Nonlinear Doppler-shifted cyclotron resonance in metals

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Nonlinear reduction of collisionless cyclotron absorption reduced the permittivity of an electron-hole plasma in cadmium so much that a new Doppler wave was observed. The new doppleron was due to a Doppler-shifted cyclotron resonance of "lens" electrons, but its circular polarization was opposite to that of the familiar doppleron. A linear plasma bleaching effect was observed also in samples of tungsten.

Nonlinear behavior of the rf impedance of a silicon plate in a static magnetic field perpendicular to its plane was investigated by us earlier.[1] We studied nonlinear characteristics of the impedance in the vicinity of the threshold for the formation of a hole doppleron. We demonstrated that the nonlinearity was due to the effect of the magnetic field $\mathscr{H}$ of an electromagnetic wave on the paths of "lens" electrons responsible for the collisionless cyclotron damping of this doppleron. The Lorentz force associated with this magnetic field modulated the velocity of electrons along the static field **H** at a frequency proportional to the square root of the wave amplitude. Such modulation of the longitudinal velocity of electrons suppressed strongly the cyclotron absorption by hole dopplerons. This increased greatly the amplitude of oscillations of the impedance in the vicinity of the hole doppleron threshold.

In the present study we found experimentally that at high amplitudes of the rf field a new wave propagated in cadmium and the excitation of this wave in the cadmium plate resulted in strong oscillations of the impedance in the case of a positive circular polarization above the range which the hole doppleron was observed. This wave appeared because the majority of electrons on the Fermi surface were trapped by the wave field and ceased to contribute the collisionless absorption process. The only electrons which were not trapped were those in the vicinity of a certain limiting point of the Fermi-surface lens. Consequently, the region of collisionless absorption was limited on the short-wavelength side, which gave rise to a new doppleron. In contrast to the familiar electron doppleron, the nonlinear doppleron was characterized by normal dispersion and was observed when the circular polarization was positive.

1. EXPERIMENTS

We investigated the surface resistance of a cadmium plate subjected to a static magnetic field **H** perpendicular to its surface. The plate was cut from a single crystal characterized by the resistance ratio 5×10^4. The hexagonal axis of the crystal was at right-angles to the plane of the plate. The method used in the resistance measurements differed from that described in Ref. 2 by the presence of an additional coil which made it possible to create a circularly polarized exciting field. The surface resistance R was determined as a function of H and the results obtained for the negative circular polarization are shown in Fig. 1a, whereas those obtained for the positive polarization are given in Fig. 1b. Suitable variation of the amplitude of the exciting field altered also the gain so that the records obtained in the linear regime practically coincided for different amplitudes. Curves 1 and 3 were obtained when the exciting alternating magnetic field was 4 Oe. Severalfold variation of the amplitude of the alternating magnetic field did not change these curves, i.e., the linear regime was obtained at these amplitudes. Oscillations of curve 1 were associated with the excitation of an electron doppleron in the plate and this was due to a Doppler-shifted cyclotron resonance (DSCR) of electrons at a limiting point of the Fermi-surface lens. Curve 3 showed no such oscillations because the amplitude of the Gantmakher-Kaner oscillations in the negative circular polarization case was two orders of magnitude less than the amplitude of the doppleron oscillations. The amplitude of the hole doppleron oscillations, due to the DSCR of holes from the Fermi-surface monster[3] observed in the same polarization, was also small. Therefore, these oscillations could not be seen in the case of curve 3. Curve 4 was obtained when the amplitude of the alternating magnetic field was 40 Oe, and curves 2 and 5 were recorded when this amplitude was 75 Oe. Curve 2 did not differ significantly from curve 1. The changes were basically as follows. The oscillations spread to higher fields and the maxima of their amplitude also shifted to higher fields; moreover, the limiting value of the period decreased somewhat at higher fields. The changes between curves 4 and 5 were more striking. First of all, in the nonlinear regime we observed singularities[1] associated with the DSCR of holes, identified by arrows on curves 4 and 5. Moreover, in the case of curve 4 in the range of fields where the electron doppleron oscillations were observed in the negative polarization, we now found two irregular maxima. Under strongly nonlinear conditions (curve 5) the singularities in the range of existence of the electron doppleron became regular. The amplitude of the oscillations exhibited by curve 5 was greater than those exhibited by curve 1. The oscillations began above the hole doppleron threshold and, as in the negative polarization case, extended to values of H which increased on increase in the amplitude of the exciting field. The period of these oscillations decreased monotonically on increase in H and reached a limiting value very close to the limit of the doppleron oscillation period in the negative polarization (compare with curve 2). Therefore, the properties of the oscillations of curve 5 indicated that a new doppleron appeared in the nonlinear case when the polarization was positive. The proximity of the limiting values of periods of oscillations of curves 2 and 5 indicated that the new doppleron was due to a DSCR of electrons which shifted beyond the cyclotron period by an amount very close to the shift of the electrons at the limiting point.

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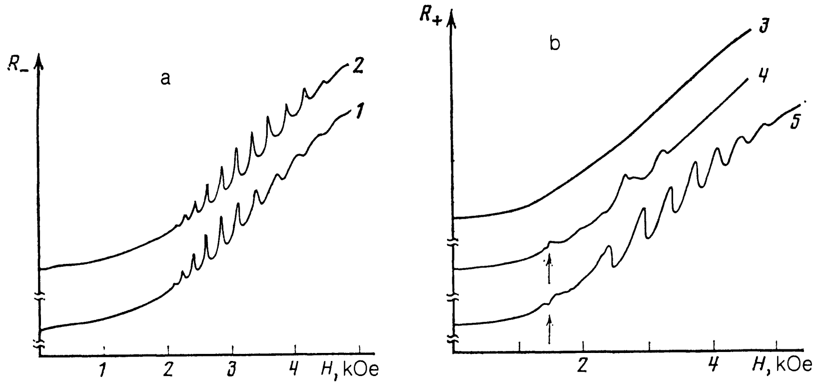

FIG. 1. Dependences of R_- (a) and R_+ (b) on the magnetic field H of frequency 32 kHz applied to a cadmium plate of thickness 1.71 mm at 1.4 K.

2. DISCUSSION

a. Electron paths. We shall assume that a monochromatic plane wave of frequency ω and with a wave vector **k** propagates along a static magnetic field **H** (z axis) directed parallel to the axis of an electron Fermi-surface lens. The electron paths can be found conveniently in a coordinate system moving along the z axis with a velocity ω/k. In this system there is no electric field of the wave and the magnetic field is equal, apart from terms of the order of $(\omega/kc)^2$, to its value in the laboratory coordinate system. In the case of the positive circular polarization the magnetic field of the wave can be written in the form

$$\mathcal{H}_x=\mathcal{H}\cos kz,\quad \mathcal{H}_y=\mathcal{H}\sin kz. \tag{1}$$

The electron paths are governed by the equation of motion

$$\dot{\mathbf{p}}=-\frac{e}{c}[\mathbf{v}(\mathbf{H}+\mathcal{H})], \tag{2}$$

where $\mathbf{v}=\partial\varepsilon/\partial\mathbf{p}$; $\varepsilon(\mathbf{p})$ is the energy of an electron with a momentum $\mathbf{p}$; $-e$ is the electron charge; c is the velocityof light; a dot above a symbol denotes differentiation with respect to time. The law of conservation of energy $\varepsilon(\mathbf{p})=\varepsilon=\text{const}$ follows from Eq. (2). In cylindrical coordinates v_z, $v_\perp$, and Φ such that $v_x=v_\perp\cos\Phi$ and $v_y=v_\perp\sin\Phi$, the equations of motion become

$$\dot{p}_z=-h\omega_c m v_\perp(\varepsilon,p_z)\sin(kz-\Phi), \tag{3}$$

$$\dot{z}=v_z, \tag{4}$$

$$\dot{p}_\perp=-\dot{p}_z v_z(\varepsilon,p_z)/v_\perp(\varepsilon,p_z), \tag{5}$$

$$\dot{\Phi}=\omega_c\left[1-h\frac{v_z}{v_\perp}\cos(kz-\Phi)\right], \tag{6}$$

where $h=\mathcal{H}/H$, $\omega_c=eH/mc$, and m is the cyclotron mass of an electron which we shall assume to be constant.

Since we are interested in small values of h, it follows that the second term of the right-hand side of Eq. (6) can be ignored and the solution of this equation becomes $\Phi=\omega_c t$.

In the linear case ($h\to 0$) the cyclotron absorption is due to electrons with a longitudinal momentum $p_z=p_{z0}$, which is described by the equation

$$kv_z(p_{z0})=\omega_c. \tag{7}$$

If $h\ll 1$, in the case of electrons close to a resonance we can assume that $p_z=p_{z0}+\delta p$. We shall ignore the quantity δp on the right-hand side of Eq. (3) and retain the term linear in δp on the right-hand side of Eq. (4). Consequently, Eqs. (3) and (4) can now be represented in the form[1]

$$\dot{s}=-\Omega\sin\alpha,\quad \dot{\alpha}=\Omega s, \tag{8}$$

where

$$s=\delta p/P,\quad \alpha=kz-\omega_c t, \tag{9}$$

$$\Omega(p_{z0})=\omega_c P(p_{z0})/m_0 v_z(p_{z0}), \tag{10}$$

$$P(p_{z0})=[hmv_\perp(p_{z0})m_0 v_z(p_{z0})]^{1/2},\quad m_0^{-1}=(\partial v_z/\partial p_z)_{p_z=p_{z0}}, \tag{11}$$

where α is the angle between the wave field and the transverse electron velocity $v_\perp$.

The system of Equations (8) is characterized by an integral $U=s^2/2-\cos\alpha=\text{const}$. Using this integral, we can transform the equation for α to

$$\dot{\alpha}=2\Omega\rho\left[1-\left(\frac{1}{\rho}\sin\frac{\alpha}{2}\right)^2\right]^{1/2}, \tag{12}$$

where

$$\rho=[(U+1)/2]^{1/2}\,\text{sign}\,s. \tag{13}$$

The solution of Eq. (12) can be expressed in terms of the Jacobi amplitude. If $|\rho|>1$, the angle α can assume any values and the motion of an electron is unbounded with respect to this angle. We shall call these transit electrons. If $|\rho|<1$ the motion of an electron is finite with respect to α and the turning points can be found from vanishing of the radicand in Eq. (12). Such electrons will be called trapped. This trapping of electrons, responsible for the cyclotron absorption, by the magnetic field of a wave is similar to the trapping of electrons by the field of longitudinal sound.[4]

Since the angle α of the trapped electrons ranges from $-\alpha_0$ to α_0, where $\alpha_0=2\arcsin|\rho|$, it follows that during one half of a period of the oscillations the wave field accelerates an electron along the z axis, whereas during the other half it slows the electron down. Consequently, the change in the electron energy in the laboratory system of coordinates is zero. Therefore, the trapping of resonance electrons by the wave field in the nonlinear regime is responsible for the disappearance of the collisionless cyclotron absorption.

b. Role of collisions. The situation is as described above if the oscillation frequency Ω exceeds the oscillation frequency ν. However, if Ω is much less than ν, then a collision "knocks out" an electron before it completes an oscillation. In this case there is no trapping of electrons by the wave field and the cyclotron absorption is the same as in the linear regime.

It is important to stress that the oscillation frequency Ω is different for different sections of the Fermi surface. In fact, according to Eqs. (10) and (11), the value of Ω^2 is proportional to the ratio $v_\perp(p_{z0})/v_z(p_{z0})$. Therefore, the frequency Ω is high if the section for resonance electrons ($p_z = p_{z0}$) is near the equator of the Fermi-surface lens where v_z is low and tends to zero in the vicinity of the limiting point of the lens characterized by $v_\perp \to 0$. We can therefore have a situation when for the majority of the values of p_{z0} the frequency Ω exceeds ν, but it is much less than ν for sections near the limiting point. Electrons belonging to such sections remain of the transit type and are responsible for the collisionless cyclotron absorption of the wave. The other electrons are trapped by the wave field and may not participate in the collisionless absorption process. In this case the nonlinearity "paralyzes" the majority of the electrons on the Fermi surface. The cyclotron absorption region is then limited not only on the long-wavelength side, as in the linear case, but also on the short-wavelength side and the wavelength interval where this absorption is observed is very narrow. Suppression of the nonlinear cyclotron absorption threshold results in a strong dispersion of the permittivity of the electron gas and creates a new doppleron. Since the Doppler shift of the frequency of this doppleron kv_z exceeds ω_c for the transit electrons, the difference $\omega_c - kv_z$ becomes negative so that the nonlocal Hall conductivity changes its sign. The result is that the field of a nonlinear doppleron rotates in the direction opposite to the rotation of electrons, i.e., this doppleron exists in the positive polarization case.

c. Model of the Fermi surface and the spectrum of electromagnetic waves. It follows from the above that a qualititative description of the wave properties of a metal in the positive polarization can be provided simply by considering the contribution of the transit electrons to the nonlocal conductivity $\sigma_+(k)$ and the contribution of the trapped electrons in the local approximation. The situation is simpler in the negative polarization case. For this polarization in the linear regime there is a doppleron due to a DSCR of electrons at the limiting point of the Fermi-surface lens. The wavelength of this doppleron is greater than the shift of the limiting-point electrons in one cyclotron period. In other words, the wave vector k is such that the cyclotron absorption condition of Eq. (7) is not satisfied by any electrons on the lens surface. It follows that the field of such a doppleron does not trap electrons and the whole electron lens contributes to the nonlocal conductivity $\sigma_-(k)$. Therefore, in the case of the negative polarization there are no such dramatic nonlinear changes in the wave properties as in the positive polarization case.

The limiting point of the lens of cadmium is nearly parabolic in the $\mathbf{H} \parallel C_6$ geometry,[3] i.e., the shape of the lens in the vicinity of its vertex is close to the shape of a paraboloid. (In the case of a paraboloid Fermi surface the longitudinal velocities v_z of all the electrons and, consequently, their displacements in one cyclotron period are the same.) The doppleron field equalizes even more strongly the longitudinal velocities of electrons. In fact, it follows from the equations of motion (3)–(5) that the wave field reduces the longitudinal velocity of electrons at the limiting point, but increases the transverse velocity. Conversely, the longitudinal velocity of those electrons which are near the end of the truncated vertex of the lens increases, whereas the transverse velocity decreases. In other words, the doppleron field alters the transit electron paths in such a way that their displacements become almost identical. Therefore, we can demonstrate the wave properties of cadmium in the nonlinear regime by considering a simple model in which the Fermi surface of electrons is a parabolic lens. The hole Fermi surface is assumed to be a corrugated cylinder. The axes of the cylinder and lens coincide with the direction of the static magnetic field $\mathbf{H}$ and with the normal to the plane of the cadmium plate. We shall assume that the maximum displacement of holes is one quarter the displacement of electrons. Then, ignoring carrier collisions, we find that the transverse nonlocal conductivity of this metal is

$$\sigma_\pm = \sigma_{xx} \pm i\sigma_{yx} = \pm \frac{nec}{H}\left[\frac{1}{1-q^2} - \frac{1}{(1-(q/4)^2)^{1/2}}\right], \quad (14)$$

where

$$q = kcp_0/eH, \quad (15)$$

p_0 is a constant with the dimensions of the momentum (this constant represents the displacement of electrons), and n is the carrier density. The first term in the square brackets of Eq. (14) describes the nonlocal Hall conductivity of electrons and the second the corresponding conductivity of holes.

The dispersion equation for an electromagnetic wave in a metal characterized by $k^2c^2 = 4\pi i\omega\sigma_\pm$ can be written conveniently in the form

$$1/\xi = \Phi_\pm(q), \quad (16)$$

$$\Phi_\pm(q) = \mp\frac{1}{q^2}\left[\frac{1}{1-q^2} - \left(1 - \frac{q^2}{16}\right)^{-1/2}\right], \quad (17)$$

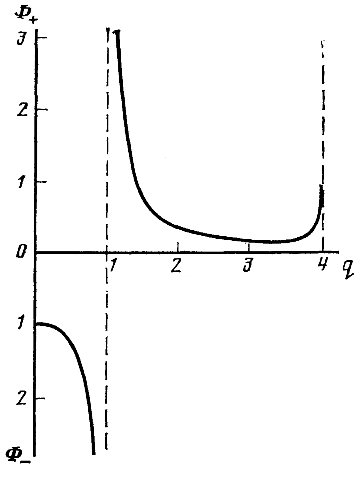

FIG. 2. Dependences of the functions $\Phi_\pm$ on H.

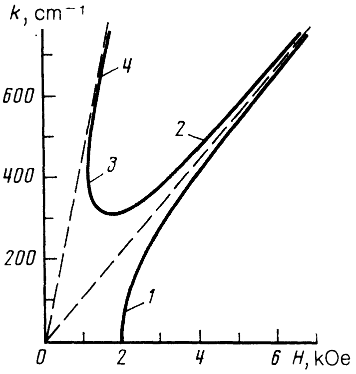

FIG. 3. Spectral curves for the circular polarizations calculated at the exciting field frequency $\omega/2\pi = 32$ kHz.

where

$$\xi = 4\pi\omega n p_0^2 c/eH^3. \tag{18}$$

We shall assume that the parameter p_0 is such that the limiting value of the period of an electron doppleron is equal to the half-sum of the limiting values of the periods of the oscillations exhibited by curves 2 and 5 in Fig. 1. This condition corresponds to $p_0 = 1.42\hbar$ Å^{-1}. We shall assume that the carrier density n is such that the threshold of an electron doppleron existing in the negative polarization is equal to the observed threshold. This is true for $n = 2\times 10^{21}$ cm^{-3}.

Since the quantity ξ^{-1} is proportional to H^3, it follows that we can solve Eq. (16) conveniently by plotting graphs of the functions $\Phi_\pm$. These graphs are shown in Fig. 2. The solutions of the dispersion equation are obtained at the points where curves 1 and 2 are intersected by horizontal lines corresponding to different values of H. The dependences of k on H for the two polarizations are shown in Fig. 3. A branch 1 describes the electron doppleron spectrum in the negative circular polarization case, whereas branches 2–4 are the spectra of an electron doppleron, a hole helicon, and a hole doppleron in the positive polarization case. We shall not be interested in branches 3 and 4. A characteristic property of spectral curves 1 and 2 is that they extend to high magnetic fields, whereas the spectrum of an electron doppleron obtained in the linear regime is limited on the high-field side. Calculations indicate that the maximum of the amplitude of oscillations of the impedance corresponding to dopplerons 1 and 2 is shifted considerably toward higher fields compared with the maximum of the amplitude of oscillations of a linear doppleron.

The nature of changes in the oscillation period due to dopplerons 1 and 2 is the most convincing proof of displacement of all the transit electrons. Curves 1 and 2 in Fig. 4 represent the calculated dependences of these periods on the field H, deduced from the spectra 1 and 2 in Fig. 3. The points in Fig. 4 represent the experimental results corresponding to curves 2 and 5 in Fig. 1. We can see that these points are strikingly close to the calculated curves.

Finally, we shall comment on the reduction in the limiting period of oscillations exhibited by curve 2 in Fig. 1, compared with the limiting period of curve 1. This reduction can be explained by the fact that the limiting-point electrons exhibiting the largest displacement are unavoidably characterized by a smaller displacement in the nonlinear regime.

Figure 1 shows the $R_+(H)$ dependences recorded when the exciting field frequency is 32 kHz. This frequency is selected in such a way that a doppleron appears most clearly in the positive polarization case (curve 5), i.e., when the number of regular oscillations is as large as possible. When the frequency is increased, the range of existence of an electron doppleron in the linear regime (corresponding to the negative polarization) shifts toward higher fields. This reduces the nonlinear effect so that a new doppleron is observed as clearly. When the frequency is reduced, the range of existence of this doppleron shifts toward weaker fields and becomes narrower. Narrowing of this range reduces the number of oscillations in the experimental curves. However, a shift toward weaker fields is accompanied by enhancement of the influence of the nonlinearity on the oscillation profile. Figure 5 shows the dependences $R_-(H)$ obtained for different amplitudes of the exciting field at a frequency 8 kHz. These curves manifest even more strongly the nonlinear changes which we discussed when comparing curves 1 and 2 in Fig. 1. Moreover, the oscillations become sharper and asymmetric. The slopes of the maxima exhibit abrupt jumps indicating resonance "flipping." In the region of such a jump the surface resistance becomes a multivalued function of H and behaves differently when the magnetic field is reduced

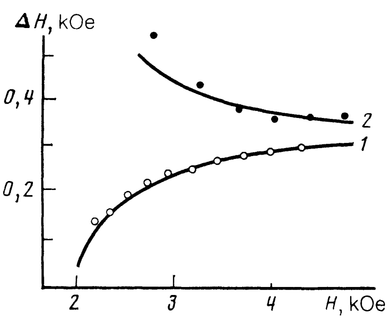

FIG. 4. Dependences of the oscillation periods for two circular polarizations on the magnetic field. The curves are calculated and the points are the experimental values ($\omega/2\pi = 32$ kHz).

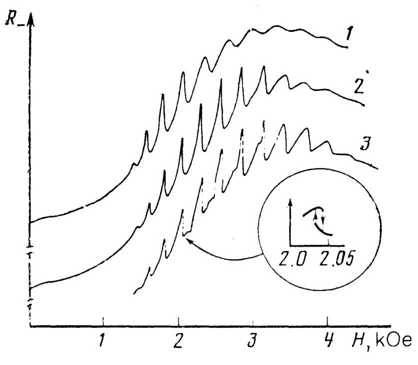

FIG. 5. Dependences $R_-(H)$ recorded for a cadmium plate of thickness 1.71 mm at a frequency of 8 kHz at 1.4 K. Curves 1, 2, and 3 were obtained using an exciting magnetic field of intensity 4, 126, and 200 Oe, respectively. The dashed parts of curve 3 represent jumps of the surface resistance. The inset shows a fragment of curve 3 on a scale extended along the H axis.

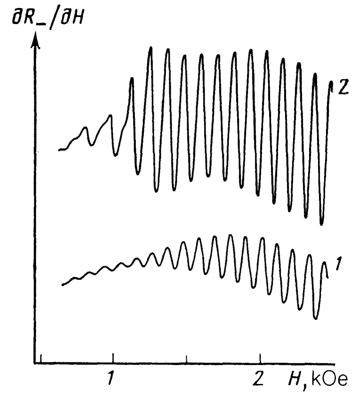

FIG. 6. Dependences of the derivative of the surface resistance $dR\ \ /dH$ obtained for a tungsten plate of thickness $d = 2.0$ mm at a frequency $\omega/2\pi = 4$ kHz at $T = 1.6$ K. Curves 1 and 2 were obtained using an exciting field of amplitude 20 and 126 Oe, respectively.

or increased. This hysteresis is demonstrated in the inset in Fig. 5 which shows, on a scale magnified by a factor of 10 in respect of H, a fragment of a record obtained for one of the extrema of curve 3.

The bleaching effect in a metal was observed in the nonlinear regime not only for cadmium but also for tungsten. Figure 6 shows the field dependence of the derivative of the surface resistance dR_{-}/dH of a tungsten plate ($\rho_{300\,K}/\rho_{4.2\,K} \approx 50\,000$) obtained in the negative circular polarization using a magnetic field directed along the normal to the surface: $\mathbf{H}\|[001]\|\mathbf{n}$. In the linear regime (curve 1) there are Gantmakher-Kaner oscillations of amplitude which increases smoothly on increase in H. In the nonlinear regime (curve 2) these oscillations are much greater than those of curve 1. In stronger magnetic fields ($H \gtrsim 4$ kOe) the amplitudes of the oscillations are practically the same in the linear and nonlinear regimes. In weak magnetic fields we can see from curve 3 that there are clear oscillations characterized by a larger period. We can assume that, as in the case of cadmiun, these oscillations are associated with the excitation of a new doppleron wave in a polarization opposite to that in which a doppleron propagates in the linear regime.

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