Electron-positron pair production and annihilation in plasma

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The probabilities of one-photon electron-positron pair production and annihilation processes in a dispersive medium with refractive index $n(\omega) < 1$ are derived and found to be different from those previously available. An improved dispersion relation is also obtained for the relativistic degenerate ultradense plasma in which these reactions can take place.

Different e^+e^- creation and annihilation mechanisms are not only of interest in themselves but have recently attracted increasing attention because of astrophysical applications. The "space laboratory" has been found to be so full of richness and variety (from the point of view of manifestations of different forms of matter and fields) that many phenomena that are practically unattainable under terrestrial conditions have become accessible in astrophysical objects. Examples include the single-photon creation and annihilation of e^+e^- pairs $\gamma \rightleftarrows e^+ + e^-$ in a medium with refractive index $n(\omega) < 1$ (Refs. 1 and 2) and also in strongly nonstationary plasmas.[3]

The $\gamma \rightleftarrows e^+ + e^-$ processes in a medium were first investigated in Refs. 1 and 2, where it was shown that these reactions could occur in ultradense plasmas with electron density $N/V \gtrsim 10^{32}$ cm^{-3}. However, the expressions for the probabilities of the $\gamma \rightleftarrows e^+ + e^-$ processes in a medium, found in Refs. 1 and 2, turn out to be incorrect. In particular, in the $n = 1$ limit, they predict nonzero probabilities for the $\gamma \rightleftarrows e^+e^-$ processes, whereas, in vacuum, these are forbidden by conservation laws. The dispersion law necessary for these processes, found in Ref. 2, is also found to be incorrect and requires separate examination because there is independent interest in the polarizability of a medium at ultrahigh densities.

In this paper, we reexamine the $\gamma \rightleftarrows e^+ + e^-$ processes in a medium and the dispersion properties of ultradense plasmas that govern the main features of these processes. Using the general rules for constructing the matrix element of a single-vertex $\gamma \to e^+ + e^-$ diagram in a dispersive medium, we find that the pair-production probability per unit time ($\hbar = c = 1$) is given by

$$W = \frac{e^2}{8\pi^2\alpha\omega n^2(\omega)} \int \frac{E_1E_2 + m^2 - p_1p_2\cos\vartheta_1\cos\vartheta_2}{E_1E_2}\,\delta(\omega - E_1 - E_2) \times \delta(\mathbf{k} - \mathbf{p}_1 - \mathbf{p}_2)\,d^3p_1\,d^3p_2. \tag{1}$$

where $E_{1,2}$ and $\mathbf{p}_{1,2}$ are, respectively, the energies and momenta of the electron and positron, $k^i(\omega,\mathbf{k})$ is the 4-momentum of the proton ($|\mathbf{k}| = n\omega$), $\vartheta_{1,2}$ is the angle between $\mathbf{k}$ and $\mathbf{p}_{1,2}$, $n(\omega)$ is the refractive index of the medium, and $\alpha = 1 + (\omega/n)dn/d\omega$. The integral with respect to p_2 can be evaluated in an elementary manner, and the integral with respect to ϑ_1 ($d^3p_1 = 2\pi p_1E_1dE_1\sin\vartheta_1 d\vartheta_1$) reduces formally to the following replacement in (1):

$$\delta(\omega - E_1 - E_2)\sin\vartheta_1\,d\vartheta_1 \to \frac{\omega - E_1}{kp_1}\left[H(E_1 - E_{min}(\omega)) - H(E_1 - E_{max}(\omega))\right],$$

where $H(\xi)$ is the Heaviside function

$$H(\xi) = \begin{cases} 1, & \xi \geqslant 0 \\ 0, & \xi < 0 \end{cases}.$$

The limits of integration over $E_1 \in [E_{\min}, E_{\max}]$

$$E_{\substack{min\\max}}(\omega) = \frac{\omega}{2} \mp \frac{n(\omega)}{2}\left[\omega^2 - \frac{4m^2}{1 - n^2(\omega)}\right]^{1/2} \tag{2}$$

are determined by the conservation laws for the $\gamma \rightleftarrows e^+ + e^-$ processes in a medium with $n(\omega) < 1$, which have the following reaction threshold:

$$\omega > 2m[1 - n^2(\omega)]^{-1/2} \tag{3}$$

[we note that, instead of (3), the ($\gamma \rightleftarrows e^+ + e^-$) reaction threshold was chosen in Refs. 1 and 2 to be ($\omega > 2m$)]. Integration with respect to the electron energy finally yields the following probability for the single-photon e^+e^- pair production in plasma:

$$W = \frac{e^2m^2}{6\pi\alpha n^2(\omega)\omega^2}\left[\omega^2 - \frac{4m^2}{1 - n^2(\omega)}\right]^{1/2} \times \left\{\frac{1}{2}\left(\frac{\omega}{m}\right)^2[1 - n^2(\omega)] + 1\right\}. \tag{4}$$

We note that, in real situations and depending on the state of the medium (we shall see later that, for the densities necessary for this process, the electron component of plasma is highly degenerate), the Pauli principle must also be taken into account and imposes additional restrictions on the $\gamma \to e^+ + e^-$ reaction.

Let us now examine the single-photon annihilation of e^+e^- pairs. The probability per unit time of the process $e^+ + e^- \to \gamma$ is given by

$$dW' = \frac{\pi e^2}{2\alpha\omega n^2(\omega)} \times \frac{E_1E_2 + m^2 - p_1p_2\cos\vartheta_1\cos\vartheta_2}{E_1E_2}\,\delta(\omega - E_1 - E_2) \times \delta(\mathbf{k} - \mathbf{p}_1 - \mathbf{p}_2)\,d^3k. \tag{5}$$

This gives the annihilation probability for a single e^+e^- pair in plasma. To obtain the total probability of annihilation of

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all the possible pairs (the initial positron with the electrons in the medium), we must determine the probability of annihilation of a positron of given energy E_2 with electrons in the medium in which they have momenta in the range $\mathbf{p}_1, \mathbf{p}_1 + d\mathbf{p}_1$:

$$dW_1 = f(|\mathbf{p}_1|)\, dW' d^3 p_1, \tag{6}$$

where $f(|\mathbf{p}_1|)$ is the distribution function of the plasma electrons. We first integrate with respect to k in (6) and then with respect to ϑ, having substituted,

$$\delta(\omega - E_1 - E_2) \sin\vartheta\, d\vartheta$$

$$\to \frac{\alpha n^2(\omega)\, \omega}{p_1 p_2} [H(E_2 - E_{min}(\omega)) - H(E_2 - E_{max}(\omega))],$$

where the quantities $E_{\min(\max)}(\omega)$ are again given by (2) and ϑ is the angle between $\mathbf{p}_1$ and $\mathbf{p}_2$.

In contrast to the pair-production process [its probability can be obtained without resorting to the explicit form of $n(\omega)$], here we must have the explicit form of the function $n = n(\omega)$ in order to be able to integrate with respect to the electron energy (ω is now a function of E_1, since $\omega = E_1 + E_2$).

We note, first, that the macroscopic refractive index gives the lower bound for the density of the medium (at least one particle within a distance of the order of $\lambda/2$, where λ is the wavelength of the γ-ray photon). Consequently, in addition to condition (3) for the reaction threshold, we have a second condition that restricts the plasma density N/V for a given γ-ray frequency:

$$\omega \leqslant \pi (N/V)^{1/3} \equiv \omega_{\text{lim}}. \tag{7}$$

From the standpoint of single-photon pair creation and annihilation in plasma, the latter must compensate the longitudinal momentum $\Delta p = [1 - n(\omega)]\hbar\omega$ transferred in these processes. Consequently, the characteristic length in the macroscopic description of the dispersion of the medium is the wavelength $\lambdabar = h/\Delta p$, that corresponds to the transferred momentum, and the condition necessary for this is $h/\Delta p > (V/N)^{1/3}$. Since $n(\omega) < 1$, this condition is satisfied automatically when (7) is satisfied.

For densities satisfying conditions (3) and (7), the electron component of the ultradense plasma is highly degenerate (the dispersion of the transverse electromagnetic waves is determined by electrons). Actually, the degeneracy temperature of the electron component of this plasma is $T_F > 10^{10}$. On the other hand, because of neutrino energy losses, the physically attainable temperatures in an equilibrium system are much lower than this: $T \ll T_F$. Since the Fermi energy at such densities is $E_F > mc^2$, we must determine the dispersion relation of the highly degenerate relativistic plasma.

By solving the self-consistent Maxwell-Vlasov equations for a collisionless relativistic isotropic plasma ($\omega\tau_{\text{tr}} \gg 1$ for the processes under investigation, where τ_{tr} is the transport collision time) in the form of a plane transverse electromagnetic wave of frequency ω, we obtain the dispersion law in the general form

$$n^2 = 1 - \frac{16\pi^2 e^2}{\omega^2 n^2} \int \frac{f(p_1)\, p_1^2}{E_1} \left[1 + \frac{1-n^2}{2} \frac{E_1}{n p_1} \ln \frac{E_1 - n p_1}{E_1 + n p_1} \right] dp_1. \tag{8}$$

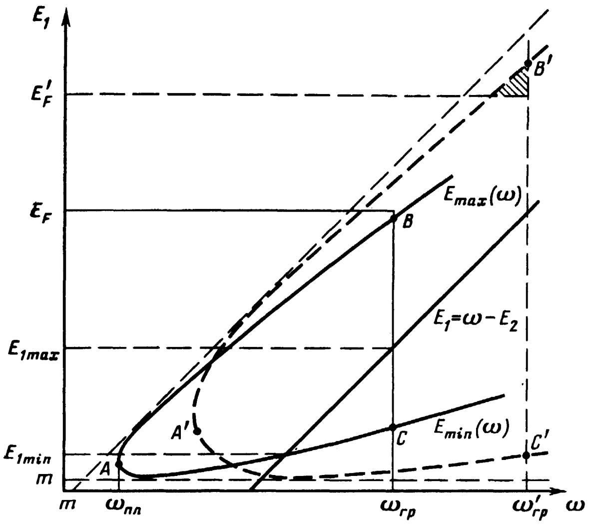

FIG. 1. Schematic representation of regions in which the reactions $\gamma \rightleftarrows e^+ + e^-$ can occur in a dispersive medium with refractive index $n(\omega) < 1$. Solid curves representing $E_{\min(\max)}(\omega)$ correspond to $36m < E_F < 150m$; for these values, the reaction $e^+ + e^- \to \gamma$ occurs in the region $ABCA$ and $\gamma \to e^+ + e^-$ is forbidden. Broken curves represent $E_{\min(\max)}(\omega)$ and correspond to $E'_F > 150m$. The reaction $\gamma \to e^+ + e^-$ can occur in the shaded region.

This equation describes the dispersion of relativistic plasma for an arbitrary electron distribution function. In principle, it is also valid for a nondegenerate (relativistic and Maxwellian) electron plasma if an equilibrium distribution with temperature $T \gtrsim T_F$ can be realized in nature. However, it is shown above that the plasma is highly degenerate for the $\gamma \rightleftarrows e^+ + e^-$ processes and, hence, the electron distribution function takes the form

$$f(p_1) = \begin{cases} 1/4\pi^3, & p_1 \leqslant p_F \\ 0, & p_1 > p_F \end{cases}. \tag{9}$$

Integrating (8) with respect to the electron momenta, and using (9), we obtain the dispersion relation for degenerate plasma, conveniently written in the inverted form $\omega = \omega(n)$:

$$\omega^2 = \frac{2e^2}{\pi} \frac{p_F^3}{E_F} \frac{1}{1-n^2} \Phi\left(n \frac{p_F}{E_F}\right),$$

$$\Phi(\beta) = \frac{1}{\beta^2} \left\{ 1 + \frac{1-\beta^2}{2\beta} \ln \left| \frac{1-\beta}{1+\beta} \right| \right\} = 2 \sum_{s=1}^{\infty} \frac{\beta^{2s-2}}{4s^2 - 1}. \tag{10}$$

It is clear from (10) that the frequency of natural oscillations of degenerate relativistic plasma is

$$\omega_{\text{pl}} = (4e^2 p_F^3 / 3\pi E_F)^{1/2}. \tag{11}$$

The frequency range corresponding to transverse waves that can propagate in this plasma, $\omega_{\text{pl}} \leqslant \omega < \infty$, can then be obtained by varying the refractive index in the range $0 \leqslant n < 1$. The parameter β in (10) then varies in the range $0 \leqslant \beta < p_F/E_F$. Analysis of the function $\Phi(\beta)$ shows that, throughout the physically admissible range $0 \leqslant \beta < 1$ ($p_F/E_F \sim 1$ at very high densitites of ultrarelativistic plasma), it varies monotonically between 2/3 and 1. Having determined $n(\omega)$ we find that the probability of annihilation of a positron of energy E_2 with all the plasma electrons is given by

$$W_1=\frac{\pi^2e^2}{p_2E_2}\int f(p_1)\left[m^2+(E_1+E_2)^2\frac{1-n^4(\omega)}{4n^2(\omega)}\right.$$
$$\left.-\frac{1-n^2(\omega)}{n^2(\omega)}E_1E_2\right][H(E_2-E_{min}(\omega))-H(E_2-E_{max}(\omega))]dE_1, \tag{12}$$

where ω must be replaced with $\omega=E_1+E_2$.

The problem now reduces to the determination of the range of variation of the energies of electrons that actually participate in the annihilation process, taking account of the limitations defined by (3), (7), and $E_1\leqslant E_F$. The situation may be clarified by defining this region graphically. The figure shows the $E_{\min(\max)}(\omega)$ curves and the $\omega=\omega_{\rm lim}=(\pi/3)^{1/3}p_F$ [see (7)] and $\omega=\omega_{\max}=E_F+E_2$ lines. The energies of the particles and of the γ-ray can vary within the region $ABCA$, and the limits of integration with respect to the electron energy, $E_{1\min}$ and $E_{1\max}$, are determined by the points at which the $E_1=\omega-E_2$ line cuts the boundaries of this region. Evaluating the integral in (12) with the dispersion law given by (10), we obtain a very unwieldy expression for the total probability of the annihilation process. However, for the admissible values of $n(\omega)$ and of the density of the medium, we have $\Phi(np_F/E_F)\approx\frac{2}{3}$ in (10), and the expression for the probability of the $e^++e^-\to\gamma$ process becomes very much simpler. The points of intersection of the line $E_1=\omega-E_2$ and the boundaries of the region $ABCA$ then correspond to

$$\omega_1=\frac{\omega_{\rm pl}}{2m^2}[\omega_{\rm pl}E_2-p_2(\omega_{\rm pl}{}^2-4m^2)^{1/2}],$$

$$\omega_2=\begin{cases}\dfrac{\omega_{\rm pl}}{2m^2}[\omega_{\rm pl}E_2+p_2(\omega_{\rm pl}{}^2-4m^2)^{1/2}], & E_2\leqslant E_{min}(\omega=\omega_{\rm lim})\\ \omega_{\rm lim}, & E_{min}(\omega=\omega_{\rm lim})<E_2<E_{max}(\omega=\omega_{\rm lim})\end{cases}. \tag{13}$$

Finally, the total probability of the annihilation process is

$$W_1=\frac{e^2}{4\pi p_2E_2}\left[\left(m^2+\frac{\omega_{\rm pl}{}^2}{2}\right)(\omega_2-\omega_1)\right.$$
$$+\frac{1}{2}\omega_{\rm pl}\left(E_2{}^2+\frac{1}{4}\omega_{\rm pl}{}^2\right)\ln\frac{(\omega_2-\omega_{\rm pl})(\omega_1+\omega_{\rm pl})}{(\omega_2+\omega_{\rm pl})(\omega_1-\omega_{\rm pl})}$$
$$\left.-\frac{E_2\omega_{\rm pl}{}^2}{2}\ln\frac{(\omega_2-\omega_{\rm pl})(\omega_2+\omega_{\rm pl})}{(\omega_1-\omega_{\rm pl})(\omega_1+\omega_{\rm pl})}\right]. \tag{14}$$

As far as the pair-production process in degenerate electron plasma with the dispersion relation (10) is concerned, the Pauli principle demands that the condition $E_1>E_F$ be satisfied and this, together with (7), substantially reduces the range of parameter values, so that the probability of the process $\gamma\to e^++e^-$ becomes practically equal to zero [the range of integration with respect to E_1 in (1) then shrinks to a point]. As the electron density increases, and $E_F\gtrsim 150m$ ($E_{\max}(\omega_{\rm lim})>E_F$, cf. the figure), a narrow region breaks off, and (7) and (13) show that the creation of a pair by a γ-ray with energy $\omega_1(E_2=E_F)<\omega<\omega_{\rm lim}$ becomes possible in this region. However, it is important to recall that this region ($\omega\simeq\omega_{\rm lim}$) lies at the limit of validity of the macroscopic concept of a refractive index of a medium (one particle within the length $\lambda/2$). Since $E_F\propto(N/V)^{1/3}$, and so is the separation between the plasma particles, an increase in density does not facilitate the pair-production process, i.e. this process is hardly possible in degenerate relativistic plasma.

Let us now determine the lower limit for the density of the medium, above which pair annihilation is possible. From the reaction threshold condition (3) and the dispersion relation (10), we obtain $\omega_{\rm pl}>2m$, which is equivalent to $E_F>(3\pi)^{1/2}m/e\approx 36m$. The electron plasma density corresponding to this value of E_F is $N/V>p_F^3/3\pi^2\approx 3\cdot 10^{34}$ cm^{-3}. Plasmas of this electron density exist in the cores of neutron stars.

Let us now estimate the probability of the process $e^++e^-\to\gamma$. Confining our attention to the annihilation of a nonrelativistic positron in electron plasma, we find that (14) becomes very much simpler:

$$W_1=\frac{e^2\omega_{\rm pl}{}^3}{8\pi m^3}(\omega_{\rm pl}{}^2-4m^2)^{1/2},\qquad p_2\ll m. \tag{15}$$

When $N/V\sim 10^{35}$ cm^{-3}, this gives $W_1\sim 10^{19}$ s^{-1}.

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