Current-stimulated quantum interference in composite NbTi–Cu superconductors

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The current-voltage characteristics were determined in a weak magnetic field (up to ~ 0.2 Oe) at 1.85 K for a sample cut from a fine-grained composite NbTi–Cu wire in such a way that the measuring current flowed at right angles to the superconducting filaments. The dependence of the critical current and the sample resistance on a magnetic field directed along superconducting filaments was oscillatory. The oscillation period was determined quite accurately by the geometric parameters of the structure of the sample. A detailed analysis of the mobility of current excitations demonstrated a strong interaction between topological vortices in the critical region near a phase transition associated with the disappearance of the equilibrium concentration of topological dipoles as a result of cooling. The experimental results exhibited full scaling invariance of the observed nonlinear effects and quantum interference.

1. INTRODUCTION

In the last decade the attention of experimentalists and theoreticians has been attracted increasingly to phase transitions of the Kosterlitz–Thouless–Berezinskiĭ type,[1,2] involving ordering of topological defects and establishment of a topological long-range order in two-dimensional systems. Below the critical temperature T_{c0} of such a transition only pairs of topological defects (vortices) can exist (in the form of coupled vortices of the opposite sign forming topological dipoles), whereas for $T > T_{c0}$ these pairs dissociate rapidly with increasing temperature. The appearance of the first free vortices at $T = T_{c0}$ destroys the topological long-range order and greatly alters the macroscopic properties of a sample. Similar phase transitions have been observed recently also in three-dimensional systems.[3,4] In the case of composite NbTi–Cu superconductors it has been established that electrical and magnetic properties of bulk samples (with sample thickness several orders of magnitude greater than the coherence length of copper and the penetration depth of the superconductor) are governed at low temperatures by two-dimensional topological excitations, the collective behavior of which depends on the amplitude of the applied magnetic field or of the measuring current, and changes qualitatively at the temperatures of topological phase transitions.

The basic phase diagram of such a system is shown in Fig. 1 for scaling-invariant quantities I/I_c and R_0/R_n, where I_c is the critical current of the system without allowance for thermal fluctuations; R_0 is the transverse resistance of a sample in the limit $I \to 0$; $R_n = R_0$ ($T \gtrsim T_{cs}$); T_{cs} is the temperature of the superconducting transition in NbTI filaments which form a regular $2D$ triangular lattice. The average energy of the interaction E_J of superconducting elements is a function of temperature and is characterized by a critical current I_c, i.e., $E_J \propto I_c$ (line A–C in Fig. 1). The behavior of the resistance (line C–D–E) is in agreement with the theory of Halperin and Nelson[5] and it is governed by fluctuations of the superconducting phase in the individual elements (C–D–F) and free vortices (D–E). The temperatures T_v and T_c^0 represent the formation of the first stable topological vortices and dipoles, respectively, as the short-range order increases due to cooling. The topological phase transition of the $3D$ system at $T = T_{c0}$ is very similar to the Kosterlitz–Thouless–Berezinskiĭ transition in $2D$ systems except that now the characteristic universal discontinuity of the critical exponent η is a function of the effective dimensionality of the sample and, therefore, of the applied current I/I_c (Ref. 4). The departure of the critical current of the system I_s (line B–E in Fig. 1) from I_c at temperatures $T_c < T < T_{c0}$ is due to an equilibrium concentration of dipoles or multipoles, which recombine completely at $T = T_c$. In addition to thermal fluctuations, topological defects are excited by the measuring current, the influence of which on the total resistance of the system increases as a result of cooling beginning from $T = T_v$ and becomes dominant at $T \leqslant T_{c0}$. When the coherence length of copper $\xi_N(T)$ reaches a value equal to the distance between neighboring superconducting filaments ($T = T_{cI}$) as a result of cooling, a sample goes over to a qualitatively new state similar to a homogeneous superconductor. The critical current I_s represents the effective coupling energy in a system of superconducting filaments subject to topological defects or the average concentration of superconducting electrons per unit cross-sectional area n_s, i.e., $I_s \propto n_s$. On the other hand, the effective critical current depends on the mobility of free vortices. Depending on the dimensions of a sample below T_{c0} the first free vortices appear at currents $I \geqslant I_m$ (line A–E in Fig. 1). In the case of strong coupling in a regular lattice of superconducting elements these topological defects remain pinned in deep potential wells at the center of a unit cell and cannot contribute to energy dissipation. A significant mobility of vortices is observed only in the range $I \geqslant I_c$ when the effective potential barrier between neighboring sites disappears.[6,7]

The application of a static magnetic field induces a certain concentration of vortices of one sign in addition to current and thermal topological excitations. In a homogeneous superconductor such vortices would form a regular triangular lattice the parameter of which is governed by the value of H. In regular superconducting structures a vortex lattice of this type interacts with the periodic potential of the system and the mobility of free vortices is governed by the commensurability or incommensurability of these two lattices. If the parameters of the lattices are commensurable, then free vortices are localized in deep potential wells and the effective

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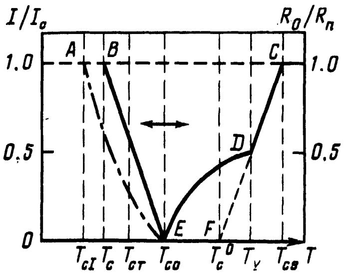

FIG. 1. Basic phase diagram.

barrier between these wells decreases considerably in the incommensurable case. Consequently, a smooth variation of the applied magnetic field can induce regular oscillations of the transport properties of such samples. Oscillations of this type have been observed for film superconducting structures at temperatures $T \gtrsim T_{c0}$ (Refs. 8–11).

The present paper reports the first results of experimental investigations of this type in the case of a three-dimensional system in the form of a composite superconductor near T_c (i.e., at $T < T_{c0}$, in contrast to the situation in the earlier investigations), where previous measurements[3,4] revealed strong anomalies of resistive behavior and magnetic susceptibility. We paid special attention to the mobility $\mu = \mu(T, I, H)$ of the vortex system because it can provide valuable information on the mechanism of the interaction of various topological defects and the nature of different topological phase transitions.

2. MEASUREMENTS AND RESULTS

Our sample was a slab of $0.9 \times 0.5 \times 0.2$ mm dimensions cut from a composite fine-grained superconducting NbTi–Cu wire in such a way that the measuring current flowed at right angles to the superconducting filaments.

This sample was subjected to an external magnetic field and oriented in such a way that this field coincided with the direction of the superconducting filaments. At a temperature $T = 1.850$ K, kept constant to within ~ 1 mK, the current-voltage characteristics were determined for different values of the external field H, which was kept constant to within $\pm 0.01\%$. A detailed description of the preparation of the samples and of the method used to determine the current-voltage characteristics was given in Ref. 4.

Figure 2 shows the current-voltage characteristics obtained in this way. The vertical axis represents the readings of a secondary instrument averaged over two direction of the measuring current. The curves are numbered in the same way as in Table I and also in Figs. 3, 4, and 7. A selected field H was applied at a temperature $T > T_{cs}$, since the behavior of the current-voltage characteristics depended strongly on the sequence in which the field and temperature were altered.

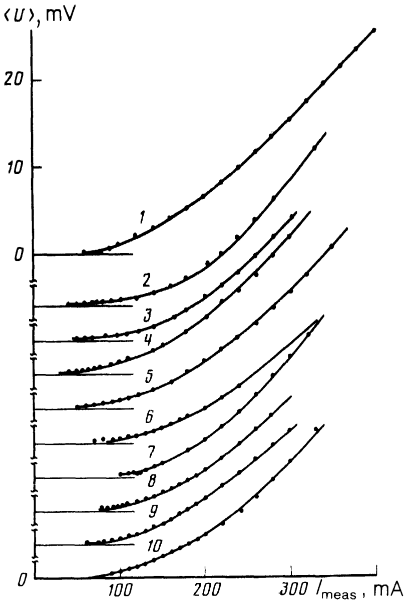

FIG. 2. Experimental current-voltage characteristics obtained for different values of the magnetic field. The curves are numbered in the same way as in Table I.

Each successive curve was thus recorded after heating the sample above T_{cs}. The values of Φ/Φ_0 listed in Table I were deduced from

$$\Phi/\Phi_0 = BS_{\text{эл}}/\Phi_0;$$

where $\Phi_0 = 2 \times 10^{-7}$ G cm^2,

$$S_{\text{el}} = 3^{1/2}(d_{\text{scf}} + a)^2/4,$$

where $d_{\text{scf}} + a = 22.5 \pm 0.5$ μm is the period of the structure of a sample; d_{scf} is the diameter of a superconducting filament; and a is the shortest distance between two neighboring filaments. The magnetic field of the earth was not cancelled in our experiments and it corresponded approximately to $\Phi/\Phi_0 \approx 4$–5. It is clear from Fig. 2 that the main features of the current-voltage characteristic (effective critical current, maximum slope of the curve) changed monotonically on increase in the magnetic field. Moreover, in this range the changes in the measuring current had practically no linear region $\partial U/\partial I = \text{const}$.

The resistance of a sample was defined as $R = U/I \sim \mu n_f$, where n_f is the number of free vortices and μ is the mobility of these vortices[3–5,7]; it is plotted in Fig. 3 for different values of the field in the form $\ln R \propto I^{-1}$. Clearly, at high values of the current all the curves could be extrapolated to a straight line with a slope that varied strongly with the magnetic field. The intersection of the rectilinear part with the Y axis gave the value of R_∞, whose dependence on H is plotted in Fig. 5. We can distinguish two types of curve in Fig. 3,

TABLE I.

	No. of curve									
	1	2	3	4	5	6	7	8	9	10
H, Oe	0	0.0291	0.0437	0.0585	0.0928	0.1160	0.1458	0.1740	0.1929	0.2315
Φ/Φ_0	0	0.320	0.481	0.643	1.021	1.276	1.604	1.914	2.122	2.548

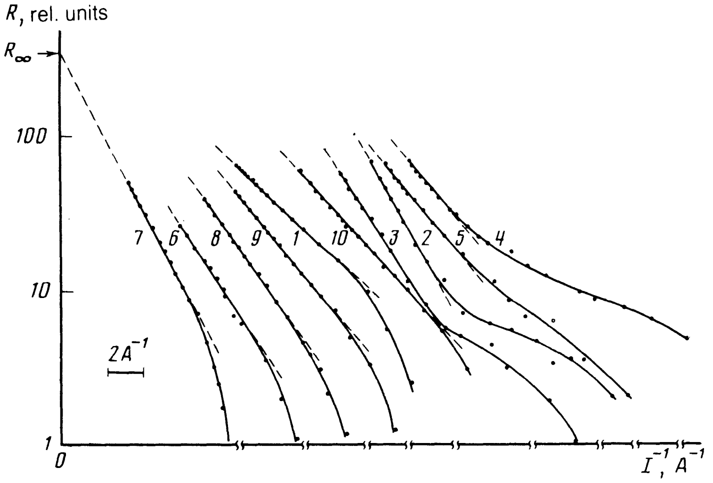

FIG. 3. Dependence of R (on a logarithmic scale) on I^{-1}. The curves are numbered in the same way as in Table I.

which differ in the range of low currents. For example, traces 2–5 exhibit an inverse bending indicating an additional contribution to the total current in the limit $I \to 0$. This difference in the behavior of the resistance in different fields H is demonstrated particularly clearly in the linear representation $R(I)$ used in Figs. 4a and 4b. In all fields there are rectilinear parts of the dependence $R(I)$, but in group b these parts are shifted toward higher resistances. Extrapolation of the rectilinear parts to the X axis gives the critical current I_c^{exp}, whose dependence on the magnetic field is also plotted in Fig. 5. It is clear from Fig. 5 that the dependences of I_c^{exp}, $\partial(\ln R)/\partial(I^{-1})$, and R_∞ on the magnetic field are qualitatively similar and oscillatory. The resistance of a sample corresponding to $I_{\text{meas}} = \text{const}$ varies in antiphase with the other three quantities but the period is still the same.

3. ANALYSIS OF EXPERIMENTAL RESULTS

We shall consider our results on the basis of an expression obtained earlier[4]:

$$R(T, I) = R_\infty c_d \exp(-a_0 I_c / I), \tag{1}$$

where I_c, R_∞, c_d, and a_0 can, in principle, depend on H. The quantity $R_\infty(H)$ can be determined experimentally (Figs. 3 and 5) and, consequently,

$$\lim_{I \gg I_c} c_d = 1,$$

so that c_d represents the relative influence of the measuring current on the vortex mobility in a given field. It is clear from Fig. 3 that when I is sufficiently high, the general behavior of the resistance as a function of the current can be described by an exponential factor and, therefore, it represents the vortex concentration and not the vortex mobility. Hence, we obtain

$$a_0 I_c = -d(\ln R)/d(I^{-1}). \tag{2}$$

Expansion of Eq. (1) as a Taylor series near a point of inflection $\mu(I) \approx \text{const}$ [in the approximation that $I_v = a_0 I_c/2$ at $I \approx I_v$] gives $R(I) \propto (I/I_c - a_0/4)$, so that the values of I_c^{exp} found by linear extrapolation of the dependence $R(I)$ to $R = 0$ (Fig. 4) are equal to $I_c(H)$ apart from $|1 - a_0/4|$ ($\sim 15\%$). We shall therefore assume that $I_c(H) \equiv I_c^{\text{exp}}(H)$. We can see from Fig. 5 that the value of a_0, deduced from Eq. (2) using the experimental values $d \ln R/d(I^{-1})$ and $I_c^{\text{exp}}(H)$, is independent of the magnetic field and its absolute value $a_0 = 4.5 \pm 0.2$ agrees well with that obtained earlier ($a_0 = 4.6$ is given in Ref. 4).

The observed oscillations of the quantities $R_\infty(H)$, $I_c(H)$, etc., as functions of the magnetic field (Fig. 5) have the period $\Delta H = 0.12 \pm 0.01$ Oe, which corresponds to the

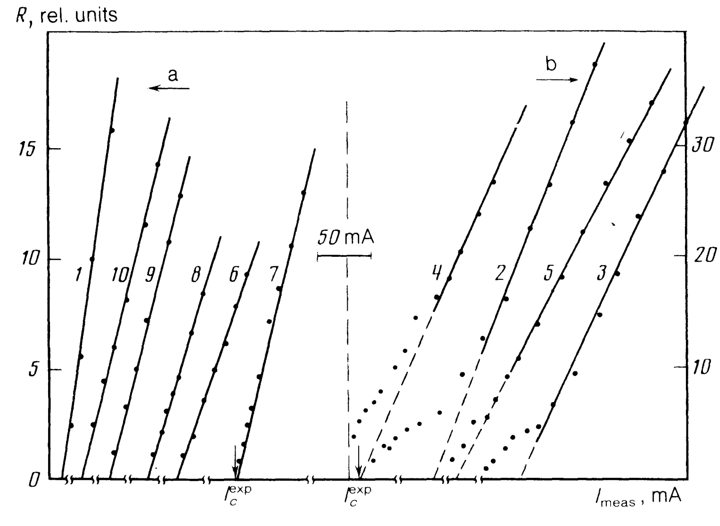

FIG. 4. Dependence $R(I)$ in the limit $I \to 0$.

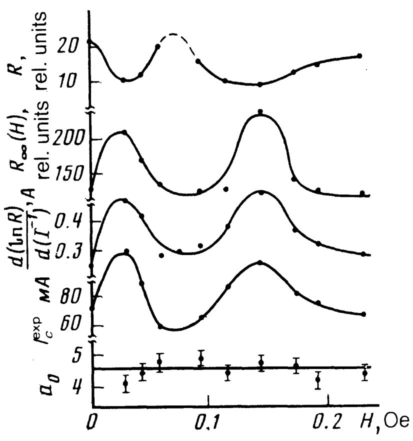

FIG. 5. Dependences of R, R_∞, $d(\ln R)/d(I^{-1})$, I_c, and a_0 on the external magnetic field [the absolute value of $d(\ln R)/d(I^{-1})$ is shown].

average dimension of a unit cell $r_0 = 20 \pm 2\ \mu$m. This is in excellent agreement with the geometric parameter of the sample, $d_{scf} + a = 22.5 \pm 0.5\ \mu$m.

The results obtained demonstrate that the main influence of the magnetic field on the resistive properties of our system is modulation of the critical current I_c (or the average coupling energy E_J) analogous to the two-dimensional structure[10] and the average mobility of vortices.

The resistance plotted in Fig. 6 as a function of the magnetic field observed for different values of the normalization parameter $I/I_c(H)$ demonstrates the interaction between current excitations and a lattice of fluxoids, which disappears when $I \leqslant I_c(H)$ (see curve 1). On the other hand, we can see from Fig. 5 that an increase in the critical current also increases R_∞ and, consequently, the mobility of vortices in ordered lattices ($\Delta\Phi/\Phi_0 = 0, 1, 2, \ldots$) is considerably higher. The ordinate X in Fig. 6 is calibrated in such a way that $\Delta\Phi(H_0) = 0$, where H_0 corresponds to the first maximum of $I_c(H)$ in Fig. 5 (all the measurements were made in the presence of a magnetic field, the accurate value of which was not determined).

Since a_0 is independent of H (see Fig. 5) and we also have[4]

$$\frac{T_c(H)}{T_{c0}(H)} = 3.2\cdot 10^{-2}\frac{I_c(T_c, H)}{I_c(T_{c0}, H)},$$

it follows that the oscillations of $I_c(H)$ should give rise to corresponding changes in the critical temperatures $T_c(H)$ and $T_{c0}(H)$, in agreement with the familiar results for film arrays.[10] The relative change in the amplitude of the oscillations $I_c(H)$ is $\Delta I_c/I_c \approx 50\%$, which again is comparable with the results obtained for two-dimensional systems.[8,9]

It follows from the above discussion that Eq. (1) describes well the experimental dependence $R(I)$ even in the presence of a magnetic field. We shall therefore use Eq. (1) to determine more accurately how the vortex mobilities depend on the current and magnetic field. The values of c_d considered as a function of I/I_c and obtained from the experimental data by means of Eq. (1) are plotted in Fig. 7a for different degrees of frustration $f = \Delta\Phi/\Phi_0$. Two features are then observed. Firstly, with the exception of curves 2–5 at low currents, all the values of $c_d(I/I_c, H)$ can be described satisfactorily by a single universal curve which demonstrates the existence of a threshold at $I = I_c(H)$ for this vortex activation mechanism. The shaded region in Fig. 7a represents the scatter of the individual curves. Secondly, in some fields (curves 2–5) this mechanism is clearly masked at low currents by a different mechanism, which depends relatively weakly on the current. Since in the range $I < I_c$ our measurements were close to the limit of sensitivity in our experiments, definite conclusions on the behavior of this mechanism in the limit $I \to 0$ could not be drawn. The functional dependence $c_d(I/I_c)$ in Fig. 7b was determined by plotting it in the form $\ln c_d \propto \ln(I/I_c - 1)$. We can see that the main mechanism of vortex motion in the range $I > I_c$ obeys the dependence

$$c_d \propto (I/I_c - 1)^n, \tag{3}$$

where $n = 0.65$–0.82, i.e., $n \approx 3/4$, and the coefficients of proportionality are the same for many values of H. This dependence is somewhat similar to the universal behavior of the resistance in a magnetic field, where the activation energy of moving vortices is proportional to $H^{2/3}$ (Ref. 12). Equation (3) differs qualitatively from that describing the mobility of single vortices[7]:

$$c_d = [1 - (I_c/I)^2]^{1/2} \tag{4}$$

and it applies to our sample at higher temperatures (see Fig. 8 for the range $T > 2.5$ K and Ref. 4). The dependences $c_d(I/I_c)$ obtained at different temperatures are plotted in Fig. 8. The mechanism described by Eq. (3) is clearly related directly to an anomaly in the vortex mobility near a topological phase transition at $T = T_c$, which reaches its maximum at $T \approx 2$ K, indicating a strong interaction of topological excitations, the collective motion of which at

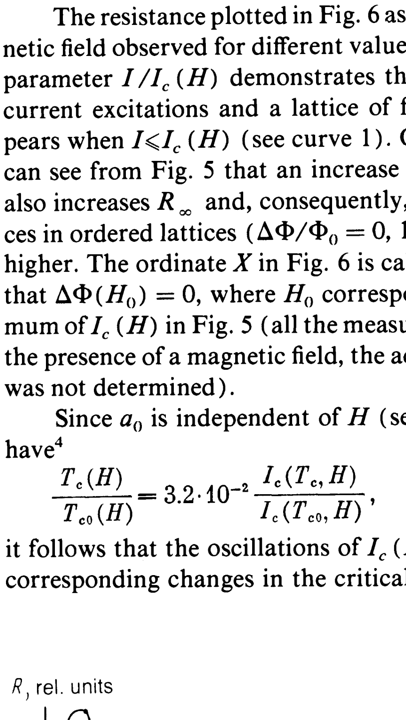

FIG. 6. Dependence of the resistances on $\Delta\Phi/\Phi_0$ for different values of I/I_c: 1) $I/I_c < 1$; 2) $I/I_c = 1.5$; 3) 2.0; 4) 3.0; 5) 4.0.

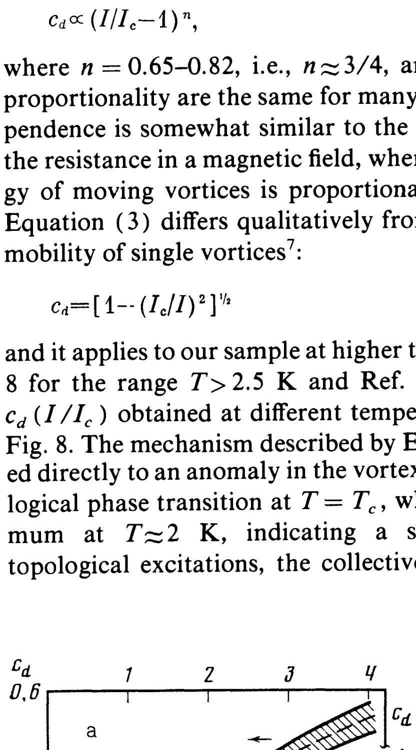

FIG. 7. a) Dependence of c_d on I/I_c with the curves numbered in the same way as in Table I. b) Dependence of c_d on $(I/I_c - 1)$.

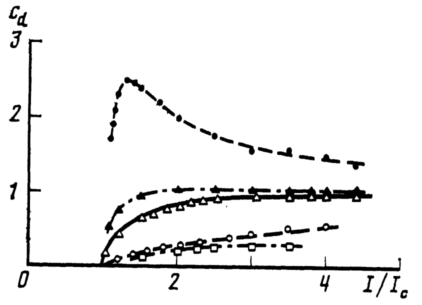

FIG. 8. Dependence of c_d on I/I_c at various temperatures: □) $T = 1.5$ K; ○) 1.85 K; △) 3.0–4.5 K; ▲) 2.5 K; ●) 2.0 K. The continuous curve represents c_d in accordance with Eq. (4).

these temperatures cannot be considered using a single approximation.

4. CONCLUSIONS

Our experimental results describing the influence of a weak magnetic field on the properties of a periodic structure near the transition point T_c confirm the conclusion of earlier investigations that topological phase transitions analogous to the Kosterlitz–Thouless–Berezinskiĭ transition are also observed in bulk samples.[3,4]

The oscillations observed for $R(H)$, $R_\infty(H)$, and $I_c(H)$ and the exact agreement of the oscillation period of a parameter of a lattice of vortices in NbTi provide the direct proof that these are the effects of quantum interference. In contrast to investigations of film structures, in which such effects have been investigated near T_{c0} (i.e., near the Kosterlitz–Thouless–Berezinskiĭ transition), our measurements were carried out near a topological transition at T_c associated with recombination of topological dipoles and with a change in the collective properties of the current-induced vortices. We obtained a universal dependence of the mobility of vortices on I and H in the form of Eq. (3), which demonstrates a strong interaction of current excitations and is clearly an important characteristic of the critical behavior of the system in this range of temperatures. Under our experimental conditions the influence of a magnetic field on the resistance of the system reduces to modulation of R_∞ and of the coupling energy $E_J(H) \propto I_c(H)$, which should also be manifested by corresponding oscillations of the critical temperatures. The general behavior of the resistance follows the dependence (1) and its scaling invariant relative to $f = \Phi/\Phi_0$ and $I/I_c(H)$ i.e., the ratio $R[f, I/I_c(f)]/R_\infty(f)$ is a universal function of these parameters. It is proposed to continue measurements of this type in a wide range of temperatures and magnetic fields in order to extend the range of results to determine more accurately the microscopic nature of the effects.

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