Electromagnetic excitation of sound in metals in the anomalous skin effect

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Electromagnetic generation of longitudinal sound in metals is theoretically investigated under conditions of nonlinear anomalous skin effect, when the main source of the acoustic oscillations is the deformation force of the electron-phonon interaction. The nonlinearity mechanism is magnetodynamic—it is due to the influence of the magnetic field of the radiowave on the dynamics of the conduction electrons. The analysis is carried out in a wide range of variation of the external-signal amplitude $\mathscr{H}$ and covers the cases of both weak and strong nonlinearity. The dependence of the excited-sound amplitude on the radiowave amplitude $\mathscr{H}$, on its frequency ω, and on the electron mean free path l are calculated. It is established that the longitudinal sound contains only even harmonics of the incident wave. The nonlinear conversion of electromagnetic energy into acoustic is not very sensitive to the sample surface state.

1. INTRODUCTION

Theoretical and experimental study of contactless generation of sound is the subject of many papers (see, e.g., Ref. 1 and the citations therein). In most of them the analysis is restricted to the linear situation, when the external magnetic wave of frequency ω has a small amplitude $\mathscr{H}$ and generates therefore in the metal acoustic oscillations of the same frequency. The effectiveness of the electromagnetic excitation of sound increases with increase of incident-signal power and mean free path l of the conduction electrons. With increase of the parameters $\mathscr{H}$ and l, however, nonlinear processes begin to develop in the sample quite rapidly. The most relevant in contemporary experiments on pure metals at low frequencies is in fact just the nonlinear regime.

Nonlinear electromagnetic generation of acoustic oscillations under normal-skin-effect conditions $l \ll \delta$ (δ is the skin-layer depth) was investigated theoretically in Refs. 2 and 3. Particular interest attaches to the anomalous skin-effect situation which is typical for metals:

$$\delta \ll l. \tag{1.1}$$

This case was considered in only one theoretical paper,[4] in which the weak-nonlinearity regime was investigated. The results of Ref. 4 demonstrate the important role of nonlinear electromagnetic processes in sound excitation. Thus, for example, in metals having a spherical Fermi surface longitudinal sound is generated only as a result of nonlinearity. The amplitude of the nonlinear sound is in this case larger by a factor $(l/\delta)^2 \gg 1$ than in the normal skin effect.

The nonlinear mechanism in metals is connected with the influence of the radiowave magnetic field on the dynamics of the electrons and by the same token on the sample conductivity. This mechanism is called magnetodynamic. The parameter b, which characterizes its effectiveness, is determined by the ratio of the mean free path l to the electron path length in the inhomogeneous magnetic field of the skin layer $(8R\delta)^{1/2}$ (see Ref. 5):

$$b=(2\mathscr{H}/h)^{1/2}, \qquad h=8cp_F\delta/el^2. \tag{1.2}$$

Here $R = cp_F/2e\mathscr{H}$ is the characteristic curvature radius of the electron trajectory in the skin-layer magnetic field, e is the absolute value of the charge, p_F is the Fermi momentum of the electron, and c is the speed of light. Let us estimate the strength of the magnetic field h in which the nonlinearity parameter $b = 1$. For typical pure metals at low temperatures $\delta \sim 10^{-3}$–10^{-4} cm and $l \sim 10^{-1}$ cm we obtain $h \sim 0.5$–5 Oe. The amplitudes $\mathscr{H}$ of the electromagnetic wave reach in experiments tens and even hundreds of Oersteds, so that both weak ($b \ll 1$) and strong ($b \gg 1$) nonlinearity can be realized in experiment.

In the case of weak nonlinearity ($b \ll 1$) the electron trajectories in the skin layer are almost straight lines slightly bent by the wave's magnetic field. Under these conditions, the nonlinear effects manifest themselves in the approximation quadratic in the amplitude $\mathscr{H}$.[4,5]

In the strong-nonlinearity regime ($b \gg 1$) the electrodynamic properties of the metal are formed by a group of trapped electrons,[5] attention to which was first called in a paper by Babkin and Dolgopolov[6] devoted to current states. This group is due to the fact that the spatial distribution of the radiowave magnetic field is of alternating sign. The trapped electrons move along the sample surface along trajectories that weave around the magnetic-field sign-reversal plane (see Fig. 1). The stay all the time in the skin layer and therefore interact most effectively with the electromagnetic wave. Thus, under conditions of strong nonlinearity, the conversion of the electromagnetic energy into acoustic should be determined by the trapped particles.

We investigate theoretically in this paper electromagnetic excitation of longitudinal sound in a wide range of variation of the radio-wave amplitude $\mathscr{H}$ (of the nonlinearity parameter b) under the normal-skin-effect conditions (1.1). The analysis is restricted to the quasistatic case

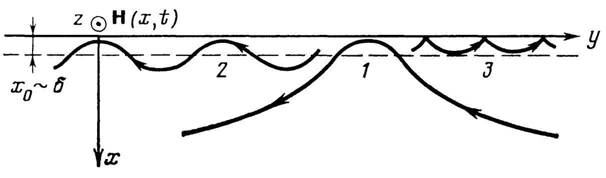

FIG. 1. Trajectories of effective electrons in the electromagnetic field of an electromagnetic wave: transiting (untrapped) (1), trapped (2), and glancing (3).

$$\omega \ll \nu, \tag{1.3}$$

when the frequency ω of the external wave is much lower than the electron relaxation frequency ν. The inequality (1.3) allows us to neglect the change of the magnetic field during the entire time of the electron free path. Regardless of the degree of nonlinearity, the main source of the generated sound is the deformation force. A detailed analysis of the dynamics of the conduction electrons in a non-uniform magnetic field yielded a general expression for the deformation force. This equation makes it possible to analyze, from a unified standpoint, the cases of both weak and strong nonlinearity. We calculate the dependence of the excited-sound amplitude on the external-signal amplitude $\mathscr{H}$, on its frequency ω, on the mean free path l, and on other parameters of the problem. We show that for any degree of nonlinearity the generated longitudinal sound contains only even harmonics (2ω, 4ω,...) of the incident wave. It is established that nonlinear electromagnetic generation of acoustic oscillations is not very sensitive to the sample surface state.

2. FORMULATION OF PROBLEM. PHYSICAL ANALYSIS OF THE PHENOMENON

1. Consider a metallic half-space with a plane monochromatic electromagnetic wave of frequency ω and amplitude $\mathscr{H}$ incident on its surface. We direct the x axis along the inward normal to the metal ($x = 0$ on the metal-vacuum boundary), and the y and z axes parallel to the vectors of the electric and magnetic components of the electromagnetic field (Fig. 1):

$$\mathbf{E}(x,\ t)=\{0,\ E(x,\ t),\ 0\}, \quad \mathbf{H}(x,\ t)=\{0,\ 0,\ H(x,\ t)\}. \tag{2.1}$$

We investigate electromagnetic generation of a longitudial sound wave in which the displacement vector is

$$\mathbf{u}(x,\ t)=\{u(x,\ t),\ 0,\ 0\}. \tag{2.2}$$

The set of equations describing this process consists of the Maxwell equations, the Boltzmann kinetic equation for the nonequilibrium increment $\chi(\partial f_F/\partial\varepsilon)$ to the electron Fermi distribution function f_F, and the elasticity-theory equations. Since the electron and ion mass ratio is small, the solution of the set of equations breaks up into two stages. We determine first the nonlinear perturbation of the electron subsystem by the external magnetic field and the distribution of the fields $E(x, t)$ and $H(x, t)$ in the metal. The kinetic equation is linearized in this case with respect to the electric field $E(x, t)$, and the nonlinearity is due to the magnetic field $H(x, t)$ and is contained in the Lorentz force. This stage constitutes essentially the problem, solved in Ref. 5, of the nonlinear anomalous skin effect.

The sound is generated because the elasticity-theory equation contains the driving force $F(x, t)$ exerted on the lattice by the conduction electrons:

$$F(x,t)=\frac{\partial}{\partial x}\left\{\int \frac{2\,d\mathbf{p}}{(2\pi\hbar)^3}\Lambda_{xx}(\mathbf{p})\frac{\partial f_F}{\partial\varepsilon}\chi\right\}. \tag{2.3}$$

In our very simple model with quadratic and isotropic electron dispersion, the component $\Lambda_{xx}(\mathbf{p})$ of the deformation-potential tensor is given by

$$\Lambda_{xx}(\mathbf{p})=-\tilde{m}(v_x^2-v_F^2/3), \tag{2.4}$$

where m is the "deformation" mass, $\mathbf{v}$ the velocity, v_F the Fermi velocity, $\mathbf{p} = m\mathbf{v}$ the momentum, and m the electron mass. In expression (2.3) for the force we have taken into account only the deformation mechanism of the electron-phonon interaction. Analysis shows that the inductive contribution to $F(x, t)$ is negligibly small under the anomalous skin effect, regardless of the degree of nonlinearity.

The equation for the acoustic oscillations can be easily solved by using the Fourier expansions of the displacement $u(x, t)$ and the force $F(x, t)$:

$$\begin{aligned} u(x,t)&=\frac{1}{\pi}\sum_{n=-\infty}^{\infty} e^{-in\omega t}\int_0^{\infty} dk\,\sin(kx)\,\tilde{u}_{n\omega}(k),\\ F(x,t)&=\frac{1}{\pi}\sum_{n=-\infty}^{\infty} e^{-in\omega t}\int_0^{\infty} dk\,\sin(kx)\,F_{n\omega}(k). \end{aligned} \tag{2.5}$$

At large distances ($x \gg l$) this solution takes the form of a superposition of plane waves:

$$u(x,t)=\sum_{n=-\infty}^{\infty} u_n \exp[in(qx-\omega t)], \qquad q=\omega/s, \qquad x\gg l, \tag{2.6}$$

where q is the wave number and s is the speed of the longitudinal sound. The amplitude of the nth harmonic is then

$$u_n=\frac{i}{n\pi\rho_0\omega s}\int_0^{\infty} dk\frac{k}{k^2-(nq)^2}F_{n\omega}(k), \qquad n\neq 0. \tag{2.7}$$

Here ρ_0 is the density of the metal. It was recognized in the derivation of (2.6) and (2.7) that the sound damping length l_s is the largest parameter of the problem. Indeed, for nonresonant electron-phonon interaction we have $l_s \gg \delta$, $l_s \sim v_F/sq \gg 1/q$ and $l_s \sim l\nu/\omega \gg l$.

The problem of electromagnetic excitation of sound reduces thus to calculation of the amplitudes u_n (2.7).

2. Before we proceed to the asymptotically exact solution, we obtain the dependence of u_n on the amplitude of the electromagnetic wave $\mathscr{H}$, starting from simple physical considerations. At low temperatures, under conditions of the quasistatic ($\omega \ll \nu$) anomalous skin effect, particular interest attaches to the case when $q\delta \ll 1$ ($\omega \ll s/\delta$). According to (2.7), in this case the amplitude of the excited n-harmonic is described by the following qualitative formula ($k \sim \delta^{-1}$):

$$|u_n|=F\delta/\rho_0\omega s. \tag{2.8}$$

From a comparison of (2.3) with the analogous expression for the current density j it is easy to obtain a relation between the deformation force F the value of j:

$$F\sim jmv_F/e\delta. \tag{2.9}$$

The current density is then

$$j=\sigma E, \qquad E\sim\mathscr{H}\omega\delta/c, \tag{2.10}$$

and the conductivity σ in the interior of the electromagnetic skin layer δ can be determined with the aid of Pippard's ineffectiveness concept.

In the strong-interaction regime ($b \gg 1$) the electrodynamics of the metal is determined by the group of trapped electrons.[5] Their relative number is of the order of $(\delta/R)^{1/2}$, so that in Pippard's mode their conductivity takes the form

$$\sigma=\sigma_0(\delta/R)^{1/2}=\sigma_0\delta b/l, \tag{2.11}$$

where σ_0 is the static conductivity of the bulk metal. From the Maxwell equations, the connection between the skin-layer depth δ and the effective conductivity σ is

$$\delta^2=ic^2/4\pi\omega\sigma. \tag{2.12}$$

Substituting (2.11) and (1.2) in (2.12) and solving the result for δ we get

$$\delta\sim(c^5p_F/4\pi^2e\mathscr{H}\omega^2\sigma_0^2)^{1/5}. \tag{2.13}$$

In accordance with (2.8)–(2.13) we get

$$|u_n|\sim\frac{\mathscr{H}}{\rho_0\omega s}\frac{R}{\delta}\propto\mathscr{H}^{6/5}\omega^{-3/5}l^{2/5}. \tag{2.14}$$

In the advanced nonlinearity regime ($b\gg1$) the amplitude of each sound-field harmonic increases thus with $\mathscr{H}$ in accordance with one and the same law. This distinguishes the situation $b\gg1$ in principle from the case of weak nonlinearity ($b\ll1$), when the nth harmonic amplitude is proportional to $b^{2n}\propto\mathscr{H}^n$ ($n=2, 4, 6,\ldots$). Note also that the amplitude of the excited sound is $R/\delta\gg1$ times larger than under normal skin-effect conditions,[2,3], when the main generation source is the inductive electron-phonon interaction mechanism.

3. DEFORMATION FORCE

The deformation force $F(x, t)$ is determined by solving the kinetic equation. Since the nonlinearity is contained there in the Lorentz force, to find $F(x,t)$ it is necessarry first to examine the dynamics of the electrons in the magnetic field $H(x, t)$ of the radiowave. Recall that our analysis is restricted to the quasistatic case. Under conditions (1.3), the motion is in a non-uniform but constant magnetic field $H(x,t)$, since the phase of the wave remains unchanged during the entire free-path time. In other words, the "electromagnetic time" t in the equations of motion and in the kinetic equation does not change and plays the role of an external parameter.

We represent the vector potential $\mathbf{A}(x, t)$ of the magnetic field in the form

$$\mathbf{A}(x,t)=\{0,A(x,t),0\},$$

$$A(x,t)=\int_0^x dx'H(x',t)\equiv H(0,t)a(x,t). \tag{3.1}$$

In the chosen gauge, the integrals of the electron motion are the free energy, equal to the Fermi energy ε_F, and the generalized momenta $p_z=mv_z$ and $p_y=-eH(0,t)X/c$. To simplify the exposition, we have introduced the function $a(x,t)$ and the conserved quantity X.

The electron motion in a plane perpendicular to the vector $\mathbf{H}(x, t)$ is described by the velocities $v_y(x)$ and $v_x(x)$:

$$v_y(x)=-\Omega[X-a(x,t)]\operatorname{sign}H(0,t),$$
$$|v_x(x)|=\Omega\{R_\perp^2-[X-a(x,t)]^2\}^{1/2}. \tag{3.2}$$

Here

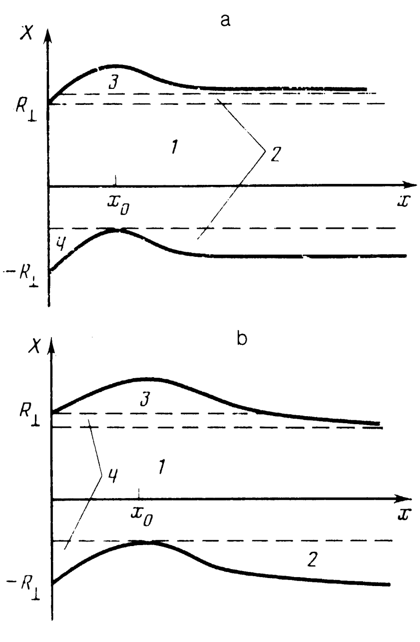

FIG. 2. Plots of $X=\pm R_\perp+a(x,t)$ in the region of existence of electron groups (see the text): 1—surface untrapped electrons; 2—bulk untrapped electrons; 3—whirling (trapped) electrons; 4—surface electrons, a—$a(\infty,t)>0$, b—$a(\infty,t)<0$.

$$\Omega=e|H(0,t)|/mc,\quad R_\perp=v_\perp/\Omega,\quad v_\perp=(v_F^2-v_z^2)^{1/2}. \tag{3.3}$$

The distinctive properties of electron motion in the field $H(x,t)$ are determined by the character of the $a(x,t)$ dependence. The electromagnetic field in the metal oscillates and is damped at a distance on the order of the skin-layer thickness δ. We assume therefore that for $x\ll\delta$ the function $a(x,t)\approx x$ reaches a maximum $a(x_0,t)$ at the point $x=x_0(t)$ where the magnetic field $H(x,t)$ vanishes, and that for $x\gg\delta$ it tends to a constant value $a(\infty,t)$. The latter can be either positive or negative (depending on the instant of time t). In addition, since the skin effect is anomalous, it follows that $a(x_0,t)\sim\delta\ll R_\perp$ and $a(\infty,t)\sim\delta\ll R_\perp$.

Figure 2 shows the region of possible values of the electron coordinate x and of the integral of motion X for $a(\infty, t)>0$ (Fig. 2a) and $a(\infty,t)<0$ (Fig. 2b). In accordance with the requirement that the radicand in (3.2) be positive, this region is bounded from below by the curve $X=-R_\perp+a(x,t)$, and from above by the curve $X=R_\perp+a(x,t)$. It is seen from Fig. 2 that, depending on the value of X, the in respect to the character of their motion the electrons break up into four groups: surface untrapped, bulk untrapped, trapped (whirling), and surface electrons (a more detailed description of the electron groups is contained in Ref. 5).

A solution of the kinetic equation must be sought for each electron group separately. For electrons having turning points in the interior of the sample, the boundary condition for the solution is the continuity of the distribution function at these points. A condition on the metal boundary $x=0$ is formulated only for surface and surface-untrapped electrons. We choose the simplest of these conditions (the Fuchs condition[7]), in which the interaction of the electrons with the sample surface is characterized by a phenomenological parameter, the probability ρ of specular reflection from the boundary ($0\leqslant\rho\leqslant1$). Ultimately, for the spatial Fourier sine transformation

$$\tilde{F}(k,t)=2\int_0^\infty dx\sin(kx)F(x,t) \tag{3.4}$$

we obtain

$$\tilde{F}(k,t)=\frac{1}{\pi}\int_0^\infty dk'\tilde{E}(k',t)\sum_{\alpha=1}^{4}C_\alpha(k,k'). \tag{3.5}$$

Here $\tilde{E}(k,t)$ is the spatial Fourier component of the electric field $E(x,t)$, namely,

$$\tilde{E}(k,t)=2\int_0^\infty dx\cos(kx)E(x,t). \tag{3.6}$$

The coefficient $C_1(k,k')$ is due to the surface untrapped electrons and is given by

$$C_1(k,k')=\frac{3\sigma_0 k}{\pi e l}\int_0^{v_F}\frac{dv_z}{v_F}\int\frac{dX}{R(t)}\int_0^\infty dx\cos(kx)$$
$$\times\frac{\Lambda_{xx}(x)}{|v_x(x)|}\int_0^\infty dx'\cos(k'x')\frac{v_y(x')}{|v_x(x')|}$$
$$\times\{\exp[-\nu|\tau(x,x')|]+\rho\exp[-\nu(\tau(0,x)+\tau(0,x'))]\}, \tag{3.7}$$

$-R_\perp+a(x_0,t)\leqslant X\leqslant R_\perp$

for $a(\infty,t)>0$, $-R_\perp+a(x_0,t)\leqslant X\leqslant R_\perp+a(\infty,t)$

for $a(\infty,t)<0$.

For the coefficient C_2 of the bulk untrapped electrons we have

$$C_2(k,k')=\frac{3\sigma_0 k}{\pi e l}\int_0^{v_F}\frac{dv_z}{v_F}\int\frac{dX}{R(t)}\int_{x_1}^\infty dx\cos(kx)$$
$$\times\frac{\Lambda_{xx}(x)}{|v_x(x)|}\int_{x_1}^\infty dx'\cos(k'x')\frac{v_y(x')}{|v_x(x')|}$$
$$\times\{\exp[-\nu|\tau(x,x')|]+\exp[-\nu(\tau(x_1,x)+\tau(x_1,x'))]\}, \tag{3.8}$$

$-R_\perp+a(\infty,t)\leqslant X\leqslant -R_\perp+a(x_0,t)$

and $R_\perp\leqslant X\leqslant R_\perp+a(\infty,t)$ for $a(\infty,t)>0$,

$-R_\perp+a(\infty,t)\leqslant X\leqslant -R_\perp+a(x_0,t)$ for $a(\infty,t)<0$.

The contribution of the trapped electrons to the density of the deformation force is described by the term

$$C_3(k,k')=\frac{3\sigma_0 k}{\pi e l}\int_0^{v_F}\frac{dv_z}{v_F}\int\frac{dX}{R(t)}\operatorname{sh}^{-1}(\nu T_{\mathrm{vol}})$$
$$\times\int_{x_1}^{x_2}dx\cos(kx)\frac{\Lambda_{xx}(x)}{|v_x(x)|} \tag{3.9}$$
$$\times\int_{x_1}^{x_2}dx'\cos(k'x')\frac{v_y(x')}{|v_x(x')|}\{\operatorname{ch}[\nu(T_{\mathrm{vol}}-|\tau(x,x')|)]$$
$$+\operatorname{ch}[\nu(T_{\mathrm{vol}}-\tau(x,x_2)-\tau(x',x_2))]\},$$

$R_\perp+a(\infty,t)\leqslant X\leqslant R_\perp+a(x_0,t)$ for $a(\infty,t)>0$,
$R_\perp\leqslant X\leqslant R_\perp+a(x_0,t)$ for $a(\infty,t)<0$.

Finally, surface electrons give rise to the coefficient

$$C_4(k,k')=\frac{3\sigma_0 k}{\pi e l}\int_0^{v_F}\frac{dv_z}{v_F}\int\frac{dX}{R(t)}[\exp(\nu T_{\mathrm{sur}})-\rho\exp(-\nu T_{\mathrm{sur}})]^{-1}$$
$$\times\int_0^{x_2}dx\cos(kx)\frac{\Lambda_{xx}(x)}{|v_x(x)|}\int_0^{x_2}dx'\cos(k'x')\frac{v_y(x')}{|v_x(x')|}$$
$$\times\{\exp[\nu(T_{\mathrm{sur}}-|\tau(x,x')|)]+\rho\exp[-\nu(T_{\mathrm{sur}}-|\tau(x,x')|)]$$
$$+\exp[\nu(T_{\mathrm{sur}}-\tau(x,x_2)-\tau(x',x_2))] \tag{3.10}$$
$$+\rho\exp[-\nu(T_{\mathrm{sur}}-\tau(x,x_2)-\tau(x',x_2))]\},$$

$-R_\perp\leqslant X\leqslant -R_\perp+a(x_0,t)$ for $a(\infty,t)>0$,

$-R_\perp\leqslant X\leqslant -R_\perp+a(x_0,t)$

and $R_\perp+a(\infty,t)\leqslant X\leqslant R_\perp$ for $a(\infty,t)<0$.

In Eqs. (3.7)–(3.10) we have introduced the notation: $R(t)=v_F/\Omega$, $\tau(x,x')$ is the time of electron motion from the point x to the point x' in the magnetic field $H(x,t)$,

$$\tau(x,x')=\int_x^{x'}dx''/|v_x(x'')|, \tag{3.11}$$

x_1 and x_2 are the turning points, $2T_{\mathrm{vol}}\equiv 2\tau(x_1,x_2)$ is the oscillation period of the trapped electrons, and $2T_{\mathrm{sur}}\equiv 2\tau(0,x_2)$ is the oscillation period of the surface electrons.

Under conditions of the anomalous skin effect ($\delta\ll l$, $\delta\ll R$) the expressions for the coefficients (3.7)–(3.10) can be simplified by replacing them with asymptotes. The main contributions to the integrals with respect to X, x, and x' are made then by the vicinities of those points where the velocity $|v_x(x)|$ is zero. These are the turning points in the integrals with respect to x and x' and the end points of the integral with respect to X, i.e., the limits of the existence regions of each electron group. The asymptotes of coefficients (3.7)–(3.10) differ substantially in the case of small external-wave amplitudes ($b\ll 1$) and large amplitudes ($b\gg 1$). We consider therefore these two cases separately.

4. WEAK NONLINEARITY (SMALL AMPLITUDES $\mathscr{H}$)

In the weak-nonlinearity regime ($b\ll 1$) all the electron groups make equal contributions to the density of the deformation force (3.5). Its asymptotic form is

$$\tilde{F}(k,t)=\frac{1}{4}\frac{\tilde{m}}{m}\frac{\sigma_0 l}{c}k\Big\{\rho k\int_0^\infty dk'\frac{k'}{(k+k')^2}\tilde{E}(k',t)\tilde{H}(k+k',t)$$
$$-2(1-\rho)\Big[kE(0,t)\int_0^\infty dx\frac{\sin(kx)}{x}\int_0^x dx''A(x'',t)$$
$$+\int_0^\infty dx\int_0^\infty dx'\frac{\cos(kx')}{(x'-x)^2}(A(x',t)(x'-x) \tag{4.1}$$
$$-\int_x^{x'}dx''A(x'',t))\frac{\partial E(x,t)}{\partial x}\Big]\Big\}.$$

Here $\widetilde{H}(k, t)$ is the spatial Fourier sine transform of the magnetic field $H(x, t)$:

$$\widetilde{H}(k,t)=2\int_0^\infty dx \sin(kx) H(x,t). \tag{4.2}$$

Expression (4.1) was obtained in the first-order approximation in the nonlinearity parameter $b\ll 1$ ($\widetilde{F}(k, t)\propto b^4$). The electric and magnetic fields $\widetilde{E}(k, t)$ and $\widetilde{H}(k, t)$ are solutions of the linear Maxwell equation and contain only first harmonics ($n = \pm 1$) of the radio wave incident on the metal. Consequently, the asymptote (4.1) has a zeroth ($n = 0$) and second ($n = \pm 2$) harmonics. Since the expansion under weak-nonlinearity conditions is in terms of the parameter $b^4\propto \mathscr{H}^2$, there are no odd harmonics ($n = \pm 1, \pm 3, \pm 5,\ldots$) in the considered longitudinal sound, while the even harmonics with $|n| > 2$ are small compared with the second ($|n| = 2$) in respect to the parameter $b^{2|n|-4}\ll 1$.

Equation (4.1) generalizes the result of Ref. 4 to include the case of arbitrary electron reflection from the sample boundary. In specular reflection ($\rho = 1$) the second term in the curly brackets of (4.1) vanishes, and the first becomes the corresponding expression of Ref. 4.

Let us determine the amplitude of the second harmonic u_2 of the generated longitudinal sound. To this end, we exclude $\widetilde{F}_{2\omega}(k)$ from (4.1) and substitute the result in (2.7). We calculate the amplitude u_2 using the equations of Ref. 8, in which the distribution of the electromagnetic field was obtained in the linear regime for an arbitrary value of the specularity parameter ρ. The final expression for u_2 is quite unwieldy. We present therefore only its asymptotic form for large and small $q\delta_a$.

If the sound wavelength q^{-1} is much larger than the depth of the skin layer δ_2 of the linear theory

$$q\delta_a\ll 1, \quad \delta_a=(c^2l/3\pi^2\sigma_0\omega)^{1/3}, \tag{4.3}$$

we have with logarithmic accuracy

$$u_2=-\frac{i}{24\pi^4\rho_0\omega s}\frac{\widetilde{m}}{m}\mathscr{H}^2\frac{l^2}{\delta_a^2}[2\rho(\varkappa_1+\varkappa_2(2q\delta_a)^2\ln(1/2q\delta_a)) + (1-\rho)(\varkappa_1-i\pi M(-1)+\varkappa_2(2q\delta_a)^2\ln(1/2q\delta_a))], \tag{4.4}$$

where

$$\varkappa_1=\int_{c-i\infty}^{c+i\infty} dz\frac{M(z)M(-z-3)}{z(z+1)}, \quad -2<c<-1$$

$$\varkappa_2=\int_{c-i\infty}^{c+i\infty} dz\, M(z)M(-z-1), \quad -2+z_0<c<1+z_0 \tag{4.5}$$

$$z_0=\pi^{-1}\arccos\rho.$$

The explicit form of the function $M(z)$ is given in Ref. 8 (see also Ref. 9). The quantities $\varkappa_1$, $\varkappa_2$, and $M(-1)$ depend smoothly only on the specularity parameter ρ. For diffuse reflection ($\rho = 0$),

$$\varkappa_1\approx 3.9e^{2\pi i/3},\ \varkappa_2\approx -29.46,\ M(-1)=(3^{1/2}\pi/4)e^{i\pi/6}. \tag{4.6}$$

In the case of specular reflection ($\rho = 1$)

$$\varkappa_1\approx 3.5e^{2\pi i/3},\quad \varkappa_2=-2\pi/3. \tag{4.7}$$

In the long-wave region (4.3) the sound amplitude $|u_2|$ increases with increase of $q\delta_a$ like $(q\delta_a)^2|\ln q\delta_a|$.

Under conditions when the sound wavelength q^{-1} is much less than the skin layer δ_a,

$$q\delta_a\gg 1, \tag{4.8}$$

the asymptotic form of u_2 is

$$u_2=\frac{1}{24\pi^5\rho_0\omega s}\frac{\widetilde{m}}{m}\mathscr{H}^2\frac{l^2}{\delta_a^2}\left[\rho\frac{\pi^4}{2q\delta_a}-65.01e^{5\pi i/6}\frac{\ln(2q\delta_a)}{(2q\delta_a)^2}\right]. \tag{4.9}$$

It follows from (4.4) and (4.9) that the character of the interaction of the electrons with the metal surface exerts no substantial influence on the process of electromagnetic sound generation in the weak-nonlinearity regime. The only exception is the region of extremely small values of the parameter ρ and short sound wavelengths:

$$\rho\ll(q\delta_a)^{-1}\ll 1. \tag{4.10}$$

The sound amplitude is proportional here to $(q\delta_a)^{-2}\ln q\delta_a$, rather than to $(q\delta_a)^{-1}$ as for $\rho\neq 0$.

5. STRONG NONLINEARITY (LARGE AMPLITUDES $\mathscr{H}$)

1. To obtain the asymptotic density of the deformation force $\widetilde{F}(k, t)$ in the case $b\gg 1$ of strong nonlinearity it is necessary to calculate the coefficient $C_3(k, k')$. We put $v_y(x) = -v_\perp \operatorname{sign} H(0, t)$ in expression (3.9) and expand $|v_x|$ in the vicinity of $X = R_\perp$, using the smallness of $a(x, t)$ compared with $R_\perp$. Changing to an integration variable $\bar{x} = X - R_\perp$, we obtain

$$C_3(k,k')=-\frac{\operatorname{sign} H(0,t)}{\pi}\frac{k\sigma_0\widetilde{m}}{el}\int_0^{v_F} dv_z \times\int_{a(\infty,t)\theta(a(\infty,t))}^{a(x_0,t)} d\bar{x}\operatorname{cth}(\nu T_{\rm vol}) \times\int_{x_1}^{x_2}dx\frac{\cos(kx)}{[a(x,t)-\bar{x}]^{1/2}}\int_{x_1}^{x_2}dx'\frac{\cos(k'x')}{[a(x',t)-\bar{x}]^{1/2}}, \tag{5.1}$$

where $\theta(x) = 1$ for $x>0$ and $\theta(x) = 0$ for $d<0$. In the asymptotic form (5.1) we have neglected the small terms $-mv_x^2$ of expression (2.4) for Λ_{xx}. The integrals with respect to x, x, and x' can be calculated by using a quadratic expansion of the "vector potential" $a(x, t)$ near the point $x = x_0(t)$. The asymptotic form of the coefficient $C_3(k, k')$ is then

$$C_3(k,k')=-2\pi\operatorname{sign}(H(0,t))\frac{k\sigma_0\widetilde{m}}{el}\int_0^{v_F} dv_z\operatorname{cth}(\nu T)\cos(kx_0) \times\cos(k'x_0)[kJ_1(k\varkappa_0)J_0(k'\varkappa_0)-k'J_0(k\varkappa_0)J_1(k'\varkappa_0)]\frac{\varkappa_0}{k^2-k'^2}. \tag{5.2}$$

Here $J_0(x)$ and $J_1(x)$ are Bessel functions of zeroth and first order; $2T$ is the limiting period of the whirling electrons with velocity v_x tending to zero and with distance $x_2 - x_1$ between the turning points, i.e.,

$$T=\pi\left(mc/ev_\perp|H'(x_0, t)|\right)^{1/2}, \tag{5.3}$$

where the prime in (5.3) marks a derivative with respect to x. The quantity $\varkappa_0 (0 \leqslant \varkappa_0 \leqslant x_0)$ is determined from the equation

$$a(x_0-\varkappa_0, t)=a(\infty, t)\theta(a(\infty, t)) \tag{5.4}$$

and is the maximum distance from the point x_0 to the turning point closest to the metal surface. For $a(\infty, t) \leqslant 0$ we have $\varkappa_0 = x_0$.

We substitute (5.2) in the expression for the deformation force (3.5). We use for the field $E(k, t)$ the result of Ref. 5. The amplitude of the nth harmonic (2.7) of the excited sound takes then the form

$$u_n=-\frac{5[1+(-1)^n]}{216\pi^6}\Gamma^4\left(\frac{1}{4}\right)\frac{\tilde{m}}{m}\frac{\mathscr{H}}{\rho_0\omega s}\frac{cp_F}{e\delta}B(nq\delta), \tag{5.5}$$

where

$$B(nq\delta)=\sum_{r=-\infty}^{\infty}[1-(-1)^r]r \times\int_{-\pi/2}^{\pi/2}\frac{d\varphi'\alpha(\varphi')}{\mu}\cos\varphi'\exp[ir\xi(\varphi')]$$
$$\times\int_{-\pi/2}^{\pi/2}d\varphi\exp[in-ir\xi(\varphi)]\frac{\varkappa_0}{\delta}\int_0^\infty d\zeta\frac{\zeta}{\zeta^3-ir}\cos\left(\zeta\frac{x_0}{\delta}\right)$$
$$\times\int_0^\infty d\eta\frac{\eta^2}{\eta^2-(nq\delta)^2}\frac{\cos(\eta x_0/\delta)}{\eta^2-\zeta^2}\left[\eta J_1\left(\eta\frac{\varkappa_0}{\delta}\right)J_0\left(\zeta\frac{\varkappa_0}{\delta}\right)-\zeta J_0\left(\eta\frac{\varkappa_0}{\delta}\right)J_1\left(\zeta\frac{\varkappa_0}{\delta}\right)\right]. \tag{5.6}$$

The functions $\alpha(\varphi)$ and $\xi(\varphi)$ are given by

$$\alpha(\varphi)=\pi\left(\mathscr{H}\delta^{-1}/|H'(x_0, \varphi)|\right)^{1/2}, \quad \varphi=\omega t,$$
$$\xi(\varphi)=\mu^{-1}\int_0^\varphi\alpha(\varphi')d\varphi', \quad \mu=\pi^{-1}\int_0^\pi\alpha(\varphi)d\varphi. \tag{5.7}$$

The penetration depth δ in the strong-nonlinearity regime is

$$\delta=\left(\frac{25\Gamma^4(1/4)\mu^2}{81(2\pi)^5}\frac{c^5p_F}{\sigma_0{}^2\omega^2e\mathscr{H}}\right)^{1/5}\infty\mathscr{H}^{-1/5}. \tag{5.8}$$

It must be noted that under conditions of the nonlinear anomalous skin effect the electromagnetic field contains only odd harmonics of the incident wave. The summation index r in (5.6) takes on therefore in fact only odd values ($r = \pm 1, \pm 3, \ldots$). The deformation force (3.5) and also the displacement vector $u(x, t)$ (2.5) are consequently periodic in time with a period π/ω and contain only even harmonics of the external electromagnetic wave. The number n of the harmonic in (2.5), (2.6), and (5.5) takes on therefore even values ($n = 0, \pm 2, \pm 4, \ldots$), and there are no odd harmonics in the excited longitudinal sound.

Equations (5.5)–(5.8) pertain the case of diffuse reflection of electrons from the metal surface, inasmuch as at $\rho = 0$ it is precisely the trapped electrons that make the principal contribution to the conversion of the electromagnetic energy into acoustic. If $\rho \neq 0$ the amplitude of the generated sound remains unchanged, since the whirling electrons do not interact with the sample boundary. Insignificant differences occur only at near-specular reflection. In that situation an important role in the sound generation is played not only by the trapped particles, also by the glancing electrons (see Fig. 1, trajectory 3), whose contribution $C_4(k, k')$ to the kinetic coefficient is of the order of $C_3(k, k')$.

2. Let us analyze the dependence of the amplitude u_n (5.5) on the value of $\mathscr{H}$ in the limiting cases of small and large values of the parameter $|n|q$. When the inequality

$$|n|q\delta \ll 1 \tag{5.9}$$

is met, the function $B(nq\delta)$ (5.6) is equal, in first-order approximation, to the constant $B(0)$. Under these conditions we obtain for the amplitude of the nth harmonic of the generated sound

$$|u_n|\sim\frac{cp_F\mathscr{H}}{\rho_0\omega se\delta}\sim\frac{\mathscr{H}^2}{\rho_0\omega s}\frac{R}{\delta}\infty\mathscr{H}^{6/5}\omega^{-6/5}l^{2/5}. \tag{5.10}$$

The result (5.10) agrees with that obtained in Sec. 2 on the basis of qualitative considerations.

In the opposite situation, when the length of the sound wave is much shorter than the skin-layer depth

$$|n|q\delta \gg 1, \tag{5.11}$$

the main contributions are made to the integral (5.6) with respect to η by the region of the large values $\eta \sim |n|q\delta \gg 1$, and to the integral with respect to φ by the interval where $\varkappa_0 = x_0$. Using the asymptotic form of the Bessel function $J_1(\eta\varkappa_0/\delta)$ for large values of the argument, we obtain from (5.6) and (5.5)

$$B(nq\delta)\sim 1/(|n|q\delta)^{1/2},$$
$$|u_n|\sim\frac{cp_F\mathscr{H}}{\rho_0\omega se\delta}\frac{1}{(q\delta)^{1/2}}\infty\mathscr{H}^{13/10}\omega^{-9/10}l^{3/5}. \tag{5.12}$$

We present in conclusion an interpolation equation for the dependence of the amplitude of the second harmonic of the excited longitudinal sound on the value of $\mathscr{H}$; this equation is valid for arbitrary degree of nonlinearity and for any value of the parameter $q\delta$:

$$|u_2|=\frac{1}{24\pi^4\rho_0\omega s}\frac{\tilde{m}}{m}\mathscr{H}^2\frac{l^2}{\delta^2}[1-\exp(-1/b)]^2G(q\delta), \tag{5.13}$$

where

$$G(x)=\left[\frac{1+x^2((x-1)\ln x+2)}{1+x^3(x-1)\ln x}\right]\left[1+\left(\frac{1+x^3}{1+x^{5/2}}\right)^{(b-1)/(b+1)}\right],$$
$$b=(\mathscr{H}el^2/4cp_F\delta)^{1/2}.$$

The electromagnetic-field penetration depth δ is determined by the solution of the equation

$$\delta=\delta_a[1-\exp(-1/b)]^{1/3}. \qquad (5.14)$$

Under conditions of weak ($b\ll 1$) and strong ($b\gg 1$) nonlinearity, the asymptotes obtained for δ from (5.14) agree, apart from a numerical coefficient, with expressions (4.3) and (5.8), respectively.

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