Resistive transition and critical fields of superconducting ceramics

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The anisotropy of the critical fields of individual grains determines the percolative nature of the superconducting transition of ceramics in a magnetic field. This result means that the broadening of the transition can be described by the field of a resistive transition, and the critical fields of a single crystal can be reconstructed. Analysis of the experimental data of N. Kabayashi *et al.* [Jpn. J. Appl. Phys. **26**, L358 (1987)] for the compound $La_{1.85}Sr_{0.15}CuO_{4-y}$ yields the values $dH_{c2}^{\perp}/dT \approx 0.25$ T/K and $dH_{c2}^{\parallel}/dT \gtrsim 3$ T/K. A study is made of the effect of composition inhomogeneities, which give rise to a positive-curvature region on the plot of $H_{c2}(T)$ for ceramics. The lower critical field H_{c1} is also determined by anisotropy effects. In the existing large-grain ceramics, H_{c1} is shown to correspond to the minimal critical field of a single crystal, $H_{c1}^{\parallel}$. The effect of the transport current on the shift of the resistive transition is discussed.

INTRODUCTION

Single crystals of superconducting cuprites which demonstrate a pronounced anisotropy in electronic properties have now been synthesized. Resistive measurements in a magnetic field yield estimates of the critical-field anisotropy [$H_{c2}^{\parallel}/H_{c2}^{\perp} = 5$–$13$ (Refs. 2 and 3) and $H_{c1}^{\perp}/H_{c1}^{\parallel} = 10$–$20$ (Ref. 4)] which stems from the layered structure of the lattice. It is nevertheless difficult to measure the anisotropy more accurately because of the poor quality of the single crystals and the large width of the resistive transitions. On the other hand, there are ceramic samples with narrow transitions (≈ 2–3 K) in the absence of a magnetic field. When a field is applied, the resistive transition in these samples becomes broader.[1,5,6] The customary method for determining $H_{c2}(T)$ is to measure the shift of the transition temperature caused by a magnetic field at a fixed resistivity level. The curves of $H_{c2}(T)$ plotted in this fashion yield dH_{c2}/dT values with a large scatter and have a positive curvature.

Our purpose in the present study was to find the relationship between the orientational disorder of anisotropy axes which is characteristic of ceramics and their observable properties. Some of the results have been published in summary form.[7]

In Sec. 1 we study the shape of the resistive transition of ceramics in a magnetic field. We attribute the experimentally observed broadening of the transition to a two-dimensional anisotropy of grains. As for polycrystalline magnetic superconductors,[8] this transition is percolative for $H \neq 0$. This result leads to the suggestion of a correct method for determining the dependence $H_{c2}(T)$ for ceramics and for finding the critical fields of a single crystal, $H_{c2}^{\parallel}$, $H_{c2}^{\perp}$.

The appearance of a special direction, associated with the magnetic field, gives rise to macroscopic anisotropy of the resistivity of a polycrystalline sample near the resistive transition. It turns out that the resistivity tensor $\rho_{ij}(H,T)$ actually depends on only the one variable $(T_c - T)/H$.

In Sec. 2 we study the effect of variations in the critical temperature on the percolation transition in a magnetic field. The results show that these variations give rise to a region of positive curvature on the curve of $H_{c2}(T)$. This region is amenable to a quantitative description. We point out that it is possible to determine the transition-temperature distribution of grains from the resistivity curves.

The results presented above pertain to the case of infinitely low measurement currents. A current of finite magnitude flowing through a sample, as in a magnetic field, reduces the concentration of the superconducting phase and shifts the curve of the resistive transition. This effect is studied in Sec. 3.

Section 4 deals with the penetration of vortices into the interior of a polycrystalline sample. We link the critical field H_{c1} found experimentally[9] with the entry of vortices, certain parts of which are oriented parallel to the anisotropy planes in each grain. This mechanism for forming the field H_{c1} operates if the typical grain size d is far greater than the penetration depth λ. We also study the opposite limiting case, $d \ll \lambda$.

In Sec. 5 we write an analytic expression for the magnetization $\mathbf{M}(\mathbf{H})$ in strong magnetic fields.

1. CURVE OF THE RESISTIVE TRANSITION IN A MAGNETIC FIELD

We consider a uniform polycrystalline sample with a sharp resistive transition at $T = T_c$. In an external magnetic field $\mathbf{H}$, this transition broadens, since the anisotropy of the field H_{c2} causes the transition temperature of each grain to be determined by its orientation with respect to the magnetic field.[7,10] As the temperature is lowered, the grains which are the first to go superconducting are those which are oriented with their layers parallel to $\mathbf{H}$, so the shift of the point at which the transition begins, $T_c - T_{on}(H)$ should be determined by the critical magnetic field parallel to the layers:

$$T_c - T_{on}(H) = H(dH_{c2}^{\parallel}/dT)^{-1}. \tag{1.1}$$

As the temperature drops further, those grains whose upper critical fields become greater than H go superconducting. This situation corresponds to an inclination of the anisotropy axes at an angle $\theta > T_c$, where the critical angle θ_c is defined by

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$$H_{c2}(T,\ \theta_c)=H. \tag{1.2}$$

It follows from simple geometric considerations that in the case of a uniform distribution of anisotropy axes the concentration of the superconducting phase in a magnetic field is

$$c_h(H,\ T)=\cos[\theta_c(H,\ T)].$$

Since the cuprites exhibit a clearly expressed type II superconductivity, the distortion of the magnetic field lines can be ignored near the resistive transition. The specific form of the function $c_h(H,T)$ is determined by the dependence $H_{c2}(T,\theta)$. When the Ginzburg-Landau equation holds, we have

$$H_{c2}^{-2}(\theta)=(H_{c2}^{\perp})^{-2}\cos^2\theta+(H_{c2}^{\parallel})^{-2}\sin^2\theta \tag{1.3}$$

and thus

$$c_h(H,T)=\left[\frac{(H_{c2}^{\parallel}/H)^2-1}{(H_{c2}^{\parallel}/H_{c2}^{\perp})^2-1}\right]^{1/2}, \tag{1.4}$$

where

$$H_{c2}^{\parallel}=(T_c-T)dH_{c2}^{\parallel}/dT,\quad H_{c2}^{\perp}=(T_c-T)dH_{c2}^{\perp}/dT$$

are the critical fields respectively parallel and perpendicular to the layers ($H_{c2}^{\parallel}\gg H_{c2}^{\perp}$). It can be seen from expression (1.4) that at the beginning of the transition (as $c_h\to 0$), c_h goes as the square root of $T_{on}(H)-T$. The length of this region is small if the anisotropy is pronounced, and at temperatures satisfying the inequality $T_c-T\gg T_c-T_{on}(H)$, the dependence $c_h(T)$ is approximately linear:

$$c_h(H,T)=\frac{T_c-T}{H}\frac{dH_{c2}^{\perp}}{dT}. \tag{1.5}$$

In the transition region, the resistivity is governed exclusively by the concentration of the superconducting phase, and, as we can see from expression (1.4), it depends on the temperature and magnetic field through the combination $(T_c-T)/H$. Figure 1 shows resistivity curves in the coordinates $[\rho/\rho_n,(T_c-T)/H]$, plotted from the data of Ref. 1. The fact that the points corresponding to a large set of values of (H,T) conform to a common curve in Fig. 1 is evidence that the model used here can be applied to actual ceramics. The probable reason for the deviation from this common curve at the beginning of the transition is the existence of variations in the composition, whose role will be discussed below.

The coherence length in the superconducting cuprites is a few tens of angstroms, far shorter than the typical size of the grains. For this reason we can ignore the Josephson tunneling between grains which are not in contact. Consequently, the resistivity vanishes only after the formation of an infinite cluster of the superconducting phase. This event happens when the concentration reaches the percolation value ($c_p\approx 0.17$; Ref. 11), which corresponds to a temperature

$$T_c(H)\approx T_c-H[c_p^2(dH_{c2}^{\perp}/dT)^{-2}+(dH_{c2}^{\parallel}/dT)^{-2}]^{1/2}. \tag{1.6}$$

In the intermediate region, the resistivity becomes anisotropic (the components of resistivity parallel and perpendicular to the magnetic field are different). In this case, anisotropy can arise for two reasons. First, the grains which are the first to go superconducting are those which are oriented with their layers parallel to the magnetic field. The result is

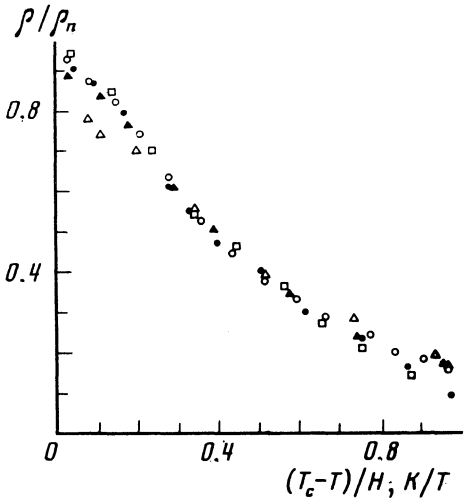

FIG. 1. Resistivity curves for $La_{1.85}Sr_{0.15}CuO_{4-y}$ (Ref. 1), plotted in reduced coordinates for several values of the magnetic field: △—5.0 T; ▲—9.7 T; ○—12.9 T; ●—18.0 T; □—20.3 T.

to disrupt the uniformity of the angular distribution of anisotropy axes. (Since the conductivity of each grain reaches a maximum parallel to the layers, this nonuniformity leads to a macroscopic anisotropy of the resistivity.) Second, layered grains generally have a flattened shape. For this reason, the grains which have gone superconducting are elongated along the magnetic field, and their contribution to the conductivity of the sample is anisotropic.

At low concentrations, the transverse ρ_t and longitudinal ρ_l components of the resistivity depend linearly on c_h:

$$\rho_{t,l}=\rho_n(1-\alpha_{t,l}c_h). \tag{1.7}$$

The constants here satisfy $\alpha_{t,l}\gtrsim 1$. They are estimated in the Appendix. When there is no anisotropy in the shape of the grains, the inequality $\alpha_l<\alpha_t$ holds. If, on the other hand, the grains are highly elongated parallel to the layers there is an anisotropy of the opposite sign: $\alpha_l>\alpha_t$. It follows from (1.5) and (1.7) that under the condition $H_{c2}^{\parallel}\gg H_{c2}^{\perp}$ the resistivity depends linearly on the temperature over a large part of the transition; this conclusion is confirmed by experimental data.[1,5,6] Near the percolation point, the anisotropy in the resistivity is small, and the resistivity is a power function of the concentration:

$$\rho\propto\rho_n(c_p-c_h)^s, \tag{1.8}$$

where $s=0.8$–0.9 (Refs. 12 and 13).

The curves of $H_{c2}(T)$ plotted from the resistivity curves are plots of H as a function of T at a fixed concentration of the superconducting phase, c. From (1.4) we find

$$H_{c2}(T)=[(H_{c2}^{\parallel})^{-2}+c^2(H_{c2}^{\perp})^{-2}]^{-1/2}. \tag{1.9}$$

It can be seen from this expression that the derivative dH_{c2}/dT decreases as the relative level at which the transition temperature is determined drops (i.e., as c increases). In particular, if the transition temperature is assumed to be the point at which the resistivity vanishes, then in the case of a pronounced anisotropy ($dH_{c2}^{\parallel}/dT\gg 6dH_{c2}^{\perp}/dT$) we find

$$(dH_{c2}/dT)_{\rho=0}=c_p^{-1}dH_{c2}^{\perp}/dT\approx 6dH_{c2}^{\perp}/dT. \tag{1.10}$$

If a sample contains pores or nonsuperconducting in-

clusions, and the superconducting phase occupies a fraction c_s of the total volume, the relation (1.10) becomes[10]

$$(dH_{c2}/dT)_{\rho=0}=(c_s/c_p)\,dH_{c2}^{\perp}/dT.$$

For porous samples, c_s can be estimated as the ratio of the actual density of the sample to the x-ray density. The slope dH_{c2}/dT thus increases with the density of the sample.

2. EFFECT OF COMPOSITION VARIATIONS

Real ceramic samples have a resistive transition which is 2–5 K wide. We attribute this width to variations in their chemical composition. We cannot rule out the possibility that the broadening of the transition in the highest-quality samples is a consequence of Aslamazov-Larkin fluctuations, but we will restrict the discussion here to the case in which the transition width is much greater than the fluctuation width. Our second assumption is that we can assume that the length scales of the variations are far larger than the correlation length. In this case the composition variations lead to a random variation of the critical temperature in space, $T_c(\mathbf{R})$. The temperature dependence of the concentration of the superconducting phase, $c_i(T)$, is determined by the critical-temperature distribution of the grains, $f(T_c)$:

$$c_i(T)=\int_T^{T_{on}} f(T_c)\,dT_c. \tag{2.1}$$

The distribution function has a width ΔT which exceeds the width of the resistive transition only by a numerical factor. We will consider here only the case in which there is no nonsuperconducting phase in the sample at $T=0$: $c_i(0)=1$. Using relations (1.4) and (2.1), we can find the concentration of the superconducting phase as a function of the temperature and the magnetic field:

$$c_i(H,T)=\int_T^{T_{on}} f(T_c)\,c_h(H,T-T_c)\,dT_c. \tag{2.2}$$

It follows from expression (2.2) that in the limit of strong magnetic fields the variations are unimportant, and we can use the relations derived in the preceding section of this paper [we need to replace T_c in those expressions by the average transition temperature $\langle T_c\rangle=\int f(T_c)\,T_c\,dT_c$)]. The overall shape of the resistivity curve and the point of the percolation transition are determined by the anisotropy of the field $H_{c2}(\theta)$ (not by the scatter in T_c) under the inequality $H\gg\Delta T dH_{c2}^{\perp}/dT$. The small region of a square-root temperature dependence of the resistivity, which we mentioned back in Sec. 1, appears in far stronger fields, $H\gg\Delta T dH_{c2}^{\parallel}/dT$.

In the opposite limit, of weak magnetic fields, $H\ll\Delta T dH_{c2}^{\perp}/dT$, we find from (2.2) a correction linear in H to the concentration of the superconducting phase:

$$c_i(H,T)-c_i(T)=-f(T)H\langle(dH_{c2}(\theta)/dT)^{-1}\rangle_\theta, \tag{2.3}$$

where the angle brackets mean an average over angles. In the case of a pronounced anisotropy we would have

$$\langle(dH_{c2}/dT)^{-1}\rangle_\theta=0.5\,(dH_{c2}^{\perp}/dT)^{-1}.$$

It follows from expression (2.3) that in weak fields the slope dH_{c2}/dT, regardless of the criterion we select for determining H_{c2}, is

$$(dH_{c2}/dT)_{H=0}=2dH_{c2}^{\perp}/dT, \tag{2.4}$$

and it does not depend on the particular functional dependence $f(T_c)$. Comparing expressions (1.10) and (2.4), we conclude that when there are variations in the critical temperature the curve of $H_{c2}(T)$ has a region of positive curvature. In nonporous samples, the slope dH_{c2}/dT increases by a factor of about three with increasing field.

In the magnetic-field interval

$$(T_{on}-T)\,dH_{c2}^{\perp}/dT<H\ll(T_{on}-T)\,dH_{c2}^{\parallel}/dT \tag{2.5}$$

expression (2.2) simplifies substantially:

$$c_i(H,T)=\frac{1}{H}\frac{dH_{c2}^{\perp}}{dT}\int_T^{T_{on}} dT_c\,f(T_c)\,(T_c-T). \tag{2.6}$$

As we mentioned back in Sec. 1, the resistivity depends linearly on the concentration c at low values of the latter. It thus follows from expression (2.6) that if the relative decrease in the resistivity is small, i.e., if $(\rho_n-\rho)/\rho_n\ll 1$, the quantity $(\rho_n-\rho)/\rho_n$ will be inversely proportional to the magnetic field in this field interval. Figure 2 shows experimental results on the temperature dependence of $H(\rho_n-\rho)/\rho_n$ for the beginning of the resistive transition ($\rho_n-\rho<0.5\rho_n$) in $La_{1.85}Sr_{0.15}CuO_{4-y}$ (Ref. 1). The good agreement between these results for a set of values of H is evidence that the broadening of the transition is nonuniform and the anisotropy of the critical fields is pronounced.

Because of this pronounced anisotropy of the critical fields, it is possible to use the resistivity curves to reconstruct the temperature dependence of the concentration, $c_i(T)$, over the entire temperature range. Specifically, differentiating (2.6) with respect to the temperature, we find

$$H\partial c_i(H,T)/\partial T=-c_i(T)\,dH_{c2}^{\perp}/dT. \tag{2.7}$$

In field interval (2.5), the temperature dependence of the derivative $d[(\rho_n-\rho)/\rho_n]/dT$ thus reproduces the functional dependence of the concentration, $c_i(T)$. Furthermore, expression (2.7) furnishes an independent method for determining $dH_{c2}^{\perp}/dT$:

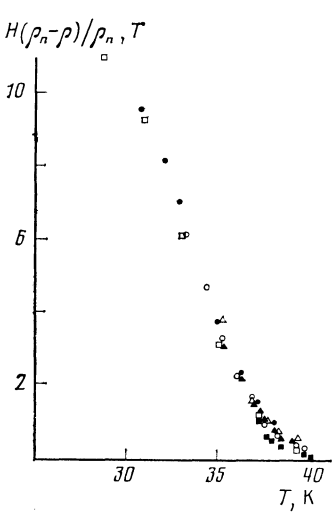

FIG. 2. Beginning of the resistive transition in $La_{1.85}Sr_{0.15}CuO_{4-y}$ (Ref. 1) in a magnetic field. The temperature region $T>35$ K corresponds to a nonuniform broadening of the transition. This figure illustrates the validity of relation (2.6). □—$H=2.0$ T (the notation is otherwise the same as in Fig. 1).

$$\frac{dH_{c2}^{\perp}}{dT}$$

$$=-H\frac{\rho_n}{\rho_n-\rho(0,T)}\frac{d}{dT}\left[\frac{\rho_n-\rho(H,T)}{\rho_n}\right] \tag{2.8}$$

Measurements of the dependence $H_{c2}(T)$ for a nonuniform sample also provide a method for reconstructing the function $c_i(T)$. Using expression (2.6), and calculating the derivative of the magnetic field with respect to the temperature at a fixed concentration of the superconducting phase, $c_i(H,T)=c$ (i.e., at a fixed value of the resistivity), we find the relation

$$(\partial H_{c2}/\partial T)_c=[c_i(T)/c]dH_{c2}^{\perp}/dT. \tag{2.9}$$

The temperature dependence of the quantity $(\varphi H_{c2}/\varphi T)$, measured at any resistivity level, thus also agrees with the functional dependence $c_i(T)$. For the temperature dependence of $H_{c2}(T,c)$ found at different resistivity levels (at different levels of the concentration c), we have the following scaling law in the interval (2.5):

$$H_{c2}(T,c)/H_{c2}(T,c')=c'/c. \tag{2.10}$$

The experimental data presently available in the literature are not an adequate basis for testing (2.7)–(2.10).

3. EFFECT OF TRANSPORT CURRENT ON THE RESISTIVE TRANSITION OF A POLYCRYSTALLINE SAMPLE IN A MAGNETIC FIELD

Critical current near the percolation threshold. At temperatures slightly below $T_c(H)$, the system of superconducting paths in a sample has a low density. The number n of superconducting channels which cross a unit cross section is determined by the correlation length of the percolation problem,[11] $\xi\propto d(c-c_p)^{-\nu}$:

$$n\propto\xi^{-2}\propto[T_c(H)-T]^{2\nu}. \tag{3.1}$$

The transport current flowing along the superconducting paths disrupts the superconductivity in those grains which are closest to the transition point. In other words, the typical critical current for a single path is a function of the different $T_c(H)-T$:

$$I_c=f(T_c(H)-T). \tag{3.2}$$

Since the critical current is determined by the grains in which the field $H_{2c}(T)$ differs only slightly from the applied field, the value of I_c is determined by the pair rupture mechanism[14]: $I_c\propto[T_c(H)-T]$. From expressions (3.1) and (3.2) we then find an expression for the critical current density:

$$j_c\propto[T_c(H)-T]^{2(\nu+1)}. \tag{3.3}$$

Since the value of this index is[11] $\nu\approx 0.9$, the value of the exponent in expression (3.3) for j_c is close to 3.8.

Effect of transport current on the resistivity near the resistive transition. The lowering of the resistivity of a polycrystalline sample near the resistive transition is governed by the appearance of superconducting grains. The current density flowing through these grains, j_s, is equal in order of magnitude to the current density in the normal matrix, j_n (for spherical superconducting inclusions we would have $j_s=3j_n$). A transport current of finite magnitude flowing through granules with a low critical current density j_c puts them in a normal state. There is accordingly a decrease in the function of the volume of the sample represented by the superconducting region, and the resistivity of the sample increases. (This effect was observed in Ref. 15 in a granular Josephson medium.) Let us find the increment in the resistivity which stems from the current density j. The critical angle θ_c, which determines possible orientations of the anisotropy axes of the superconducting grains, is given by a relation which is a generalization of Eq. (1.2):

$$j_c(H,\theta_c,T)=j_s. \tag{3.4}$$

Since we have $j_c=0$ on the line of the superconducting transition, Eq. (3.4) becomes (1.2) in the case $j=0$. We also seek the correction $\delta\rho$ in the low-current limit. The superconductivity is disrupted by the current only in the grains with angles θ close to θ_c, and we can use the limiting expression for j_c as a function of $[H_{c2}(\theta,T)-H]$ for the pair-rupture mechanism:

$$g[H_{c2}(\theta_c,T)-H]^2=j, \tag{3.5}$$

where $g\sim e\xi^3/\hbar\lambda^2$ (e is the charge of an electron). We will be interested below only in the shape of the functional dependence $\delta\rho(j)$, so we will draw no distinction between j_s and the average transport current density j. We will furthermore ignore all the numerical coefficients. It follows from expression (3.5) that the quantity H in (1.4) and (1.5) should be replaced by $H+(j/g)^{1/2}$ at a nonzero measurement current j. In the linear part of the transition, where Eqs. (1.5) and (1.7) hold, the correction to the resistivity satisfies

$$\delta\rho\propto[\rho_n-\rho(H,T)]j^{1/2}/H. \tag{3.6}$$

4. THE FIELD H_{c1}

The critical field H_{c1} is determined by the energy of the noninteracting vortex filaments, which we know are dominated by superconducting currents flowing around the normal core of the vortex.[16] The characteristic region "occupied" by the currents of one vortex is determined by the penetration depth λ of a weak magnetic field. The effective penetration depth λ_e for ceramic samples generally depends on the relation between λ and the typical grain size d, so an increase in λ as $T\to T_c$ leads to a change in the magnetic-field screening conditions. Let us estimate the critical field H_{c1} for two limiting relations between $\lambda(T)$ and d.

The case $d\gg\lambda_\perp(T)$. If an individual grain is highly anisotropic, energy considerations favor a position of the vortex in the plane of the layers. The direction of a vortex in an easy plane is determined by the conditions for its transition to neighboring grains. We assume that a vortex passes through a sample without leaving the easy plane of each of the grains which it intersects. The shape of a vortex filament is far from rectilinear, and the field H_{c1} exceeds the minimum single-crystal field $H_{c1}^{\parallel}$ by a numerical factor $\beta\gtrsim 1$, which characterizes the relative elongation of the vortex:

$$H_{c1}=\beta\frac{\phi_0}{4\pi\lambda_\parallel\lambda_\perp}\ln\left(\frac{\lambda_\parallel\lambda_\perp}{\xi_\parallel\xi_\perp}\right)^{1/2}, \tag{4.1}$$

where ϕ_0 is the magnetic flux quantum.

The case $d\ll\lambda_\perp(T)$. In this case the currents which screen the magnetic field include a large number of disor-

iented grains. The slowly decreasing magnetic field interacts with rapidly moving currents (which move over distances on the order of d). This separation of scales makes it possible to reduce the problem of finding the effective penetration depth to the problem of the effective conductivity of a polycrystalline medium.[17]

The equation for the average vector potential $\langle \mathbf{A} \rangle$ in the gauge div $\mathbf{A} = 0$ is

$$\Delta \langle A_i \rangle + \left\langle \lambda_{ij}^{-2} \left(\frac{c\hbar}{2e} \frac{\partial \varphi}{\partial x_j} - A_j \right) \right\rangle = 0, \tag{4.2}$$

where

$$\lambda_{ij}^{-2} = \lambda_{\parallel}^{-2}(\delta_{ij} - n_i n_j) + \lambda_{\perp}^{-2} n_i n_j,$$

$\mathbf{n}$ is a randomly directed anisotropy vector, and φ is the phase of the order parameter. The energy of a vortex is determined by the average magnetic field at a distance $R_{\perp} \sim d$ from the core of the vortex:

$$\varepsilon = (\phi_0/8\pi) \langle H(R_{\perp} \sim d) \rangle. \tag{4.3}$$

[In the case of a pronounced anisotropy, the component of the vortex energy which comes from the distance interval $\xi \ll R_{\perp} \ll d$ is smaller than (4.3) to the extent that the parameter $\lambda_{\parallel}/\lambda_{\perp}$ is $\ll 1$.] It follows from Eq. (4.2) that the average vector potential varies over distances of order λ. The screening current

$$j_i = \frac{c}{4\pi \lambda_{ij}^2} \left(\frac{c\hbar}{2e} \frac{\partial \varphi}{\partial x_j} - A_j \right) \tag{4.4}$$

obeys the equation div $\mathbf{j} = 0$ and varies over distances $d \ll \lambda$, so its average value is determined by the "smooth" part of the expression $(c\hbar/2e)\nabla\varphi - \mathbf{A}$:

$$\langle \mathbf{j} \rangle = \frac{c}{4\pi \lambda_e^2} \left(\frac{c\hbar}{2e} \nabla \langle \varphi \rangle - \langle \mathbf{A} \rangle \right). \tag{4.5}$$

In the vortex problem, the average phase is $\langle \varphi \rangle = \gamma$ (γ is the polar angle). Equations (4.4) and (4.5) are equivalent to the equations for the conductivity of a polycrystalline medium[17] [the quantity $(c\hbar/2e)\nabla\langle\varphi\rangle - \langle \mathbf{A} \rangle$ corresponds to an external electric field, $(c/4\pi)\lambda_{ij}^{-2}$ corresponds to the local conductivity, and $c/4\pi\lambda_e^2$ corresponds to the effective conductivity of the medium]. Since the anisotropy of λ_{ij}^{-2} is the same as the anisotropy of σ_{ij}, we reach the conclusion that λ_e^{-2} depends on $\lambda_{\perp}^{-2}$ and $\lambda_{\parallel}^{-2}$ just as the effective conductivity σ_e of this polycrystalline sample depends on $\sigma_{\perp}$ and $\sigma_{\parallel}$. In particular, in the highly anisotropic case ($\lambda_{\parallel} \ll \lambda_{\perp}$) we find $\lambda_e^{-2} = \nu \lambda_{\parallel}^{-2}$, where $\nu < 1$. In the effective-medium approximation (see the Appendix), we have $\nu \approx 0.52$. Substituting (4.4) into (4.2), we find an equation for $\langle \mathbf{A} \rangle$ which is the same as the London equation for an isotropic superconductor. We thus find the following expression for the field H_{c1}:

$$H_{c1} = \frac{\phi_0}{4\pi \lambda_e^2} \ln \frac{\lambda_e}{d}.$$

In the case of a pronounced anisotropy, the field H_{c1} is smaller than the maximum single-crystal field $H_{c1}^{\perp}$ to the extent that the numerical parameter is small:

$$H_{c1} \approx 0{,}52 \frac{\phi_0}{4\pi \lambda_{\parallel}^2} \ln \frac{\lambda_{\parallel}}{d}. \tag{4.6}$$

In the Ginzburg-Landau approximation we would have $\lambda(T) \propto (T_c - T)^{-1/2}$. If the size of the grains satisfies $d \gg \lambda_{\perp}(0)$, then two linear regions with quite different slopes should be seen on the plot of H_{c1} versus $T_c - T$: Near the transition temperature [$\lambda_{\parallel}(T) > d$], relation (4.6) holds, while at relatively low temperatures [$\lambda_{\perp}(T) < d$], expression (4.1) holds. It should be noted, however, that the first of these intervals becomes narrower as the ratio $\lambda(0)/d$ decreases, and in the experiments which have been carried out ($d \sim 10^{-4}$ cm, $\lambda \sim 10^{-5}$ cm) this interval was not observed.

5. MAGNETIZATION OF A POLYCRYSTALLINE SAMPLE IN STRONG MAGNETIC FIELDS

For type II superconductors with a Ginzburg-Landau parameter $\varkappa \gg 1$, the magnetization falls off rapidly at fields $H > H_{c1}$, and the condition $4\pi M \ll H$ holds over a broad interval of magnetic fields. In this case, the magnetic induction is approximately the same as the magnetic field, and for an isotropic distribution of anisotropy axes the magnetization of the polycrystalline samples, $M(H)$, is equal to the single-crystal magnetization $M(H,\theta)$, averaged over angles:

$$M(H) = \int_0^{\pi/2} M(H, \theta) \sin\theta \, d\theta. \tag{5.1}$$

To evaluate the integral (5.1) we use the approximation[1)]

$$4\pi M(H, \theta) = -[H_{c2}(\theta) - H]/2\beta\varkappa^2(\theta), \quad H_{c2}(\theta) > H, \tag{5.2}$$

which become exact in the limit[14] $H \to H_{c2}$. In (5.2) we have

$$\varkappa^{-2}(\theta) = \varkappa_{\perp}^{-2} \cos^2\theta + \varkappa_{\parallel}^{-2} \sin^2\theta, \quad \varkappa_{\parallel}/\varkappa_{\perp} = H_{c2}^{\parallel}/H_{c2}^{\perp},$$

and $\beta \approx 1.16$ is the Abrikosov parameter for a triangular vortex array.[16] Averaging over angles, we find the following expression for the magnetization of the polycrystalline sample:

$$4\pi M(H) = -\frac{1}{2\beta\varkappa_{\parallel}^2} \left(\frac{\varkappa_{\parallel}^2}{\varkappa_{\perp}^2} - 1 \right)^{-1/2} \left\{ \frac{H_{c2}^{\parallel}}{2} \left[\frac{H_{c2}^{\parallel}}{H} \left(\frac{(H_{c2}^{\parallel})^2}{H^2} - 1 \right) + \ln \left(\frac{H_{c2}^{\parallel}}{H} + \left(\frac{(H_{c2}^{\parallel})^2}{H^2} - 1 \right)^{1/2} \right) \right] - \frac{H}{3} \left(\frac{(H_{c2}^{\parallel})^2}{H^2} - 1 \right)^{1/2} \left(2 + \frac{(H_{c2}^{\parallel})^2}{H^2} \right) \right\}. \tag{5.3}$$

This expression simplifies in two limiting cases:

$$4\pi M(H) = \begin{cases} [\varkappa_{\perp}/\beta\varkappa_{\parallel}^3 (2H_{c2}^{\parallel})^{1/2}] (H_{c2}^{\parallel} - H)^{1/2}, & (H_{c2}^{\parallel} - H)/H_{c2}^{\parallel} \ll 1 \\ -(H_{c2}^{\perp})^3/12\beta\varkappa_{\perp}^2 H^2, & H_{c2}^{\perp} < H \ll H_{c2}^{\parallel} \end{cases}. \tag{5.4}$$

The asymptotic functional forms of $M(H)$ are given in Ref. 10. The reason for the fairly strong dependence $M(H)$ in the interval from $H_{c2}^{\perp}$ to $H_{c2}^{\parallel}$ is the increase in the concentration of the superconducting phase with decreasing magnetic field. The curve of the generalized susceptibility $\chi(H) = dM/dH$ versus the magnetic field has a slope change at $H = H_{c2}^{\perp}$ because of the transition of the entire polycrystalline sample into the superconducting state. The jump in the second derivative d^2M/dH^2 can be found from (5.1) and (5.2):

$$(d^2M/dH^2)_{H = H_{c2}^{\perp} + 0} - (d^2M/dH^2)_{H = H_{c2}^{\perp} - 0} = (8\pi\beta\varkappa_{\perp}^2 H_{c2}^{\perp})^{-1}. \tag{5.5}$$

In real samples, this jump is blurred by inhomogeneities.

CONCLUSION

We have examined several superconducting properties of a polycrystalline sample in a model which considers only the anisotropy and the scatter in the values of T_c of the various grains. Despite the simplicity of this model, it gives a satisfactory description of the changes observed experimentally in the shape of the resistive transition over a broad range of magnetic field.[1,5] We can accordingly use (1.1) and (1.10) to evaluate the upper critical fields for $La_{1.85}Sr_{0.15}CuO_{4-y}$ from the experimental results on polycrystalline samples[1]: $dH_{c2}^{\perp}/dT \approx 0.25$ T/K, $dH_{c2}^{\parallel} dT \gtrsim 3$ T/K.

Measurements of the field H_{c1} involve an analysis of the behavior of the magnetization of the sample, $M(H)$, in weak fields. The quantity $M(H)$ turns out to be very sensitive to the quality of the samples, particularly their porosity. In the highest-quality ceramics, however, the field $H_{c1} = 800$–900 Oe (Ref. 18) is found to be approximately the same as the minimum value of the field H_{c1} for single crystals: $H_{c1}^{\parallel} = 800$ Oe (Ref. 4) (the compound $YBa_2Cu_3O_{7-y}$). This result agrees with the results of Sec. 4 [see (4.1)].

In this model we have taken account of the formation of a superconducting state in the interior of grains, and we have assumed that the conductivity through the contacts between grains is direct. An explanation of the various fine points of the transition thus goes beyond the scope of this model. The long tail on the $R(T)$ curve at low temperatures which is sometimes observed falls in the category of these fine points.[6] One possible reason for the tail is that a small fraction of the resistivity of a sample is determined by S–I–S contacts, in which the Josephson junction is suppressed by a magnetic field. Also unclear are whether nonuniform superconducting states can form near the surfaces of grains and the role which these states would play. It can be shown, however, that for polycrystalline samples there is no field H_{c3} associated with the external surface. An infinite superconducting cluster forms in the interior before it does at the surface.

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APPENDIX

The lowering of the resistivity as the concentration increases, described by (1.7), stems primarily from the appearance of superconducting inclusions in the normal matrix. In addition, there is a component, linear in c, which comes from the change in the effective conductivity of the normal matrix itself. The coefficients α_t and α_l are the principle values of the tensor α_{ij}. The superconducting-inclusion component of this tensor, α_{ij}^{I}, is

$$\alpha_{ij}{}^{\mathrm{I}} = \langle n_{ij}{}^{-1} \rangle, \tag{A1}$$

where n_{ij} is the depolarization-coefficient tensor,[19] which is determined by the shape of the superconducting inclusions. If the grains have typical dimensions $d_{\perp}$ and $d_{\parallel}$ in the directions along the anisotropy axis and in the basal plane, then we can estimate α_{ij}^{I} from the depolarization coefficients for an ellipsoid with an eccentricity $\eta = (d_{\parallel}^2/d_{\perp}^2 - 1)^{1/2}$:

$$\alpha_t{}^{\mathrm{I}} = \eta^3/(1+\eta^2)(\eta - \operatorname{arctg}\eta), \quad \alpha_l{}^{\mathrm{I}} = 2/(1-\alpha_t{}^{-1}). \tag{A2}$$

For superconducting inclusions of arbitrary shape, the inequality $(2\alpha_t + \alpha_l)/3 \gtrsim 3$ holds.

If $d_{\parallel} \approx d_{\perp}$, the difference between the longitudinal resistivity of the sample and the transverse resistivity arises because of a nonuniformity in the angular distribution of the anisotropy axes of the nonsuperconducting grains. To determine the corresponding component $\alpha_{ij}^{\mathrm{II}}$ of the tensor α_{ij}, we need to find the effective conductivity of the normal matrix, σ_{ij}^{e}. This quantity is determined by the equations (Ref. 17, for example)

$$\sigma_{ij}{}^{e} = \langle \sigma_{ij} \rangle - \langle \sigma_{in} \partial_n \psi_j \rangle, \tag{A3a}$$

$$\partial_i \sigma_{in} \partial_n \psi_j = \partial_i \sigma_{ij}. \tag{A3b}$$

Here the vector ψ_j is determined by $\varphi = \mathbf{rE} + \psi\mathbf{E}$, where $\mathbf{E}$ is the external field, and φ is the potential of the resultant electric field in the sample. When the conductivity of the grains is highly anisotropic we would have $\sigma_{ij} = \sigma_{\parallel}(\delta_{ij} - n_i n_j)$, where $\mathbf{n}$ is a random unit vector distributed in the region $c < |n_z| < 1$. Equations (A3) cannot be solved exactly. To estimate the conductivity and its anisotropy, we used the effective-medium approximation. In this approximation, the random conductivity σ_{in} on the left side of Eq. (A3b) is replaced by its effective value σ_{in}^{e}:

$$\sigma_{in}{}^{e} \partial_i \partial_n \psi_j = \partial_i \sigma_{ij}. \tag{A3c}$$

Equations (A3a) and (A3c) constitute a self-consistent set of equations for determining σ_{in}^{e}. In the case $d_{\parallel} \approx d_{\perp}$ we find

$$^1/_3(2\sigma_t + \sigma_l) = [(3+3^{1/2})/9]\sigma_{\parallel} \approx 0.52\sigma_{\parallel}, \quad \alpha_l{}^{\mathrm{II}} = -2\alpha_t{}^{\mathrm{II}} \approx -0.77. \tag{A4}$$

[1)] Expression (5.2) determines the magnetization component parallel to the magnetic field. The angular average of the component $M(H)$ which is perpendicular to the field is zero.

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