Sound vibrations and bulk viscosity of a mixture of light and heavy gases

V. A. Alekseev and P. G. Fedorov

P. N. Lebedev Physics Institute, USSR Academy of Sciences

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It is shown that sound damping in a mixture of gases differing greatly in mass is determined almost completely by bulk viscosity, for which analytic expressions are obtained. The theoretically described strong dispersion of the sound velocity in such a system is in good agreement with experiment.

INTRODUCTION

In Ref. 1, devoted to measurement of the sound velocity in a helium-xenon mixture, it was observed that the character of the frequency dependence of the sound velocity changes jumpwise on going through the relative-concentration region $c = N_1(\mathrm{He})/(N_1(\mathrm{He}) + N_2(\mathrm{Xe})) \approx 0.5$, where $N\,[\mathrm{cm}^{-3}]$ is the number of particles per unit volume of gas. The sound velocity decreases with increase of frequency at $c < 0.5$ and increases at $c > 0.5$. An approximately similar behavior of the sound velocity was predicted theoretically in Ref. 2. Its authors solved numerically the equations of two-temperature hydrodynamics with phenomenologically introduced coefficients describing the dynamics of equalization of the mixture-component velocities and temperatures of and with three usual transport coefficients describing the thermal conductivity and viscosity of the considered mixture.

We develop in the present paper a kinetic approach to the description of the frequency dependence of the sound velocity. It shows that in the transition concentration region $c \approx 0.5$ the frequency dependence of the sound velocity is determined in fact by only one transport coefficient—the rate of momentum exchange between the mixture components. The value of this coefficient is reliably determined by comparing the theoretical experimental frequency dependences of the sound velocity.

Analytic equations are obtained for the frequency dependence of the sound velocity and for the bulk viscosity.

§1. KINETIC APPROACH

For a kinetic description of the propagation of sound vibrations in a mixture of two gases, we represent the component distribution function in the form

$$f_k(\mathbf{r}, \mathbf{v}, t) = N_k\varphi_k(\mathbf{v})\,(1+\chi_k(\mathbf{r}, \mathbf{v}, t)), \quad k=1,\, 2, \tag{1}$$

where $\varphi_{1,2}$ are Maxwellian distribution functions, $N_{1,2}$ are the densities of gases 1 and 2, respectively, and $|\chi| \ll 1$. We seek next the functions χ in plane-wave form:

$$\chi(\mathbf{r}, \mathbf{v}, t) = \psi(\mathbf{v})\exp(i\mathbf{q}\mathbf{r} - i\omega t). \tag{2}$$

Substituting (1) in the transport equations for the distribution functions $f_1(r,v,t)$, $f_2(r,V,t)$ and using (2), we obtain by the usual linearization procedure[3] the following system of equations:

$$\begin{gathered} -i\omega\psi_1(\mathbf{v}) + i\mathbf{q}\mathbf{v}\psi_1(\mathbf{v}) + \hat{K}_{12}(\psi_1(\mathbf{v}),\, \psi_2(\mathbf{V})) = 0, \\ -i\omega\psi_2(\mathbf{V}) + i\mathbf{q}\mathbf{V}\psi_2(\mathbf{V}) + \hat{K}_{21}(\psi_2(\mathbf{V}),\, \psi_1(\mathbf{v})) = 0. \end{gathered} \tag{3}$$

The linearized operator $\hat{K}_{12}$ describes elastic collisions of the particles of gas 1 both with one another and with particles of gas 2; it takes the form

$$\begin{aligned} &\hat{K}_{12}(\psi_1(\mathbf{v}), \psi_2(\mathbf{V})) \\ &\quad = N_1 \int W_{11}(\mathbf{v}, \mathbf{v}', \mathbf{v}_1, \mathbf{v}_1')\,\varphi_1(\mathbf{v}')\,[\psi_1(\mathbf{v}) + \psi_1(\mathbf{v}') \\ &\quad - \psi_1(\mathbf{v}_1) - \psi_1(\mathbf{v}_1')]\,d\mathbf{v}'\,d\mathbf{v}_1\,d\mathbf{v}_1' + N_2 \int W_{12}(\mathbf{v}, \mathbf{V}, \mathbf{v}_1, \mathbf{V}_1)\,\varphi_2(\mathbf{V}) \\ &\qquad \times [\psi_1(\mathbf{v}) + \psi_2(\mathbf{V}) - \psi_1(\mathbf{v}_1) - \psi_2(\mathbf{V}_1)]\,d\mathbf{V}\,d\mathbf{V}_1\,d\mathbf{v}_1. \end{aligned} \tag{4}$$

Here N_1W_{11} is the number of $v\cdot v' \to v_1\cdot v_1'$ transitions per unit time in collisions of the first-gas particles with one another, and N_2W_{12} is the number of analogous transitions in collisions of particles of different species. The operator $\hat{K}_{21}$ is obtained from K_{12} by the interchange $1\leftrightarrow 2$.

To continue the description it is convenient to introduce the two-component vector $\psi = (\psi_1, \psi_2)$, define the scalar product by the relation[4]

$$(\psi, \mathbf{g}) = \frac{N_1}{N}\int \psi_1 g_1 \varphi_1(\mathbf{v})\,d\mathbf{v} + \frac{N_2}{N}\int \psi_2 g_2 \varphi_2(\mathbf{V})\,d\mathbf{V}, \quad N_1 + N_2 = N$$

and introduce the operators $\hat{A}$ and $\hat{K}$:

$$\hat{A}\psi = \begin{pmatrix} iqv_x\psi_1 \\ iqV_x\psi_2 \end{pmatrix}, \quad \hat{K}\psi = \begin{pmatrix} \hat{K}_{12}(\psi_1, \psi_2) \\ \hat{K}_{21}(\psi_2, \psi_1) \end{pmatrix},$$

which enable us to rewrite the system (3) in compact form:

$$-i\omega\psi + \hat{A}\psi + \hat{K}\psi = 0. \tag{5}$$

The x axis is chosen along the direction of the vector $\mathbf{q}$. Solution of Eq. (5) yields the dispersion dependence of $\hat{q}$ on ω.

It can be shown that the introduced operator $\hat{K}$ is Hermitian and that its eigenvalues are non-negative. Four of the eigenfunctions of the operator $\hat{K}$ correspond to a zero eigenvalue $\lambda = 0$:

$$\begin{gathered} \mathbf{F}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \mathbf{F}_2 = \frac{1}{(N_1N_2)^{1/2}}\begin{pmatrix} N_2 \\ -N_1 \end{pmatrix}, \\ \mathbf{F}_3 = \frac{1}{(\mu T)^{1/2}}\begin{pmatrix} m_1v_x \\ m_2V_x \end{pmatrix}, \quad \mathbf{F}_4 = \left(\frac{2}{3}\right)^{1/2}\begin{pmatrix} \dfrac{m_1v^2}{2T} - \dfrac{3}{2} \\ \dfrac{m_2V^2}{2T} - \dfrac{3}{2} \end{pmatrix}, \end{gathered} \tag{6}$$

as is easily verified by direct substitution in (4). In Eq. (6), $\mu = cm_1 + (1-c)\,m_2$ and T is the gas temperature in energy units. The first of these functions reflects the fact that the functions (1) remain equilibrium distribution functions when the total number of particles is changed but the percentages of the mixture components are preserved. The action of the operator $\hat{K}$ on the functions F_3 and F_4 is zero in view of the momentum and energy conservation laws. In a mixture of gases with not greatly disparate masses all the remaining eigenfunctions correspond in order of magnitude of eigenvalues λ equal to the gas-kinetic frequency of the collisions, $\lambda \sim Nv\sigma_{gk}$, which exceeds noticeably in the hydrodynamic limit both the sound frequency and the quantity qv_x. This means that these eigenfunctions relax to the equilibrium value in times much shorter than ω^{-1}. The distribution function corresponding to weakly damped vibrations of such a system is therefore described by the linear combination of the functions (6) which diagonalizes the operator $\hat{A}$.

In the case of gases with highly disparate masses, the times of energy and momentum transfer from one gas to the other, which are of the order of $(Nv\sigma_{gk} m_1/m_2)^{-1}$, turn out to be much longer than the mean free path time $(Nv\sigma_{gk})^{-1}$. This means that besides the functions (6) the operator $\hat{K}$ has also other eigenfunctions that correspond to small eigenvalues $\lambda \ll Nv\sigma_{gk}$. These functions can be written in the form of a linear combination of the functions

$$\Phi_1=\left(\frac{N}{N_1}\right)^{1/2}\begin{pmatrix}1\\0\end{pmatrix},\quad \Phi_2=\left(\frac{N}{N_1m_1T}\right)^{1/2}\begin{pmatrix}m_1v_x\\0\end{pmatrix},$$
$$\Phi_3=\left(\frac{2}{3}\frac{N}{N_1}\right)^{1/2}\begin{pmatrix}\dfrac{m_1v^2}{2T}-\dfrac{3}{2}\\0\end{pmatrix}, \tag{7}$$
$$\Phi_4=\left(\frac{N}{N_2}\right)^{1/2}\begin{pmatrix}0\\1\end{pmatrix},\quad \Phi_5=\left(\frac{N}{N_2m_2T}\right)^{1/2}\begin{pmatrix}0\\m_2V_x\end{pmatrix},$$
$$\Phi_6=\left(\frac{2}{3}\frac{N}{N_2}\right)^{1/2}\begin{pmatrix}0\\\dfrac{m_2V^2}{2T}-\dfrac{3}{2}\end{pmatrix},$$

since this combination makes it possible to construct both functions of the type (6), which describe the vibrations of the entire mixture as whole, and functions that describe vibrations of each gas separately.

Substituting a linear combination of the functions (7) in Eq. (5), which is an equation for the eigenvalues $\lambda = i\omega$ of the operator $\hat{A} + \hat{K}$, we obtain a secular equation for the connection between q and ω:

$$\begin{aligned}
&\omega^6+i\omega^5\left[G_2\frac{\mu}{m_2}+G_3\right]\\
&-\omega^4\left[\frac{5}{3}(qu_1)^2+\frac{5}{3}(qu_2)^2+G_2G_3\frac{\mu}{m_2}\right]\\
&-i\omega^3\Big[(qu_1)^2\left(\frac{5}{3}\frac{N_1m_1}{N_2m_2}G_2+\frac{5}{3}\frac{N_1}{N}G_3+\frac{N_2}{N}G_3\right)\\
&+(qu_2)^2\left(\frac{5}{3}G_3\frac{N_2}{N}+\frac{5}{3}\frac{N_2}{N}G_2+\frac{N_1}{N}G_3\right)\Big]\\
&+\omega^2\Big\{\left(\frac{5}{3}qu_1qu_2\right)^2\\
&+\frac{N_2}{N}G_2G_3\Big[(qu_1)^2\frac{N_1m_1}{N_2m_2}\left(\frac{5}{3}\frac{N_1}{N_2}+1\right)\\
&+(qu_2)^2\left(\frac{5}{3}+\frac{N_1}{N_2}\right)\\
&+\frac{4}{3}qu_1\,qu_2\frac{N_1}{N_2}\left(\frac{m_1}{m_2}\right)^{1/2}\Big]\Big\}+i\omega\frac{5}{3}(qu_1)^2(qu_2)^2G_3=0.
\end{aligned} \tag{8}$$

Here $u_1 = (T/m_1)^{1/2}$ and $u_2 = (T/m_2)^{1/2}$ are the mean thermal velocities of the molecules, while G_2 and G_3 are given by

$$\begin{aligned}
G_2&=\frac{Nm_1}{T}\int W_{12}(\mathbf{v},\mathbf{V},\mathbf{v}',\mathbf{V}')\,v_x(v_x-v_x')\\
&\qquad\times\varphi_1(\mathbf{v})\varphi_2(\mathbf{V})\,d\mathbf{v}\,d\mathbf{V}\,d\mathbf{v}'\,d\mathbf{V}'\\
&=\frac{1}{3}\frac{Nm_1}{T}\int v^3\varphi_1(v)\sigma_{tr}(v)\,dv\equiv\frac{1}{3}\frac{Nm_1}{T}\langle v^3\sigma_{tr}\rangle\approx\frac{1}{3}Nv\sigma_{tr},\\
G_3&=\frac{N}{6T^2}\int W_{12}(\mathbf{v},\mathbf{V},\mathbf{v}',\mathbf{V}')\,(m_1v^2-m_1v'^2)\varphi_1(\mathbf{v})\varphi_2(\mathbf{V})\\
&\qquad\times d\mathbf{v}\,d\mathbf{V}\,d\mathbf{v}'\,d\mathbf{V}'\approx Nv\sigma_{tr}\frac{m_1}{m_2},
\end{aligned} \tag{9}$$

where σ_{tr} is the transport cross section for the scattering of the light particles by the heavy ones. It is taken into account in (8) that the result of the action of the operator K on the functions

$$\mathbf{F}_3=\left(\frac{N_1m_1}{N\mu}\right)^{1/2}\Phi_2+\left(\frac{N_2m_2}{N\mu}\right)^{1/2}\Phi_5$$

and $$\mathbf{F}_4=\left(\frac{N_1}{N}\right)^{1/2}\Phi_3+\left(\frac{N_2}{N}\right)^{1/2}\Phi_6$$

from the set (6) is zero, $\hat{K}\mathbf{F}_{3,4} = 0$. Therefore the only matrix elements of the operator $\hat{K}N/N_2$ which differ from zero in the basis (7) are:

$$K_{22}=(\Phi_2|\hat{K}|\Phi_2)=G_2,\quad K_{33}=(\Phi_3|\hat{K}|\Phi_3)=G_3,$$
$$K_{55}=\frac{N_1m_1}{N_2m_2}G_2,\quad K_{66}=\frac{N_1}{N_2}G_3,$$
$$K_{25}=-\left(\frac{N_1m_1}{N_2m_2}\right)^{1/2}G_2,\quad K_{36}=-\left(\frac{N_1}{N_2}\right)^{1/2}G_3.$$

The quantities $(m_1/m_2)G_2$ and G_3, which characterize respectively the rate of particle momentum and energy exchange in the light and heavy gases, are of the same order of magnitude. The hindrance to the momentum and energy transfer is manifested by the presence of the factor m_1/m_2 and leads to the appearance of bulk viscosity. It will be shown below that at not too great a difference between the concentrations of the light and heavy gases this bulk viscosity is much larger than the shear viscosity obtained when all the remaining eigenfunctions of the operator $\hat{K}$ are included in the analysis. In the upshot, it is just this excess of the bulk over the shear viscosity which justifies the description of the mixture vibrations in the basis of the functions (7).

§2. HYDRODYNAMIC APPROACH

Vibrations in a mixture of light and heavy gases admit of a hydrodynamic description in which, however, it must be recognized that the component temperatures and velocity directions can differ.[2] This situation is described by the following equations:

$$\frac{\partial n_1}{\partial t}+\frac{\partial}{\partial x}(n_1\bar{u}_1)=0,$$

$$
\begin{aligned}
&n_1m_1\frac{\partial \bar{u}_1}{\partial t}=-\frac{\partial}{\partial x}(n_1T_1)+B(\bar{u}_2-\bar{u}_1),\\
&\frac{\partial}{\partial t}(n_1T_1)=-\frac{5}{3}\frac{\partial}{\partial x}(n_1\bar{u}_1T_1)+Q(T_2-T_1),\\
&\frac{\partial n_2}{\partial t}+\frac{\partial}{\partial x}(n_2\bar{u}_2)=0,\\
&n_2m_2\frac{\partial \bar{u}_2}{\partial t}=-\frac{\partial}{\partial x}(n_2T_2)-B(\bar{u}_2-\bar{u}_1),\\
&\frac{\partial}{\partial t}(n_2T_2)=-\frac{5}{3}\frac{\partial}{\partial x}(n_2\bar{u}_2T_2)-Q(T_2-T_1).
\end{aligned}
\tag{10}
$$

Here n_1, n_2, T_1, T_2, $\bar{u}_1$, and $\bar{u}_2$ are respectively the concentrations, temperatures, and directional velocities of components 1 and 2. The terms proportional to the differences $\bar{u}_2-\bar{u}_1$ and T_2-T_1 describe the equalization of the velocities and temperatures of the mixture components, while the constants B and Q determine the rate of this equalization.

The system (10) is linearized in the standard manner:

$$
\begin{aligned}
&n_k=N_k[1+\tilde{N}_k\exp(iqx-i\omega t)],\quad T_k=T[1+\tilde{T}_k\exp(iqx-i\omega t)],\\
&\bar{u}_k=u_k\tilde{u}_k\exp(iqx-i\omega t),\\
&u_k=(T/m_k)^{1/2},\quad k=1,\,2,\quad \tilde{N}_k,\,\tilde{T}_k,\,\tilde{u}_k\ll 1.
\end{aligned}
\tag{11}
$$

Substituting (11) in (10) and retaining only the terms linear in the variables $\tilde{N}_k$, $\tilde{u}_k$, and $\tilde{T}_k$ we obtain for them a system of equations that can be solutions if a determinant that coincides with (8) after substitution

$$B=(N_1N_2m_1/N)G_2,\quad Q=(N_1N_2/N)G_3$$

is equal to zero.

Note that no such equality is observed in Ref. 5, where the hydrodynamic equations did not take into account the equalization of the component temperatures, and the dispersion equation corresponding to (8) contains errors.

Thus, the foregoing kinetic analysis enables us to express the phenomenological constants introduced into the hydrodynamics in terms of the elastic cross sections for scattering of particles of one gas by those of the other. One of these constants could have been obtained also by another method. The quantity $M=B\bar{u}_2/N_2$ has the physical meaning of the force exerted by the immobile gas 1 on one particle of gas 2 that moves with velocity u_2. Substituting for G_2 the expression from (9), we obtain for the value of this force

$$M=\frac{1}{3}N\frac{N_1m_1}{N}\langle v^3\sigma_{tr}\rangle\,\bar{u}_2\frac{m_1}{T}=\frac{1}{3}N_1\frac{m_1^2}{T}\langle v^3\sigma_{tr}\rangle\,\bar{u}_2,$$

which coincides with the one given in Ref. 3. We were unable to obtain a similar expression for G_3.

The presence of the terms $B(\bar{u}_2-\bar{u}_1)$ and $Q(T_2-T_1)$ in (10) and their connection with G_2 and G_3 from (9) leads to the conclusion that their presence causes the bulk viscosity in the mixture. The microscopic equations obtained for B and Q allow us to state that this bulk viscosity is much larger than the shear viscosity not accounted for in (10). The possibility of neglecting the corresponding terms in (10), which we shall prove rigorously below, leads in fact to the much simpler form of (10) compared with the employed two-temperature hydrodynamics equations,[2] in which the influences of the shear viscosity and thermal conductivity are retained. As a result, Eqs. (10) and (8) can be solved analytically, whereas the hydrodynamic equations in Ref. 2 were investigated only numerically.

§3. VIBRATIONAL MODES

Introducing the notation $a=m_1/m_2$, $u_1=v$, $u_2=va^{1/2}$, we rewrite Eq. (8) in the form of a biquadratic equation for the variable qv:

$$
\begin{aligned}
&(qv)^4\frac{5}{3}a\left[\frac{5}{3}\omega+iG_3\right]\\
&\quad-(qv)^2\omega\frac{5}{3}\left[\omega^2(1+a)+i\omega G_3\left(1+a-\frac{2}{5}\frac{\mu}{m_2}\right)\right.\\
&\quad\left.+i\omega G_2a-aG_2G_3\right]+\omega^3\left[\omega+iG_2\frac{\mu}{m_2}\right][\omega+iG_3]=0.
\end{aligned}
\tag{12}
$$

Solutions of this equation give the dependence of the wave vector $q=q'+iq''$ on the vibration frequency. The form of the functions $q(\omega)$ depends substantially on the values of the parameters aG_2 and G_3, and they can be simplified in the two limiting cases $\omega\ll aG_2$ and $\omega\gg aG_2$.

In the low-frequency limit $\omega\ll aG_2$, when the light and heavy gases have time to exchange momentum energy during a vibration period, we obtain

$$q_{1,2}=\pm\frac{\omega}{v_c}\left[1+i\frac{\omega}{2G_2a}\frac{c(1-c)}{ca+1-c}\right],\quad v_c=v\left(\frac{5}{3}\frac{m_1}{\mu}\right)^{1/2},\tag{13}$$

$$q_{3,4}=\pm 2^{-1/2}\frac{(\omega G_2)^{1/2}}{v}(i-1).\tag{14}$$

The second of these solutions corresponds to a strongly damped vibration for which $q''/q'=1$. The solution (13) attenuates weakly over a wavelength $q''/q'\sim\omega/aG_2\ll 1$ and propagates at the velocity v_m of the sound in the mixture. It is easy also to verify that this solution corresponds to vibrations of both gases, since it is a linear combination of all six functions (7). The sound damping (13) determines the bulk viscosity

$$\zeta=\frac{\rho v_c^2}{G_2a}\frac{c(1-c)}{ca+1-c},\quad \rho=N_1m_1+N_2m_2,$$

which depends on the percentage composition of the mixture and reaches at $c\approx 1-a$ a maximum equal to $\zeta=\frac{1}{2}\rho v_c^2(1-a/2)/aG_2$ ($1-a=0.97$ in the case of an He-Xe mixture). In the limiting case $N_1\gg N_2$ expression (13) coincides with that obtained in Ref. 4.

We consider now the relation between the contribution made to the sound damping by the bulk viscosity contained in (13) and the shear viscosity not included in Eqs. (8) and (10).

In the case of the gas He and Xe of interest to us, it is known that their shear viscosities (in the pure gases) differ by only a few times (see, e.g., Ref. 6). The damping due to the shear viscosity in the mixture can therefore be estimated at

$$q_c''=\frac{2}{3}\frac{\omega}{v_c}\frac{\eta\omega}{\rho v_c^2},$$

where, in order of magnitude, $\eta - v_c/3N\sigma tr$. Recognizing further that $Nv\sigma_{tr}\sim 3G_2$, we obtain ultimately for q_c''

$$q_c''\approx\frac{\omega}{v_c}\frac{4}{27}\frac{\omega}{2G_2}\frac{v^2}{v_c^2}\left(\frac{m_1}{\mu}\right)^{1/2}.\tag{15}$$

Stipulating that the damping in (13), due to bulk viscosity, exceed the damping due to shear viscosity (15) by, say, seven times, we find that this takes place in the concentration interval $0.17 < c < 1 - a^2 \approx 0.99$. This condition means in fact that the shear viscosity of the mixture can be disregarded when the relative concentration of the He exceeds 20%.

We consider now the limiting case of high frequencies. For $\omega \gg aG_2$ the solution of (12) is

$$q_{1,2}^{2} = \frac{\omega^2}{v_л^{2}} + i\frac{\omega G_2}{v_л^{2}}(1-c), \qquad v_л = \left(\frac{5}{3}\right)^{1/2} v, \tag{16}$$

$$q_{3,4} = \pm\frac{\omega}{v_л a^{1/2}} + \frac{i(2G_3/5+aG_2)}{v_л a^{1/2}}\frac{c}{2}. \tag{17}$$

Since the inequality $\omega \ll aG_2$ must always be met (only then is the hydrodynamic approach valid), the mode (16) is weakly damped only if

$$G_2(1-c) \ll \omega \ll G_2, \tag{18}$$

i.e., in the case $0 < 1 - c \ll 1$, corresponding to a high relative concentration of the light gas. The solution corresponding to (16) is in this case a linear combination of the functions Φ_1, Φ_2, and Φ_3 from the set (7), i.e., describes only the vibrations of the light gas, a fact reflected also by their propagation velocity. The solution described by the mode (17) is made up of the functions Φ_4, Φ_5, and Φ_6, i.e., it describes vibrations of only the heavy gas with velocity $v_h = v_L a^{1/2}$. As seen from (17), this solution is always weakly damped. It must be borne in mind, however, that this solution was obtained without allowance for the shear viscosity which does not lead to a substantial increase of the damping only when the heavy-gas hydrodynamics is valid with a large margin, i.e., when the following additional condition is met:

$$aG_2 \ll \omega \ll N_2 u_2 \sigma_{tr} \approx (1-c) G_2 a^{1/2}. \tag{19}$$

In the specific case of an He-Xe mixture, $a^{1/2} = 0.17$, this condition is met without margin, so that the damping of the mode (17), due to the unaccounted-for shear viscosity, is always appreciable. Thus, for example, putting $c = 0.3$ and $\omega = 0.07G_2 > aG_2 = 0.03G_2$, we find that the frequency of the heavy sound is only half the collision frequency, i.e., the sound is damped over a distance of the order of the wavelength.

Combining inequalities (18) and (19), we get the condition for simultaneous propagation of weakly damped vibrations of the light (16) and heavy (17) gases at high frequencies $\omega \gg aG_2$. Since $a^{1/2} < 1$, the right-hand side of (20) is smaller than the left-hand side, from which it is clear that inequalities (18) and (19) are not satisfied simultaneously even if they are relaxed, i.e., when the "much less than" sign is replaced by "less than." This means that at all frequencies $\omega > aG_2$ and at arbitrary concentrations one of the modes is damped over distances shorter than the wavelength. This conclusion is the diametric opposite of the conclusion of Ref. 5, whose Eq. (3) actually asserts the possibility of simultaneous propagation of two weakly damped vibrations in light and heavy gases.

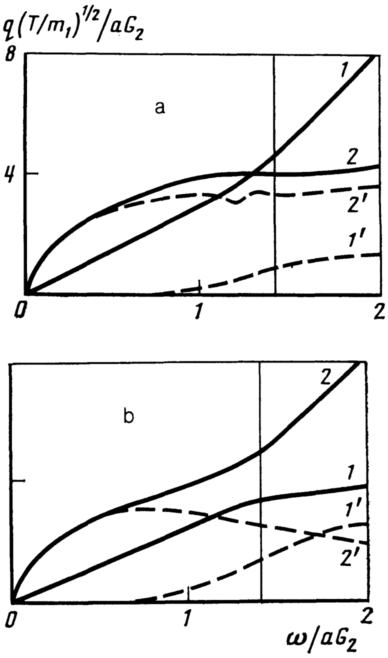

FIG. 1. Real (q', curves 1 and 2) and imaginary (q'', curves 1' and 2") parts of the wave vector vs the frequency ω in units of aG_2/v and aG_2, respectively, at $c = 0.4$ (a) and $c = 0.5$ (b).

In the intermediate frequency region $\omega \sim aG_2$ the roots of Eq. (12) cannot be simplified and the character of their frequency dependence is illustrated in Fig. 1. Figure 1a shows plots of $q'(\omega/aG_2)$ and $q''(\omega/aG_2)$ at a relative concentration $c = 0.4$. Curve 1 for $\omega \ll aG$ describes sound propagation with velocity v_m in the mixture. At $\omega \sim aG_2$ the slope of this curve changes, and at $a \gg aG_2$ its slope corresponds to propagation of sound with velocity v_h in a heavy mixture. The damping of this vibration (curve 1') increases with frequency and remains relatively small in the considered region ($q''/q' < 1/3$). Curve 2 describes strongly damped vibration. At $c < 0.4$ the situation remains similar to that shown in Fig. 1a, the only difference being that curves 2 and 2' are even closer to each other.

At a relative concentration $c \approx 0.5$ the situation changes radically. Curve 1 in the region $\omega \sim aG_2$ changes slope as before (see Fig. 1b), but in the region $\omega > aG_2$ its slope corresponds to the sound velocity v_L not in the heavy but in the light gas. This situation is realized further on in the entire concentration region $0.5 < c < 1$.

As seen from Fig. 1, at least one of the modes is strongly damped at all frequencies.

§4. COMPARISON WITH EXPERIMENT

Sound propagation in an He-Xe mixture was experimentally investigated in Ref. 1. The dependence of the sound velocity on the gas pressure p was measured at constant frequency $\omega = 1.005$ MHz and at different relative concentrations of the mixture. The measurement results were presented in the form of plots of the sound velocity vs the parameter ω/p. It was observed that the sound velocity $v_s = v_s(\omega/p)$ increases with increase of ω/p at a relative concentration $c > 0.5$, and decreases at $c < 0.5$. Let us examine the relation between these results and the theory expounded above.

At low frequencies (small values of the parameter ω/p) the sound velocity is determined from Eq. (13): $v_s = v_c = (5T/3\mu)^{1/2}$ and is independent of frequency. This case corresponds to the plateau on Fig. 2, which is located in the region of low frequencies $\omega < aG_2$ up to which Eq. (13) is valid. As seen from Fig. 1, in the region $\omega \sim aG_2$ an abrupt

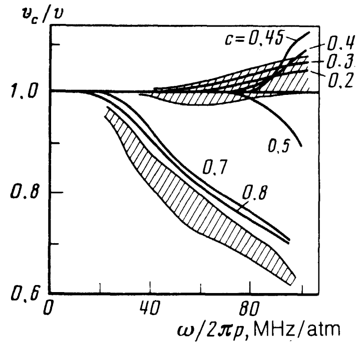

FIG. 2. Sound velocity vs frequency at various relative concentrations of the light gas. The solid curves describe the dependence obtained from Eq. (18). The shaded regions contain the points corresponding to the experimental data of Ref. 1. The upper and lower regions describe the dispersion of the sound velocity at relative concentrations $0.2 < c < 0.5$ and $0.7 < c < 0.8$, respectively.

change takes place in the dependence of the sound velocity on the frequency (curve 1), as is indeed observed in experiment.[1] In that reference, the results of which are shown in Fig. 2, the corresponding frequency $\omega = \omega_c$ was assumed to be $= 70$ MHz/atm. We put therefore $aG_2/p = 70$ MHz/atm. If it is assumed in (9) that the transport scattering cross section σ_{tr} is independent of the relative velocity v, we obtain $G_3 = (3m_1/m_2)G_2$. One can hardly expect allowance for the dependence of σ_{tr} on v to lead to a substantial change of this relation. On the other hand, it was verified that changing G_3 by a factor of two leads to a practically unnoticeable change of the presented theoretical curves. We therefore put henceforth $G_3 = (3m_1/m_2)G_2$ throughout.

The sound velocity was measured in Ref. 1 up to values $\omega/p = 100$ MHz/atm. The corresponding regions of ω/aG_2 are located in Fig. 1 to the left of the vertical straight lines.1) As seen from Fig. 1, in the frequency region $\omega \sim aG_2$ in which the sound velocity changes radically, the damping of both modes is appreciable, but the damping of the mode corresponding to curve 1 is at least several times smaller than the damping of the vibration described by curve 2. Since the total damping of the sound in experiment 1 was strong enough, it is natural to assume that the detector recorded in fact only wave 1.

The sound velocity corresponding to the vibrations described by curves 1 of Fig. 1 is compared in Fig. 2 with the measured velocity at different relative light-gas concentrations. It can be seen that the agreement with experiment is quite satisfactory. A similar comparison for the case $aG_2 = 100$ MHz/atm shows a noticeable discrepancy between the theoretical and experimental curves, and leads to the conclusion that the value of aG_2 assumed in Fig. 2 is accurate enough.

Equation (13), which shows the presence of bulk viscosity in the considered mixture is valid, with sufficient accuracy, up to values $\omega \lesssim aG_2$, as demonstrated by direct comparison of the damping given by this equation with curve 1′ of Fig. 1. For $\omega = aG_2$ and, e.g., $c = 0.5$, it can be seen from (13) that the sound is damped over a distance of four wavelengths. So large a damping of the sound may turn out to be very useful when such mixtures are used as working media in experiments on wave-front reversal by the method of stimulated Rayleigh scattering. Thus, for example, in the mixture $N_1(\mathrm{He}) = N_2(\mathrm{Xe}) = 3\cdot 10^{20}$ cm^3 we obtain for the reciprocal time $\tau^{-1} = \omega^2/2G_2a$ of the onset of stimulated Rayleigh scattering ($\omega = qv_m = 6\cdot 10^0$ s^{-1}, $q = 10^5$ cm^{-1}, $v_m = 3\cdot 10^4$ cm/s, $G_2a = 2\pi\cdot 100$ MHz/atm$\cdot$10 atm) the values $\tau^{-1} = 1.5\cdot 10^9$ s^{-1} and $\tau = 10^{-9}$ s. In pure xenon with $N_2(\mathrm{Xe}) = 3\cdot 10^{20}$ cm^{-3} we get under these conditions $\tau^{-1} = \omega^2/3G_2 = 6\cdot 10^7$ s^{-1}, which makes it impossible to use pulses of duration shorter than $\tau = 0.5\cdot 10^{-7}$ s.

1) Note that the experimentally investigated maximum sound frequencies exceed aG_2 by only 30%, so that the region $\omega \gg aG_2$ in which the modes (16) and (17) appear is not reached.

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