Effect of dissipation on the characteristics of small-area tunnel junctions: Application of the polaron model

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A comparison of the expressions for the effective action reveals an analogy between the polaron model and the quantum mechanical model for tunnel junctions between normal metals. The current-voltage characteristic and impedance of a low-capacitance tunnel junction can be calculated by the polaron model in the Feynman approximation for arbitrary values of the tunnel conductance (dissipation) and temperature.

INTRODUCTION

Interest has increased recently in the properties of tunnel junctions with extremely small capacitances C at low temperatures $k_B T \lesssim E_Q \equiv e^2/2C$ (Refs. 1–4). In these junctions the tunneling of a single electron can appreciably alter the voltage across the junction capacitance, thereby significantly changing the tunneling conditions for the other electrons. By correlating the tunneling events of the individual electrons, this mechanism gives rise to coherent one-electron effects such as Coulomb blockade and single-electron oscillations, among others.[1,2] These were analyzed in Refs. 1, 2 by using perturbation theory with the tunnel dissipation

$$\alpha_T = \pi^{-2} G_T R_Q \tag{1}$$

as the small parameter (here G_T is the tunnel conductance of the junction and $R_Q = 2\pi\hbar/4e^2 \approx 6.5$ kΩ is the quantum unit of resistance). However, this procedure yielded only the estimate $\alpha_T \gtrsim 1$ for suppression of correlated tunneling and onset of Coulomb blockade.

It was shown subsequently[3–7] that the equilibrium characteristics and dynamic properties of tunnel junctions are sensitive to increases in α_T, and also to the linear dissipation parameter $\alpha_s = \pi^{-2} G_s R_Q$ due, for example, to shunting of the junction by the ohmic conductance G_s. The current voltage (I-V) characteristic for a junction at low currents was calculated in Ref. 4 for arbitrary α_s and small $\alpha_T \ll 1$, and the results were used to estimate the threshold for the onset of Coulomb blockade in this case also. It was shown in Refs. 5 and 6 that in the opposite limit $\alpha_T \gg \max(\alpha_s, 1)$, Coulomb blockade occurs only for exponentially small currents $I \sim \exp(-2\pi^2 \alpha_T)$, i.e., it is almost totally suppressed by dissipation. The behavior of the I-V characteristic at high currents is determined by incoherent tunneling, whose probability was calculated in Ref. 5. Finally, the case $\alpha_T \ll \alpha_s$ of strong junction shunting was considered in Ref. 7, where the conductance and I-V characteristic were calculated.

Unfortunately, only the static properties (conductance, I-V characteristic) were calculated in Refs. 3–7, where the Feynman path integral formalism was employed. It is also important to analyze the dynamics of one-electron processes in low-capacitance tunnel junctions with nonzero dissipation. The present paper is concerned with deriving and analyzing an approximate equation for the dynamics.

2. THE POLARON HAMILTONIAN AND THE EQUATION OF MOTION

A tunnel junction with $\alpha_s = 0$ can be described using the effective action[8]

$$S[\varphi] = \int_0^\beta d\tau \left(\frac{C}{8e^2} \dot{\varphi}^2 - \frac{I}{2e} \varphi \right) - \int_0^\beta d\tau \int_0^\beta d\tau' \alpha(\tau - \tau') \cos\left[\frac{1}{2} (\varphi(\tau) - \varphi(\tau')) \right], \tag{2}$$

$$\alpha(\tau) = \alpha_T \left[\frac{\pi/\beta}{\sin(\pi t/\beta)} \right]^2, \tag{3}$$

where $\hbar = 1$, $\beta = 1/k_B T$. The action for the polaron problem is given by an expression with a similar structure.[9] The two coincide exactly if one formally considers a one-dimensional polaron, in which an "electron" in an external electric field E interacts with "phonon modes" having identical wave vectors $\pm k$ but different frequencies ω_n. The Hamiltonian for such a system is

$$H = \frac{p^2}{2m} - E(t) x + \sum_n G_n \sum_{k=\pm 1} (a^+_{n,-k} + a_{n,k}) e^{ikx} + \sum_n \omega_n \sum_{k=\pm 1} a^+_{n,k} a_{n,k}, \tag{4}$$

where the electron momentum and coordinate p, x correspond to the charge Q on the junction capacitance and its canonical conjugate φ, respectively:

$$p = Q/e, \quad x = \varphi/2 \tag{5}$$

(we also have $p = V/V_0$, where V is the junction voltage and $V_0 = e/C$). The magnitude of the "external electric field" E is proportional to the current I across the junction, and the electron mass is proportional to the junction capacitance:

$$E = I/e, \quad m = C/e^2. \tag{6}$$

The coupling coefficients G_n determine the magnitude of the dissipation and the form of the function $\alpha(\tau)$ ($\Sigma_n \ldots \to \int d\omega \rho(\omega) \ldots$):

$$\alpha(\tau) = \int_0^\infty d\omega\, \rho(\omega) G^2(\omega) \left(e^{-\omega\tau} + e^{\omega(\tau-\beta)}\right) / \left(1 - e^{-\omega\beta}\right), \tag{7}$$

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which for α given by (3) corresponds to

$$\rho(\omega) G^2(\omega) = \alpha_T \omega f_c(\omega). \tag{8}$$

Here the function $f_c(\omega)$ is obtained by cutting off the phonon spectrum at high frequencies. For definiteness we will assume that $f_c(\omega) = \omega_c^2/(\omega^2 + \omega_c^2)$, where the cutoff frequency ω_c is assumed to be much higher than all the characteristic frequencies of the problem.

The above analogy has a simple physical interpretation—a change of $\Delta p = k = \pm 1$ in the momentum of an electron interacting with a phonon corresponds to the tunneling of a single electron across the junction: $\Delta Q = \pm e$, while the change in the number of phonons corresponds to the response of the system of conduction electrons to the discrete tunneling. All the results in Ref. 1 can be derived from the Hamiltonian (4) by perturbation theory in the limit $\alpha_T \ll 1$.

The polaron mobility was calculated for arbitrary α_T and T by Thornber and Feynman,[10] who employed path-integral techniques and approximated the Green functional by a Gaussian. A simpler operator technique was later employed for the same purpose by Bogolyubov,[11] using an approximate linear Hamiltonian (both methods lead to the same result). The basic idea behind the operator method[1] is to calculate the expectations

$$\langle \exp(ikx(t)) \exp(-ikx(t')) \rangle \tag{9}$$

of the Heisenberg operators in the equation of motion for the case of a model Hamiltonian H_L, obtained from the exact Hamiltonian (4) by replacing the term containing the exponential by the linear expression

$$\sum_n G_n \sum_{k=\pm 1} ikx(\overset{+}{a}_{n,-k} + a_{n,k}) + c^2 x^2, \quad c^2 = 2\sum_n \frac{G_n^2}{\omega_n} \tag{10}$$

(the term c^2x^2 must be included to preserve translational invariance[11]). It is important to note that in spite of this simplification, the method does treat the strong exponential dependence of the expectations (9) on x. In addition, since the electron becomes spatially localized as the dissipation α_T increases, it is perfectly legitimate to approximate the exponential by (10) for $\alpha_T \gg 1$.

For $\alpha_T \ll 1$, however, the approximation gives results that differ somewhat from the predictions of perturbation theory, which are exact in the limit $\alpha_T \to 0$ (Refs. 1, 2). This is because the calculation of the expectations (9) using the linear Hamiltonian H_L neglects the self-consistent change in the momentum distribution function $W(p)$, which is assumed to be Gaussian. Nevertheless, this assumption is valid at high temperatures $k_B T \gg E_Q$ or for strong shunting $\alpha_T \ll \alpha_s$.

Applying Eq. (11) to the present case, we have

$$\frac{dp}{dt} - E(t) = -\int_0^t d\tau \sin\left(\int_\tau^t \frac{p(t')}{m} dt'\right) \mathscr{F}(t-\tau), \tag{11}$$

$$\mathscr{F}(t) = 4\,\mathrm{Im}[G(t)\exp(if_1(t) - f_2(t))], \tag{12}$$

$$f_1(t) = \frac{1}{4\pi\alpha_T}(1 - e^{-\gamma t}), \tag{13a}$$

$$f_2(t) = \frac{\alpha_T}{m^2}\int_{-\infty}^{\infty} d\omega \frac{1}{\omega(\omega^2+\gamma^2)} \mathrm{cth}\left(\frac{\beta\omega}{2}\right)(1 - \cos\omega t), \tag{13b}$$

$$\gamma = 2\pi\alpha_T/m, \tag{13c}$$

$$G(t) = \alpha_T \int_0^\infty d\omega\, \omega \frac{\omega_c^2}{\omega^2 + \omega_c^2}\left[\mathrm{cth}\left(\frac{\beta\omega}{2}\right)\cos\omega t + i\sin\omega t\right]. \tag{14}$$

We will analyze the equation of motion (11) for several special cases.

3. THE VOLTAGE-CURRENT CHARACTERISTIC AND THE IMPEDANCE

For a constant external field $E(t) = \mathrm{const}$, Eq. (11) takes the form

$$E(p) = \mathrm{Im}\left(F\left(\omega = \frac{p}{m}\right)\right)$$
$$= 2\pi\alpha_T \frac{p}{m} - 4\alpha_T \int_0^\infty dt \sin\left(\frac{p}{m}t\right)\left[\frac{\pi/\beta}{\mathrm{sh}(\pi t/\beta)}\right]^2$$
$$\times \sin\{f_1(t)\}\exp\{-f_2(t)\}, \tag{15}$$

where $F(\omega)$ denotes the Fourier transform of the function $\mathscr{F}(t)$:

$$F(\omega) = \int_0^\infty dt\, e^{i\omega t}\mathscr{F}(t). \tag{16}$$

Relation (15) gives the equation for the I-V characteristic of the tunnel junction (Fig. 1). The first term on the right in (15) gives the ohmic component of the conductance, the second gives the nonlinear component. At zero temperature the I-V characteristics for large and small α_T are given by

$$\varepsilon = p\left\{1 - \frac{1}{2\pi^2\alpha_T}\left[\frac{1}{2}\ln\left(1 + \left(\frac{2\pi\alpha_T}{p}\right)^2\right) + \frac{2\pi\alpha_T}{p}\mathrm{arctg}\left(\frac{p}{2\pi\alpha_T}\right)\right]\right\}, \tag{17}$$

$$2\pi\alpha_T\exp(-2\pi^2\alpha_T) \ll p, \quad 2\pi^2\alpha_T \gg 1;$$

$$\varepsilon = p - \sum_{k=\pm 1}\frac{k}{2}\left\{\left|p + \frac{k}{2}\right|\Phi(q_k) + 2\left(\frac{\Delta}{\pi}\right)^{1/2}\exp(-q_k^2)\right\}, \tag{18}$$

$$q_k = \left|p + \frac{k}{2}\right|(2\Delta^{1/2})^{-1}, \quad \Delta \approx -\frac{\alpha_T}{2}\ln(6.23\alpha_T), \quad 2\pi^2\alpha_T \ll 1,$$

where Φ is the error integral, $\varepsilon = E/\gamma = I/I_0$, $I_0 = G_T e/C$.

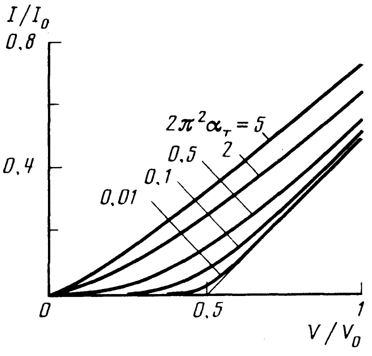

FIG. 1.

The last expression is particularly simple in the limit $\alpha_T \to 0$:

$$\varepsilon=0, \quad 0<p<1/2, \tag{19a}$$

$$\varepsilon=p-1/2, \quad p>1/2. \tag{19b}$$

For the reasons discussed above, for small currents $\varepsilon \lesssim 1$ expression (19) gives a rather crude approximation to the true I-V characteristic calculated in Ref. 1 for $\alpha_T \to 0$; however, the two results coincide when $\varepsilon \gg 1$. The I-V characteristic also obeys (19b) in the limit $\alpha_T \gg 1$. The treatment thus again predicts a shift in the asymptotic portion of the I-V characteristic by $\Delta V = e/2C$, from the ohmic line $I = G_T V$ toward higher voltages. As α_T increases the Coulomb blockade region (19a) shrinks (Fig. 1), and in the limit $\alpha_T \gg 1$ we find $p_{CB} \propto \exp(-2\pi^2\alpha_T)$, in agreement with the result in Ref. 5.

The nonlinear portions of the I-V characteristic become less distinct with heating. For small α_T $(2\pi^2\alpha_T \ll 1)$ the smearing-out becomes considerable at temperatures $k_B T \gtrsim E_Q$. For large α_T $(2\pi^2\alpha_T \gg 1)$ the Coulomb blockade region becomes fuzzy at much lower temperatures $k_B T \gtrsim E_Q \exp(-2\pi^2\alpha_T)$. Due to the smallness of the Coulomb blockade region and its instability relative to temperature fluctuations, it will probably not be possible to observe or exploit the stationary Coulomb blockade effect in junctions with large tunnel conductance $G_T R_Q \gtrsim 1$.

If a weak harmonic signal $E(t) = E_\omega e^{-i\omega t}$ is applied to the junction, with $E_\omega \to 0$, the complex conductance $G(\omega)$ of the junction can be found from Eq. (11):

$$\frac{G(\omega)}{G_T} = \frac{1}{2\pi i\alpha_T}\left[m\omega + \frac{F(\omega)-F(0)}{\omega}\right]. \tag{20}$$

In particular, we find

$$\frac{\mathrm{Re}\, G(\omega)}{G_T} = \frac{\mathrm{Im}\, F(\omega)}{2\pi\alpha_T\omega} \tag{21}$$

for the real part of the conductance, where $\mathrm{Im}\, F(\omega)$ is given by Eq. (15).

4. INFLUENCE OF THE OHMIC CONDUCTANCE OF THE JUNCTION

The ohmic conductance G_s of the junction can be found by adding a linear term H_s of the form (10) to the Hamiltonian (4); the coefficients G_n satisfy (8) with α_T replaced by α_s. The shunting gives rise to an additional term $-2\pi\alpha_s p(t)/m$ on the left in the equation of motion (11), and α_T must be replaced by α_Σ in (13), where $\alpha_\Sigma = \alpha_T + \alpha_s$. We note that adding the linear term H_s to the Hamiltonian can only improve our approximation, since the expectation (9) is now evaluated using the more realistic model Hamiltonian $H_L + H_s$. For $\alpha_T \ll \alpha_s$ the approximation is exact to first order in α_T/α_s. In particular, to first order in α_T/α_s expression (15) for the I-V characteristic (with α_T replaced by α_Σ in the first term, and p/m in the integral set equal to $E/2\pi\alpha_\Sigma$) agrees exactly with the result in Ref. 7 found by path integration. Moreover, in the limit $\alpha_T \ll \alpha_s \ll 1$ expression (19) approaches the I-V characteristic calculated numerically in Ref. 12, even for $\alpha_T/\alpha_s \lesssim 0.3$.

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