Resonant transition radiation in boundary plasma layers

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(Submitted 4 May 1987)
Zh. Eksp. Teor. Fiz. **94**, 95–105 (March 1988)

An effect of considerable order of magnitude, enhancement of resonant transition radiation in strongly inhomogeneous layers with a nonlinear profile, was observed on the boundary of an overdense plasma. The greatest intensity enhancement, compared with nonresonant radiation, is obtained in this case for nonrelativistic charges. In the relativistic resonance region, the radiation has a frequency-angular spectrum that differs qualitatively from the spectrum of the nonresonant radiation, and can exceed the latter in intensity if the collisions are weak. Structural effects in transition radiation are investigated in boundary layers of an equilibrium or inverted plasma.

1. INTRODUCTION

The idea of transition radiation, which has by now become classical, and its applications were developed mainly by considering the model of a charge crossing an ideally abrupt interface of two media with different dielectric constants ε_1 and ε_2. In the time since the publication of the pioneering paper,[1] the laws governing this radiation have been thoroughly investigated (see, e.g., Refs. 2 and 3). The known criterion for the use of an idealized abrupt boundary is smallness of the boundary-layer thickness d compared with the length L_f ($d \ll L_f$) over which the radiation is produced.[3–5] If the opposite inequality ($d \gg L_f$) holds, the transition radiation is exponentially small. These conclusions, obtained for dielectric transition layers, can generally speaking not be extended to include the case of an inhomogeneous plasma, in which appearance of plasma resonance points with $\varepsilon \approx 0$ is possible.[6–9] A manifestation of the peculiarities of the resonant transition radiation (RTR) that is produced in this case is, in particular, that RTR from a gradual layer can exceed the radiation from an abrupt discontinuity of a plasma having the same maximum plasma frequency. Up to now, however, RTR was investigated only for smoothly inhomogeneous monotonic layers[6–9] ($\omega d/c \gg 1$, where ω is the frequency) or for a homogeneous layer in vacuum.[10] The class of structural effects that depend on the plasma inhomogeneity profile in the vicinity of the resonance point was thereby excluded in fact from consideration. It will be shown below that the structural effects become most noticeable in the case of thin (boundary) plasma layers ($\omega d/c \ll 1$). The qualitative differences of the laws governing the RTR in linear layers and in layers of higher order (having an extremum or an inflection of the density)[1)] offer promise of an appreciable (in order of magnitude) increase of the RTR intensity in boundary plasma layers with complicated profiles, which are produced, in particular, in decay of the free boundary of a non-isothermal plasma[14,15] when powerful electromagnetic radiation acts on the plasma,[16,17] and also in film island structures (Refs. 18 and 19).[2)] New applications are uncovered here by the fact that the most appreciable (compared with a linear layer) turns out to be the enhancement of the RTR for nonrelativistic charges, particularly for intersection of a strongly inhomogeneous plasma with a modulated electron beam (for example, in systems of the type used in Ref. 22).

2. ITERATION REPRESENTATION FOR THE FIELDS IN A BOUNDARY LAYER

Transition radiation of a charge q moving with constant velocity v in a direction along which the properties of the medium change (along the x axis) is described by the equation for the amplitude $B(x;\omega,\mathbf{k}_\perp)$ of the magnetic field of the spatiotemporal harmonic $\exp(i\omega t - i\mathbf{k}_\perp\cdot\mathbf{r}_\perp)$ (Ref. 7):

$$\varepsilon\frac{d}{dx}\left(\frac{1}{\varepsilon}\frac{dB}{dx}\right)+\frac{\omega^2}{c^2}(\varepsilon-\gamma^2)B=F(x) \tag{1}$$

with a source $f(x) = (q\gamma\omega/2i\pi^2c^2)\exp(-i\omega x/v)$. Here $\gamma = ck_\perp/\omega$ and $\varepsilon(x)$ is the dielectric constant of the medium, assumed homogeneous outside a layer of thickness $d = d_1 + d_2$, i.e., at $x < -d_1$ and $x > d_2$: $\varepsilon(x < -d_1) = \varepsilon_1$, $\varepsilon(x > d_2) = \varepsilon_2$. It is assumed that the inhomogeneous plasma layer is thin enough so that

$$\frac{\omega d}{c} \ll 1,\quad \frac{\omega}{c}\left|\int_{-d_1}^{d_2} dx\,\varepsilon(x)\right| \ll 1, \tag{2}$$

and has a zero, or arbitrary order n, of the dielectric constant in the interval $-d_1 < x < d_2$.

Outside the inhomogeneity region of the medium, the solution of Eq. (1) can be written, with allowance for the radiation condition, in the form

$$B(x)=\begin{cases} C_1\exp[ig_1(x+d_1)]+B_1^q(x), & x<-d_1 \\ C_2\exp[-ig_2(x-d_2)]+B_2^q(x), & x>d_2 \end{cases}, \tag{3}$$

where

$$g_{1,2}=\begin{cases} (\omega/c)(\varepsilon_{1,2}-\gamma^2)^{1/2}, & \gamma^2<\varepsilon_{1,2} \\ -i(|\omega|/c)(\gamma^2-\varepsilon_{1,2})^{1/2}, & \gamma^2>\varepsilon_{1,2} \end{cases},$$

$C_{1,2}$ are the unknown amplitudes of the free fields, and $B^q_{1,2}$ is the forced solution (the field of the charge q in a homogeneous medium):

$$B^q_{1,2}(x)=\frac{iq\gamma\beta^2/2\pi^2\omega}{1-\beta^2(\varepsilon_{1,2}-\gamma^2)}\exp\left(-i\frac{\omega x}{v}\right),\quad \beta=\frac{v}{c}. \tag{4}$$

By virtue of the conditions (2), the field $B(x)$ can be determined approximately within the inhomogeneous layer $-d_1 < x < d_2$. To this end, it is convenient to rewrite Eq. (1) in integral form

$$B(x)=C_3+C_4I(x)+\int_{-d_1}^{x} dx'[I(x)-I(x')]$$

$$\times\left[\frac{\omega^2}{c^2}\left(\frac{\gamma^2}{\varepsilon}-1\right)B+\frac{F}{\varepsilon}\right], \tag{5}$$

where $C_{3,4}$ are arbitrary constants,

$$I(x)=\int_{-d_1}^{x} dx'\,\varepsilon(x'),$$

and the solution can be sought in the form of an iteration series in terms of the small parameter $(\omega/d/c)^2 \ll 1$:

$$B(x)=B^{(0)}(x)+B^{(1)}(x)+\ldots. \tag{6}$$

The first term takes the form

$$B^{(0)}(x)=C_3+C_4I(x)+\int_{-d_1}^{x} dx'[I(x)-I(x')]\frac{F(x')}{\varepsilon(x')}, \tag{7}$$

and the succeeding terms can be obtained from the iteration formula

$$B^{(n+1)}(x)=\frac{\omega^2}{c^2}\int_{-d_1}^{x} dx'[I(x)-I(x')]\left(\frac{\gamma^2}{\varepsilon}-1\right)B^{(n)}(x'). \tag{8}$$

We call attention to the fact that in view of the presence of poles in the integrands, the integrals themselves in the iteration formulas (7) and (9) may be not small for layers of higher order (with $n>1$) even if condition (2) is met. It is therefore correct to discard the inessential part of the series (6) only in the final equations for the radiation-field amplitudes.

3. CALCULATION OF THE RADIATION FIELDS

The derivative of the field $B(x)$, which is needed to solve the boundary-value problem, is defined by the relation

$$\frac{1}{\varepsilon}\frac{dB}{dx}=C_4+\int_{-d_1}^{x} dx'\left[\frac{\omega^2}{c^2}\left(\frac{\gamma^2}{\varepsilon}-1\right)B+\frac{F}{\varepsilon}\right]. \tag{9}$$

The constants C_{1-4} are obtained from the boundary conditions

$$\{B\}=0,\quad \left\{\frac{1}{\varepsilon}\frac{dB}{dx}\right\}=0,$$

which follow from Eq. (1) integrated in small vicinities of the points $x=-d_1$ and $x=d_2$. As a result, discarding small terms of order $\omega d/c$ and higher, we obtain the following representation for the free-wave amplitudes outside the layer

$$C_{1,2}=\Big\{-\beta^{-1}[\varepsilon_2^{-1}B_2^q(d_2)-\varepsilon_1^{-1}B_1^q(-d_1)]$$
$$\pm\frac{cg_{2,1}}{\omega\varepsilon_{2,1}}[B_2^q(d_2)-B_1^q(-d_1)]$$
$$+i\gamma^2B_{1,2}^q(\mp d_{1,2})J_1+\frac{q\gamma}{2\pi^2\omega}J_2\Big\}\left[\frac{c}{\omega}\left(\frac{g_1}{\varepsilon_1}+\frac{g_2}{\varepsilon_2}\right)-i\gamma^2J_1\right]^{-1} \tag{10}$$

in terms of the dimensionless resonant parameters

$$J_1=\frac{\omega}{c}\int_{-d_1}^{d_2}\frac{dx}{\varepsilon(x)},\quad J_2=\frac{\omega}{c}\int_{-d_1}^{d_2}dx\frac{\exp(-i\omega x/v)}{\varepsilon(x)}. \tag{11}$$

The general equations (10) and (11) enable us to investigate the RTR regularities in thick structures of various types, viz., in a plasma with a condensed layer (e.g., a soliton), in inhomogeneous plasma films located in vacuum, in semiconductor sandwiches, in MIS structures, etc. We shall consider hereafter, for the sake of argument, RTR in an inhomogeneous layer on an interface of vacuum with a overdense plasma ($\varepsilon_1=1$, $\varepsilon_2<0$).[3] For a particle entering the plasma, the spectral-angular density of the radiation emitted backwards (into the vacuum) is expressed by the equation

$$w(\omega,\theta)=\frac{d^2W}{d\omega\,d\theta}=8\pi^3\frac{\omega^2}{c}\sin\theta\cos^2\theta|C_1|^2,$$
$$\omega>0,\ 0<\theta<\frac{\pi}{2}, \tag{12}$$

in terms of the amplitude C_1 of Eq. (10), for which we must put $\gamma=\sin\theta$ and $cg_1/\omega=\cos\theta$ (θ is the angle between the emitted radiation and the normal to the plasma surface).

While general enough with respect to both the parameter β and the layer profile, Eqs. (10) and (12) are not illustrative enough for the analysis of the spectral and angular characteristics of the RTR. It is more expedient for this purpose to write separate equations for nonrelativistic ($\beta\ll1$) and relativistic ($\beta\sim1$) charges. In the former case, neglecting terms of order β^2, we get

$$w(\omega,\theta)$$
$$=\frac{2q^2\cos^2\theta}{\pi c\sin\theta}\left|\frac{J_2+i\beta[\exp(i\omega d_1/v)-\varepsilon_2^{-1}\exp(-i\omega d_2/v)]}{J_1+[\varepsilon_2^{-1}(\sin^2\theta-\varepsilon_2)^{1/2}+i\cos\theta]/\sin^2\theta}\right|^2. \tag{13}$$

The simplifications in the latter case are brought about by the fact that the radiation energy density can be expressed only in terms of the parameter J_1. The result is

$$w(\omega,\theta)=\frac{2q^2\cos^2\theta}{\pi c\sin\theta}(1-\beta^2\cos^2\theta)^{-2}\left|\frac{(1-\beta^2)J_1+i\beta(1-\varepsilon_2^{-1})[1-\beta^2-i\beta(\sin^2\theta-\varepsilon_2)^{1/2}][1-i\beta(\sin^2\theta-\varepsilon_2)^{1/2}]^{-1}}{J_1+[\varepsilon_2^{-1}(\sin^2\theta-\varepsilon_2)^{1/2}+i\cos\theta]/\sin^2\theta}\right|^2 \tag{14}$$

The nonresonant terms proportional to β and β^2 were retained to illustrate the limiting transition to an ideally abrupt boundary ($d\to0, J_{1,2}\to0$), when (13) and (14) coincide with the corresponding equations in Ref. 3. Transition radiation in front of a relativistic particle emerging from a plasma into a vacuum can be described by the same equation with β replaced by $-\beta$, i.e., the resonant contribution remains the same as for backward radiation in the case of a particle entering the plasma. The transition radiation of a nonrelativistic particle, as seen from (11) and (13), no longer has this symmetry. To analyze this circumstance, and also for a comparative estimate of the resonance effect in boundary layers of various types, it is necessary to specify the profile of the layer.

4. STANDARD BOUNDARY LAYER OF GENERAL FORM. COMPARISON WITH RTR IN A LINEAR LAYER

Without loss of generality, we place the plasma-resonance point that is essential for the considered effect at the origin ($x = 0$), and model the $\varepsilon(x)$ dependence in its vicinity by a set of power-law functions of the form $\varepsilon(x) = \pm (x/l)^n + \varepsilon_0 - i\nu$, where the $n = 1, 2, 3 \ldots$ is the order of the layer, $l \lesssim d_{1,2}$ the scale, ε_0 the parameter of frequency detuning from resonance at $x = 0(|\varepsilon_0| \ll 1)$, and ν the relative frequency of the collisions ($\nu \ll 1$). The use of standard layers of this type makes it possible to investigate the laws governing the resonant transition radiation as a function of the profile of Re $\varepsilon(x)$ near the resonance point, including the case of layers with resonant inflection point (for odd n), for layers with resonant extrema of the dielectric constant (for even n). It makes it also possible to track the limiting transition to a steplike boundary (as $n \to \infty$).

If Re $\varepsilon(x)$ goes through zero linearly ($n = 1$, lower sign of the $\varepsilon(x)$ dependence), the integrands in Eqs. (11) contain one pole located at the point $x = l(\varepsilon_0 - i\nu)$ in the lower half-plane. The contribution of this pole is of the same order of magnitude as the principal values of the integrals:

$$J_1^{(1)} = \frac{\omega l}{c}\left(-\int_{-d_1}^{d_2} \frac{dx}{x} + i\pi\right) = \frac{\omega l}{c}\left(-\ln\frac{d_2}{d_1} + i\pi\right), \tag{15}$$

$$J_2^{(1)} = \frac{\omega l}{c}\left(-\int_{-d_1}^{d_2} dx \frac{\exp(-i\omega x/v)}{x} + i\pi\right)$$

$$\times \exp\left[-\omega l\left(\frac{\nu}{|v|} + i\frac{\varepsilon_0}{v}\right)\right], \tag{16}$$

so that $|J_{1,2}^{(1)}| \sim \omega d/c \ll 1$, just as in the absence of plasma resonance. As a result, no qualitative singularities will appear in the transition radiation compared with the nonresonant case. On the other hand, the corrections necessitated by the resonance appear, as follows from (13) and (14), only in transition layers of thickness on the order of the formation length $d \gtrsim L_f \sim |\omega/v \pm \omega(\varepsilon_{1,2} - \gamma^2)^{1/2}/c|^{-1}$. In sufficiently abrupt (boundary) layers, by virtue of inequalities (2), this condition can be met either for a slowly moving charge ($\beta \lesssim \omega d/c$) or for a strongly transcritical plasma half-space ($|\varepsilon_2| \gtrsim (\omega d/c)^{-2}$). In the former case, nonresonant as well as resonant transition radiation is vanishingly small, ($w(\omega, \theta) \propto (\omega d/c)^2$), and in the latter the resonant corrections are noticeable only for glancing angles of the radiation ($\pi/2 - \theta \lesssim \omega d/c$).

RTR exhibits a qualitative unique feature in boundary layers of higher order (with $n > 1$). The distinctive features of layers with resonant inflections (extrema) of the density are formally connected with the locations of the poles of the integrands in Eqs. (11) on both sides of the real axis (at the points $x = l[\mp(\varepsilon_0 - i\nu)]^{1/n}$). This leads to a fast growth of the integrals $J_{1,2}$ that characterize the polarizability of the layer. Physically, on the other hand, this increase is due to the qualitative broadening of the region of resonant swelling of the longitudinal component of the electric field ($E_x(x) \propto \varepsilon^{-1}(x)$ by a factor $\nu^{(1-n)/n} \gg 1$ (Ref. 23) compared with a linear layer ($n = 1$). The foregoing allows us to speak of a principally enhanced RTR in boundary layers with $n > 1$.

5. ENHANCED RTR. CUBIC LAYER

The differences between the laws governing the RTR for linear and nonlinear boundary layers with close profiles $\varepsilon(x)$ (i.e., the effects of the fine structure of the latter) become most pronounced in the presence of an inflection point ($n = 3$).

Taking into account in the calculation of the integrals (11) only the resonant contribution from the poles of the function $\varepsilon^{-1}(x)$ (the nonresonant terms are of the order of $\omega d/c$), we arrive in this case at the following expressions for the parameters $J_{1,2}$:

$$J_1^{(3)} = \frac{\omega l}{c}\frac{2\pi i/3}{|\tilde{\nu}|^{2/3}\exp[i(2\varphi - \pi)/3]}, \tag{17}$$

$$J_2^{(3)} = \frac{\omega l}{c}\frac{2\pi i/3}{|\tilde{\nu}|^{2/3}\exp(i\cdot 2\varphi/3)}\left\{\exp\left[i\frac{\omega l}{v}|\tilde{\nu}|^{1/3}\exp\left(\frac{i\varphi}{3}\right)\right]\right.$$

$$\left.-\left[1 - \exp\left(\frac{i\pi}{3}\right)\right]\exp\left[i\frac{\omega l}{v}|\tilde{\nu}|^{1/3}\exp\left(i\frac{\varphi + 2\pi}{3}\right)\right]\right\} \quad (v > 0), \tag{18}$$

determined by the modulus $|\tilde{\nu}|$ and argument $\varphi = \arg\tilde{\nu}(0 < \varphi < \pi)$ of the generalized detuning parameter $\tilde{\nu} = i\nu - \varepsilon_0$. It can be seen from (17) and (18) that the parameters $J_{1,2}^{(3)}$ are estimated as ratios of two small quantities $\omega l/c$ and $|\tilde{\nu}|^{2/3}$, so that these parameters may be not small even in thin layers if $|\tilde{\nu}| \lesssim (\omega l/c)^{3/2}$. It is just in the last case that enhancement is realized relative to a monotonic (linear) RTR layer.

A general property of RTR, which distinguishes it from nonresonant radiation, is the weak dependence of the RTR characteristics on the relativistic parameter β. As seen from Eqs. (13)–(18), this dependence manifests itself only in a decrease of the RTR in two limiting cases: ultrarelativistic velocities ($1 - \beta^2 \ll 1$) and very low velocities v, when the resonance parameters J_2 become substantially smaller than J_1 (estimates of the corresponding values of β depend on the profile of the layer and will be presented below). In the remaining regions of values of β the spectral energy density of the enhanced RTR (in contrast to the RTR in the linear layer, where always $w_{\rm RTR} \lesssim q^2/c)(\omega l/c)^2$) turns out to be of the same order as the spectral energy density of the nonresonant transition radiation of the relativistic particles on an ideally abrupt boundary: $w \sim q^2/c$ if $\theta \neq 0, \pi/2$. Thus, in the nonrelativistic region the enhanced RTR is substantially larger (in order of magnitude) than the resonant transition radiation whose spectral energy density, for $\theta \neq 0, \pi/2$, can be estimated at $w \sim \beta^2 q^2/c$. The enhanced-RTR energy is concentrated in this case in a narrow spectral interval of relative width $|\omega - \omega_0|/\omega_0 \lesssim (\omega_0 l/c)^{3/2}$ near the plasma frequency ω_0 in the inflection of the concentration (see Fig. 1a). The RTR angle spectrum, in contrast to the nonresonant case, is closely related to the frequency spectrum (what is similar, naturally, is the absence of radiation at $\theta = 0, \pi/2$). As seen from (13) and (14), the enhanced RTR passes as $\nu \to 0$ along the angle cone $\theta \sim (c|\varepsilon_0|^{2/3}/\omega_0 l)^{1/2}$, which is narrower and is closer to the x axis the closer the frequency ω to ω_0. The maximum radiation intensity also increases in this case and its estimate as $\omega \to \omega_0$ is $w \sim (q^2/c)(\omega_0 l/c|\tilde{\nu}|^{2/3})^{1/2}$. In other words, in a weakly collisional plasma ($\nu \ll (\omega_0 l/c)^{3/2}$), in a narrow frequency band ($|\varepsilon_0| \ll (\omega_0 l/c)^{3/2}$), the enhancement of the RTR exceeds substantially in intensity

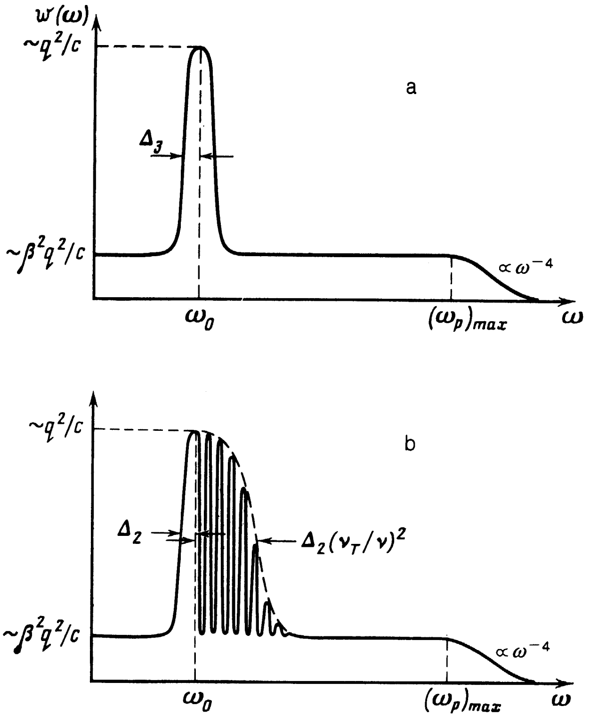

FIG. 1. Spectral energy density of transition radiation of a nonrelativistic charge in a boundary plasma layer: a—with inflection of the density profile, b—with a parabolic density well at $v \ll v_T$ (ω_0 is the plasma frequency at the inflection or minimum point, $(\omega_p)_{\max}$ is the maximum plasma frequency of the layer, $\Delta_3 \sim \omega_0(\omega_0 l/c)^{3/2}$, $\Delta_2 \sim \omega_0(\omega_0 l/c)^2$).

the nonresonant radiation of not only nonrelativistic but also of relativistic particles.

An investigation of expressions (17) and (18) makes it possible to formulate the following sufficient conditions for the extinction of RTR in a cubic layer:

$$\omega l/c \ll |\tilde{\nu}|^{2/3}, \tag{19}$$

$$\beta \ll (\omega l/c)\,|\tilde{\nu}|^{1/3} \sin\frac{\varphi+\pi\mp\pi}{3}. \tag{20}$$

The first inequality corresponds physically to suppression of the resonant peak in the inflection of the concentration, on account of collisions and detuning ($J_{1,2} \to 0$). To interpret the second condition, we consider it first in the case of small detunings ($|\varepsilon_0| \lesssim \nu$), when $\sin(\varphi/3) \approx \sin[(\varphi+2\pi)/3] \sim 1$, so that we have in lieu of (20)

$$v/\omega \ll l\nu^{1/3}. \tag{21}$$

The right-hand side of inequality (21) is equal to the width of the region of resonant swelling of the field in the cubic layer.[23] In other words, the sufficient condition (21) for RTR extinction is of the interference type and points to a suppression of the effect ($J_2 \to 0$) in the case when the charged particle negotiates during the field period a distance much shorter than the width of the resonance region. Condition (20) has the same meaning also at large detunings ($|\varepsilon_0| \gg \nu$) or if either $\sin(\varphi/3) \approx \nu/(3|\varepsilon_0|) \ll 1$ (if $\varepsilon_0 < 0$) or $\sin[(\varphi+2\pi)/3] \approx \nu/(3\varepsilon_0) \ll 1$ (if $\varepsilon_0 > 0$), so that condition (20) can be rewritten in the form

$$v/\omega \ll l\nu^{1/3}(\nu/|\varepsilon_0|)^{2/3}. \tag{22}$$

Here, too, the right-hand side of the inequality is equal to the resonant-region width, but substantially narrowed compared with the case $|\varepsilon_0| \lesssim \nu$ in view of the displacement of the resonant peak of the field to one of the branches of the cubic parabola. As a result, the interference quenching of the RTR comes into play at much lower velocities v.

The foregoing allows us to introduce the following estimates for the internal $l_{in}^{(3)}$ and external $l_{ex}^{(3)}$ scales of the cubic layers in which enhanced RTR will be effectively generated:

$$l_{in}^{(3)} \sim (c/\omega_0)\,|\tilde{\nu}|^{2/3}, \quad l_{ex}^{(3)} \sim (c/\omega_0)\,|\tilde{\nu}|^{2/3}(\beta/\nu). \tag{23}$$

This shows that the necessary condition for the existence of the enhanced RTR in a cubic layer has the simple and clear form

$$\beta \gtrsim \nu. \tag{24}$$

At $\beta \approx \nu$ the effect is selective with respect to the scale l; this can be used for diagnostics of semi-inhomogeneous plasma formations.

The use of cubic standard layers permits a substantial refinement of the notions related to the use of an idealization of an abrupt boundary, especially for nonrelativistic charges. In fact, as seen from (13) and (14), the standard condition for going to the limit in the approximation of an ideally abrupt boundary[3–5]

$$d \ll L_f \tag{25}$$

does not take into account the resonant contribution, which is essential for $n > 1$, to the transition radiation and must be replaced in a cubic layer by the much more stringent condition[4)]

$$l \ll \nu^{2/3} L_f. \tag{26}$$

On the other hand, the standard condition $l \gg L_f$ for a smooth transition layer, when the nonresonant radiation is exponentially small, is still insufficient to suppress the RTR from the density inflection, since this condition is fully compatible with the inequality $l < l_{ex}^{(3)}$.

6. BOUNDARY LAYERS OF GENERAL FORM. RADIATION SYMMETRY

In layers with other $n > 1$ the main laws governing the enhanced RTR remain qualitatively the same as for a cubic layer. The width of the region of resonant swelling of the field in such layer is estimated (for $|\varepsilon_0| \lesssim \nu$) at $l\nu^{1/n}$ (Ref. 23), and the parameters $J_{1,2}$ are of the order of

$$|J_{1,2}^{(n)}| \sim (\omega l/c)\,|\tilde{\nu}|^{(1-n)/n}. \tag{27}$$

Enhanced RTR is thus realized under the condition $|\tilde{\nu}| \lesssim (\omega l/c)^{n/(n-1)}$. Estimates of the internal and external scales of the layers that generate RTR effectively are obtained in the same manner as for a cubic layer, and lead to the following results:

$$l_{in}^{(n)} \sim (c/\omega_0)\,|\tilde{\nu}|^{(n-1)/n}, \quad l_{ex}^{(n)} \sim (c/\omega_0)\,|\tilde{\nu}|^{(n-1)/n}(\beta/\nu). \tag{28}$$

It must be borne in mind, however, that for layers of symmetric form (even n) the narrowing of the resonance region for large detunings ($|\varepsilon_0| \gg \nu$) takes place only for definite signs of ε_0: for $\varepsilon_0 < 0$ in layers with a plasma-density well and for $\varepsilon_0 > 0$ in layers with a density hump. For arbitrary signs of ε_0 the correct estimate for $l_{ex}^{(n)}$ is not (28) but

$$l_{ex}^{(n)} \sim (c/\omega_0)\,|\tilde{\nu}|^{(n-1)/n}(\beta/|\tilde{\nu}|). \tag{29}$$

This equation must be used for both symmetric and asymmetric layers at $n \gg 1$, when the contribution to the integrals (11) from the resonance peaks of the field on the slopes of the layer is small compared with the contribution from the plateau section of thickness of order $l\,|\tilde{\nu}|^{1/n}$ at the center of the layer.

It is easily seen from (28) and (29) that the necessary condition for the existence of enhanced RTR does not depend on the order of the layer and retains the form (24).

As already noted in Sec. 3, the RTR symmetry in vacuum, for particles entering or leaving the plasma, is independent of the type of layer only if $\beta \sim 1$. The differences between the indicated cases for nonrelativistic motion of the charge are determined by the difference between the values of the parameter J_2 for different particle-velocity directions, and are therefore absent only for symmetric layers (even n), when it suffices to reverse the sign of β in (13). For example, for a layer with a parabolic density well ($n = 2$, lower sign of the $\varepsilon(x)$ dependence) the form of the parameter J_2 is symmetric with respect to the sign of v:

$$J_2^{(2)} = -\frac{\pi(\omega l/c)}{|\tilde{\nu}|^{1/2}\exp(i\varphi/2)}\exp\left[-\frac{\omega l}{|v|}|\tilde{\nu}|^{1/2}\exp(i\varphi/2)\right], \tag{30}$$

whereas when a particle goes off to the vacuum from a cubic layer the expression for the corresponding resonance parameter

$$J_2^{(3)} = \frac{2\pi i}{3}\frac{\omega l/c}{|\tilde{\nu}|^{2/3}\exp[i(2\varphi-\pi)/3]} \times\exp\left[i\frac{\omega l}{|v|}|\tilde{\nu}|^{1/3}\exp\left(i\frac{\varphi+\pi}{3}\right)\right] \tag{31}$$

differs from (18). This difference is immaterial for low detunings, so long as $|\tilde{\nu}|^{1/2}\omega l/|v| \ll 1$. When the inverse inequality holds, expression (31) decreases very rapidly, whereas according to (18) quenching of the RTR at large detunings ($|\varepsilon_0| \gg \nu$) takes place only under the much stronger condition (22). Thus, in layers of odd order the enhanced RTR has an anisotropy of a type similar to that in a gradual layer.[6-9]

As the transition of $\mathrm{Re}\,\varepsilon(x)$ through zero becomes flatter, i.e., with increase of n, the differences between symmetric and asymmetric layers vanish. In particular, as $n \to \infty$, when the boundary layer contains a uniform section (plateau of thickness $2l$, the parameters $J_{1,2}$ take the form

$$J_1 = \frac{2(\omega l/c)}{\varepsilon_0 - i\nu}, \tag{32}$$

$$J_2 = \frac{2(\omega l/c)}{\varepsilon_0 - i\nu}\frac{\sin(\omega l/v)}{\omega l/v}, \tag{33}$$

and are invariant to change of sign of the particle velocity. As a result, the interference quenching of the RTR has no anisotropy, and the quenching itself is realized at $\omega l/|v| \gg 1$.

If the boundary layer contains several density inflections or extrema with close plasma frequencies, their contributions to the integrals (11) are additive. The foregoing results can therefore be used in RTR calculations for boundary layers having quite complicated profiles.

7. RTR IN INVERTED PLASMA LAYERS

In inverted plasma layers, which can be phenomenologically described by an enhancement parameter $\nu < 0$, the presence of plasma resonance in an inflection (extremum) of the density leads to a divergence, invariant to the sign of the velocity, of the spectral-angular energy density $w(\omega, \theta)$ of the transition radiation when the expression

$$J_1 + [\varepsilon_2^{-1}(\sin^2\theta - \varepsilon_2)^{1/2} + i\cos\theta]/\sin^2\theta = 0 \tag{34}$$

in the denominator of Eq. (13) or (14) vanishes. Equation (34) coincides with the condition for giant amplification of a TM wave by an inverted plasma layer, and determines the frequencies and angles for which $w(\omega, \theta) \to \infty$. In the case of a linear passage of $\mathrm{Re}\,\varepsilon(x)$ through zero ($n = 1$), the divergence of $w(\omega, \theta)$ turns out to be quite sensitive to the profile of the layer outside the resonance region. At a constant density gradient in the transition layer, when $d_2/d_1 = -\varepsilon_2$ and $J_1 = -(\omega l/c)[\ln(-\varepsilon_2) + i\pi]$, there is no divergence of $w(\omega, \theta)$. If, however, the parameters ε_2 and $d_{1,2}$ are independent (layer with strongly varying density gradient), the effect is possible for glancing radiation angles ($\theta \approx \pi/2 - \pi\omega l/c$) under conditions of strong transcriticality of the plasma half-space ($|\varepsilon_2| \gtrsim (\omega l/c)^{-2}$).

We consider here by way of example the case of a plateau-like ($n \to \infty$) density section of thickness $2l$, located in a transition layer in the region of strong overdensity ($|\varepsilon_2| \to \infty$). Then $J_1 = 2(\omega l/c)(\varepsilon_0 - i\nu)^{-1}$, and the condition (34) reduces to the relations

$$\varepsilon_0 \approx 0, \quad \cos\theta \approx \frac{\nu}{4\omega l/c} + \left(\frac{\nu^2}{(4\omega l/c)^2} + 1\right)^{1/2}. \tag{35}$$

For small detuning ($|\varepsilon_0| \ll |\nu|$), the energy density on the cone defined by (35) is estimated at $w(\omega, \theta) \propto (\nu/\varepsilon_0)^2 \gg 1$.

The layer natural modes of the form $\exp i[\omega(1 + i\gamma/2)t - hy - g_{1,2}x]$ are described by the equation

$$g_1 = -2ih^2 l/\tilde{\nu}_a \quad (\tilde{\nu}_a = \varepsilon_0 - i(\nu - \gamma)). \tag{36}$$

An investigation of (36) demonstrates the instability of the bulk natural modes inside the cone (with a growth rate $\gamma = \nu + 2(\omega l/c)\cdot\sin^2\theta/\cos\theta$) and of the surface modes of the layer (with a growth rate $\gamma = \nu$). In other words, the effect of divergence on the cone is accompanied by emission of a weakly directional background.

8. RTR IN A PLASMA WITH SPATIAL DISPERSION

The results above are directly applicable to a plasma in which the spatial dispersion is suppressed by collisions ($\nu \gg \nu_T = (r_D/l)^{2n/(n+2)}$, where r_D is the electron Debye radius[12,23]). They can be generalized to include the case of a heated plasma with $\nu \ll \nu_T$ in the absence of spatial plasmon resonances (i.e., for density profiles having a hump or inflection of not too high an order n) by replacing ν by ν_T in the expressions for $J_{1,2}$. On the other hand, the presence in the layer of trapped Langmuir oscillations leads, just as in the problem of absorption of a TM wave,[12,25] to formation of a multifrequency structure of the spectrum of the enhanced RTR with intervals on the order of ν_T between the components (see Fig. b).

9. CONCLUSION

Thus, the presence of inflection points or extrema of the $\varepsilon(x)$ profile in the boundary layer leads to the appearance, at the corresponding plasma frequencies, of peaks of enhanced (in order of magnitude) RTR against a nonresonant background. For nonrelativistic charges, substantial gain can be obtained not only in the spectral density $w(\omega, \theta)$ (see above) but also in the total energy

$$W = \int d\omega \int d\theta \, w(\omega, \theta).$$

Indeed, as shown by analysis, the ratio of the energy $W_{\rm RTR}$ of the enhanced RTR to the energy W_0 of the nonresonant transition radiation ($W_0 \sim \beta^2(q^2/c) \times (\omega_p)_{\max}$, Ref. 3) can be estimated at

$$W_{\rm RTR}/W_0 \sim (\omega_0 l/c)^{n/(n-1)} \beta^{-2} \omega_0/(\omega_p)_{max}. \qquad (37)$$

This shows that $W_{\rm RTR}$ exceeds W_0 substantially at $\beta^2 \ll (\omega_0 l/c)^{n/(n-1)}$.

Of course, structural effects similar to enhanced RTR appear also in moving boundary layers. The laws governing this transition scattering, however, are beyond the scope of the present article.

[1)]There are earlier reports of peculiarities of screening[11] and absorption[12,13] of TM waves by plasma layers of this type.

[2)]The strong increase of RTR in thin silver films, observed in Refs. 20 and 21, was explained[10] for a model of an homogeneous plasma layer.

[3)]There is obviously no resonant radiation in the region of high frequencies, where $\varepsilon_2 > 0$.

[4)]Although for $\nu^{2/3} L_f \lesssim l \ll l_{in}^{(3)}$ the RTR is weak ($w \ll q^2/c$), it is nonetheless comparable with the nonresonant transition radiation. It is clear thus that the condition $l \ll l_{in}$ is insufficient for a change to the approximation of an ideally abrupt boundary.

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Translated by J. G. Adashko