Asymptotic forms of three-spin correlations and properties of the pair spin Green function of cubic ferromagnets above T_c in a magnetic field

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The asymptotic properties of the three-point dynamical vertex of isotropic ferromagnets in the critical exchange region above T_c are determined theoretically. It is shown that these properties are directly related to the sum rule for the transverse (with respect to the magnetic field **H**) pair Green function G_{+-} and to the law of conservation of the component of the total spin of the magnet along the direction of **H**. The asymptotic forms of G_{+-} are investigated. It is shown that the results of experiments to investigate three-point correlations in iron by means of polarized-neutron scattering agree with the theory proposed.

INTRODUCTION

The critical dynamics of cubic ferromagnets with allowance for exchange and dipole forces at temperatures above the Curie point in a magnetic field H was considered in Ref. 1. In this paper it was shown that in the presence of a magnetic field, as in the case $H = 0$ as well,[2] the critical behavior of ferromagnets has a different character in the exchange region of τ and H $(4\pi\chi(\tau,H) \ll 1)$ and in the dipole region of τ and H $(4\pi\chi(\tau,H) \gg 1)$, where χ is the uniform magnetic susceptibility and $\tau = (T - T_c)T_c^{-1}$. In each of these regions expressions were obtained for dynamical correlation functions in limiting cases—the hydrodynamic $[q \ll R_c^{-1}(\tau,H)]$ and critical $[q \gg R_c^{-1}(\tau,H)]$ asymptotic forms in weak and strong fields (q is the momentum of the fluctuation and R_c is the correlation length). Although a considerable part of Ref. 1 was phenomenological, a number of it results had a clear physical interpretation and, in essence, did not require further justification. Amongst these results are the expressions for the transverse and longitudinal (with respect to **H**) pair Green functions in the hydrodynamic exchange region, which are essentially a consequence of the conservation of the component of the total spin of the magnet along the direction of the field. In the same region perturbation theory was used to calculate the dependences on τ and H of the dipolar damping in the uniform limit, and this made it possible for the authors of Ref. 3 to determine the spin-diffusion coefficients for the ferromagnets $CdCr_2Se_4$ and $CdCr_2S_4$ from magnetic-resonance data.

In other limiting cases, however, the analysis of the dynamics was based on a number of additional hypotheses. First amongst these was an assumption about the asymptotic behavior, in the critical region, of the three-spin correlator, which is the coefficient in the linear term of the expansion of the pair Green function in H.

It should be said that considerable attention has been paid recently[4–6] to the investigation of higher spin correlation functions of ferromagnets in the fluctuation paramagnetic region. But, whereas their static properties have been well studied, at least theoretically,[7] the investigation of their dynamical behavior is only just beginning. We recall that for $H = 0$, above T_c, there are static spin correlators of even order only. The odd correlators are purely dynamical and vanish in the static limit by virtue of the symmetry of the system under time reversal.[8]

The three-spin dynamical correlator is the simplest higher correlator. Some of its general properties in zero field were considered in Ref. 8. In Ref. 1 the behavior of the three-spin Green function G_3 and of the corresponding vertex $\mathcal{T}_3$ arising in linear order of the expansion in H of the pair Green function was analyzed in Ref. 1. The basic hypothesis that was adopted concerned the dependence of $\mathcal{T}_3$ on q in the critical region $q \gg \varkappa = R_c^{-1}$. The point is that in static scaling theory[7] a so-called correlation-coalescence rule has been formulated[9] that makes it possible to establish the dependence of static vertices on the momenta in conditions when these momenta differ strongly in magnitude. Analogous results were obtained in Ref. 10 in the language of the algebra of fluctuating quantities. In Ref. 9, in which the Ising model was considered, the asymptotic forms of the simplest vertex Γ_0, generated by the correlator $K(\mathbf{r},\mathbf{r}_1,\mathbf{r}_2) = \langle \varepsilon_\mathbf{r}\sigma_{\mathbf{r}_1}\sigma_{\mathbf{r}_2}\rangle$ ($\varepsilon_\mathbf{r} = \sum_i J_{\mathbf{r}\mathbf{r}_i}\sigma_\mathbf{r}\sigma_{\mathbf{r}_i}$ is the energy density, $J_{\mathbf{r}\mathbf{r}_i}$ is the interaction potential, and $\sigma_\mathbf{r} = \pm 1$) were established for the first time. The asymptotic and $\mathcal{T}_0$ were found by comparing the behavior of K in limiting cases with the behavior of critical correlators with known properties. For example, $K \propto \langle \varepsilon_\mathbf{r}\varepsilon_{\mathbf{r}_1}\rangle$ if $|\mathbf{r} - \mathbf{r}_{1,2}| \gg |\mathbf{r}_1 - \mathbf{r}_2| \sim a$, and $K \propto \langle \sigma_\mathbf{r}\sigma_{\mathbf{r}_2}\rangle$ if $|(\mathbf{r}_2 - \mathbf{r})\cdot\mathbf{r}_1| \gg |\mathbf{r} - \mathbf{r}_1| \sim a$, where a is a quantity of the order of the lattice constant. The asymptotic forms of other vertices, e.g., the four-point vertices, were found to be related in a simple manner to the asymptotic forms of $\mathcal{T}_0$. In particular, if in a vertex other than $\mathcal{T}_0$ one of the entering momenta q is large in comparison with the other momenta and with $\varkappa$, the dependence on this momentum can be separated in the form of a universal factor q^{ρ_0}, where $\rho_0 = \nu^{-1} - 1 - \eta$ (ν is the correlation-length index and η is the Fisher parameter). In Ref. 1, following Ref. 11, it was assumed that this result can also be carried over to the dynamical vertex $\mathcal{T}_3$ (absent in the static theory), i.e., it was assumed that $\mathcal{T}_3 \propto q^\rho$ for $q \gg \varkappa$, where $\rho = \rho_0 = 1/2$, since for a ferromagnet $\nu = 2/3$ (here and below, we shall disregard the small index η). However, since the vertex $\mathcal{T}_3$ is not related to $\mathcal{T}_0$ there are no theoretical grounds for such a hypothesis, and, as we shall see, it turns out to be unsound.

In the present paper we consider in detail the asymptotic properties of $\mathcal{T}_3$ in the critical exchange region. Here we shall start from a sum rule formulated below for the pair Green function in a field, the dynamical-scaling hypothesis,[12] and also physical reasoning similar to that which was used in Ref. 9 in the determination of the asymptotic forms

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of $\mathcal{T}_0$. As a result it is found that $\rho = 3/2$ for $\mathcal{T}_3$. In deriving this result we shall use the commutation relations for the spin operators; therefore, the dynamical vertex possesses new asymptotic properties, independent of those of the static vertices. These new properties turn out to be closely connected with the conservation of the component of the total spin of the magnet along the direction of **H**, as is natural for a dynamical vertex. On the basis of this analysis we investigate the behavior in the limiting cases of the transverse (with respect to **H**) pair Green function.

As was shown in Refs. 13 and 14, three-spin correlations can be studied by means of polarized-neutron scattering in magnets situated in a magnetic field. In Ref. 15 reliable quantitative experimental results were obtained in the exchange region. The first section is devoted to analyzing these results and comparing them with the conclusions of the proposed theory.

1. SPIN CORRELATIONS IN THE EXCHANGE REGION IN THE PRESENCE OF A MAGNETIC FIELD

Since, as has been noted, the vertex $\mathcal{T}_3$ arises as a result of the linear term of the expansion of the pair Green function in H, we shall discuss the general properties of this Green function

$$G_{\alpha\beta}(\mathbf{q},\omega)=i\int_0^\infty dt\, e^{i\omega t}\langle[S_{\mathbf{q}}^{\alpha}(t),S_{-\mathbf{q}}^{\beta}(0)]\rangle. \qquad (1)$$

Here

$$S_{\mathbf{q}}^{\alpha}=N^{-1/2}\sum_j \exp(i\mathbf{q}\mathbf{r}_j)S_j^{\alpha},$$

where s_j^{α} is a component of the atomic spin with coordinate $\mathbf{r}_j$ and N is the number of magnetic atoms of the sample. The averaging is performed over the states of magnet describable by a Hamiltonian in which exchange forces and the interaction with an external field are taken into account. The dynamics of the fluctuations in this case is determined by three independent functions, for which, in the coordinate system with the z axis along **H** and with $S^{\pm} = (S^x \pm iS^y)/\sqrt{2}$, it is convenient to choose G_{+-}, G_{-+}, and G_{zz}. We shall represent these functions in the form

$$G_{\lambda}(q,\omega,H)=G_{0\lambda}(q,H)\Phi_{\lambda}(q,\omega,H), \qquad (2)$$

where $G_{0\lambda}$ are the static Green functions, the properties of which are well known,[7,9] and Φ_{λ} are the dynamical form factors, normalized by the condition $\Phi_{\lambda} = 1$ at $\omega = 0$.

From the invariance of the equations of motion under time reversal it follows that[16]

$$\Phi_{+-}(\omega,H)=\Phi_{-+}(\omega,-H), \quad \Phi_{+-}^{*}(\omega,H)=\Phi_{+-}(-\omega,-H),$$
$$\Phi_{zz}(\omega,H)=\Phi_{zz}(\omega,-H), \quad \Phi_{zz}^{*}(\omega,H)=\Phi_{zz}(-\omega,H). \qquad (3)$$

By substituting Φ_{+-} in the form of a sum of the even $(\Phi_{+-}^{(+)})$ and odd $(\Phi_{+-}^{(-)})$ parts in H, for each of them we have

$$\mathrm{Re}\,\Phi_{+-}^{(\pm)}(\omega,H)=\pm\,\mathrm{Re}\,\Phi_{+-}^{(\pm)}(-\omega,H),$$
$$\mathrm{Im}\,\Phi_{+-}^{(\pm)}(\omega,H)=\mp\,\mathrm{Im}\,\Phi_{+-}^{(\pm)}(-\omega,H). \qquad (4)$$

We note that $iG_{+-}^{(-)} = G_{xy} = -G_{yx}$ and $G_{+-}^{(+)} = G_{xx} = G_{yy}$. Because they are even in H, the functions $\Phi_{+-}^{(+)}$ and Φ_{zz} transform under a change of sign of ω in the same way as Φ for $H=0$. In the static limit they are finite and equal to unity. For $H=0$ both these functions go over into Φ in zero field, and the first terms of their expansion in H are proportional to H^2.

In its turn, because it is odd in H, the imaginary part of $\Phi_{+-}^{(-)}$ is an even function of ω while the real part is odd. For $H\to 0$ the function $\Phi_{+-}^{(-)} \propto H$. In addition, it is purely dynamical and, since the static susceptibilities are even in H, vanishes in the static limit. For the function G_{+-} with new symmetry properties that has appeared in the presence of a field we have a sum rule that follows for (1) and (4):

$$\frac{1}{i\pi}\int_{-\infty}^{\infty} G_{+-}(q,\omega)\,d\omega=\frac{1}{\pi}\int_{-\infty}^{\infty} G_{0\perp}(q)\,\mathrm{Im}\,\Phi_{+-}^{(-)}(q,\omega)\,d\omega=\langle S_z\rangle. \qquad (5)$$

We shall analyze the expansion of G_{+-} in H. Since the quantity $\langle[S_q^{+}(t),S_{-q}^{-}(0)]\rangle$ contains a dependence on H both in the time evolution and in the statistical averaging, we stress that the integral in (5) is nonzero only if the dependence of the statistical weight on H is preserved.

As in Ref. 1, we shall make use of the relationship of the expansion of G_{+-} in H to the vertex parts. In a weak field $(g\mu H \ll T_c(\varkappa a)^{5/2})$ in the approximation linear in H we have

$$-G_{+-}^{(-)}(q,\omega,H)=g\mu HG_3(q,\omega)$$
$$=g\mu HG_0(\varkappa)G^2(q,\omega)\mathcal{T}_3(q,\omega). \qquad (6)$$

As has been established, $\mathrm{Im}G_3$ is an even function of ω and $\mathrm{Re}G_3$ is an odd function of ω, and in the static limit $G_3=0$. The same is true for $\mathcal{T}_3$, since $G(\omega)$ possesses the same symmetry properties as $G_{zz}(\omega)$ (3) and is finite at $\omega=0$. Both the scaling dimension and the ordinary dimensionality of $\mathcal{T}_3$ are found (with allowance for the dimension of the field) from (6): $|\mathcal{T}_3| \sim T_c(\varkappa a)^{3/2}$. The order of magnitude of $\mathcal{T}_3$ for $q\sim\varkappa$ and $\omega\sim\Omega(\varkappa) = T_c(\varkappa a)^{5/2}$, where $\Omega(\varkappa)$ is the characteristic energy of the critical fluctuations, is thereby determined. As a result, in accordance with dynamical scaling,[12] we have

$$\mathcal{T}_3(q,\omega)=T_c(\varkappa a)^{3/2}\tilde{\gamma}_3[q/\varkappa,\ \omega/\Omega(\varkappa)], \qquad (7)$$

with $\tilde{\gamma}_3(x,0)=0$ and $|\tilde{\gamma}_3(1,1)|\sim 1$.

We shall assume, in analogy with the static case,[9] that in the critical region factorization of the momentum dependence occurs[1]:

$$\mathcal{T}_3(q,\omega)=T_c(qa)^{\rho}(\varkappa a)^{-\rho+3/2}\gamma_3(\omega/\Omega(q)), \quad q\gg\varkappa, \qquad (8)$$

where $\gamma_3(0)=0$ and $|\gamma_3(x)|\sim 1$ for $x\sim 1$.

We shall show first how one can determine the value of ρ in the dynamical case, using the ideas of the physical reasoning of Ref. 9. We shall consider the behavior of G_3 and $\mathcal{T}_3$ in the region of large momenta $q\sim a^{-1}$ and large frequencies $\omega\sim T_c$. For these values of the arguments, $|G(q,\omega)|\sim T_c^{-1}$, and from (6) we find

$$|G_3(q,\varkappa,\omega)|\sim|G_3(\varkappa)|\propto|\mathcal{T}_3(\varkappa)|G_0(\varkappa).$$

We shall consider G_3 in the $(\mathbf{r},t)$-representation, which can be obtained by expanding $G_{+-}(\mathbf{r}_1-\mathbf{r}_2,t)$ in H. Then terms from the expansion of the statistical weight and $S_{\mathbf{r}_1}^{+}(t)$ arise. First we shall estimate the former, which is the sum over $\mathbf{r}_3$ of the quantities

$$\langle [S_{\mathbf{r}_1}{}^+(t),\ S_{\mathbf{r}_2}{}^-(0)]S_{\mathbf{r}_3}{}^z(0)\rangle = R_3(\mathbf{r}_1,\ \mathbf{r}_2,\ \mathbf{r}_3,\ t).$$

In the Fourier transform of each of these quantities large momenta $q\sim a^{-1}$ correspond to configurations with closely spaced spins $S^+_{\mathbf{r}_1}$ and $S^+_{\mathbf{r}_2}$, and large frequencies correspond to small times. We may expect that the dependence on the distance R_3 for $|\mathbf{r}_3-\mathbf{r}_{1,2}|\gg|\mathbf{r}_1-\mathbf{r}_2|\sim a$ over short times will not differ from the dependence of the correlator $\langle S^z_{\mathbf{r}_1}S^z_{\mathbf{r}_3}\rangle$, which is obtained from R_3 at $t=0$ by means of the commutation relation $[S^+_{\mathbf{r}_1},S^-_{\mathbf{r}_1}]=S^z_{\mathbf{r}_1}$. As a result, in the momentum representation, $|\mathscr{T}_3(\varkappa)|G_0(\varkappa)\propto G_0(\varkappa)$, and, consequently, the power $\rho=3/2$.

Similar reasoning can be adduced in the case when $S^+_{\mathbf{r}_1}$ and $S^-_{\mathbf{r}_2}$ are nearest neighbors ($\mathbf{r}_1\neq\mathbf{r}_2$). We shall expand R_3 to second order in t. In this expansion there is a term

$$t^2\sum_i J_{\mathbf{r}_1\mathbf{r}_i}J_{\mathbf{r}_i\mathbf{r}_2}\langle(S_{\mathbf{r}_1}{}^zS_{\mathbf{r}_i}{}^zS_{\mathbf{r}_2}{}^z)S_{\mathbf{r}_3}{}^z\rangle,$$

where $J_{\mathbf{r}\mathbf{r}_i}$ is the exchange interaction of nearest neighbors. In the case when $|\mathbf{r}_{1,2}-\mathbf{r}_3|\gg\mathbf{r}_1-\mathbf{r}_2|\sim a$, the product $S^z_{\mathbf{r}_1}S^z_{\mathbf{r}_i}S^z_{\mathbf{r}_2}$ inside $\langle\ldots\rangle$ can be replaced by one spin, e.g., $S^z_{\mathbf{r}_1}$ (exactly as in the static case[9]). Then

$$R_3(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,t)\sim t^2\sum_i J_{\mathbf{r}_1\mathbf{r}_i}J_{\mathbf{r}_i\mathbf{r}_2}\langle S_{\mathbf{r}_1}{}^zS_{\mathbf{r}_3}{}^z\rangle,$$

which, for $t\sim T_c^{-1}$, leads to the previous result for R_3, and hence also for $\mathscr{T}_3$.

We see that by means of the commutation relations it is possible to "project" the dynamical critical correlator onto the static critical correlator. In other words, the use of the commutation relations makes it possible to extend the algebra of fluctuating operators and to obtain a dynamical rule of coalescence of correlations. Here, as in the static theory, it is not necessary to introduce any new critical indices. Finally, we note that is precisely the above term that makes a contribution to (5), since it has arisen from the expansion of the statistical weight in H.

We shall estimate the contribution to G_3 from the term associated with the dependence of $S^+_q(t)$ on H. We note that if in the statistical weight we omit H then $\Phi_{+-}(\omega)=\Phi(\omega-\omega_0)$, where Φ is the dynamical form factor for $H=0$. Then the contribution to G_3 from this term for $\omega\sim\Omega(q)$ is proportional to $(q^2\Omega(q))^{-1}$ and has order of smallness $\varkappa^2q^{-2}$ in comparison with the contribution considered above. It is not surprising that the given term does not give a contribution to the sum rule. Thus,

$$\mathscr{T}_3(q;\ \omega)=T_c(qa)^{3/2}\gamma_3[\omega/\Omega(q)],\quad q\gg\varkappa, \tag{9}$$

where $\gamma_3(0)=0$ and $|\gamma_3(x)|\sim1$ for $x\sim1$.

This result is not obvious in advance. For example, in the static theory the simplest vertex $\mathscr{T}_0(\mathbf{q},p)$, determined above, has different asymptotic forms depending on the relative magnitudes of q and p (the momentum $\mathbf{q}$ corresponds to $\mathbf{r}$ in the Fourier transform $K=\langle\varepsilon_\mathbf{r}\sigma_{\mathbf{r}_1}\sigma_{\mathbf{r}_2}\rangle$).[9] The vertex $\mathscr{T}_0\propto q^{2-1/\nu}$ for $q\gg\max(p,\varkappa)$, i.e., depends only on the large momentum. But if $p\gg\max(q,\varkappa)$, then $\mathscr{T}\propto p^{2-1/\nu}\{p/\max(q,\varkappa)\}^{\alpha/\nu}$, where $\alpha=2-3\nu$ is the specific-heat index and $\mathscr{T}_0$ remains dependent on the small momentum of $\varkappa$. The difference in the asymptotic forms corresponds to the possibility of "projecting" K on to $\langle\sigma_\mathbf{r}\sigma_{\mathbf{r}_2}\rangle$ in the former case and on to $\langle\varepsilon_\mathbf{r}\varepsilon_{\mathbf{r}_1}\rangle$ in the latter case. As we shall see, the similar behavior of the simplest vertices $\mathscr{T}_0$ and $\mathscr{T}_3$ (the dependence on only the large momentum) obtains only in the case when the asymptotic form of $\mathscr{T}_0$ can be attributed to the behavior of the pair correlator of the spins. Furthermore, the static vertex $\mathscr{T}_4$, which appears in the expansion of the pair correlator in H^2 in a weak field, depends on $\varkappa$ for $q\gg\varkappa$: $\mathscr{T}_4(\mathbf{q},\mathbf{q},0)\propto q^{-1+1/\nu}\varkappa^{2-1/\nu}$ (Ref. 9). This result is a consequence of the correlation-coalescence rule, which makes it possible to relate the asymptotic form of $\mathscr{T}_4$ to the behavior of $\mathscr{T}_0(0,0)\propto\varkappa^{2-1/\nu}$.

We now determine the asymptotic form of $\gamma_3(x)$ for $x\gg1$. Its first term is found from the requirement that $G^{(-)}_{+-}$ be independent of q, which is a consequence of the existence of uniform precession in the field. In fact, from the hydrodynamic expressions[1] for G_{+-} in a weak field it follows that $G^{(-)}_{+-}=-\langle S_z\rangle\omega^{-1}$ for $\omega\gg\max(g\mu H,Dq^2)$, where D is the spin-diffusion coefficient. Since this result does not depend on q, it remains valid in the critical region too. From this, substituting into (6) the known asymptotic form $G(q,\omega)\propto q^2(iT_c/\omega)^{8/5}$ $[\omega\gg\Omega(q)]$ (Ref. 17), we find

$$\gamma_3{}^{[1]}[\omega/\Omega(q)]\sim i[\omega/i\Omega(q)]^{11/5},\quad \omega\gg\Omega(q). \tag{10}$$

This expression corresponds to the asymptotic form of $\mathrm{Re}G_3$. The behavior of $\mathrm{Im}G_3$ is determined by the next term $\gamma_3^{[2]}$ of the expansion of γ_3. Inasmuch as the contribution corresponding to the uniform precession has already been separated out, we must expect that $G_3^{[2]}\propto q^2$, as is the case for the asymptotic form of G for $H=0$. As a result, $|\gamma_3^{[2]}(x)/\gamma_3^{[1]}(x)|\sim x^{-4/5}(x\gg1)$, and

$$-G^{(-)}_{+-}(q,\ \omega)=g\mu HG_3(q,\ \omega)$$

$$=\frac{g\mu HG_0(\varkappa)}{\omega}\left\{1+ic\left(\frac{i\Omega(q)}{\omega}\right)^{4/5}\right\},\quad \omega\gg\Omega(q). \tag{11}$$

Here c is a real number. The phase in (11) has been chosen to correspond to the fact that $iG^{(-)}_{+-}$ is real on the imaginary upper semiaxis of ω, as is not difficult to establish, starting from the analytic properties of the Green function and the symmetry (3). It is clear that the physical considerations outlined above are in need of rigorous justification. Since at the present time there is no critical-dynamics theory starting from a microscopic Hamiltonian, we shall suggest what are, in our view, direct arguments [based on an analysis of the sum rules (5)] in support of the results obtained [above all, for the power ρ in (8)].

Comparing (5) and (6), (8) and taking into account that in the approximation linear in H we have $\langle S_z\rangle=g\mu HG_0(\varkappa)$, we immediately find $\rho=3/2$, if the principal contribution to the integral is given by values $\omega\sim\Omega(q)$. Before convincing ourselves of the latter, we shall see how the sum rule works in the hydrodynamic region. It is easily verified that the expression for G_{+-} in a weak field[1]:

$$G_{+-}(q,\ \omega)=\frac{-g\mu H+iDq^2}{\omega-g\mu H+iDq^2}G_0(\varkappa)$$

satisfies (5), and for $Dq^2\gg g\mu H$ (when the dynamical form factor can be expanded in H) the characteristic values of ω in the integral (5) are $\sim Dq^2$. We shall show that for $q \gg \varkappa$ the principal contribution to (5) is determined by values $\omega \sim \Omega(q)$. If this is not so, there remains in the problem just one frequency scale (T_c) that can make the necessary contribution to the sum rule. However, the expression for $\mathrm{Im}G_{+-}^{(-)}$ yields in this region the following estimate:

$$\mathrm{Im}\, G_{+-}^{(-)}(q,\omega) \sim \frac{(qa)^2}{T_c}\langle S_z\rangle g^{(-)}\left(\frac{\omega}{T_c}\right), \quad \omega \gtrsim T_c, \quad qa \ll 1, \tag{12}$$

where $|g^{(-)}(x)| \sim 1$ for $x \sim 1$, and the factor q^2, which arises from the conservation law, ensures that the contribution from $\omega \sim T_c$ in (5) is small. The complete derivation of the expression (12) requires the use of the equations of motion and an analysis of the graphs for the Green function of the operators $\dot{S}_q^{\pm}(t)$, which lies outside the scope of the present paper. Therefore, we shall elucidate only the origin of the factor q^2. Using the fact that the interaction with H commutes with the exchange Hamiltonian $\mathscr{H}_{\mathrm{ex}}$, and also the properties of the operator S_j^+, integrating (1) by parts we find

$$G_{+-}(\mathbf{q},\omega) = -\frac{\langle S_z\rangle}{\omega-\omega_0+i\delta} - \frac{1}{\omega-\omega_0+i\delta} L_{+-}(\mathbf{q},\omega),$$

$$L_{+-}(\mathbf{q},\omega) = \int_0^\infty \exp\{i(\omega-\omega_0)t\}\langle[\dot{S}_{\mathbf{q}}^+(t), S_{-\mathbf{q}}^-(0)]\rangle dt. \tag{13}$$

Here

$$\omega_0 = g\mu H, \quad \dot{S}_{\mathbf{q}}^+(t) = \exp(-i\mathscr{H}_{ex}t) S_{\mathbf{q}}^+ \exp(i\mathscr{H}_{ex}t).$$

It is clear that $\mathrm{Im}G_{+-}^{(-)}(\mathbf{q},\omega)$ for $|\omega| \gtrsim T_c \gg \omega_0$ is determined by $\mathrm{Im}L_{+-}^{(-)}(\mathbf{q},\omega)$ in (13). Since $\mathscr{H}_{\mathrm{ex}}$ conserves the components of the total spin of the magnet, $\dot{S}_{\mathbf{q}}^+(t)$ vanishes as $\mathbf{q} \to 0$, and it is this which leads to the appearance of the factor q^2 in (12).

An important check on the estimate (12) is provided by its compatibility with the known inequality $\omega \mathrm{Im}G_{+-}(q,\omega) \geqslant 0$ (Ref. 18), which ensures that the energy of a circularly polarized oscillating magnetic field being absorbed by the magnet is positive. In fact, neglecting the small, field-induced corrections (proportional to H^2 in a weak field) to $G_{xx} = G_{yy} = G$, we obtain

$$\omega\, \mathrm{Im}\, G_{+-}(q,\omega) = \omega(\mathrm{Im}\, G(q,\omega) + \mathrm{Im}\, G_{+-}^{(-)}(q,\omega)) \geqslant 0. \tag{14}$$

Since $\omega \mathrm{Im}G(q,\omega) \geqslant 0$, and $\mathrm{Im}G(\omega)$ is an odd function of ω while $\mathrm{Im}G^{(-)}(\omega)$ is an even function of ω, from (14) we find

$$|\mathrm{Im}\, G(q,\omega)| \geqslant |\mathrm{Im}\, G_{+-}^{(-)}(q,\omega)|. \tag{15}$$

It follows from an analysis[17] of the asymptotic forms of $G(q,\omega)$ that

$$\mathrm{Im}\, G(q,\omega) \sim \frac{(qa)^2}{T_c} g\left(\frac{\omega}{T_c}\right), \quad \omega \gtrsim T_c, \quad qa \ll 1, \tag{16}$$

where the factor q^2 arises from the conservation of the total spin of the magnet, and $g(\omega/T_c) \sim 1$ for $\omega \sim T_c$. It can be seen from a comparison of (16) and (12) that the inequality (15) is satisfied for $\omega \sim T_c$ in the entire fluctuation region, since $\langle S_z\rangle \ll 1$. If, however, the factor $(qa)^2$ in (12) is omitted, then even in a weak field for $\omega \sim T_c$ in the critical region ($q \gtrsim \varkappa$) we can find ranges of parameter values

$$(\varkappa a)^2 \leqslant (qa)^2 \ll [g\mu H/\Omega(\varkappa)](\varkappa a)^{1/2} \sim \langle S_z\rangle$$

for which (15) is violated. This shows once again that for small q in (5) a contribution of the order of $\langle S_z\rangle$ from values $\omega \sim T_c$ cannot arise.

Thus, the necessary contribution to (5) is given only by values $\omega \sim \Omega(q)$. Inasmuch as the hydrodynamic expressions for G_{+-} (Ref. 1) satisfy (5), the given sum rule in the whole fluctuation region has the same force as the well-known relation in zero field:

$$\int_{-\infty}^{\infty} \mathrm{Im}\, G(q,\omega)\,\omega^{-1}\, d\omega = G_0(q). \tag{17}$$

We shall now determine the asymptotic form of $G_{+-}^{(-)}(q,\omega)$ for $\omega \gg (q)$. Since in (5) the characteristic values of ω are $\sim \Omega(q)$, the function $\mathrm{Im}G_{-}^{(-)}(q,\omega) \propto \omega^{-(1+\xi)}$ ($\xi > 0$) for $\omega \gg \Omega(q)$. As can be seen from (13)

$$\mathrm{Im}\, G_{+-}^{(-)}(q,\omega) = \omega^{-1}\, \mathrm{Im}\, L_{+-}^{(-)}(q,\omega) \quad \text{for} \quad \omega \gg \Omega(q) \gg \omega_0$$

and, consequently $\mathrm{Im}L_{+-}^{(-)}(q,\omega) \propto \omega^{-\xi}$. (In the critical region we can set $\omega_0 = 0$ throughout, since here the main H-odd part of G_{+-} and L_{+-} is determined by the dependence of the statistical weight on H). Using the spectral representation for L_{+-}, it is not difficult to convince oneself that $\mathrm{Re}L_{+-}^{(-)}(\omega)$ falls off no more slowly than $\mathrm{Im}L_{+-}^{(-)}(\omega)$ with increase of ω. As a result, from (13) we find $G_{+-}^{(-)}(\omega) = \langle S_z\rangle \omega^{-1}$. The next term of the asymptotic form of $G_{+-}^{(-)}$, which determines the power ξ in $\mathrm{Im}L_{+-}^{(-)}$, can be found by considering the joining with the asymptotic form of $G_{+-}^{(-)}(\omega)$ in the hydrodynamic region. By a method analogous to that used in Ref. 17, it can be shown that in the hydrodynamic region the second asymptotic term of the expansion of $G_{+-}^{(-)}(\omega)$ for $q \ll \varkappa$ and $\omega \gg \Omega(\varkappa)$ is proportional to $[HG_0(\varkappa)/\omega]q^2\omega^{-4/5}$, where the factor q^2 is a consequence of the conservation law. From the matching of the asymptotic forms at $q = \varkappa$ we arrive at $\xi = 4/5$, i.e., at (11).

We note that $\mathrm{Im}G_{+-}^{(-)}(q,\omega)$ falls off more rapidly than $\mathrm{Im}G(q,\omega) \propto q^2\omega^{-8/5}$ with increase of ω. This ensures that the inequality (15) is fulfilled for the asymptotic forms.

Furthermore, as we shall see, the dependence of the expressions (6) and (11) on τ and H in a weak field has turned out to be due entirely to the factor $g\mu HG_0(\varkappa)$. It is understandable that in the general case it is simply replaced by $\langle S_z\rangle$, i.e., the formulas (6), (9), and (11) determine the behavior of $G_{+-}^{(-)}$ in the leading order in $\langle S_z\rangle$. We shall estimate the next terms. From (6) and (9) it can be seen that for $\omega \sim \Omega(q)$ the ratio $|G_{+-}^{(-)}/G| \sim \langle S_z\rangle/(qa)^{1/2}$ [in a strong field, $(\varkappa_H/q)^{1/2}$]. The quantity $G_{+-}^{(-)}$ is expanded in $\langle S_z\rangle$ and the second term of the expansion is of order of smallness $\langle S_z\rangle^2/qa$ ($\varkappa_H/q$, in a strong field) in comparison with the first term.

We now consider the relationship of the results obtained for G_3 to the dynamical form factor Φ_{+-} of the pair Green function. We shall need some of the properties of the dynamical form factor, in particular, in the discussion of the experimental results. Following Ref. 19, we write G_{+-} in the form

$$G_{+-}(q,\omega,H) = G_{0\perp}(q,H)\frac{-\langle S_z\rangle G_{0\perp}^{-1}(q,H) + i\Gamma_{+-}(q,\omega,H)}{\omega - \langle S_z\rangle G_{0\perp}^{-1}(q,H) + i\Gamma_{+-}(q,\omega,H)}. \tag{18}$$

Here G_{01} is the transverse (with respect to **H**) static Green function. In the dynamical form factor (18) two quantities, describing different physical processes, have been separated out. The term $\langle S_z\rangle G_{01}^{-1}$ that does not depend on ω corresponds in the general case to nonuniform precession. The quantity Γ_{+-}, which may be called the relaxation coefficient, has a nonzero value at $\omega = 0$, the real and imaginary parts of which correspond to the damping and frequency shift of the mode with momentum q. The quantity Γ_{+-} falls off at large frequencies. Its symmetry properties follow from (3) and (4). The character of the dependence of Γ_{+-} on its arguments is determined by the character of the interaction of the dynamical modes in this system.

As is well known, in Ref. 19 there is a general expression for the relaxation coefficients. However, it is not very suitable for analysis, since the projection operator appears in a nonlinear manner in the time evolution of Γ. It can be shown that, even in the case $H = 0$, this feature of the time evolution of Γ cannot be fully taken into account in the only microscopic approach to the dynamics (see, e.g., Ref. 6), based on analysis of the expressions for the kinetic coefficients, and the well-known difficulties of this approach remain unsurmounted.[20] In zero field, problems arise in the self-consistent determination of the dependence of Γ on q for $q \gtrsim \varkappa$ and $\omega = 0$. It can be shown that in a magnetic field these problems are mainly associated with the analysis of the critical region. Therefore, we have considered the expansion of G_{+-} in H, without using the representation (18). It now remains to convince ourselves that the expression (18) agrees with the results obtained above, and to determine the behavior of the H-dependent part of Γ_{+-} in the critical region.

Expanding G_{+-} in quantities odd in H, we have, in the critical region,

$$G_{+-}^{(-)}(q,\omega,H) = -\omega\,\frac{\langle S_z\rangle G_0^{-1}(q) + i\Gamma_{+-}^{(-)}(q,\omega,H)}{(\omega + i\Gamma(q,\omega))^2}\,G_0(q) + \dots, \tag{19}$$

where Γ is the zero-field relaxation coefficient, with known properties,[17] and $\Gamma_{+-}^{(-)}$ is the H-odd part of Γ_{+-}. In (19) for $G_{+-}^{(-)}$ we have given only the term proportional to H for $H \to 0$.

If in (19) we retain only the term with $\langle S_z\rangle G_0^{-1}(q)$, then, by comparing (19) with (6) and (9), we can easily convince ourselves that the expressions (19) and (6) for $G_{+-}^{(-)}$ are consistent for $\omega \lesssim \Omega(q)$. Since $\Gamma(q,\omega)$ decreases with frequency, we also have coincidence of the leading asymptotic terms of the expressions (19) and (11) for $\omega \gg \Omega(q)$. The function $\Gamma_{+-}^{(-)}$ can be analyzed conveniently by representing it, in first order in $\langle S_z\rangle$, in the form

$$i\Gamma_{+-}^{(-)}(q,\omega,H) = \langle S_z\rangle G_0^{-1}(q)\,\psi\left(\frac{\omega}{\Omega(q)}\right). \tag{20}$$

Here $|\psi(0)| = |\mathrm{Re}\,\psi(0)| \sim 1$ and $|\psi| \sim 1$ for $\omega \sim \Omega(q)$. Comparing (19) and (20) with the second asymptotic term in (11), for the asymptotic form of ψ we find

$$\psi\left(\frac{\omega}{\Omega(q)}\right) \sim (qa)^2\left(\frac{iT_c}{\omega}\right)^{4/5}, \quad \omega \gg \Omega(q). \tag{21}$$

We stress that only when the term $\Gamma_{+-}^{(-)}$ is taken into account in (19) will the dynamical form factor $G_{+-}^{(-)}$ (19) be consistent with the asymptotic form of $\mathrm{Im}\,G_3(q,\omega)$ in (11). Thus, the second asymptotic term in (11) is determined by the asymptotic form of $\Gamma_{+-}^{(-)}(q,\omega)$.

2. ANALYSIS OF EXPERIMENTAL RESULTS

The temperature dependence of the cross section for small-angle scattering of polarized neutrons in the paramagnetic phase of Fe in a magnetic field was studied in Ref. 15. The problem consisted in investigating the asymptotic properties of the three-spin correlations in the critical region. Polarized neutrons were used for this purpose, since the dependence of the neutron-scattering cross section on the initial neutron polarization is determined by the function $G_{+-}^{(-)}$ (Ref. 13). As a result, in ferromagnets in a weak field the cross section for magnetic scattering through angle θ is found to be proportional to[13]

$$\sigma(\theta,\tau) = \sigma_0(\theta,\tau) + \sigma_{P_0}(\theta,\tau) \propto \int \frac{d\omega}{\omega}\frac{k'}{k}\,\{\mathrm{Im}\,G(q,\omega) + g\mu(\mathbf{He})(\mathbf{P}_0\mathbf{e})\,\mathrm{Im}\,G_3(q,\omega)\}, \tag{22}$$

where $\omega = E' - E$ is the energy transfer, $\mathbf{q} = \mathbf{k}' - \mathbf{k}$ is the momentum transfer, $\mathbf{k}$, $\mathbf{k}'$ and E, E' are the momenta and energies of the neutrons before and after scattering, $\mathbf{P}_0$ is their initial polarization, and $\mathbf{e} = \mathbf{q}q^{-1}$.

It follows from the formula (22) that in the scattering scheme depicted in Fig. 1 there is asymmetry in the scattering plane:

$$\Delta\sigma(\theta,\tau) = \sigma_{P_0}(\theta,\tau) - \sigma_{P_0}(-\theta,\tau) \propto P_0 H \int \frac{k'}{k} e_x e_z\,\mathrm{Im}\,G_3(q,\omega)\,\frac{d\omega}{\omega}. \tag{23}$$

We recall that here the oddness of $\mathrm{Im}\,G_3(q,\omega)\omega^{-1}$ is compensated by the odd part of the factor $(k'/k)e_x e_z$—above all, by the odd part of e_z, which is proportional to ω in the quasielastic approximation. Thus, by studying the dependence of $\Delta\sigma$ on θ and τ one can elucidate the properties of G_3.

The authors of Ref. 15 investigated not only $\Delta\sigma$ but also σ_0 (the part of the cross section that does not depend on P_0), determined in a weak field by the first term in (22). We shall consider first the results obtained for the cross section in Ref. 15, in which $\sigma_0(\tau)$ was studied for fixed θ. As is well known, the paramagnetic critical cross section of elastic scattering of neutrons in ferromagnets is well described by the Ornstein–Zernike formula (for data pertaining to Fe, see Refs. 21–23 and 6):

$$\sigma_0^{-1}(\theta,\tau) \propto (k\theta)^2 + \varkappa^2, \quad \varkappa = \varkappa_0\tau^{\nu}. \tag{24}$$

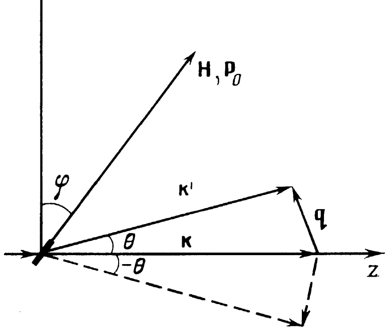

FIG. 1. Kinematic scheme of the polarized-neutron scattering.

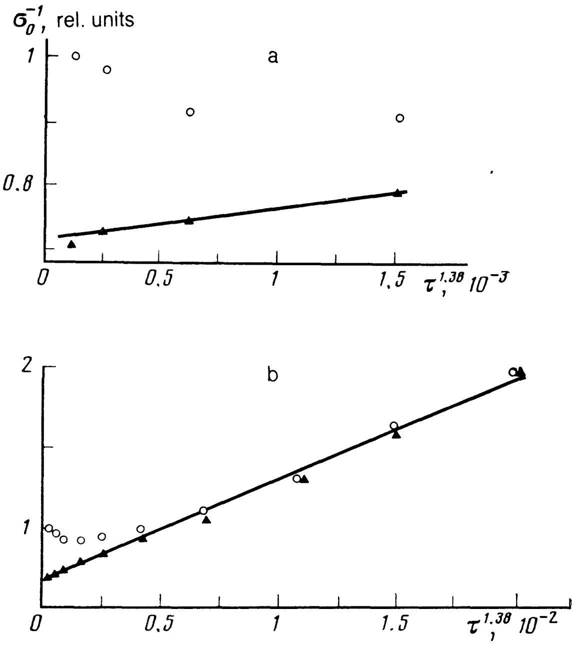

FIG. 2. Reciprocal of the polarization-independent part of the neutron-scattering cross section as a function of $\varkappa^2 \propto \tau^{1.38}$:○—the experimental data of Ref. 15, ▲—the same data after allowance for the inelasticity of the scattering; a) the region $k\theta > \varkappa$ (the critical region, $\max(\varkappa/k\theta) \cong 0.4$); b) the entire region of the measurements ($\max(\varkappa/k\theta) \cong 1.4$).

In the case of Fe, $\varkappa_0 = 1.1$ Å^{-1} and $\nu = 0.69$. We note that in the actual experiments a small spread of values of $\varkappa_0$ and ν is observed ($\Delta\varkappa_0/\varkappa_0 = \pm 0.1$, $\Delta\nu/\nu = \pm 0.03$). Here we give the parameter values that were used in the experimental work that gave the results used below.

Figure 2 shows the dependence of σ_0^{-1} on $\varkappa^2$ from Ref. 15. It can be seen that in the region $\varkappa < k\theta$ the temperature dependence of σ_0^{-1} does not agree with (24). The main reason for the discrepancy is known.[24] It is due to the partial inelasticity of the scattering, i.e., to the fact that in scattering through a given angle the dependence of q on ω turns out to be important ($q \cong k\theta$ for $\omega = 0$). Since $G(q,\omega) = G_0(q)\Phi(q,\omega)$ the cross section acquires an additional temperature dependence due to the τ-dependence of Φ, and, as a result, differs from the elastic limit (24), which is proportional to $G_0^{-1}(q)$. To see this, one introduces an inelastic correction to σ_0 in the following manner.[24,25] Using the explicit form of G, with $q = q(\omega)$ determined by the kinematics of the scattering, one calculates the cross section from (22). One then divides the experimental cross section by the calculated cross section and multiplies by the elastic limit. If inelasticity is the problem, the data corrected in this manner should be described by the expression (24). To allow for inelasticity, $(\mathrm{Im}G)/\omega$ was written in the form

$$\frac{1}{\omega}\,\mathrm{Im}\,G(q,\omega) \propto \frac{1}{q^2+\varkappa^2}\,\frac{\Gamma(q,\varkappa)}{\omega^2+\Gamma^2(q,\varkappa)}. \tag{25}$$

Here Γ is the half-width of the distribution, and, as usual, was represented in the form

$$\Gamma(q,\varkappa) = \Gamma_0 q^{5/2} f(\varkappa/q), \quad f(0) = 1. \tag{26}$$

The value $\Gamma_0 = 130$ MeV·Å$^{5/2}$ and the function $f(x)$ for Fe, both of which are known from the experimental work of Ref. 26, in which $\varkappa_0 = 1.1$ Å^{-1} and $\nu = 0.69$ [see (24)], were used. The data, corrected in the manner described above, are depicted in Fig. 2. It is clear from this figure that inelasticity is indeed the cause of the nonmonotonic behavior of $\sigma_0^{-1}(\tau)$. We note that the minimum in the experimental dependence $\sigma_0^{-1}(\tau)$ arises because of the comparatively steep decrease of $f(x)$ with increase of x on the interval $0 < x \lesssim 1$.

Above, we selected for $\Phi(q,\omega)$ the very simple approximation (25). From experiments on inelastic scattering in Fe (Ref. 27) it is known that for $\omega > \Gamma$ a slight deviation of Φ from the Lorentz formula is observed. However, in the present case the use of this formula is justified, since, first, in the conditions of the given experiment ($k\theta = 0.1$ Å^{-1}, $E = 3.55$ MeV), with allowance for $q = q(\omega)$ we always have $\omega \leqslant \Gamma(q,\varkappa)$, and, secondly, the form (25) was in fact precisely the form used to specify G in Ref. 26 in the determination of the Γ (26) that we have used.

We turn to the analysis of the data for the asymmetry $\Delta\sigma$. In Ref. 15 the asymptotic properties of G_3 in the critical region were established by studying the dependence of $\Delta\sigma$ on τ for $k\theta > \varkappa$ in a weak field. We note that in the formula (23) the integrand differs only by the factor $(k'/k)e_x e_z/\omega$ from the corresponding expression in (5). If the scattering is quasielastic, then $k' \approx k$, $e_x \approx 1$, and $e_z \approx \omega/2E\theta$, and, by virtue of (5), $\Delta\sigma \propto G_0(\varkappa) \propto \tau^{-2\nu}$. Thus, in the quasielastic limit the dependence of $\Delta\sigma$ on τ (and, if the field is not weak, its dependence on H as well) is dictated by the sum rule: $\Delta\sigma \propto \langle S_z \rangle$. In the general case, however, the situation with the asymmetry is similar to that analyzed above for the cross section, and the proportionality to $G_0^{(\varkappa)}$ is not the only reason for the dependence of $\Delta\sigma$ on τ. From $\Delta\sigma$ it is necessary to separate out the factor $G_0(\varkappa)$ and to consider the dependence on τ of the remaining integral over ω in (23). For this, by making use of (6), (19), and (20), we write $\mathrm{Im}G_3/G_0(\varkappa)$ in the form

$$F(q,\omega) = \frac{\mathrm{Im}\,G_3(q,\omega)}{G_0(\varkappa)} \propto \frac{\Gamma(q,\varkappa)\,\omega^2}{[\omega^2+\Gamma^2(q,\varkappa)]^2}. \tag{27}$$

The question of the applicability of this expression for $\mathrm{Im}G_3(q,\omega)$ was in fact elucidated experimentally in Ref. 28, in which the dependence on ω of the entire part of σ proportional to P_0 ($\sigma_{P_0}(\omega)$) was investigated for $k\theta = 0.1$ Å^{-1} and $\Delta T = 30°$. The dependence $\sigma_{P_0}(\omega)$ was described on the basis of the expressions (22) and (27) in the simplest, quasielastic approximation ($q = k\theta$), and the quasielastic value of Γ was found from the experimental spectrum. In Fig. 3 we give a comparison of the dependence $\sigma_{P_0}(\omega)$ from Ref. 28 with that calculated using the formulas (22) (without the integration over ω) and (26), (27) with allowance for the dependence $q(\omega)$. In Fig. 3 the errors in the fitting curve correspond to the experimental uncertainty $\Delta\Gamma/\Gamma = \pm 0.1$ in Ref. 28. As can be seen from Fig. 3, the two functions are in comparatively good agreement, and thus we have an experimental indication of the applicability of the expression (27) for $\mathrm{Im}G_3(q,\omega)$. We do not give the experimental spectrum from Ref. 28 directly, since to describe this spectrum it is necessary also to introduce the resolving function of the spectrometer.

Since the factor (27) depends on τ it is necessary to take this dependence into account in the analysis of $\Delta\sigma(\tau)$. We have done this by analogy with analysis of $\sigma_0(\tau)$. The integral in (23) was calculated with $F(q,\omega)$ from (27), $\Gamma(q,\varkappa)$

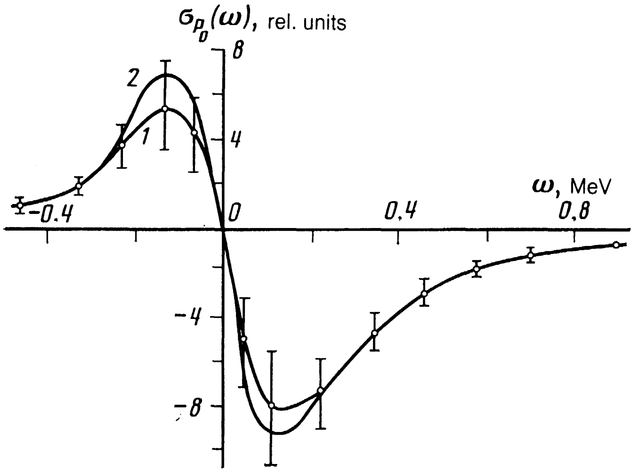

FIG. 3. Polarization-dependent part of the neutron-scattering cross section as a function of the energy transfer for $\varphi = 22°$, $\theta = 4.5°$, $k\theta = 0.1$ Å^{-1}, and $\Delta T = T - T_c = 30°$: 1) fitting curve from Ref. 28; 2) function calculated using the expressions (22) and (26), (27).

from (26), and $q = q(\omega)$. The experimental data for $\Delta\sigma$ were divided by the calculated integral and the temperature dependence of the corrected data was found. It is clear that only the dependence determined in this way can be compared meaningfully with $\tau^{-2\nu}$. Finally, it is also important to take into account the demagnetization in the scattering system. In the formulas (22) and (23) H is the internal field in the sample, which is related to the external field H_{ext} by the well-known relation

$$H = H_{ext}[1 + N_0 4\pi\chi(\tau)]^{-1}, \tag{28}$$

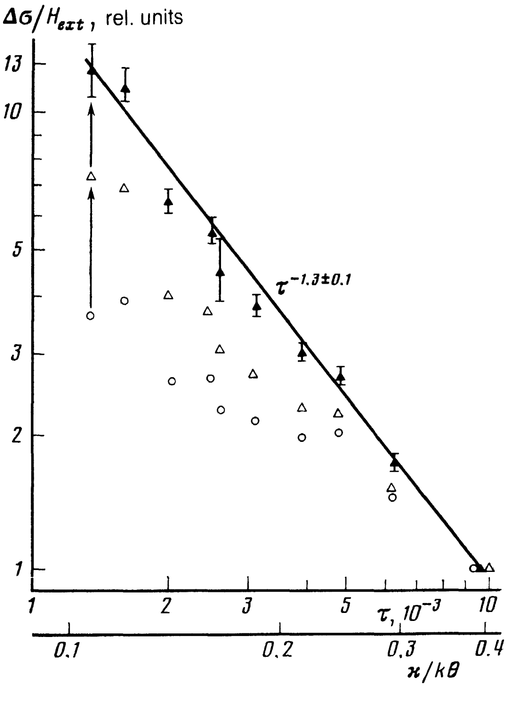

FIG. 4. Dependence of the scattering asymmetry $\Delta\sigma$, divided by $H_{ext} = 22$ Oe, on τ in the region $k\theta > \varkappa$ (the critical region): O—experimental data of Ref. 15, Δ—the same data with allowance for the demagnetization in the scattering system; ▲—data with allowance for the demagnetization and the inelasticity of the scattering.

where N_0 is the demagnetizing factor. Calculations show that $N_0 = 0.1 \pm 0.02$ in the conditions of the experiment of Ref. 15. The corrections for the demagnetization were introduced by multiplying the data by $[1 + N_0 4\pi\chi(\tau)]$.

Figure 4 shows the dependence of the asymmetry $\Delta\sigma$, divided by H_{ext}, on τ. It can be seen that after allowance for the demagnetization and the inelastic correction the temperature dependence of $\Delta\sigma$ agrees with the expected dependence $\tau^{-1.38}$. The fact that the error in the index is comparatively large despite the high statistical accuracy of the data is explained by the appreciable uncertainty in the value of the demagnetizing factor.

Next, the authors of Ref. 15 studied the $\Delta\sigma(\tau)$ dependence in a wider range of temperatures and at large values of the external field ($H_{ext} \cong 2650$ Oe, $\Delta T \geqslant 13.5$ K). In this region, to judge from the data of Ref. 29, the field still remained weak in the static sense, and $\langle S_z \rangle = g\mu H G_0(\varkappa)$. However, the quantity $\langle S_z \rangle G_0^{-1}(q)$ in (18) is comparable with $\Gamma(q,\varkappa)$. In these conditions it is natural to try to determine the term proportional to H in the expansion of $\Gamma_{+-} = \Gamma + \Gamma_{+-}^{(-)}$ in (18). The asymmetry will now be determined by the expression (23) with $-\operatorname{Im} G_3 H$ replaced by $\operatorname{Im} G_{+-}^{(-)}$. For the latter, from (18) we find

$$\begin{aligned} \operatorname{Im} G_{+-}^{(-)}(q,\omega) &= G_0(q)\omega^2\Gamma(q,\varkappa)\Delta(q,\varkappa,H) \\ &\times\{[\omega-\Delta(q,\varkappa,H)]^2+\Gamma^2(q,\varkappa)\}^{-1}\{[\omega+\Delta(q,\varkappa,H)]^2 \\ &\quad +\Gamma^2(q,\varkappa)\}^{-1}, \\ \Delta(q,\varkappa,H) &= \frac{g\mu H G_0(\varkappa)}{G_0(q)} + i\Gamma_{+-}^{(-)}(q,\varkappa,H) \\ &= g\mu H\left\{\frac{q^2}{\varkappa^2}(1+c_0)+1\right\}. \end{aligned} \tag{29}$$

Here, according to (20), $\Gamma_{+-}^{(-)}$ in a weak field can be represented in the form $i\Gamma_{+-}^{(-)} = c_0 g\mu H q^2/\varkappa^2$, which is valid for $q > \varkappa$. Analysis of the experimental data for $\Delta\sigma$ in accordance with (23) and (29) with $k\theta = 0.1$ Å^{-1} and $1.35 \leqslant \Delta T \leqslant 50$ K made it possible to determine the value

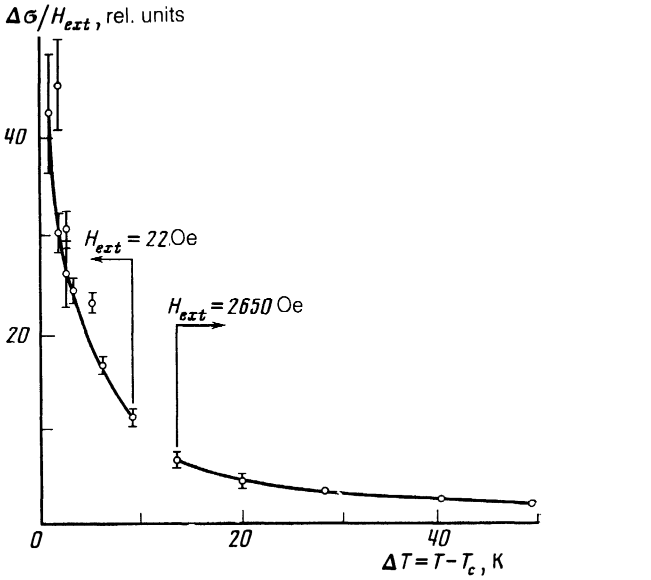

FIG. 5. Temperature dependence of the normalized scattering asymmetry for $k\theta = 0.1$ Å^{-1} and $\varphi = 22°$. The solid line corresponds to the calculation based on the expression (29).

$c_0 = 0.17 \pm 0.08$ (see Fig. 5). This result is extremely interesting, since $1 + c_0 \cong 1$, and, consequently, for G_{+-}, at least in a weak field, in a wide temperature range $0.7 \leqslant k\theta/\varkappa \leqslant 9$ the simple expression (18) with $\Gamma_{+-} \cong \Gamma$ is applicable.

We now compare the experimentally known values of the ratios $\Delta\sigma/2\sigma_0$ with the calculated values. These ratios are equal to $(3.25 \pm 0.15)\cdot 10^{-2}$ for $\varphi = 22°$ and $(4.8 \pm 0.3)\cdot 10^{-2}$ for $\varphi = 35°$, for $\Delta T = 28$ K, $\theta = 4.5°$, and $H_{\rm ext} = 2650$ Oe. The calculated values $3.3\cdot 10^{-2}$ and $4.5\cdot 10^{-2}$ agree well with the experimental values.

In conclusion it must be said that in Ref. 15 the reason for the deviation of the temperature dependence of the cross section from the Ornstein–Zernlike formula were not in fact elucidated. In the interpretation of the dependence of $\Delta\sigma$ on τ the dependence of the factor (27) on the temperature was not taken into account. At the same time, in Ref. 28 these same authors demonstrated the existence of such a dependence. Thus, the interpretation of the results for $\Delta\sigma$ in these two papers, was, at least, inconsistent.

In the present paper we have succeeded in explaining the temperature dependence of the cross section and asymmetry from a unified point of view. In our view, the theory presented is in good agreement with the results obtained in Ref. 15 from an investigation of the asymmetry of the scattering of polarized neutrons in Fe.

In conclusion we note the following. The experimental data were analyzed above in the framework of the purely exchange approximation, and forces giving rise to relaxation of the total moment of the ferromagnet were not taken into account at all. It is not difficult to convince oneself that in the present case the influence of these forces is entirely unimportant. As was discovered in Ref. 26, in Fe the interactions that do not conserve the total spin lead to a correction to Γ (26) describable by the empirical expression $\Gamma_c = B\varkappa/\{1 + (q/q_c)^4\}$, where $B = 0.075$ MeV·Å and $q_c = 0.03$ Å^{-1}. The maximum relative correction to Γ that arises in the conditions of the experiment of Ref. 15 for $q = k\theta = 0.1$ Å^{-1} and $\max\varkappa \cong 0.14$ Å^{-1} is completely insignificant: $\max(\Gamma_c/\Gamma) \cong 4\cdot 10^{-4}$. As a result, the effect of the interactions that do not conserve the total spin is within the error bars of the experiment of Ref. 15 and does not impinge at all on the quantitative analysis given above.

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