# Relaxation of the nuclear magnetization of amorphous ferromagnets

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An investigation has been made of the influence of elementary tunnel excitations known as twolevel systems, present in amorphous substances, on nuclear relaxation. The cases of nonmagnetic and magnetic two-level systems are discussed and in both cases the Hamiltonians of the interaction of nuclear spins with these systems are derived. A calculation is made of the macroscopic relaxation of nuclei allowing for spin diffusion. The proposed nuclear relaxation mechanisms are compared with the familiar two-magnon case.

### **1. INTRODUCTION**

The currently accepted explanation of low-temperature properties of amorphous solids is based on the representation of structural excitations of a special kind known as twolevel systems. These representations were developed in Refs. 1 and 2 specifically for amorphous insulators. According to these ideas, an amorphous system has specific atoms or groups of atoms which can be at one of two levels and which undergo phonon-assisted tunneling transitions between these levels.

It has been found that the presence of two-level systems is specific not only to amorphous insulators but also to other disordered systems such as glassy metals,<sup>3</sup> and semiquantum liquids.<sup>4</sup> It was postulated in Refs. 5 and 6 that atoms forming two-level systems exist in amorphous ferromagnets and influence strongly the transport properties of these substances. Atoms forming these systems can be magnetic or nonmagnetic. When the tunnel states are formed by magnetic atoms, they can modulate the exchange interaction energy, the dipole energy, and the magnetic anisotropy energy. When atoms in two-level systems are nonmagnetic, they can participate in the establishment of an indirect exchange interaction and a crystal field, and can also modulate the exchange interaction and magnetic anisotropy energies.

In the case of amorphous ferromagnets it is quite natural to assume that the tunnel transitions between the states of two-level systems are induced not only by phonons but also by spin waves. The relaxation time of two-level systems interacting with spin waves and the decay of spin waves due to their scattering by such systems, as well as the magnetic contribution to the thermal conductivity (which in a certain range of temperatures is greater than the phonon contribution) are calculated in Ref. 5. Thus the coupling of a magnon system to two-level systems in amorphous ferromagnets can sometimes be the dominant effect. One would expect the scattering to have a considerable influence also on the nuclear spin–lattice relaxation (SLR).

Our aim was to investigate the influence of two-level systems on nuclear relaxation in amorphous ferromagnets.

#### 2. NONMAGNETIC TWO-LEVEL SYSTEMS

We shall first consider the case when the atoms forming two-level systems are nonmagnetic. Obviously, in this case there is no direct interaction between nuclear spins and these systems. However, there may be indirect coupling. In fact, nuclear spins interact with magnons and the latter in turn interact with two-level systems. This is the reason for the indirect coupling of nuclear spins to two-level systems.

We shall now derive the Hamiltonian for the indirect interaction of nuclear spins with two-level systems. We shall consider a ferromagnet with easy-axis anisotropy. It is shown in Ref. 5 that the Hamiltonian of an amorphous ferromagnet describing the processes of interaction of spin waves with two-level systems in any (including a noncollinear or canted ferromagnet) is

$$V_{i} = \sum_{\alpha i \mathbf{k}} D_{i}^{\alpha} (a_{\mathbf{k}}^{*} \Phi_{\mathbf{k}}^{*i} e^{-i\mathbf{k}\mathbf{R}_{i}} + a_{\mathbf{k}} \Phi_{\mathbf{k}}^{i} e^{i\mathbf{k}\mathbf{R}_{i}}) (S_{i\alpha}^{*} + S_{i\alpha}^{-})$$
  
+ 
$$\sum_{\alpha i \mathbf{k}} F_{i}^{\alpha} (a_{\mathbf{k}}^{*} \Phi_{\mathbf{k}}^{*i} e^{-i\mathbf{k}\mathbf{R}_{i}} + a_{\mathbf{k}} \Phi_{\mathbf{k}}^{i} e^{i\mathbf{k}\mathbf{R}_{i}}) S_{i\alpha}^{i\alpha},$$
$$D_{i}^{\alpha} = D_{i} \frac{\Delta_{0\alpha}}{E_{\alpha}}, \quad F_{i}^{\alpha} = F_{i} \frac{(E_{\alpha}^{2} - \Delta_{0\alpha}^{2})^{\prime h}}{E_{\alpha}},$$

where  $E_{\alpha} = (\Delta_{\alpha}^2 + \Delta_{\alpha}^2)^{1/2}$  is the energy of two-level systems;  $\Delta_{\alpha}$  is the asymmetry parameter;  $\Delta_{0\alpha}$  is the tunnel energy of two-level systems (the definitions of these quantities can be found in Ref. 1);  $a_k^+$  and  $a_k$  are the operators of creation and annihilation of magnons in states with an energy  $\varepsilon_k = \lambda + \beta k^2$  and with a wave vector **k** ( $\lambda$  is the gap parameter in the magnon spectrum and  $\beta$  is the exchange parameter); and  $S^{z(\pm)}$  are the components of the pseudospin of two-level systems. The meaning and definitions of the other quantities occurring in  $V_1$  can be found in Ref. 5.

We shall write down the Hamiltonian of an amorphous ferromagnet subjected to a static magnetic field:

$$H = H_0 + V_1 + V_2 + V_3,$$

$$H_0 = -\hbar\omega_0 \sum_i I_i^z + \sum_k \varepsilon_k a_k^+ a_k + \sum_\alpha E^\alpha S_\alpha^z,$$

$$V_2 = \sum_{\alpha i \mathbf{k} \mathbf{k}'} C_i^\alpha a_k^+ a_{\mathbf{k}'} (S_{i\alpha}^+ + S_{i\alpha}^-) \exp\{-i(\mathbf{k} - \mathbf{k}')\mathbf{R}_i\},$$

$$V_3 = B \sum_{i\mathbf{k}} (I_i^- a_k^+ \exp(-i\mathbf{k}\mathbf{R}_i) + I_i^+ a_k \exp(i\mathbf{k}\mathbf{R}_i)),$$

where  $H_0$  is the ground-state Hamiltonian of the nuclear spins, the magnon system, and two-level systems ( $\omega_0$  is the NMR frequency); and  $V_3$  is the operator representing the interaction of nuclear spins with magnons (*B* is the hyperfine structure constant). In contrast to Ref. 5, where no allowance was made for two-magnon processes of interaction with two-level systems (the operator  $V_2$ ), we included this interaction (we shall show below that this interaction can sometimes dominate relaxation of the nuclear magnetization).

The Hamiltonian describing the indirect interaction of nuclear spins with two-level systems is derived in the same way as in Ref. 7. The result is

$$\begin{split} H_{\text{eff}} &= H_1 + H_2 + H_3, \\ H_1 &= -B \sum_{ij\alpha\mathbf{k}} \frac{F_j^{\alpha} S_{j\alpha^z}}{\varepsilon_{\mathbf{k}}} \left( I_i^{-} \Phi_{\mathbf{k}}^{-j} \exp\left(-i\mathbf{k}\mathbf{R}_{ij}\right) \right) \\ &+ I_i^{+} \Phi_{\mathbf{k}}^{*j} \exp\left(i\mathbf{k}\mathbf{R}_{ij}\right) \right), \\ H_2 &= B \sum_{ij\alpha\mathbf{k}\mathbf{k}'} \frac{C_j^{\alpha}}{\varepsilon_{\mathbf{k}} - \hbar \omega_0} \left( I_i^{-} a_{\mathbf{k}}^{+} S_{j\alpha^-} \exp\left(-i\mathbf{k}\mathbf{R}_{ij} - i\mathbf{k}'\mathbf{R}_j\right) \right) \\ &+ I_i^{+} a_{\mathbf{k}} S_{j\alpha^+} \exp\left(i\mathbf{k}\mathbf{R}_{ij} + i\mathbf{k}'\mathbf{R}_j\right) \right), \\ H_3 &= -B \sum_{ij\alpha\mathbf{k}} \frac{D_j^{\alpha}}{\varepsilon_{\mathbf{k}} - \hbar \omega_0} \left( I_i^{-} S_{j\alpha^+} \Phi_{\mathbf{k}}^{-j} \exp\left(-i\mathbf{k}\mathbf{R}_{ij}\right) \\ &+ I_i^{+} S_{j\alpha^-} \Phi_{\mathbf{k}}^{*j} \exp\left(i\mathbf{k}\mathbf{R}_{ij}\right) \right). \end{split}$$

The operators  $H_1$ ,  $H_2$ , and  $H_3$  describe different relaxation mechanisms, but not all these mechanisms are equally effective. It should be pointed out straight away that the relaxation mechanism described by the operator  $H_3$  is not very effective. In fact,  $H_3$  describes a process of reversal of the nuclear spin and of the pseudospin of two-level systems when a quantum of the Zeeman energy of the nuclei should be absorbed by two-level systems. Clearly, this process involves the two-level systems for which the splitting energy is equal to a quantum of the Zeeman energy of the nucleus. Clearly, the number of such two-level systems is relatively small, so that this relaxation mechanism is ineffective.

Summing over the magnon states in the expressions for  $H_1$  and  $H_2$ , we obtain

$$H_{i} = -B \sum_{ij\alpha} F_{ij}^{\alpha} S_{j\alpha}^{z} (I_{i}^{+} + I_{i}^{-}),$$

$$H_{2} = B \sum_{ij\alpha\mathbf{k}} C_{ij}^{\alpha} (I_{i}^{-} a_{\mathbf{k}}^{+} S_{j\alpha}^{-} \exp(-i\mathbf{k}\mathbf{R}_{j}) + I_{i}^{+} a_{\mathbf{k}} S_{j\alpha}^{+} \exp(i\mathbf{k}\mathbf{R}_{j})),$$

$$F_{ij}^{\alpha} = F_{j}^{\alpha} \frac{V}{4\pi R_{ij}\beta} \exp\left\{-\left(\frac{\lambda}{\beta}\right)^{\prime b} R_{ij}\right\},$$

$$C_{ij}^{\alpha} = C_{j}^{\alpha} \frac{V}{4\pi R_{ij}\beta} \exp\left\{-\left(\frac{\lambda}{\beta}\right)^{\prime b} R_{ij}\right\}.$$

We can see that the constants of the interaction of nuclear spins with pseudospins of two-level systems depend on the distance  $R_{ij}$  between them.

Applying the Kubo expression for the nuclear SLR time, we can readily obtain an expression for the time representing direct relaxation of nuclei located at a distance R from two-level systems and characterized by a splitting energy E and by a tunnel parameter  $\Delta_0$ . Omitting the simple calculations of the direct relaxation times, described by the interaction operators  $H_1$  and  $H_2$ , respectively, we quote the results directly

$$\frac{1}{T_1^{(1)}(R)} = \frac{B^2 F^2 V^2}{32\pi^2 \beta^2 \hbar^2} \frac{(E^2 - \Delta_0^2) \tau \exp\{-2(\lambda/\beta)^{1/\epsilon} R\}}{E^2 \operatorname{ch}^2(E/2T)(1 + \omega_0^2 \tau^2) R^2}, (1)$$

$$\frac{1}{T_1^{(2)}(R)} = \frac{B^2 C^2 V^3}{8\pi\hbar\beta^3} \left(\frac{E-\lambda}{\beta}\right)^{1/2} \frac{\Delta_0^2 \exp\left\{-2\left(\lambda/\beta\right)^{1/2} R\right\}}{E^2 \operatorname{sh}\left(E/2T\right) R^2} . (2)$$

Clearly, this nuclear relaxation mechanism described by the operator  $H_2$  involves only those two-level systems for which the splitting energy is higher than the gap parameter in a magnon spectrum  $(E > \lambda)$ .

Equation (1) is derived on the assumption that the pseudocorrelation function is Lorentzian with a correlation time  $\tau$ :

$$\langle \delta S^z \delta S^z(t) \rangle = \langle (\delta S^z)^2 \rangle f_s(t) = \frac{e^{-t/\tau}}{4 \operatorname{ch}^2(E/2T)}$$

It follows from Eqs. (1) and (2) that the time constant for direct relaxation of nuclei depends on the distance from the two-level systems in accordance with the law

$$\frac{1}{T_{i}(R)} \propto \frac{e^{-\gamma R}}{R^{2}},$$

where  $\gamma = 2(\lambda /\beta)^{1/2}$ , which demonstrates that the nuclei located close to two-level systems reach an equilibrium with the lattice faster than the more distant nuclei. This gives rise to a spin temperature gradient and induces spin diffusion.<sup>9</sup>

Ignoring the power-law dependence of the nuclear SLR time on the distance in Eqs. (1) and (2), compared with the exponential dependence, we find that  $T_1^{(1)}(R)$  and  $T_1^{(2)}(R)$  are described by

$$\frac{1}{T_1^{(1)}(R)} = \frac{1}{T_{01}} e^{-\gamma R},$$
(1')

$$\frac{1}{T_1^{(2)}(R)} = \frac{1}{T_{02}} e^{-\gamma R},$$
(2')

where the values of  $T_{01}$  and  $T_{02}$  can be readily obtained by comparing Eqs. (1) and (2) with Eqs. (1') and (2').

The density of the magnetization of those nuclei which are in the "sphere of influence" of the two-level systems can be described by the following diffusion equation<sup>10</sup>:

$$\frac{\partial \mathbf{m}(R)}{\partial t} = D\Delta \mathbf{m}(R) - \frac{\mathbf{m}(R) - \mathbf{m}_0}{T_1(R)},$$
(3)

where the dependence  $T_1(R)$  is given by Eqs. (1') or (2'); D is the diffusion coefficient of nuclear spins governed by their dipole-dipole interactions with one another;  $\Delta$  is the Laplace operator;  $m_0$  is the equilibrium value of the nuclear magnetization density.

Although the operators  $H_1$  and  $H_2$  describe physically different relaxation mechanisms, the distance dependence of the direct relaxation time associated with  $H_1$  and with  $H_2$  is the same, so that both relaxation cases can be considered at the same time.

The problem we are treating is mathematically equivalent to that of nuclear relaxation due to local electron centers discussed in Ref. 11, so that we shall apply directly the expressions obtained in Ref. 11 for the macroscopic relaxation time of the nuclear magnetization. It is shown in Ref. 11 that in the case of slow diffusion the macroscopic relaxation time is

$$\frac{1}{T_{i}} = 4\pi n D \frac{1}{\gamma} \ln\left(\frac{1}{\gamma^{2}} D T_{o}\right), \qquad (4)$$

where n is the concentration of two-level systems per unit volume and

$$\frac{1}{T_{i}(R)}=\frac{1}{T_{o}}e^{-\gamma R}.$$

In the opposite case of fast diffusion the macroscopic relaxation time is independent of the diffusion coefficient, but is governed by the direct relaxation time  $T_1^*$  of the nuclei nearest to two-level systems and is described by

$$\frac{1}{T_{1}} = \frac{8\pi n}{\gamma^{3}} \frac{1}{T_{1}}.$$
(5)

The experimental results for the macroscopic relaxation time can be described by averaging Eqs. (4) and (5) over the states of two-level systems via the probability of the distribution of the parameters of such systems:

$$dw(E,\Delta_0) = \overline{P} \frac{E}{\Delta_0} \frac{dE \, d\Delta_0}{\left(E^2 - \Delta_0^2\right)^{\frac{1}{2}}}.$$
(6)

By definition, dw is the probability that two-level systems have a splitting energy and a tunnel parameter within the intervals E, E + dE and  $\Delta_0, \Delta_0 + d\Delta_0$ . The average value of any function  $\varphi(E, \Delta_0)$  can be found from

$$\overline{\varphi} = \overline{P} \int_{0}^{E_{m}} dE \int_{\Delta_{0} \min}^{E} \varphi(E, \Delta_{0}) \frac{Ed\Delta_{0}}{\Delta_{0} (E^{2} - \Delta_{0}^{2})^{\frac{1}{2}}}.$$

Here,  $\Delta_{0 \min}$  is the minimum value of the tunnel parameter,  $\overline{P}$  is the density of states of two-level systems, and  $E_m$  is the maximum value of the splitting energy of these systems.

Averaging over the macroscopic SLR time, which is due to the interaction  $H_1$  in the case of fast diffusion, we obtain

$$\frac{1}{T_{i}} = \frac{nF^{2}V^{2}B^{2}e^{-\imath \alpha}T^{2}\overline{P}\alpha_{i}}{2\gamma^{3}h^{2}\pi\beta^{2}a^{2}K\Delta_{0\,\min}^{2}},$$

$$\alpha_{i} = \int_{0}^{\infty} \frac{x \operatorname{sh} x}{\operatorname{ch}^{3}x} dx;$$
(7)

$$\frac{1}{T_{1}} = \frac{2^{\gamma_{a}} n F^{2} V^{2} B^{2} e^{-\tau_{a}} T^{5/2} \overline{p} \alpha_{2}}{\gamma^{3} \hbar^{2} \pi \beta^{2} a^{2} K' \Delta^{2}_{0 \text{ min}}},$$
(8)

$$\alpha_2 = \int_{0}^{\infty} \frac{x^{s/s} \operatorname{sh} x}{\operatorname{ch}^{s} x} \, dx.$$

The systems of equations (7) and (8) are derived on the assumption that the temperature satisfies the conditions  $T \gg \hbar \omega_0$  and  $T \gg \Delta_{0 \min}$ , and that the correlation time  $\tau$  of two-level systems representing their coupling to magnons, satisfies the inequality  $\omega_0 \tau \ll 1$ . The correlation time  $\tau$  for the case when the dipole-dipole interactions are important has the form<sup>5</sup>

$$\frac{1}{\tau} = K \left(\frac{\Delta_0}{E}\right)^2 E \operatorname{cth} \frac{E}{2T}$$

[Eq. (7) corresponds to this case], but if the dipole–dipole interactions can be ignored, then  $\tau$  is given by<sup>5</sup>

$$\frac{1}{\tau} = K' \left(\frac{\Delta_0}{E}\right)^2 E^{\eta} \operatorname{cth} \frac{E}{2T}.$$

In the case of slow diffusion it follows from Eq. (4) that in averaging the temperature functions occur in the logarithm, so that the temperature dependence of  $T_1$  will be weak in the case of slow diffusion.

We shall now give the expression for the macroscopic relaxation time in the case of the interaction described by  $H_2$ . In the case of fast diffusion simple calculations yield

$$\frac{1}{T_{1}^{(2)}} = \frac{2^{1/2}nB^{2}C^{2}V^{3}e^{-\gamma a}\overline{P}T^{3/2}\alpha_{3}}{\hbar\beta^{1/2}a^{2}},$$

$$\alpha_{3} = \int_{0}^{\infty} \frac{x^{1/2}dx}{\sinh x},$$
(9)

(it is assumed here that  $T > \lambda$ ). However, in the case of slow diffusion (exactly as pointed out above) the temperature dependence of the relaxation time is weaker than in the case of fast diffusion.

## 3. MAGNETIC TWO-LEVEL SYSTEMS

We shall now consider the case when the atoms forming the two-level systems are magnetic. Then, in addition to those mechanisms of the interaction of nuclear spins with pseudospins of the two-level systems discussed above, there is also a dipole-dipole interaction between the magnetic moments of atoms in two-level systems, on the one hand, and nuclear spins, on the other; the constant of this interaction fluctuates as a result of transitions of atoms between the states of the two-level systems and this naturally gives rise to an additional channel of relaxation of nuclei. The fluctuation part of the Hamiltonian of the dipole-dipole interaction of nuclear spins located at points  $R_i$  with the moments of atoms in two-level systems located at points  $R_{\alpha}$  leads to relaxation of nuclei and, in the representation in which the Hamiltonian of the two-level systems is diagonal, it can be written in the form

$$H_{4} = \sum_{i\alpha} L_{i\alpha} (I_{i}^{+} + I_{i}^{-}) S_{\alpha}^{2}$$
(10)

(terms of the  $I_i^{\pm} S_{\alpha}^{\mp}$  type are not included because the relaxation processes are ineffective);

$$L_{i\alpha} = l_{i\alpha} \frac{(E_{\alpha}^2 - \Delta_{0\alpha}^2)^{\gamma_i}}{E_{\alpha}}, \quad l_{i\alpha} = \frac{a_0}{R_{i\alpha}^4} \gamma_I \gamma_s^{\alpha} = \frac{\delta}{R_{i\alpha}^4};$$

 $\gamma_I$  and  $\gamma_s^{\alpha}$  are the gyromagnetic ratios of the nuclear spins and atoms in two-level systems, respectively;  $a_0$  is the distance between the positions of a tunneling particle;  $\mathbf{R}_{i\alpha}$  $= \mathbf{R}_i - \mathbf{R}_{\alpha}$ .

The interaction  $H_4$  decreases strongly on increase in the distance and, therefore, it is clear that those nuclei which are located close to two-level systems will relax relatively rapidly, whereas in the case of the distant nuclei we can expect, as above, spin diffusion.

The direct relaxation time of nuclei located at distances R from two-level systems, characterized by the parameters E and  $\Delta_0$ , due to the interaction  $H_4$  is described by

$$\frac{1}{T_1(R)} = \frac{\delta^2 (E^2 - \Delta_0^2) \tau}{8R^3 \hbar^2 E^2 \operatorname{ch}^2 (E/2T) (1 + \omega_0^2 \tau^2)}$$
(11)

(as before, this expression is derived from the Kubo equation). The magnetization density of the nuclei is once again described by Eq. (3) with  $T_1(R)$  taken from Ref. 11.

The equation for the magnetization now differs from that investigated by Khutsishvili<sup>9</sup> because of the dependence  $T_1 = T_1(R)$ . In Ref. 9 it is assumed that the rapidly relaxing centers are paramagnetic impurities fixed in space and fluctuations of the spin of these impurities are responsible for nuclear relaxation. Therefore, the dependence of the direct relaxation time on the distance is  $T_1 \propto R^6$ . However, in our case fluctuations of the magnetic field at the nuclei are due to spatial jumps of atoms forming magnetic two-level systems and, therefore, as shown above, we have  $T_1 \propto R^8$ .

An analysis of Eq. (3) with  $T_1(R)$  from Eq. (11) will be made in a manner similar to that adopted by Khutsishvili.<sup>9</sup> The central-symmetric stationary solution of the diffusion equation satisfying the boundary conditions stating that at R = 0, we have  $m(0) = m_0$  and at  $R = \infty$ , we have  $m(\infty) = m_1 (m_1$  are certain densities of the nuclear magnetization far from the two-level systems, which is kept constant by, for example, an external field) is

$$m(R) = m_0 - m_0 R^{-\frac{1}{2}} [AI_{-\frac{1}{6}}(\sqrt{\frac{1}{2}}/3R^3) - BI_{\frac{1}{6}}(\sqrt{\frac{1}{2}}/3R^3)],$$

where A and B are constants of integration,  $I_p(x)$  is a modified Bessel function, and

 $v = R^{s}/DT_{n}(R)$ .

Following Ref. 9, we can show that in our case the nuclear system is also characterized by a single relaxation time which in the case of slow diffusion is given by the expression

$$1/T_1 = 4\pi n D b, \tag{12}$$

where the quantity

$$b = 2^{2/3} \pi \nu^{1/6} / 3^{1/3} \Gamma(7/6) \Gamma(1/6) \approx 0.84 \nu^{1/6}$$

with the dimensions of length has the following physical meaning: for the nuclei which are inside a sphere of radius b with the center at a magnetic two-level system the direct diffusion is more important than the spin diffusion, whereas in the case of nuclei outside this sphere the spin diffusion process is more important.

However, in the case of fast diffusion the macroscopic relaxation time is governed by the direct relaxation time  $T_n(a)$  of the nuclei closest to the two-level systems and it is described by

$$\frac{1}{T_{i}} = \frac{n}{n_{I}} \frac{1}{T_{n}(a)},$$
(13)

where  $n_1$  is the concentration of nuclear spins in a sample.

Averaging over the states of the two-level systems in Eqs. (12) and (13), we obtain:

a) in the case of slow diffusion

$$\frac{1}{T_{i}} = 3.5\pi n D \left( \delta^{2}/D \right)^{1/6} K^{1/6} \overline{P} T^{5/6} \alpha_{4}, \tag{14}$$

where

$$\alpha_{4}=\int_{0}^{\infty}\frac{x^{-1/4}(\operatorname{sh} x)^{1/4}}{(\operatorname{ch} x)^{1/2}}dx;$$

b) in the case of fast diffusion,

$$\frac{1}{T_1} \approx \frac{n}{n_I} \frac{\delta^2 \overline{P} T^2}{4\hbar^2 a^8 K \Delta_{0\min}^2}.$$
(15)

Equations (14) and (15) are derived assuming that the correlation time  $\tau$  is determined by  $\lambda \ge \hbar \omega_0$ . We can see that the temperature dependences of the macroscopic relaxation time are quite different for the cases of slow and fast diffusion.

#### 4. DISCUSSION OF RESULTS

We shall now consider the effectiveness of the proposed nuclear SLR mechanisms in amorphous ferromagnets, in

which we allowed for the influence of the two-level systems, compared with those nuclear relaxation mechanisms which are discussed in the literature when dealing with the relaxation in ferromagnets. It is known<sup>10</sup> that if the gap parameter in a magnon spectrum satisfies the inequality  $\lambda \ge \hbar \omega_0$ , the contribution of one-magnon processes to SLR of the nuclei is forbidden by the law of conservation of energy, so that at low temperatures it is necessary to allow for two-magnon processes (the contribution of many-magnon processes is small at low temperatures). Allowance for two-magnon processes in SLR of nuclei leads to an almost quadratic temperature dependence of the relaxation rate

$$\frac{1}{T_{\rm II}} = \frac{2}{15\pi} \left(\frac{\hbar^2 \gamma_I \gamma_*}{a^3}\right)^2 \frac{a^3 T^2}{\hbar (2\lambda\beta)^{\frac{1}{2}}} \ln \frac{T}{\lambda}.$$
 (16)

Here,  $\gamma_s$  is the gyromagnetic ratio of electrons.

The same temperature dependence can be deduced from our relaxation mechanisms for the case of fast diffusion [Eq. (15)]. The ratio of the SLR rate of nuclei due to their interaction with magnetic two-level systems in the case of fast diffusion to the SLR rate reached as a result of twomagnon processes is given by the expression

$$\frac{T_1^{-1}}{T_{11}^{-1}} \sim \left(\frac{a_0}{a}\right)^2 \frac{(\lambda\beta)^{3/\epsilon} \bar{P} \ln (T/\lambda)}{\hbar K \Delta_0^2 \min a^3} \left(\frac{\gamma_\alpha}{\gamma_s}\right)^2.$$
(17)

Substituting here the constant K taken from Ref. 5 and described by

$$K = \hbar^{-1} \alpha^{-\frac{3}{2}} D^2 (4\pi \mu_0 M_0)^{-\frac{1}{2}}$$

 $(M_0$  is the saturation magnetization) and using typical values of the parameters in Eq. (17),  $\lambda \sim 10^{-17}$  erg,  $\beta \sim 10^{-27}$  erg,  $D \sim 10^{-20}$  erg, and  $\Delta_{0 \min} \sim 10^{-18}$  erg, we can easily show that for reasonable concentrations of two-level systems we obtain

$$\frac{1}{T_{\rm I}}\gg\frac{1}{T_{\rm II}},$$

i.e., the relaxation mechanism we propose is more effective than the familiar well-known two-magnon relaxation mechanism. In the case of slow diffusion it follows from Eqs. (14) and (16) that in the investigated range of temperatures the rate of relaxation given by Eq. (14) is higher than the rate of relaxation allowing for two-magnon processes.

We shall now consider the case of nonmagnetic twolevel systems. In this case the ratio of  $T_1^{-1}$  from Eq. (17) to  $T_{II}^{-1}$  from Eq. (16) is

$$\frac{T_1^{-1}}{T_{11}^{-1}} = \frac{n}{n_I} \frac{B^2}{(\hbar^2 \gamma_s \gamma_I / a^3)^2} \frac{\beta^{3/2} (4\pi \mu_0 M_0)^{1/2} \overline{P}}{a^8 \Delta_{0 \text{ min}}^2 e^{-\gamma a}}.$$
 (18)

We can easily see that in this case we have  $T_1^{-1} > T_{II}^{-1}$ , i.e., at low temperatures the dominant role in the SLR of nuclei in amorphous ferromagnets is played by tunnel elementary excitations in the form of two-level systems.

It therefore follows from estimates that if the concentration of two-level systems is reasonable, the proposed nuclear relaxation mechanisms will predominate.

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