Sound viscosity in media in thermodynamic disequilibrium

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Media in thermodynamic disequilibrium are considered. It is shown that negative second viscosity can exist in a large class of such media and leads to instability of sound waves.

Gasdynamic theory of sound absorption takes into account two viscosity coefficients, shear viscosity and second (bulk) viscosity. Second viscosity is manifested in processes accompanied by a change of the volume of the medium, and is due to the finite time needed to establish thermodynamic equilibrium as the sound wave propagates. If the period of the sound wave is of the same order as the time to establish equilibrium (the relaxation time τ), contraction and expansion of the medium will involve an appreciable energy dissipation due to irreversibility of the processes. This dissipation is determined by the second viscosity and can exceed substantially the sound damping to friction, i.e., to shear viscosity. It is shown in Refs. 1 and 2 that the second-viscosity coefficient is defined as

$$\mu=(c_\infty^2-c_0^2)\tau c_{v0}/2c_{v\infty}, \tag{1}$$

where $c_\infty=(c_{p\infty}T_0/c_{v\infty}m)^{1/2}$, and $c_0=(c_{p0}T_0/c_{v0}m)^{1/2}$ are the velocities of the high-frequency ($\omega\gg 1/\tau$) and low-frequency ($\omega\ll 1/\tau$) sound, while $c_{p\infty}$, $c_{v\infty}$, and c_{p0}, c_{v0} are the heat capacities of the high- and low-frequency sound at constant pressure and volume.

The velocity c_∞ of a sound wave propagating in a medium under thermodynamic equilibrium is always higher than the velocity c_0 (Ref. 3), and the second viscosity is positive, meaning sound absorption. Interest attaches to a medium in which a thermodynamic disequilibrium is maintained even in the stationary regime. Such are, for example, laser media, a glow-discharge plasma, media with chemical reactions, etc. It is known that in such media Rayleigh instability of sound can set in,[4] whereby the sound can become amplified under suitable phase relations between the oscillating energy source and the acoustic perturbations. The rather appreciable number of studies devoted to finding the amplification coefficient of sound in media with nonequilibrium excitation of the internal degrees of freedom, chemically active media, or weakly ionized plasma with electron temperature higher than that of the heavy particles[5–7] use an equally large number of expressions for the instability growth rate. This makes their practical use difficult and obscures the common character of the amplification mechanism. We show in the present paper that the growth rate of the acoustic instability in such nonequilibrium media can be expressed in unified form by introducing the second-viscosity coefficient in the form (1). The generalized amplification condition is the change of sign (reversal) of the second-viscosity coefficient. The connection between sound amplification in gases and the reversal of the second-viscosity sign was first indicated in Refs. 12 and 13 as applied to media with nonequilibrium excitation of the internal degrees of freedom of the molecules.[7,10,11] It will be shown below that reversal of second viscosity is possible in media with an energy source and in the absence of nonequilibrium population of the internal levels of the molecules.

Consider sound propagation in a gas containing a stationary energy source. This source can comprise discharge electrons that heat the gas by colliding with its molecules, absorbed light energy, chemical reactions, and others. The heat removal may be by transverse propulsion of the gas. The initial set of equations, in the absence of dissipative processes due to shear viscosity and heat conduction, is of the form

$$\frac{d\rho}{dt}+\rho\nabla\mathbf{v}=0, \quad P=\rho T/m, \tag{2}$$
$$\rho\frac{d\mathbf{v}}{dt}=-\nabla P, \quad c_v\frac{dT}{dt}-\frac{T}{\rho}\frac{d\rho}{dt}=Q,$$

where ρ, P, T, and $\mathbf{v}$ are the density, pressure, temperature, and velocity of the gas, m the molecule mass, c_v is the specified heat of the equilibrium degrees of freedom per molecule at constant volume, and Q is the source power per molecule of the medium.

For one-dimensional perturbation of the form $\sim\exp(-i\omega t+ikx)$, the linearized system of equations (2) leads to the dispersion relation

$$k^2=\frac{\omega^2 m}{T_0}\frac{c_v-\hat{Q}_0(-i\omega\tau_Q)^{-1}}{c_p-(\hat{Q}_0-\check{Q}_0)(-i\omega\tau_Q)^{-1}}, \tag{3}$$

where c_p is the heat capacity of the equilibrium degrees of freedom at constant pressure

$$\hat{Q}_0=\partial\ln Q_0/\partial\ln T_0, \quad \check{Q}_0=\partial\ln Q_0/\partial\ln\rho_0,$$

T_0 and ρ_0 are the unperturbed values of the corresponding quantities, $Q_0=Q(T_0,\rho_0)$, and $|\tau_Q|=T_0/|Q_0|$ is the characteristic time of the source.

It is correct to express the perturbations of the parameters of the medium in the form of plane waves if the amplification (absorption) α at the wavelength λ is small, i.e., if

$$\alpha\lambda\ll 2\pi \tag{4}$$

or, taking (3) into account, if

$$|(c_v\check{Q}_0+\hat{Q}_0)\omega\tau_Q|\ll|c_v c_p\omega^2\tau_Q^2+\hat{Q}_0(\hat{Q}_0-\check{Q}_0)|. \tag{5}$$

If k^2 has a negative real part, the inequality (5) must be reversed. According to (3), $\mathrm{Re}\,k^2<0$ if

$$\hat{Q}_0(\hat{Q}_0-\check{Q}_0)<-c_v c_p\omega^2\tau_Q^2.$$

Under condition (5) we obtain from (3) for $k=k'+ik''$

$$k'=\omega(m/T_0)^{1/2}[c_v c_p\omega^2\tau_Q^2+\hat{Q}_0(\hat{Q}_0-\check{Q}_0)]^{1/2}$$
$$[c_p^2\omega^2\tau_Q^2+(\hat{Q}_0-\check{Q}_0)^2]^{-1/4}, \tag{6}$$

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$$k''=-{}^1/_2\omega^2(m/T_0)^{1/2}\tau_Q(c_v\check{Q}_0+\hat{Q}_0)$$
$$\times[c_p^2\omega^2\tau_Q^2+(\hat{Q}_0-\check{Q}_0)^2]^{-1/2}[c_vc_p\omega^2\tau_Q^2+\hat{Q}_0(\hat{Q}_0-\check{Q}_0)]^{-1/2}. \quad (7)$$

We consider the two limiting cases of high and low frequencies. At high frequencies $\omega|\tau_Q|\gg|\hat{Q}_0-\check{Q}_0|/c_p$, $(|\hat{Q}_0(\hat{Q}_0-\check{Q}_0)|/c_pc_v)^{1/2}$ it follows from (6) and (7) that

$$c_\infty\equiv\omega/k'=(c_pT_0/c_vm)^{1/2}, \quad (8)$$

$$\alpha_\infty\equiv k''=-(c_v\check{Q}_0+\hat{Q}_0)/2c_\infty\tau_Qc_pc_v, \quad (9)$$

where α_∞ is the amplification coefficient of the high-frequency sound.

For $\theta|\tau_Q|\ll|\hat{Q}_0-\check{Q}_0|/c_p$, $(|Q_0(\hat{Q}_0-\check{Q}_0)|/c_pc_v)^{1/2}$ we have

$$c_0=[(T_0/m)(\check{Q}_0-\hat{Q}_0)/(-\hat{Q}_0)]^{1/2}, \quad (10)$$

$$\alpha_0=-\omega^2\tau_Q(c_v^0\check{Q}_0+\hat{Q}_0)/2c_0(\hat{Q}_0-\check{Q}_0)\hat{Q}_0, \quad (11)$$

where α_0 is the low-frequency sound amplification coefficient. Note that in the case $\hat{Q}_0=0$ the condition (4) is not met for low frequencies, so that no sound wave propagates in this frequency band.

Expressions (9) and (11) can be written in the usual form for the absorption coefficient if second viscosity exists in the medium

$$\alpha_0=\omega^2\mu/c_0^3,\ \alpha_\infty=\mu c_{v0}^2/c_{v\infty}^2c_\infty^3\tau_Q^2, \quad (12)$$

where

$$\mu=-c_\infty^2c_v\tau_Q(c_v\check{Q}_0+\hat{Q}_0)/2c_p\hat{Q}_0^2. \quad (13)$$

At $c_{v\infty}=c_v$, $c_{v0}=-Q_0$, $c_{p0}=\check{Q}_0-\hat{Q}_0$, and $\tau=\tau_Q$ expression (13) takes the form (1). The amplification condition, according to (12), is $\mu<0$ or

$$Q_0(c_v\check{Q}_0+\hat{Q}_0)>0. \quad (14)$$

For chemically active media, the energy source can be represented in the form $Q=Hk(T)\rho_1\rho_2/\rho$, where k and H are the rate and enthalpy of the chemical reaction ($H<0$ and $H>0$ for endothermic and exothermic reactions, respectively), and ρ_1 and ρ_2 are the densities of the reagents. The amplification coefficients α_0 and α_∞ (12) coincide then with the acoustic-instability growth rates given in Refs. 7 and 8. Amplification is possible for both exothermic and endothermic reactions if condition (14) is met. For endothermic reactions, the condition (14) means that the energy absorption should be smaller at the maxima of the sound wave than at the minima. Of course, to maintain the medium stationary, additional heat removal is necessary at $Q_0<0$.

Another example of a medium in which sound amplification was extensively investigated is a weakly ionized plasma.[5,6] Here Q_0 is the electron energy loss in collisions with heavy particles. It is assumed in Refs. 5 and 6 that $\hat{Q}_0=0$, so that the condition (4) is not met. For the high-frequency band, the coefficient α_∞ defined by Eq. (12) coincides, apart from the notation, with the corresponding amplification coefficient obtained in these papers.

If the source power is consumed initially in excitation of the internal (say, vibrational) degrees of freedom, the system (2) must be supplemented by the relaxation equation

$$\frac{d\varepsilon_k}{dt}=\frac{\varepsilon_k^{eq}-\varepsilon_k}{\tau_k(T,\rho)}+Q,$$

where ε_k and ε_k^{eq} are the reserve of the vibrational energy and its equilibrium value per molecule, and τ_k is the vibrational relaxation time. In addition, account must be taken of the change of ε_k in the equation for the energy conservation law (in analogy with Ref. 13). As a result, the dispersion relation (3) is transformed into

$$k^2=\frac{\omega^2m[c_v+c_{kv}-\hat{Q}_0(-i\omega\tau_Q)^{-1}-i\omega\tau_kc_v]}{T_0[c_p+c_{kp}-(\hat{Q}_0-\check{Q}_0)(-i\omega\tau_Q)^{-1}-i\omega\tau_kc_p]}, \quad (15)$$

where

$$c_{kv}=\left(\frac{\partial\varepsilon_k}{\partial T}\right)_v=c_k+\frac{\varepsilon_k^0-\varepsilon_{eq}^0}{T_0}\hat{\tau}_k,$$

$c_{kp}=(\partial\varepsilon_k/\partial T)_p=c_{kv}+(\varepsilon_k^0-\varepsilon_{eq}^0)/T_0$ are the low frequency vibrational heat capacities, c_k is the equilibrium vibrational heat capacity, ε_k^0 and ε_{eq}^0 are the stationary values of ε_k and ε_{eq}, and $\tau_k=\partial\ln\tau_k/\partial\ln T_0$.

According to (15), the perturbations of the energy source $\hat{Q}_0$ and $\check{Q}_0$ come into play primarily for sound of frequency

$$\omega^2\ll\left|\frac{(c_v+c_{kv})\check{Q}_0+\hat{Q}_0(1+c_{kp}-c_{kv})}{\tau_Q\tau_k(c_pc_{kv}-c_vc_{kp})}\right|.$$

In this case the instability growth rate is determined as before by (12) and (1), but subject to the substitutions $c_{v\infty}=c_v+c_{kv}$, $c_{p\infty}=c_p+c_{kp}$.

At higher frequencies, the influence of the source can be neglected. The amplification coefficient is again determined by (12) and (1), but with $c_{p0}=c_p+c_{kp}$, $c_{v0}=c_v+c_{kv}$, $\tau=\tau_k$, as shown in Ref. 13. For these frequencies, the second viscosity and the corresponding amplification of the sound are due to the relaxational properties of the medium itself and to its degree of excitation, and do not depend on the method used for stationary maintenance of the nonequilibrium energy distribution.

We have considered so far gaseous media. We show in conclusion that negative sound viscosity can exist also in solids. Consider an isotropic solid with active centers whose excitation is not in equilibrium. If the longitudinal sound-wave frequencies are such that elasticity theory is valid (i.e., the medium can be regarded as homogenous and continuous), the initial system of equations describing the sound propagation in the direction of the x axis can be written in the form

$$\frac{\partial\mathscr{E}}{\partial t}+c_u\frac{\partial T}{\partial t}+3K\alpha_TT_0\frac{\partial^2u}{\partial t\,\partial x}=0,$$
$$\rho_0\frac{\partial^2u}{\partial t^2}-B\frac{\partial^2u}{\partial x^2}+3K\alpha_T\frac{\partial T}{\partial x}=0, \quad (16)$$
$$\frac{\partial\mathscr{E}}{\partial t}=-\frac{\mathscr{E}}{\tau_0}+\frac{\mathscr{E}_0}{T_0\tau_0}\hat{\tau}_0T,$$

where $\mathscr{E}$ is the energy stored in the active centers per cm^3 (we assume its equilibrium value to be zero), τ_0 is the relaxation time, $\tau_0=\partial\ln\tau_0/\partial\ln T_0$, u is the displacement of the continuous medium in the x direction, K is the hydrostatic-compression modulus, B is the elastic modulus of the plane wave, and c_u is the heat capacity at constant u per unit volume (the analog of the specific heat at constant volume), and α_T is the thermal-expansion coefficient. If the relaxation time depends on the strain in the medium or if the nonequi-

librium excitation source power depends on the temperature and on the strain, they must also be taken into account in the initial system.

For one-dimensional perturbations $u \sim \exp(ikx - i\omega t)$, the dispersion equation corresponding to (16) takes the form

$$k^2 = \frac{\omega^2 \rho_0}{B} \frac{c_{v0} - c_{v\infty} i\omega\tau_0}{c_{p0} - c_{p\infty} i\omega\tau_0}, \tag{17}$$

where $c_{v\infty} = c_u$, $c_{p\infty} = c_u + 9\alpha_T^2 T_0 K^2/B$ is the specific heat at constant volume and pressure of the high-frequency sound ($\omega\tau_0 \gg c_{p0}/c_{p\infty}$, $c_{v0}/c_{v\infty}$); $c_{v0} = c_{v\infty} + \mathscr{E}_0\hat{\tau}_0/T_0$, $c_{p0} = c_{p\infty} + \mathscr{E}_0\hat{\tau}_0/T_0$ are the corresponding low-frequency specific heats.

It follows from (17) that the amplification coefficients of high- and low-frequency sound are determined by (12), where the sound velocities are $c_0^2 = c_{p0}B/\rho_0 c_{v0}$, $c_\infty^2 = c_{p\infty}B/\rho c_{v\infty}$, and the second viscosity is

$$\mu = \frac{9\tau_0 \mathscr{E}_0 \hat{\tau}_0 K^2 \alpha_T^2}{2c_{v0}^2 \rho_0} = \frac{\tau_0 (c_\infty^2 - c_0^2) c_{v\infty}}{2c_{v0}},$$

i.e., in full accord with (1). For $\hat{\tau} < 0$ the second viscosity is negative. Since, however, the thermal-expansion coefficient of a solid is small, and the specific heat is large, the amplification connected with the negative second viscosity will be very small at realistic energy inputs to the nonequilibrium degrees of freedom.

The sound instability due to the Rayleigh mechanism can thus be caused, in a rather large class of media that are not in thermodynamic equilibrium, by negative second viscosity. This unified approach makes it possible not only to find the generalized amplification condition ($\mu < 0$), but also to simplify considerably the investigation of the nonlinear evolution of acoustic pulses and beams, and also their interaction with electromagnetic waves in nonequilibrium media. This is due to the possibility of using, in a number of cases, the corresponding equations for equilibrium media, but with reversal of the sign of the second-viscosity coefficient, as can be seen from Refs. 12–14.

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