Thermoelectric effects in mesoscopics

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We calculate the mesoscopic contribution to the thermoelectric coefficients of a sample of small size (a microjunction of two bulky samples). We show that this contribution can exceed the regular term and can determine by the same token the magnitude of the effect. The thermoelectric power can in this case be either positive or negative, and should have an irregular dependence on the experimental conditions (on the electron chemical potential and on the magnetic field). We calculate the correlation functions of the thermoelectric coefficients and of the thermoelectric coefficient and conductivity.

I. INTRODUCTION

It has become clear in the last few years that at low temperatures the properties of conductors of small size, such as microjunctions, are determined to a considerable degree by quantum interference effects.[1-6] These effects are called mesoscopic, to emphasize that they are produced in all small samples, but reflect for each sample the individual properties that depend on the location of the impurities in the sample. This non-averaging of the properties leads, for example, to a complicated irregular dependence of the microjunction resistance R on the chemical potential μ of the electrons. If $e^2R/\hbar \ll 1$, the scale of the variations is $\delta R \sim e^2R^2/\hbar$, and the characteristic $\Delta\mu$ that leads to a change of order R is itself of the order of $\Delta\mu \sim \hbar/\tau_f$, where τ_f is the time of flight of the electron through the microjunction. If the microjunction takes the form of a bridge of length L between massive shores, then $\tau_f \sim L^2/D$, where D is the electron diffusion coefficient.

To identify the experiments in which mesoscopics is most pronounced, it is necessary apparently to bear in mind either phenomena that do not occur at all if mesoscopics is not taken into account, or are such for which the mesoscopic contributions are larger than the regular ones that correspond to averaging over the impurity locations. The advantage of mesoscopic contributions over the regular ones is that they are greatly altered by a relatively small change of the parameters, such as the electron energy or the chemical potential. Therefore, if the quantity of interest to us is determined by the derivative with respect to energy or by some other parameter, the mesoscopic contribution to this quantity will be large. An example of how a large derivative with respect to a parameter makes the mesoscopic contribution to the differential conductance of a microjunction in the nonlinear region larger than the regular contribution is given in Ref. 7.

We examine in the present article thermoelectric effects for which mesoscopics may turn out to be decisive for the above reason. To be specific, we consider a microjunction between two bulky conductors, in the form of a bridge of length L and a cross section S ($S \ll L^2$). The electric current I and the heat flux Q through the junction are determined by the voltage U across the junction and by the temperature difference ΔT between the shores. At small U and ΔT this connection is linear[8]:

$$I = \frac{1}{R}(U - \alpha\Delta T), \tag{1}$$

$$Q = \beta U - \gamma\Delta T, \tag{2}$$

where R is the electric resistance, α the thermoelectric power, $\beta = \alpha T/R$ by virture of the Onsager relation, and γ is connected with the thermal resistance $R_T = (\gamma - \alpha^2 T/R)^{-1}$. Sivan and Imry[9] called attention to a possible violation of the Onsager relations. We believe that these relations are not violated for a two-contact measurement circuit, and we have verified this fact within the framework of the principal approximation, in the parameter $\varepsilon_F\tau/\hbar \gg 1$, which we assume in this paper.

At low temperatures, all the kinetic coefficients in (1) and (2) are determined by scattering of electrons having an energy close the Fermi energy μ (Ref. 10, §78), with

$$\alpha = \frac{\pi^2 T}{3e}\frac{d\ln R(\mu)}{d\mu}.$$

The resistance R depends on μ, first because the scattering cross section and the density of states depends on the energy, and second because the interference (mesoscopic) correction changes by an amount of order $e^2R^2/\hbar$ when μ changes by $\hbar/\tau_f$. Therefore

$$R(\mu) = R_0(\mu) + \frac{e^2}{\hbar}R_0^2 f_R\left(\frac{\mu\tau_f}{\hbar}, \frac{T\tau_f}{\hbar}\right), \tag{3}$$

$$\alpha = \frac{\pi^2 T}{3e}\left[\frac{d\ln R_0}{d\mu} + \frac{e^2}{\hbar}R_0\frac{\tau_f}{\hbar}f_R'\right]. \tag{4}$$

It is clear from (4) that the mesoscopic contribution to the thermoelectric power

$$\alpha \sim \frac{1}{e}\frac{e^2R_0}{\hbar}\begin{cases} T\tau_f/\hbar & \text{if} \quad T\tau_f/\hbar \ll 1 \\ (T\tau_f/\hbar)^{-1/4} & \text{if} \quad T\tau_f/\hbar \gg 1 \end{cases}, \tag{5}$$

can exceed considerably the regular contribution $\alpha \sim T/e\mu$, It turns out thus that the thermoelectric power of the microjunction is relatively large and can be positive as well as negative. The onset of a large thermoelectric power leads to violation of the Wiedemann-Franz law, since the heat flux due to the presence of the electric current produced by the tem-

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perature gradient becomes comparable with the heat flux due directly to the temperature difference ΔT. Violation of the Wiedemann-Franz law in the presence of a strong energy dependence of the permeability was noted in Ref. 11. In analogy with the procedure used for the electric resistance R, we can investigate the correlation functions for variation of the magnetic field H or of the chemical potential[1)] μ. In this case

$$\langle\delta R(0,0)\,\delta R(\Delta\mu,\Delta H)\rangle=\left(\frac{e^2R_0}{\hbar}\right)^2 F_R\left(\frac{\Delta\mu\tau_f}{\hbar},\frac{\Delta HLw}{\Phi_0}\right),$$

$$\langle\delta\alpha(0,0)\,\delta\alpha(\Delta\mu,\Delta H)\rangle=\frac{1}{e^2}\left(\frac{e^2R_0}{\hbar}\right)^2 F_\alpha\left(\frac{\Delta\mu\tau_f}{\hbar},\frac{\Delta HLw}{\Phi_0}\right), \tag{6}$$

w is the width of the junction, and $\Phi_0=2\pi\hbar c/e$. Thus, $\alpha(\mu)$ or $\alpha(H)$ is a reproducible irregular dependence. How are $\alpha(\mu,H)$ and $R(\mu,H)$ correlated? It turns out that the cross correlation function

$$\langle\delta\alpha(0,0)\,\delta R(\Delta\mu,\Delta H)\rangle=\frac{1}{e}\left(\frac{e^2}{\hbar}R_0\right)^2 F_{\alpha R}\left(\frac{\Delta\mu\tau_f}{\hbar},\frac{\Delta HLw}{\Phi_0}\right) \tag{7}$$

is such that at $\Delta\mu=0$ the function $F_{\alpha R}=0$, i.e., the $\alpha(H)$ and $R(H)$ dependences are not correlated for equal μ. For $\Delta\mu\sim\hbar/\tau_f$, however, the correlator $F_{\alpha R}(H)$ is a function of H which is of the order of unity at $H=0$ and falls to zero at large H; consequently, $\alpha(H)$ and $R(H,\Delta\mu)$ correlate with each other.

By analogy with the electric resistance, one can investigate the thermoelectric effects also at high temperature drops ΔT or at high junction voltages U. The heat flux Q as a function of the voltage U is an irregular function with a characteristic correlation length $U_c\sim\hbar/e\tau_f$. At $U\gg U_c$, just for an electric current,[7] $|\delta Q|\sim(UU_c)^{1/2}$. An increase of ΔT leads not only to an increase of the thermal current $I(\Delta T)$ but also to irregular oscillations. The most important nonlinear effect, in our opinion, is that $\alpha(H)$ and $R(H)$, which are not correlated in the linear region, turn out to be correlated if, for example, the resistance is measured at a finite voltage.

We present below the calculation results which were just described qualitatively. In Sec. 2 we derive general equations for the correlators of the current I and the heat flux Q; these relations are valid both in the linear and nonlinear regimes. These equations are used in Sec. 3 to calculate the correlators for small U and ΔT. The nonlinear effects are the subject of Sec. 4.

2. EXPRESSIONS FOR THE CORRELATORS

We are interested in the correlation functions

$$K_{JJ'}(T_1,T_2;T_1',T_2';U,U';\mu,\mu')$$
$$=\langle J(T_1T_2,U\mu)J'(T_1'T_2',U'\mu')\rangle$$
$$-\langle J(T_1T_2,U\mu)\rangle\langle J'(T_1'T_2',U'\mu')\rangle, \tag{8}$$

where each subscript, J or J', can correspond to one of two fluxes, I or Q; T_1 and T_2 are the temperatures of the junction shores, U is the voltage, and μ is the chemical potential of the conduction electrons. The correlator calculation which is valid in the nonlinear region is best carried out by the Keldysh diagram technique, as in Ref. 7. The technique involves the retarded, advanced, and Keldysh Green's functions $G_\varepsilon^R(\mathbf{r},\mathbf{r}')$, $G_\varepsilon^A(\mathbf{r},\mathbf{r}')$ and $G_\varepsilon^K(\mathbf{r},\mathbf{r}')$, which are connected at equilibrium by the relation

$$G_\varepsilon^K=[G_\varepsilon^R-G_\varepsilon^A][1-2n(\varepsilon/T)],$$

where $n(\varepsilon/T)$ is the Fermi distribution function. After averaging over the impurity arrangement,[12] $\langle G^R\rangle$ and $\langle G^A\rangle$ assume the standard forms,[12] and $\langle G^K\rangle$ satisfies the equation

$$D\nabla^2\langle G_\varepsilon^K(\mathbf{r},\mathbf{r}')\rangle=0 \tag{9}$$

with boundary conditions

$$\langle G_\varepsilon^K(\mathbf{r},\mathbf{r})\rangle|_{x=0,L}=2\pi i\nu\{1-2n[(\varepsilon-\mu\pm eU/2)/T_{1,2}]\}. \tag{10}$$

Here ν is the density of states on the Fermi level, $D=v_F^2\tau/3$ is the diffusion coefficient, and τ is the free-path time. The electric current I and the heat flux Q can be expressed in terms of G_ε^K:

$$\mathbf{I}=-i\frac{e\hbar}{2m}\int_{-\infty}^{+\infty}d\varepsilon\int dS(\nabla-\nabla')G_\varepsilon^K(\mathbf{r},\mathbf{r}')|_{\mathbf{r}'=\mathbf{r}}, \tag{11}$$

$$\mathbf{Q}=-i\frac{\hbar}{2m}\int_{-\infty}^{+\infty}d\varepsilon(\varepsilon-\mu)\int dS(\nabla-\nabla')G_\varepsilon^K(\mathbf{r},\mathbf{r}')|_{\mathbf{r}=\mathbf{r}'}. \tag{12}$$

Averaging (11) and (12) over the locations of the impurities and using (9) and (10), we obtain expressions for $\langle I\rangle$ and $\langle Q\rangle$, the current and heat flux through the junction:

$$\langle I\rangle=\frac{eS}{L}\int_{-\infty}^{+\infty}d\varepsilon\,\nu(\varepsilon)D(\varepsilon)\Delta n(\varepsilon), \tag{13}$$

$$\langle Q\rangle=\frac{S}{L}\int_{-\infty}^{+\infty}d\varepsilon(\varepsilon-\mu)\nu(\varepsilon)D(\varepsilon)\Delta n(\varepsilon),$$

$$\Delta n(\varepsilon)=n[(\varepsilon-\mu-eU/2)/T_1]-n[(\varepsilon-\mu+eU/2)/T_2]. \tag{14}$$

From (13) and (14), assuming U and T_2-T_1 to be small, we obtain the equations of the linear transport theory for R, R_T, α, and β (Ref. 8).

The correlation functions $K_{JJ'}$ are determined by the contributions of the diagrams shown in Fig. 1. The thick dot of each diagram corresponds to a current vertex ($e\mathbf{p}/m$ for the electric current and $(\varepsilon-\mu)\mathbf{p}/m$ for the heat flux), the stubs correspond to the function $\langle G^K(r,r)\rangle$, and the ladders to two-particle Green's functions—cooperons $P^{(C)}$ and diffusion $P^{(D)}$, which satisfy the relations

$$\{-i[\omega+\mu-\mu'+e\varphi(\mathbf{r})-e\varphi'(\mathbf{r})]/\hbar-D\partial_{D,C}^2\}P_\omega^{(D,C)}(\mathbf{r},\mathbf{r}')$$
$$=2\pi\nu\delta(\mathbf{r}-\mathbf{r}'), \tag{15}$$

where $\varphi(\mathbf{r})$ is the electric potential connected with the voltage on the junction by the relation $\varphi(L)-\varphi(0)=U$,

$$\partial_D=\frac{\partial}{\partial\mathbf{r}}-\frac{e}{c\hbar}(\mathbf{A}-\mathbf{A}'),\quad \partial_C=\frac{\partial}{\partial\mathbf{r}}-\frac{e}{c\hbar}(\mathbf{A}+\mathbf{A}'),$$

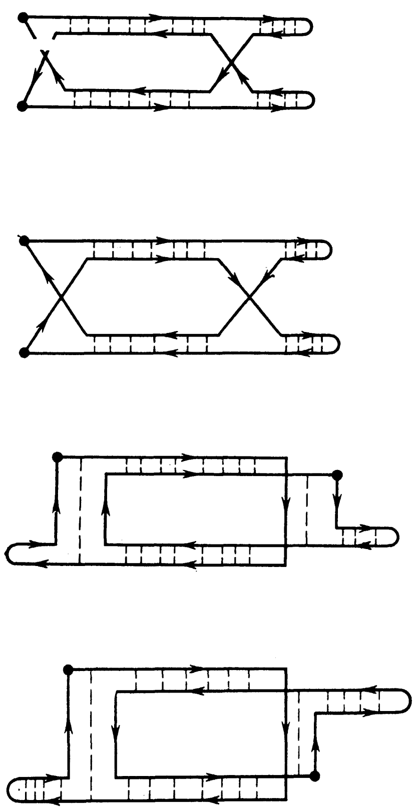

FIG. 1.

and $\mathbf{A}$ is the vector potential of the magnetic field. On the lateral surface of the conducting channel, $P^{(D,C)}$ satisfy the conditions

$$(\mathbf{n}\partial_{D,C})P_\omega^{(D,C)}(\mathbf{r},\mathbf{r}')=0, \tag{16}$$

and for $x=0$ and L, the condition

$$P_\omega^{(D,C)}(\mathbf{r},\mathbf{r}')\,|_{x=0,L}=0. \tag{17}$$

The sums of the contributions of all the diagrams of Fig. 1 are

$$K_{II}(T_1,T_2,U,\mu;T_1',T_2',U',\mu')$$
$$=\frac{1}{(2\pi)^4}\left(\frac{eSD}{\nu\hbar L^2}\right)^2\iint\limits_0^L dx_1\,dx_2\iint\limits_{-\infty}^{+\infty} d\varepsilon\,d\varepsilon'\,\mathscr{H}_{\varepsilon-\varepsilon'}(x_1,x_2)\,\Delta n\,\Delta n', \tag{18}$$

$$K_{QQ}=\frac{1}{(2\pi)^4}\left(\frac{SD}{\nu\hbar L^2}\right)^2\iint\limits_0^L dx_1\,dx_2\iint\limits_0^\infty d\varepsilon\,d\varepsilon'\,\mathscr{H}_{\varepsilon-\varepsilon'}(x_1,x_2)$$
$$\times(\varepsilon-\mu)(\varepsilon'-\mu')\,\Delta n\,\Delta n', \tag{19}$$

$$K_{QI}=\frac{e}{(2\pi)^4}\left(\frac{SD}{\nu\hbar L^2}\right)^2\iint\limits_0^L dx_1\,dx_2\iint\limits_0^\infty d\varepsilon\,d\varepsilon'\,\mathscr{H}_{\varepsilon-\varepsilon'}(x_1,x_2)$$
$$\times(\varepsilon-\mu)\,\Delta n\,\Delta n'. \tag{20}$$

We have introduced here the notation

$$\mathscr{H}_\omega(x_1,x_2)=|P_\omega^{(D)}(x_1,x_2)|^2+|P_\omega^{(C)}(x_1,x_2)|^2$$
$$+{}^1/_2\,\mathrm{Re}\{[P_\omega^{(D)}(x_1,x_2)]^2+[P_\omega^{(C)}(x_1,x_2)]^2\}, \tag{21}$$

and $\Delta n'$ is obtainable from Δn by making the substitutions $\varepsilon\to\varepsilon'$, $\mu\to\mu'$, $U\to U'$, $T_{1,2}\to T'_{1,2}$.

Equations (18)–(20) together with (11) and (12) describe the dependences of the fluxes I and Q on T_1, T_2, and U. Actually, the experiment can be performed with I or Q specified rather than $T_{1,2}$ and U. Thus, for example, if the shores of the microjunction are not shorted by an external circuit, a potential difference U is produced between them, given by the relation

$$I=I(T_1,T_2,U)=0.$$

Since the function $I(U)$ can be nonmonotonic,[7] the function $U(T_1,T_2)$ can turn out to be non-single-valued and various nonlinear phenomena are possible (hysteresis, relation oscillations of the voltage and temperature, and others). A detailed treatment of these phenomena calls for analysis of the processes in the external circuit, and will not be dealt with here. This possibility, however, must be borne in mind when an experiment is planned.

3. LINEAR THERMOELECTRIC EFFECTS

For $T_{1,2}=T\pm\Delta T/2$, $\Delta T\ll T$, and small U, the fluxes I and Q are linear in U and ΔT. The coefficients R, α, and β are given under these conditions by

$$\langle g\rangle=\langle 1/R\rangle=e^2\nu DS/L,$$

$$\langle\beta\rangle=-\frac{\pi^2T^2}{12e}\frac{d\ln R}{d\mu},\qquad \langle\alpha\rangle=\langle\beta\rangle/T\langle g\rangle. \tag{22}$$

The fluctuations of g and β can be determined by using (18)–(20):

$$\langle\delta g\delta\beta\rangle=\langle\alpha\rangle\langle(\delta g)^2\rangle T, \tag{23}$$

$$\langle(\delta\beta)^2\rangle=\{\langle\alpha\rangle^2\langle(\delta g)^2\rangle+\langle g\rangle^2\langle(\delta\alpha)^2\rangle\}T. \tag{24}$$

The fluctuations of the conductance $\langle(\delta g)^2\rangle$ were calculated in Refs. 4 and 6, and Eq. (24) can be rewritten by using the relation $\beta=g\alpha T$, which is valid also without averaging. Therefore $\langle\delta g\delta\alpha\rangle=0$,[2)] and

$$\langle\delta\alpha\,\delta g\rangle\sim\lim_{\Delta\to\infty}\int\limits_0^\Delta(\partial g^2/\partial\mu)\,d\mu/\Delta=0.$$

$$\langle(\delta\alpha^2)\rangle=\left(\frac{eR_0}{\pi^2\hbar}\right)^2\left(\frac{TL^2}{D\hbar}\right)^2\Phi\left(\frac{TL^2}{D\pi^2\hbar}\right),\qquad\Phi(0)=0.55. \tag{25}$$

A plot of $\langle[\delta\alpha(T)^2]\rangle$ is shown in Fig. 2. It is clear from (24) and (25) that for $T\ll D\hbar/L^2$ and $(e^2R/\hbar)(L^2/D\hbar)>d\ln R/d\mu$ the fluctuation of α can become larger than $\langle\alpha\rangle$.

The inequality $\langle(\delta\alpha)^2\rangle>\langle\alpha\rangle^2$ is manifested by the fact that the real thermoelectric power can have either sign, and is furthermore very sensitive to variation of the magnetic field H. The correlation function of such a dependence is given in the Appendix for $HLw\gg\Phi_0\gg\Delta HLw$:

$$K_g(H,\Delta H)=\langle g(H+\Delta H)\,g(H)\rangle-\langle g(H+\Delta H)\rangle\langle g(H)\rangle$$
$$=\langle[\delta g(H,T)]^2\rangle\begin{cases}\dfrac{\pi^4}{4}[\varphi^{-3}\operatorname{cth}\varphi+\varphi^{-2}\operatorname{sh}^{-2}\varphi-2\varphi^{-4}], & T\ll\dfrac{D\hbar\pi^2}{L^2},\\ f_{2g}\left(\varphi^2\dfrac{D\hbar}{TL^2}\right), & \dfrac{\pi^2D\hbar}{L^2}\ll T\ll\dfrac{D\hbar\pi^2}{s},\end{cases} \tag{26}$$

$$K_\alpha(H,\Delta H)=\langle\alpha(H+\Delta H)\,\alpha(H)\rangle-\langle\alpha(H+\Delta H)\rangle\langle\alpha(H)\rangle$$
$$=\langle[\delta\alpha(H,T)]^2\rangle\begin{cases}\dfrac{\pi^8}{32}\left[2\dfrac{\operatorname{ch}^2\varphi}{\varphi^4\operatorname{sh}^4\varphi}+\dfrac{5}{\varphi^5\operatorname{sh}^2\varphi}+\dfrac{5\operatorname{cth}\varphi}{\varphi^7}+\dfrac{4\operatorname{cth}\varphi}{\varphi^5\operatorname{sh}^2\varphi}\right.\\ \left.-\dfrac{2}{3\varphi^4\operatorname{sh}^2\varphi}-\dfrac{16}{\varphi^8}\right], & \dfrac{D\hbar\pi^2}{L^2}\gg T\\ f_{2\alpha}\left(\varphi^2\dfrac{D\hbar}{TL^2}\right), & \dfrac{D\pi^2\hbar}{L^2}\ll T\ll D\pi^2\hbar/s,\end{cases}$$
$$\varphi^2=(e\Delta HwL/c\hbar)^2/12. \tag{27}$$

The functions f_{2g} and $f_{2\alpha}$ were determined numerically and are plotted in Fig. 3. The change of the scale of the magnetic field on going from low to high temperature was first noted in Ref. 13.

The thermoelectric power α and the conductance g at different H are uncorrelated, as before:

$$\langle\alpha(H)g(0)\rangle=\langle\alpha(H)\rangle\langle g(0)\rangle.$$

The situation is quite different for MIS structures, in which the electron chemical potential μ can be varied:

$$K_\alpha(\Delta\mu)=\langle\alpha(\mu+\Delta\mu)\alpha(\mu)\rangle-\langle\alpha(\mu+\Delta\mu)\rangle\langle\alpha(\mu)\rangle$$
$$=\langle[\delta\alpha(\mu)]^2\rangle f_{\alpha\alpha}(\Delta\mu L^2/\pi^2D\hbar), \tag{28}$$

$$K_{\alpha g}(\Delta\mu)=\langle\alpha(\mu+\Delta\mu)g(\mu)\rangle-\langle\alpha(\mu+\Delta\mu)\rangle\langle g(\mu)\rangle$$
$$=0.36[\langle(\delta\alpha)^2\rangle\langle(\delta g)^2\rangle]^{1/2}f_{\alpha g}(\Delta\mu\Delta^2/\pi^2D\hbar). \tag{29}$$

Plots of the functions $f_{\alpha\alpha}(x)$ and $f_{\alpha g}(x)$ are shown in Fig. 4. Thus, the $\alpha(\mu)$ dependence has the form of mesoscopic fluctuations of the same type as the fluctuations of $g(\mu)$ (Ref. 6), but larger in relative amplitude and possibly with the sign reversed. In contrast to the fluctuations with change of the magnetic field, the functions $\alpha(\mu)$ and $g(\mu)$ do not correlate when μ and μ' are equal, but for unequal μ and μ' a correlation does appear and is a maximum at $|\mu-\mu'|=0.47\pi^2D\hbar/L^2$.

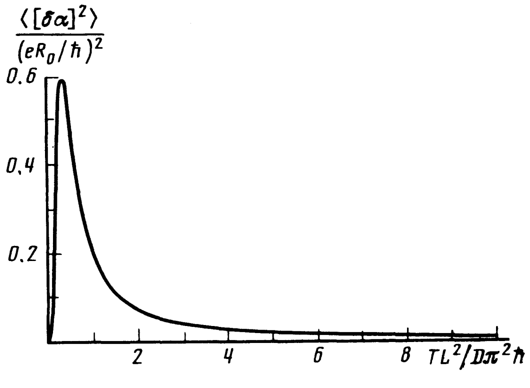

FIG. 2. Square of the mesoscopic thermoelectric power $\langle(\delta\alpha)^2\rangle$, normalized to $(eR_0/\hbar)^2$ as a function of the temperature $TL^2/\pi^2D\hbar$ for $TS/\pi^2D\hbar\ll1$.

4. NONLINEAR EFFECTS

Under real experimental conditions it may be convenient to measure the thermoelectric effects not only for small ΔT and U but also for large ones. Equations (15) and (18)–(21) permit in principle to consider mesoscopic effects also in the nonlinear region. Let us examine qualitatively the main distinctive features that appear in this case. If the junction shore temperatures are equal, the correlator of the heat fluxes is calculated in the same way as the current-voltage characteristic.[7] At high voltage, therefore, the heat flux Q oscillates irregularly as a function of the voltage U, and reverses sign. The "period" of such oscillations is $U_c\sim\hbar/\tau_fe$.

In another version of the experiment, specified temperatures T_1 and T_2 are maintained on the shores of the open-circuited junction. The zero total current I through the junction consists in this case of the conduction current U/R_0, the thermocurrent $-\langle\alpha\rangle(T_1-T_2)/R_0$, and the mesoscopic term δI in which the thermocurrent cannot be separated from the conduction current:

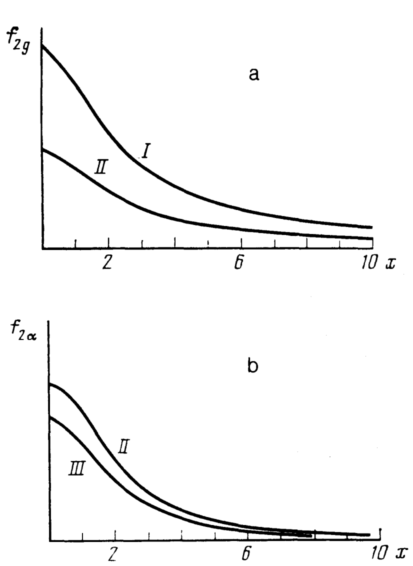

FIG. 3. The functions f_{2g} (a) and $f_{2\alpha}$ (b) calculated for $TL^2/\pi^2D\hbar=3$ (*I*), 5 (*II*), 6(*III*).

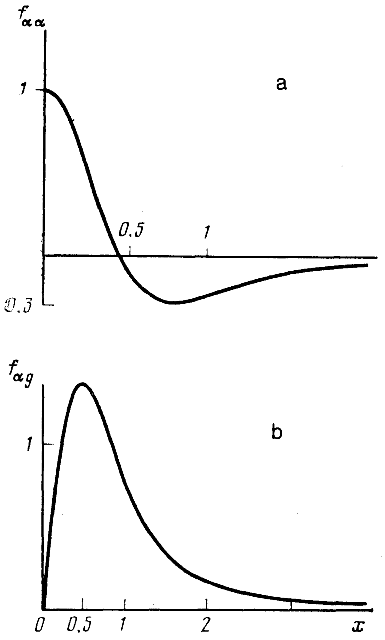

FIG. 4. The functions $f_{\alpha\alpha}$ (a) and $f_{\alpha g}$ (b).

$$I=\frac{U}{R_0}-\frac{\langle\alpha\rangle}{R_0}(T_1-T_2)+\delta I(U,T_1,T_2). \tag{30}$$

For $U<U_c$ we have

$$U\left(1+R_0\frac{\partial\delta I}{\partial U}\right)=\langle\alpha\rangle(T_1-T_2)-\delta I(0,T_1,T_2). \tag{31}$$

The parentheses in the left-hand of (31) contain R_0 multiplied by the increment to the differential conductivity of the junction. If this product is not small, the difficulties noted at the end of Sec. 2 are encountered. If this danger is neglected, we get

$$U=\langle\alpha\rangle(T_1-T_2)+\delta U,$$

$$\langle\delta U\rangle=0,\quad \langle\delta U\delta U'\rangle=R_0^2K_{II}(T_1,\ T_2,\ T_1',\ T_2',\ 0,\ 0). \tag{32}$$

As noted in the Introduction, the thermoelectric power is determined by the derivative $d\ln R(\mu)/d\mu$. Under conditions of a mesoscopic sample it must be borne in mind that the result contains R averaged over the energies in a layer of thickness T near the Fermi level. With change of temperature, the mean value and its derivative fluctuate. Therefore if the temperature difference $|T_1-T_2|\ll\hbar/\tau_f$ is fixed and the thermoelecric power α is measured as a function of T_1, one can expect irregular oscillations with a characteristic temperature period of order $\hbar/\tau_f$. Finally, we note that if, for example, the conductivity is measured in the nonlinear region of the voltages ($g=I/U$ or $g_D=\partial I/\partial U$), then g (g_D) correlates with the thermolectric power α if the magnetic field and the chemical potentials are equal:

$$\langle\delta g(H)\delta\alpha(H)\rangle\neq 0.$$

This correlation is a maximum at $U\sim U_c$.

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APPENDIX

Let us find expressions for $K_g(H,H+\Delta H)$ and $K_\alpha(H,H+\Delta H)$ in the case when the junction takes the form of a bridge whose length L is much larger than the width $w(L\gg w)$ and the magnetic field is strong ($eHw^2/c\hbar\gg 1$), so that the cooperon contribution is suppressed and the regular dependence of $\langle(\delta\alpha^2)(H)\rangle$ on H can be neglected. Let also

$$e\Delta Hw^2/\hbar c\ll 1. \tag{A.1}$$

Then, using the Landau gauge $\Delta\mathbf{A}=(-Hy,0,0)$, we can write down equations for the diffusion and for its boundary conditions, in the form

$$\left[D\left(-i\frac{\partial}{\partial x}+\frac{e}{c\hbar}\Delta Hy\right)^2-D\left(\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}\right)-i\omega\right]P_\omega^{(D)}(\mathbf{r},\mathbf{r}')$$

$$=2\pi\nu\delta(\mathbf{r}-\mathbf{r}'), \tag{A.2}$$

$$\partial P^{(D)}/\partial y,\qquad \partial P^{(D)}/\partial z|_s=0;\qquad P^{(D)}|_{x=x'=0,L}=0. \tag{A.3}$$

Bearing in mind the inequality (A.1) and the boundary conditions (A.3), we assume thet $P^{(D)}$ depends only on x and x'. Averaging (A.2) over y, we obtain

$$\left[-D\frac{\partial^2}{\partial x^2}+D\frac{e^2w^2}{12c^2\hbar^2}(\Delta H)^2-i\omega\right]\bar{P}^{(P)}(x,x')=2\pi\nu S\delta(x-x'). \tag{A.4}$$

All the equations that follow contain the integral

$$J=\iint d\mathbf{r}\,d\mathbf{r}'\{|P^{(D)}|^2+\operatorname{Re}(P^{(D)})^2/2\},$$

which is equal to

$$J=2\pi^2\nu^2S^2\sum_{n=1}^{\infty}\left(3-\frac{2\omega^2L^4}{\pi^4D^2}\right)\times\left\{\left[\frac{\pi^2Dn^2}{L^2}+\frac{De^2w^2(\Delta H)^2}{12c^2\hbar^2}\right]^2+\omega^2\right\}^{-1}. \tag{A.5}$$

At $T\ll\pi^2D\hbar/L^2$ each of the terms in the sum of (A.5) can be expanded in powers of $\omega^2L^2/D^2\pi^4$. Retaining the first two terms, we have

$$J=J_0+J_1\frac{\omega^2L^4}{\pi^4D^2}, \tag{A.6}$$

where

$$J_0=\frac{3}{2}\left(\frac{\nu\pi SL^2}{D}\right)^2\left(\frac{1}{\varphi^2\operatorname{sh}^2\varphi}+\frac{\operatorname{cth}\varphi}{\varphi^3}-\frac{2}{\varphi^4}\right),$$

$$\varphi^2=\frac{1}{12}\left(\frac{e\Delta HwL}{c\hbar}\right)^2,$$

$$J_1=-5\left(\frac{\nu SL^2}{\pi D}\right)^2\left\{\frac{1}{8}\frac{\operatorname{ch}^2\varphi}{\varphi^4\operatorname{sh}^4\varphi}+\frac{5}{16}\varphi^{-6}\operatorname{sh}^{-2}\varphi+\frac{5}{16}\varphi^{-7}\operatorname{cth}\varphi-\frac{1}{24}\varphi^{-4}\operatorname{sh}^{-2}\varphi+\frac{1}{4}\varphi^{-5}\operatorname{sh}^{-2}\varphi\operatorname{cth}\varphi-\varphi^{-8}\right\}. \tag{A.7}$$

The correlator $K_g(H,H+\Delta H)$ is determined by the first term of the expansion (A.6) and is equal to

$$K_g=\frac{\pi^4}{4}\langle\delta g(H,T)\rangle\left\{-\frac{2}{\varphi^4}+\frac{\operatorname{cth}\varphi}{\varphi^3}+\frac{1}{\varphi^2\operatorname{sh}^2\varphi}\right\}, \tag{A.8}$$

and the second term of the expansion is important in the calculation of $K_\alpha(H,H+\Delta H)$. As a result we have

$$K_\alpha(H,\Delta H)=\langle[\delta\alpha(H,T)]^2\rangle\frac{\pi^8}{32}\left\{\frac{2\operatorname{ch}^2\varphi}{\varphi^4\operatorname{sh}^4\varphi}+\frac{5}{\varphi^6\operatorname{sh}^2\varphi}+\frac{5\operatorname{cth}\varphi}{\varphi^7}+\frac{4\operatorname{cth}\varphi}{\varphi^5\operatorname{sh}^2\varphi}-\frac{2}{3\varphi^4\operatorname{sh}^2\varphi}-\frac{16}{\varphi^8}\right\}. \tag{A.9}$$

At high temperatures $T\gg\pi^2D\hbar/L^2$ the expansion (A.6) is insufficient. If at the same time $T\gg\pi^2D\hbar/w^2$, the integral J is nonetheless determined by the sum (A.5), which is equal to

$$J=2\left(\frac{\pi\nu SL^2}{D}\right)^2\left[3S_1-2\left(\frac{\omega L^2}{\pi^2D}\right)^2S_2\right], \tag{A.10}$$

where

$$S_1=\sum_{n=1}^{\infty}\{[(\pi n)^2+\varphi^2]^2+a^4\}^{-1},\quad S_2=\sum_{n=1}^{\infty}\{[(\pi n)^2+\varphi^2]^2+a^4\}^{-2},\quad a^2=\frac{\omega L^2}{D}, \tag{A.11}$$

$$S_1=\frac{1}{2R^3\sin\theta}\frac{\cos\theta/2\sin(2R\sin\theta/2)+\sin\theta/2\operatorname{sh}(2R\cos\theta/2)}{\operatorname{ch}(2R\cos\theta/2)-\cos(2\sin\theta/2)}-\frac{1}{2(\varphi^4+a^4)}$$

$$S_2=\frac{1}{8R^7}\frac{[(1/\sin\theta/2+\sin3\theta/2)\sin(2R\sin\theta/2)+(1/\cos\theta/2-\cos3\theta/2)\operatorname{sh}(2R\sin\theta/2)]}{\operatorname{ch}(2R\cos\theta/2)-\cos(2R\sin\theta/2)}+\frac{1}{4R^6}\frac{\cos\theta[1-\operatorname{ch}(2R\cos\theta/2)\cos(2R\sin\theta/2)]+\sin\theta\sin(2R\sin\theta/2)\operatorname{sh}(2R\cos\theta/2)}{[\operatorname{ch}(2R\cos\theta/2)-\cos(2R\sin\theta/2)]^2}-\frac{1}{2(\varphi^4+a^4)^2}. \tag{A.12}$$

Here $R=(\varphi^4+a^4)^{1/4}$ and $\theta=\arctan(a^2/\varphi^2)$. We obtain finally

$$K_g(H,\Delta H)=\langle[\delta g(H,T)]^2\rangle f_{2g}\left(\varphi^2\frac{D\hbar}{TL^2}\right), \tag{A.13}$$

$$K_\alpha(H,\Delta H)=\langle[\delta\alpha(H,T)]^2\rangle f_{2\alpha}\left(\varphi^2\frac{D\hbar}{TL^2}\right). \tag{A.14}$$

The functions $f_{2g}(x)$ and $f_{2\alpha}(x)$ were obtained by numerically integrating over the energies, and their plots are contained in the main text.

[1)]The chemical potential in MIS structures can be varied by varying the gate voltage V_g.
[2)]This equation follows, for example, from the fact that $\alpha\sim\partial g/\partial\mu$, and by virtue of the periodicity we have

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