with a repetition rate equal to the natural frequency of the balance and a phase shift φ, is given (when $Q \gg 1$) by

$$a_{i+1}-a_i \approx \frac{2P_0}{K}\cos\varphi - a_i\frac{\pi}{Q}, \qquad (2)$$

where a_i is the amplitude of the ith oscillation cycle, K is the stiffness of the oscillator, and Q is its quality factor. Depending on the value of φ, oscillations of the balance will either be amplified or attenuated.

We used Eq. (2) and the observed amplitudes to calculate the gravitational torque acting on the test mass for an optimal choice of phase φ:

$$P_0 = \frac{K}{2(a_{av}+a'_{av})}\left[\frac{a_N-a_1}{N-1}a_{av}' + \frac{a_{N'}'-a_1'}{N'-1}a_{av}\right], \qquad (3)$$

where unprimed quantities refer to amplified oscillations of the balance (a total of N periods), and primed quantities refer to damped oscillations (N' periods total), and

$$a_{av} = \frac{1}{N}\sum_{i=1}^{N} a_i, \qquad a'_{av} = \frac{1}{N'}\sum_{j=1}^{N'} a_j'.$$

Calculating the torque for both amplified and damped oscillations enabled us, for example, to eliminate the Q of the torsional balance from the calculations, thereby reducing the final error. Equation (2) is approximate, but with the balance actually used ($Q \gg 1$) it yields an error that is small compared with the overall measurement error.

The change in the total torque acting on the balance as mass B was moved resulted from interactions between that mass and the needle to which it was affixed, test mass A, the second mass of the balance, the balance arm, and the mirror. The gravitational attraction between masses A and B were the dominant factor, the remainder contributing less than 5%. From the measured torques, we calculated the ratio $[F(r_1)/F(r_2)]_{\exp}$. Measurements were made for two values of the distance between the centers of spheres A and B,

$$r_1 = 3.773 \pm 0.040 \text{ mm}, \quad r_2 = 6.473 \pm 0.040 \text{ mm}.$$

Measurements were carried out on eleven occasions. Statistical processing yielded the result

$$[F(r_1)/F(r_2)]_{\exp} = 2.94 \pm 0.045. \qquad (4)$$

The quoted error is at the 1σ level, and is the product of fluctuations in the amplitude of balance oscillations caused by seismic noise.

STATISTICAL AND SYSTEMATIC MEASUREMENT ERRORS

The measured values of the gravitational force acting on the balance at two different positions of the mass B were compared with the calculated ratio $[F(r_1)/F(r_2)]_{\text{calc}}$ arrived at by applying Newton's law. Since we measured a ratio of forces, the principal error in the calculated value of this ratio was due to inaccuracies in the measured mutual separation of the interacting masses. The comparatively large magnitude of the relative error may be due to the smallness of the measured distances themselves, as well as the presence of a number of partitions between the interacting masses.

Another source of error in the computed force ratio may perhaps be the contribution to the overall force coming from the attraction between the mirror, the balance arm, and the second mass comprising the balance, as well as the needle supporting the movable mass B. This error was dominated by inaccuracies in the measured masses and the dimensions of the balance, and amounted to at most 10% of the total error in the computed ratio. The value obtained for the quoted distances was

$$[F(r_1)/F(r_2)]_{\text{calc}} = 2.93 \pm 0.03. \tag{5}$$

In carrying out this experiment, it was necessary to minimize those effects that might mimic deviations from Newton's law. Such effects include nonlinearities in the transduction from mechanical oscillation of the balance to an electrical signal, i.e., dependence of the conversion factor on the amplitude of angular oscillations of the balance. Measures were taken in this regard to maximize the linearity of the capacitive displacement transducer and electronic amplifiers. Furthermore, in measuring at the closest and farthest positions of mass B, the balance was driven to approximately the same oscillation amplitude, thereby significantly reducing nonlinearities.

Equation (2) determines the precision to which the phase difference between the motion of mass B and that of the balance must be held constant, seismic noise induces errors in this phase. The amplitude to which oscillations of the balance had to drop before the active drive was turned on was chosen on the basis of the required signal-to-noise ratio, so as to assure the necessary accuracy in the determination of the phase of balance oscillations.

Besides gravitational interactions between the masses, magnetic interactions due both to magnetization of the masses in the earth's magnetic field and the presence of ferromagnetic impurities in the balls could also contribute. Test measurements were made to determine the magnetic properties of the masses employed in the experiment. In addition, the forces between masses placed in an external 10-Oe magnetic field induced by a solenoid were measured. These tests made it clear that magnetic interactions between the masses in fields of the order of magnitude of the terrestrial field were due mainly to induced magnetic moments. The error engendered was less than the systematic errors.

The size of the systematic error in the measured ratio $[F(r_1)/F(r_2)]$ induced by the above effects were therefore at most 0.5%.

DISCUSSION OF RESULTS

Comparing the experimentally determined values of the force ratio (4) between the masses at two different distances with the calculated values (5) based on Newton's law, we find that there is no significant discrepancy between these quantities at the 1σ level:

$$[F(r_1)/F(r_2)]_{\text{exp}} - [F(r_1)/F(r_2)]_{\text{calc}} = (1.0 \pm 5.4) \cdot 10^{-2}. \tag{6}$$

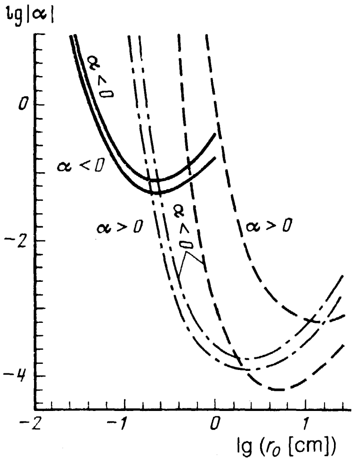

FIG. 2. Bounds on feasible values of the parameters r_0 and α given by the present experiment (solid curves), as compared with results from Ref. 7 (dashed curves) and Ref. 9 (dash-dot curves).

This means that we detected no deviation from Newton's Law of Gravitation in the present experiment.

If we start with the assumption that the gravitational potential is described by Eq. (1), then (6) provides bounds in the parameters r_0 and α. It is then necessary to take account of the fact that the distance r separating the centers of the interacting masses is comparable to their radii R_1 and R_2, and that the masses cannot be considered point masses. The gravitational potential energy between the two spherical masses A and B can be calculated by integrating Eq. (1) over their volumes:

$$V_{AB}(r) = -G_\infty \frac{m_A m_B}{r}\left[1 + \alpha\beta_1\beta_2 \exp\left(-\frac{r}{r_0}\right)\right] \tag{7}$$

where

$$\beta_i = \frac{3}{2}\left(\frac{r}{R_i}\right)^3\left[\left(\frac{R_i}{r} - 1\right)\exp\frac{R_i}{r} + \left(\frac{R_i}{r} + 1\right)\exp\left(-\frac{R_i}{r}\right)\right], \quad i = 1, 2.$$

Figure 2 shows the bounds placed on feasible values of the parameters α and r_0 by the present experiment at the 1σ confidence level; the figure also show corresponding curves derived elsewhere.[7,9] These bounds on α and r_0 enable one to narrow the range of possible masses of the hypothetical particles associated with the correction of Newtonian gravitation arising in supergravity models (for example, see Refs. 3, 8, 16).

The work cited in Refs. 10–15 has recently aroused interest in the possible connection between the coefficient α in the expression for the modified gravitational potential (1) and the atomic structure of the interacting matter. Indeed, r_0 has been assumed to lie in the range 10^2–10^3 m, although there is no theoretical prerequisite for such an assumption. The coefficient α may be written in the form[15]:

$$\alpha = q_1 q_2 \xi, \quad q_i = cB_i/\mu_i + (1-c)I_{zi}/\mu_i, \quad i = 1, 2,$$

where charge q_i is a linear combination of the baryon number B_i/μ_i and the projection of the nuclear isospin I_{zi}/μ_i normalized to unit mass of the interacting objects, $\mu_i = m_i/$

m_H is the atomic mass measured in units of the mass of the hydrogen atom, and ξ is a universal interaction constant. If we assume that $[B/\mu]_W = 1.008093$, $[B/\mu]_{Pt} = 1.008009$, $[I_z/\mu]_W = 0.19675$, and $[I_z/\mu]_{Pt} = 0.20208$, the experimental estimates for α make it possible to derive corresponding values of ξ for small values of r_0.

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