Energy relaxation of the electron-phonon system of a semiconductor under static and dynamic conditions

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An analysis is made of the rate of cooling of a carrier plasma in accordance with a simple model in which carriers are assumed to transfer their energy to *LO* phonons and these in turn undergo anharmonic decay. Analytic expressions are obtained for the cooling rate and these predict a considerable difference between the energy loss rates under static and dynamic conditions.

Cooling of an electron-hole plasma in semiconductors after excitation with high-power short light pulses is a topical subject (see, for example, Refs. 1–4). The main result of the published studies is that the cooling time increases considerably on increase in the plasma concentration. The reasons for the slowing down of cooling are screening of the electron-phonon interaction[5–7] and heating of optical phonons.[8]

We shall consider the conditions under which the screening is unimportant and slowing down of the plasma cooling rate is due to the fact that optical phonons establish an equilibrium with the plasma in a certain range of wave vectors Δq.

We shall consider a model in which a plasma interacts only with longitudinal optical phonons. These phonons in turn decay into two acoustic phonons because of the anharmonicity. The interval Δq is defined by the inequality $\tau_{qc} < \tau_a$, where τ_a is the anharmonic decay time of a *LO* phonon and τ_{qc} is the decay time due to the interaction of this phonon with the plasma. At low plasma concentrations this inequality is not obeyed for any value of q, but when the concentration increases sufficiently so that the minimum value of τ_{qc} becomes less than τ_a, an interval Δq is established and the cooling time begins to lengthen. High concentrations are defined as those for which $\tau_{qc\,\min}$ is much less than τ_a. The boundary separating low and high concentrations depends weakly on temperature (it decreases somewhat as a result of cooling) and it amounts to $\sim 10^{16}$ cm^{-3} for GaAs.

At high plasma concentrations and temperatures ($T_c \gtrsim \hbar\omega$, where ω is the *LO* phonon frequency) the interval Δq is wide and its upper limit is of the order of $(mT_c)^{1/2}/\hbar$. Under these conditions the cooling process is dominated by phonons near the upper limit of Δq. In the case of these phonons the screening is static. Estimates indicate that the contribution of the plasma to the permittivity (q_D^2/q^2, where q_D is the Debye wave vector) is small up to concentrations of 5×10^{18} cm^{-3} (in GaAs), so that up to these concentrations we can ignore the screening effect.

At high plasma concentrations and low temperatures ($T_c < \hbar\omega$) the interval Δq is narrow and lies close to $q_m = (m\omega/\hbar)^{1/2}$. Under these conditions the screening is largely dynamic and the plasma contribution to the permittivity (estimated at temperatures $T_c > 50$ K) is small up to concentrations of $\sim 5\times 10^{17}$ cm^{-3} (in the case of GaAs). This determines the concentration up to which we can ignore the screening effect and the mixing of phonon-plasmon oscillations. We shall assume that the plasma electrons and holes are characterized by the same temperature and, as usual (unless otherwise specified), the plasma will be regarded as nondegenerate. The main contribution to the cooling comes from holes so that one can frequently ignore the electron contribution and the electron degeneracy at low temperatures does not alter significantly the rate of energy losses.

In fact, a Maxwellian carrier distribution cannot be established in the cooling time at all the concentrations under discussion,[9] but the effect can be ignored because the main attention is on high plasma concentrations when the cooling time is long.

The rate of energy losses of a plasma was shown in Ref. 1 to be less under dynamic conditions than in the static case. This may be explained also by the theory developed below and it is due to the fact that under dynamic conditions the rate of energy losses affects only the plasma, whereas in the static case the plasma and phonons are involved. A similar situation was found in Ref. 10 on the basis of numerical calculations.

We shall postulate (bearing in mind GaAs) that the interaction with optical phonons is of the polarization type. However, it is known that there is also the deformation interaction of holes with transverse optical phonons. It is weaker and can be analyzed as described below, but we shall ignore it.

We shall establish an analytic formula for the dependence of the rate of cooling on the parameters of a material, its temperature, and plasma concentration in the two main temperature ranges ($T_c \gtrsim \hbar\omega$, $T_c < \hbar\omega$). All the illustrative numerical calculations will be made for GaAs.

The results will be compared with the experimental data and shown to be in good agreement with the latter. This applies to the rate of energy losses under quasistatic conditions, the difference between the rate of energy losses under static and dynamic conditions, and the relative participation of electrons and holes in the energy losses.

1. PRINCIPAL EQUATIONS

Under the above assumptions the plasma energy balance is described by the equation

$$\frac{dE}{dt} = \int \frac{d^3q}{(2\pi)^3} \hbar\omega_q [W_{qc}^- N_q - W_{qc}^+ (N_q+1)] + G, \qquad (1)$$

where E is the energy per unit volume of the plasma; q is the phonon wave vector; $\hbar\omega_q$ is the phonon energy; N_q is the occupation number of a phonon state with a wave vector q;

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W_{qc}^- and W_{qc}^+ are the probabilities of the absorption and emission of one phonon by the plasma in a unit time; G is the energy reaching a unit volume of the plasma per unit time from an external source.

The occupation numbers N_q can in turn be described by the equation

$$dN_q/dt=(N_q+1)W_q^+-N_qW_q^-, \quad (2)$$

where W_q^- and W_q^+ are the total probabilities of the annihilation and creation of a phonon per unit time. We shall allow for two channels of the change in N_q, for the interaction with the plasma, and for the anharmonic interaction with acoustic lattice vibrations.

We thus find that $W_q^\pm = W_{qc}^\pm + W_{qa}^\pm$, where the second term is due to the interaction with acoustic phonons.

Equations (1) and (2) are self-evident and well-known (see for example Ref. 4), but the conclusions which can be drawn from them have not yet been sufficiently clear.

It follows from the principle of detailed equilibrium that the relationship between the probabilities of forward and reverse processes are described by

$$\frac{W_{qc}^+}{N_{qc}}=\frac{W_{qc}^-}{N_{qc}+1}=\frac{1}{\tau_{qc}}, \quad \frac{W_{qa}^+}{N_{qa}}=\frac{W_{qa}^-}{N_{qa}+1}=\frac{1}{\tau_{qa}}, \quad (3)$$

where N_{qc} and N_{qa} are the equilibrium phonon occupation numbers at the plasma temperature T_c and at lattice temperature T_a, respectively (more exactly, these are the occupation numbers of longitudinal acoustic phonons), given by

$$N_{qc}=(\exp(\hbar\omega_q/T_c)-1)^{-1}, \quad N_{qa}=(\exp(\hbar\omega_q/T_a)-1)^{-1}.$$

The expressions in Eq. (3) introduce τ_{qc} and τ_{qa} for the decay times of phonons with a wave vector q due to the interaction with charge carriers and with acoustic phonons. The first interaction makes the phonon temperature approach T_c and the second interaction causes it to approach T_a. Using the relationships in Eq. (3), we can rewrite Eqs. (1) and (2) in the form

$$\frac{dN_q}{dt}=-\left(\frac{1}{\tau_{qc}}+\frac{1}{\tau_a}\right)(N_q-N_q^{(0)}), \quad N_q^{(0)}=\frac{\tau_{qc}N_{qa}+\tau_{qa}N_{qc}}{\tau_{qc}+\tau_{qa}}, \quad (4)$$

$$\frac{dE}{dt}=\int\frac{d^3q}{(2\pi)^3}\frac{\hbar\omega_q}{\tau_{qc}}(N_q-N_{qc})+G. \quad (5)$$

Equations (4) and (5) determine the kinetics of cooling of the coupled systems comprising the plasma and the phonons.

The values of q of importance in this problem are considerably less than the reciprocal lattice vector, so that we can ignore the dispersion of optical phonons. Consequently, we shall omit the index q in ω_q, N_{qa}, N_{qc}, and τ_{qa} and use $\omega_q \equiv \omega$, $N_{qc}\equiv N_c$, $N_{qa}\equiv N_a$, and $\tau_{qa}\equiv\tau_a$.

The time τ_a was calculated in Ref. 11. This time depends on the lattice temperature T_a and it follows from Ref. 11 that

$$\tau_a(T_a)=\tau_a(0)\,\mathrm{th}(\hbar\omega/4T_a). \quad (6)$$

The value of τ_a can be determined experimentally from the Raman scattering. According to Ref. 12 and 13, $\tau_a = 7$ ps at $T_a = 77$ K and $\tau_a = 5$ ps at $T_a = 300$ K for GaAs. According to Ref. 14, $\tau_a(0) \lesssim 30$ ps.

In the illustrative calculations for GaAs we shall use $\tau_a(0) = 21$ ps, which according to Eq. (6) gives $\tau_a(300\ \mathrm{K}) \simeq 7$ ps.

The time τ_{qc} is due to electrons and holes:

$$\tau_{qc}^{-1}=\tau_{qe}^{-1}+\tau_{qh}^{-1}.$$

We shall now give the expression for τ_{qh} (a similar expression applies to τ_{qe}).

In the case of a nondegenerate plasma irrespective of the nature of the interaction, we can write down[15]

$$\tau_{qh}=\tau_{qh}^*\exp\left\{\frac{m_h\omega^2}{2T_cq^2}+\frac{\hbar^2q^2}{8m_hT_c}\right\}. \quad (7)$$

When the interaction with longitudinal optical phonons is of the polar (Fröhlich) type, we have

$$\tau_{qh}^*=q^3/n\nu_h, \quad \nu_h=2(2\pi)^{3/2}\eta\omega e^2m_h^{1/2}\,\mathrm{sh}(\hbar\omega/2T_c)/\varepsilon_\infty\hbar T_c^{1/2}. \quad (8)$$

The following notation is used in the above expressions: ε^∞ is the high-frequency permittivity of the investigated crystal; $\eta = 1-\omega_t^2/\omega^2$; ω_t is the frequency of a transverse optical phonon; n is the electron density. In the case of GaAs we shall assume that $\eta = 0.15$, $\omega = 5.8\times10^{13}$ s^{-1}, $m_h = 0.5m_0$ and $m_e = 0.07m_0$. Equation (8) ignores a factor (amounting to about 2) obtained when an allowance is made for the complex structure of the valence band.[16]

The dependences of τ_{qh} on q are very different at high ($T_c \gtrsim \hbar\omega$) and low ($T_c \ll \hbar\omega$) temperatures.

At high temperatures there is a wide range of values of q where $\tau_{qh}\approx\tau_{qh}^*$. This interval of the effective interaction of phonons with holes is defined by the inequality $q_{\min} < q < q_{\max}$, where

$$q_{min}\approx(m_h\omega^2/2T_c)^{1/2}, \quad q_{max}\approx(8m_hT_c/\hbar^2)^{1/2}.$$

The value of τ_{qh} rises exponentially outside this interval. The minimum τ_{qh} is obtained near the lower limit of this interval, it depends weakly on temperature, and it is approximately equal to

$$\tau_{qh\,min}\approx0.05\varepsilon_\infty m_h\omega/\eta e^2n. \quad (9)$$

In the case of GaAs, we have $\tau_{qh\ \min} = 4\cdot10^{17}/n$, if time is measured in picoseconds and n is in reciprocal cubic centimeter. For electrons, we have $\tau_{qe\ \min} = 0.6\cdot10^{17}/n$, so that the minimum value of the resultant time τ_{qe} is governed by the interaction of phonons with electrons.

On increase in the concentration, when $\tau_{qc\ \min}$ becomes less than τ_a, the phonons with certain values of the wave vectors assume the plasma temperature under steady-state conditions [second formula in Eq. (4)]. The cooling of the plasma due to interaction with such phonons occurs only in the course of their decay. In the case of GaAs this occurs when $n \gtrsim 10^{16}$ cm^{-3}. A further increase in the concentration widens the range of values of q where the phonons are heated and its upper limit q_0 is now given by $\tau_{q_0c} = \tau_a$. Figure 1 shows the dependence of $q_0/q_{\max}$ on the density n when $\tau_a = 7$ ps. The electron degeneracy is allowed for by means of a formula which generalizes Eq. (7) to the case of degenerate gas.[17] Clearly, details of the electron-phonon interaction are important only if $q \gtrsim q_0$. A calculation of the contribution of free carriers to the permittivity shows that for these values of q the contribution is small compared with ε_∞ and this is true up to concentrations of $n \sim 5\times10^{18}$ cm^{-3}. Therefore, up to

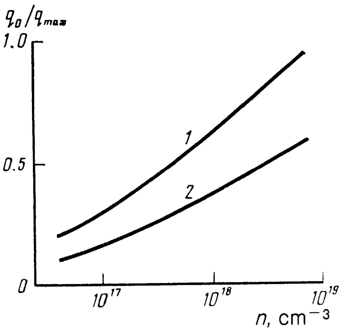

FIG. 1. Dependence of the maximum momentum of heated phonons q_0 the plasma concentration at $T_a = 300$ K: 1) $T_c = 500$ K; 2) $T_c = 1000$ K.

these concentrations, the screening of the electron-phonon interaction and mixing of plasmon-phonon modes does not play a significant role in the plasma cooling process. It should also be noted that in the case of large values of n for $q \gtrsim q_0$ the main contribution to the energy losses by the plasma comes from holes. For example, when the concentration is 10^{18} cm^{-3}, the ratio τ_{q_0e}/τ_{q_0h} amounts to 32, 9, and 4, respectively, at temperatures 300, 500, and 1000 K.

At low temperatures ($T_c \ll \hbar\omega$), we find that τ_{qh} has a sharp minimum at $q = q_m \approx (2m_h\omega/\hbar)^{1/2}$ and we can see from

$$\tau_{qh\ min} = 0.2\frac{\varepsilon_\infty m_h\omega}{\eta e^2 n}\left(\frac{T_c}{\hbar\omega}\right)^{1/2} \tag{10}$$

that in this case $\tau_{qh\,\min}$ decreases as a result of lowering of the temperature T_c, so that at low plasma temperatures the phonon heating begins at somewhat lower plasma concentrations than at high temperatures. We must bear in mind however that at low plasma temperatures the electrons first become degenerate and then this happens also to holes. Moreover, at low temperatures an important role in plasma cooling is played by acoustic phonons and we are ignoring the interaction with these phonons.

2. BEHAVIOR OF A PLASMA AND PHONONS UNDER QUASISTEADY CONDITIONS. STEADY-STATE RELAXATION TIME

If the characteristic time of a process is considerably longer than τ_a, we can assume that the instantaneous value of N_q is identical with $N_q^{(0)}$. Substituting this value in Eq. (5), we obtain the plasma energy balance under quasisteady conditions:

$$\frac{dE}{dt} = -nJ_{st}+G, \quad J_{st}=\frac{(N_c-N_a)\hbar\omega}{n}\int\frac{d^3q}{(2\pi)^3}\frac{1}{\tau_a+\tau_{qc}}. \tag{11}$$

Here, J_{st} is the steady-state rate of plasma cooling per one electron-hole pair. At low concentrations, when the minimum value of τ_{qc} is greater than τ_a, we can ignore τ_a in the denominator and a calculation of the integral [using Eqs. (7) and (8)] yields the usual expression for the energy losses experienced by a plasma because of the Fröhlich interaction

$$J_{st}=J_F=(N_c-N_a)\frac{\hbar\omega}{2\pi^2}(\nu_e+\nu_h)K_0\left(\frac{\hbar\omega}{2T_c}\right). \tag{12}$$

where K_0 is a modified Bessel function. [It should be noted that if $\beta \ll 1$, then $K_0(\beta/2) = \ln(2.24/\beta)$.] It follows from Eq. (12) that GaAs is characterized by cooling rates of 230 and 500 meV/ps at $T_c = 500$ and 1000 K, respectively, when $T_a = 300$ K. If $T_c = 100$ K and $T_a = 0$, we obtain $J_F = 20$ meV/ps. An allowance for the complex structure of the valence band (mentioned above) yields values which are half those just quoted.

The steady-state relaxation time of the plasma energy can be defined as follows:

$$\tau_{st}=(E_c-E_a)/nJ_{st}, \tag{13}$$

where E_c and E_a are the values of the plasma energy at temperatures T_c and T_a, respectively. In the case of a nondegenerate plasma we have $E_c - E_a = 3n(T_c - T_a)$ and, if we use Eq. (12), we find that τ_{st} is independent of the concentration and for GaAs it is 0.36 and 0.22 ps for $T_c = 1000$ and 500 K, respectively, when $T_a = 300$ K. If $T_c = 100$ K and $T_a = 0$, Eqs. (12) and (13) give $\tau_{st} = 1.3$ ps.

An increase in the plasma concentration reduces $\tau_{qc\,\min}$ to a value below τ_a and in a certain range of values of Δq the phonons become heated and the interaction with them ceases. Then, the loss rate J begins to depend on the concentration (it decreases on increase in n). Figures 2 and 3 show the cooling rate J_{st} and the steady-state cooling time τ_{st}, calculated from Eqs. (11) and (13) using Eq. (7) and generalized to the case of carrier degeneracy at temperatures $T_c = 500$ and 1000 K and $T_a = 300$ K. The results of these calculations can be approximated by analytic formulas. This can be done by ignoring the contribution of electrons and assuming that $q_{\min} < q < q_{\max}$ in the range $\tau_{qc} = \tau_{qh}^*$ and $\tau_{qc} = \infty$ outside this interval. Then, the integral in Eq. (11) is readily calculated and in the case of nondegenerate holes, we obtain

$$J_{st}=(N_c-N_a)\frac{\hbar\omega\nu_h}{6\pi^2}\ln\frac{\tau_a n\nu_h+q_{max}^3}{\tau_a n\nu_h+q_{min}^3}. \tag{14}$$

The approximation represented by Eq. (14) is reasonable if $q_{\max} \gg q_{\min}$, i.e., it is reasonable when $T_c > \hbar\omega$ and $m_h \gg m_e$. A comparison with the results of numerical calculation given in Fig. 2 shows that Eq. (14) describes well the steady-state rate of cooling throughout the investigated range of concentrations and temperatures in this figure.

At low concentrations we find that Eq. (12) follows from Eq. (14), apart from a small factor in the logarithm and apart from the electron contribution. These two contributions partly compensate one another. At high concentrations Eq. (14) gives

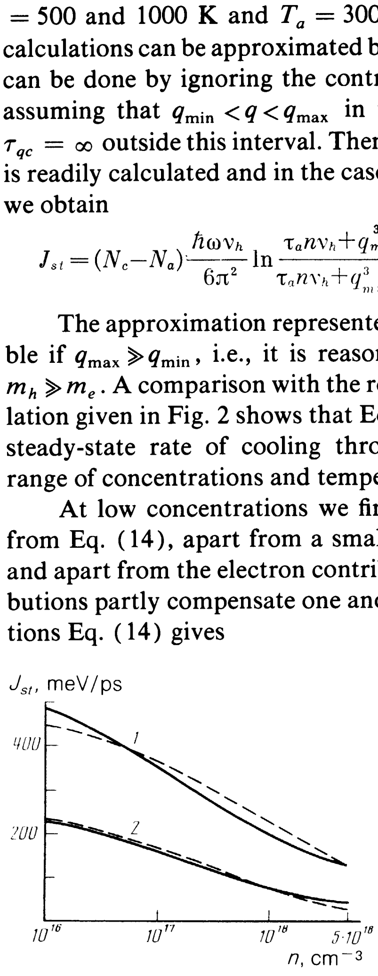

FIG. 2. Steady-state rate of cooling per one electron-hole pair plotted as a function of the plasma concentration. The continuous curves are calculated as a result of numerical integration of Eq. (11) and the dashed curves are calculated using the approximate formula (14). $T_a = 300$ K; 1) $T_c = 1000$ K; 2) $T_c = 500$ K.

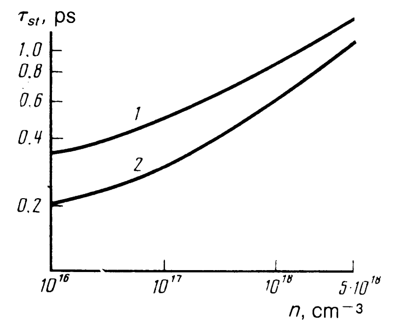

FIG. 3. Steady-state cooling rate plotted as a function of the plasma concentration. The continuous curves are calculated using Eqs. (11) and (13). $T_a = 300$ K; 1) $T_c = 1000$ K; 2) $T_c = 500$ K.

$$J_{st}=\frac{N_c-N_a}{n}\frac{\hbar\omega}{6\pi^2}(q_{max}^3-q_{min}^3)\frac{1}{\tau_a}. \tag{15}$$

This expression has a simple physical meaning: the energy is lost by a plasma only because of decay of optical phonons in the wave-vector interval between $q_{\max}$ and $q_{\min}$ (phonon bottleneck). Under these conditions the rate of losses per electron-hole pair is inversely proportional to the concentration. Under the conditions of Figs. 2 and 3, this regime has not yet been attained.

We shall now consider low temperatures characterized by $\hbar\omega \gg T_c$. Figure 4 gives the dependence of J_{st} on n for $T_c = 100$ or 50 K, when $T_a = 0$ K; the calculations are made using Eq. (11). A comparison of Figs. 2 and 4 demonstrates that the reduction in the cooling rate on increase in the plasma concentration becomes larger on lowering of the plasma temperature.

A good approximation to Eq. (11) can be achieved at low temperatures if we bear in mind that in this case, for q close to q_m, we have

$$\tau_{qh}=\tau_{qh\,min}\exp[\hbar^2(q-q_m)^2/2m_hT_c]$$

and the integral of Eq. (11) can be estimated as the product of the volume of a spherical layer of radius q_m and of thickness

$$\Delta q=2(m_hT_c/\hbar^2)^{1/2}(\ln \tau_a/\tau_{qh\,min})^{1/2}$$

and the integrand taken at $\tau_{qc} = 0$. We then obtain

$$J_{st}=\frac{1}{\tau_a}\frac{2^{1/2}m_h\omega^2(N_c-N_a)}{\pi^2 n}\left(\frac{2m_hT_c}{\hbar^2}\ln\frac{\tau_a}{\tau_{qh\,min}}\right)^{1/2}, \tag{16}$$
$$\tau_{qh\,min}\ll\tau_a,\qquad T_c\ll\hbar\omega,$$

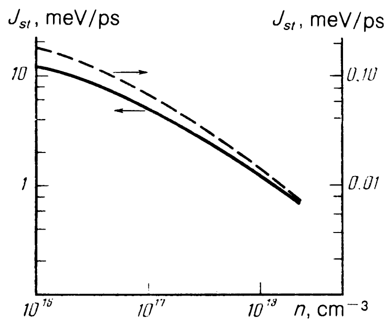

FIG. 4. Steady-state cooling rate plotted as a function of the plasma concentration on the basis of numerical calculations made using Eq. (11) for $T_a = 0$ K. The continuous curve corresponds to $T_c = 100$ K and the dashed curve to $T_c = 50$ K.

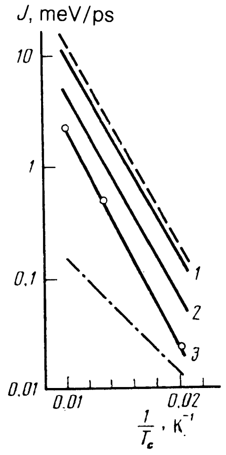

FIG. 5. Comparison of the steady-state and dynanic cooling rates as a function of the plasma temperature. The dashed line represents J_F Eq. (12)] and the continuous curves represent J_{st} Eq. (11)]: 1) $n = 10^{16}$ cm^{-3}; 2) $n = 2\times10^{17}$ cm^{-3}; 3) $n = 4\times10^{17}$ cm^{-3}. The symbols represent the values of J_{st} calculated for $n = 4\times10^{17}$ cm^{-3} using the approximate formula (16) and the chain curve gives $J_{\rm dyn}$ [Eq. (26)] for $n = 4\times10^{17}$ cm^{-3}.

where $\tau_{qh\,\min}$ is given by Eq. (10). The system (16) is in good agreement with numerical calculations based on Eq. (11) (see Fig. 5) in the range where $T_c < \hbar\omega$ and $\tau_{qh\,\min} < \tau_a$. It shows that at low temperatures the cooling rate per one pair is inversely proportional to the concentration. We can therefore have the phonon bottleneck regime and nJ_{st} is equal to, in accordance with Eq. (16), the product of τ_a^{-1} and the energy of heated phonons (i.e., of phonons with the values of q in the spherical layer mentioned above). It is interesting to note that according to Eq. (16), the dependence of J_{st} on the nature of the interaction of electrons and holes with phonons is very weak at low temperatures and for high plasma concentrations.

3. DYNAMIC REGIME. WEAK HEATING

We shall now consider relaxation of the plasma and phonon energy after the beginning of pumping. This process is described by Eqs. (4) and (5) when $G = 0$. If the heating is weak, so that T_c differs little from T_a, the system of equations (4) and (5) can be linearized and we can substitute the time dependence of the solutions in the form $\exp(-\lambda t)$ (λ^{-1} represents a dynamic relaxation time). Then, Eq. (4) yields

$$\Delta N_q=\Delta N_c\tau_a^*/(\tau_a^*+\tau_{qc}), \tag{17}$$

where $\Delta N_q = N_q - N_a$, $\Delta N_c = N_c - N_a = \Delta T_c \hbar\omega N_c(1 + N_c)/T_c^2$, and the notation $\tau_a^* = \tau_a/(1-\lambda\tau_a)$ and $\Delta T_c = T_c - T_a$ is employed. Substituting Eq. (17) into Eq. (5), we obtain the following equation for λ:

$$\lambda=\frac{\hbar\omega\Delta N_c}{\Delta E}\int\frac{d^3q}{(2\pi)^3}\frac{1}{\tau_a^*+\tau_{qc}}, \tag{18}$$

where $\Delta E = E(T_c) - E(T_a) = C\Delta T_c$ and C is the specific heat of the plasma. We note that τ_{st}^{-1} [τ_{st} is the steady-state energy relaxation time used in Eq. (13)] is equal to the right-hand side of Eq. (18) provided we replace τ_a^* with τ_a. This side of Eq. (18) is real in two ranges of λ: 1) $\lambda < \tau_a^{-1}$ and 2) $\lambda > \tau_a^{-1} + \tau_{qc\,\min}^{-1}$ In both cases it falls monotonically on increase in λ. In the former case it falls from τ_{st}^{-1} at $\lambda = 0$

to zero for $\lambda \to \tau_a^{-1}$, whereas in the latter case it falls from infinity in the case $\lambda = \tau_a^{-1} + \tau_{qc\,\min}^{-1}$ to the value τ_F defined by Eq. (13) if $J = J_F$. Consequently, Eq. (18) has two real roots $\lambda_2 < \tau_a^{-1}$ representing a slow process and $\lambda_1 > \tau_F^{-1}$ representing a fast process.

If initially the plasma is heated by a short light pulse and the phonons are cold, the relaxation process occurs in two stages: first the plasma cools and phonons are heated for a short time λ_1^{-1}, and then the plasma and phonons cool slowly in a time λ_2^{-1}. In the case of low plasma concentrations the time for the fast process is $\lambda_1^{-1} \approx \tau_F$, and that for the slow process is $\lambda_2^{-1} \approx \tau_a$. Under these conditions, we find that $\lambda_2^{-1} > \tau_{st}$.

When the concentration is reduced, the fast process accelerates ($\lambda_1^{-1} \to \tau_{qc\,\min}$) and the slow process slows down, and also the time constant of the slow process tends to τ_{st} (which also increases).

Overheating of the plasma at the beginning of the slow process ΔT_{c2} can be estimated roughly as the initial overheating $\Delta T(0)$ multiplied by the ratio of the specific heat of the plasma to the total specific heat of the plasma and overheated phonons. Moreover, we can calculate ΔT_{c2} by solving the linearized equations (4) and (5) using the Laplace transformation [subject to the initial conditions $\Delta N_q = 0$, and $\Delta T_c = \Delta T_c(0)$]

$$\Delta T_c(t) = \frac{\Delta T_c(0)}{2\pi i} \times \int \exp st \left[s + \frac{\hbar\omega \Delta N_c}{\Delta E} \int \frac{d^3q}{(2\pi)^3} \frac{1}{\tau_a^*(s) + \tau_{qc}} \right]^{-1} ds$$

and calculating the asymptotic behavior of $\Delta T_c(t)$ from a residue at a point $s = -\lambda_2$:

$$\Delta T_{c2} = \Delta T_c(0) \left[1 + \frac{\hbar\omega \Delta N_c}{\Delta E} \int \frac{d^3q}{(2\pi)^3} \frac{\tau_a^{*2}}{(\tau_a^* + \tau_{qc})^2} \right]^{-1}. \quad (19)$$

In this expression we can now substitute $\tau_a^* = \tau_a/(1 - \lambda_2 \tau_a)$.

We shall consider an approximate solution of Eq. (18) for the slow process and we shall do this separately for high and low temperatures. At high temperatures we shall employ a model used to derive Eq. (14) and we shall assume that $\tau_a^* \gg q_{\max}^3/n v_h$ (which is justified), which yields

$$\lambda_2 = \frac{1}{\tau_a} \frac{q_{max}^3}{q_{max}^3 + 18\pi^2 n}. \quad (20)$$

In the case of GaAs we have $q_{\max}^3/18\pi^2 = 6\cdot 10^{19} (T_c/1000\text{ K})^{3/2}$ so that at all temperatures throughout the investigated range and for all the plasma concentrations under discussion the dynamic cooling time λ_2^{-1} is slightly longer than τ_a.

At low temperatures we shall obtain an estimate similar to that which yields Eq. (16). Then, the equation for λ_2 can be reduced to ($T_c < \hbar\omega$).

$$\lambda_2 = \frac{1}{\tau_a} \frac{a}{(1+a)}, \quad a = \frac{4\omega^3}{3\pi^2 n} \left(\frac{m_h}{T_c} \right)^{3/2} \left(\ln \frac{\tau_a^*}{\tau_{qh\,min}} \right)^{1/2} e^{-\hbar\omega/T_c}. \quad (21)$$

In the case of GaAs, we have

$$a = \frac{2\cdot 10^{19}}{n} \left(\ln \frac{\tau_a^*}{\tau_{qh\,min}} \right)^{1/2} \left(\frac{\hbar\omega}{T_c} \right)^{3/2} e^{-\hbar\omega/T_c},$$

where n is measured in reciprocal cubic centimeters. The value of a is small and λ_2^{-1} exceeds greatly τ_a only at the lowest temperatures.

We shall show later that the dynamic cooling time may be considerably longer than τ_a if the heating is not too weak.

4. DYNAMIC REGIME. SLOW RELAXATION STAGE

We shall bear in mind that during the slow relaxation stage the characteristic time is longer than τ_a, so that an expression can be obtained for the plasma energy losses during the stage when the heating is weak. We shall do this by obtaining N_q from Eq. (4) and substituting it into Eq. (5).

We then obtain (for $G = 0$)

$$\frac{dE}{dt} = -n \left(J_{st} + \frac{1}{n} \int \frac{d^3q}{(2\pi)^3} \frac{\hbar\omega\tau_a}{\tau_a + \tau_{qc}} \frac{dN_q}{dt} \right). \quad (22)$$

Since the phonons are cooled during the slow stage, it follows that $dN_q/dt < 0$ and, consequently, the plasma cooling rate during the second stage of the dynamic process is less than the rate of cooling under static conditions. Using the slowness of the relaxation process, we substitute $N_q = N_q^{(0)}$ on the right-hand side of Eq. (22) [compare with the second formula in Eq. (4)], which gives

$$dN_q/dt = (dN_q^{(0)}/dT_c)(dT_c/dt).$$

Equation (22) can now be written in the form

$$\frac{dE}{dt} = -nJ_{dyn}, \quad J_{dyn} = J_{st}/(1+A), \quad (23)$$

where we have introduced the dynamic rate of plasma cooling $J_{\rm dyn}$ per one electron-hole pair and the quantity A is defined by

$$A = \frac{\hbar\omega}{C} \int \frac{d^3q}{(2\pi)^3} \frac{\tau_a}{\tau_{qc} + \tau_a} \frac{dN_q^{(0)}}{dT_c}, \quad (24)$$

where C is the heat capacity of the plasma.

We shall consider in greater detail the range of low plasma temperatures and assume that $T_a = 0$. We can then postulate that

$$\frac{dN_q^{(0)}}{dT_c} \approx \frac{\tau_a}{\tau_a + \tau_{qc}} \frac{dN_c}{dT_c} \approx \frac{\hbar\omega}{T_c^2} \frac{\tau_a}{\tau_a + \tau_{qc}} e^{-\hbar\omega/T_c}.$$

Using these approximations, which give Eq. (16), we find that A is described by the following expression:

$$A = \frac{4}{3\pi^2 n} \left(\frac{\hbar\omega}{T_c} \right)^{3/2} \left(\frac{m_h\omega}{\hbar} \right)^{3/2} \left(\ln \frac{\tau_a}{\tau_{qc\,min}} \right)^{1/2} e^{-\hbar\omega/T_c}. \quad (25)$$

It should be noted that A is indeed identical with the quantity a which is used to describe the relaxation time during the slow stage of the dynamic process under weak overheating conditions. (The only difference is the replacement of τ_a^* with τ_a in the logarithm.) Apart from the lowest temperatures, we find that $A \gg 1$, so that $J_{\rm dyn} \ll J_{st}$. Equation (23) for $J_{\rm dyn}$ can be deduced with the aid of Eqs. (16) and (25) to show

$$J_{dyn} = \frac{3T_c^2}{2\tau_a \hbar\omega} \left(\frac{N_c - N_a}{N_c} \right) \frac{A}{1+A}. \quad (26)$$

It should be noted that in the case of weak heating Eq. (26) is identical with the expression $3\Delta T_c \lambda_2/2$, where λ_2 is given by Eq. (21) after replacement of a with A.

It should also be noted that if $A \gg 1$, the dynamic cooling rate is independent of the plasma concentration [see Eq.

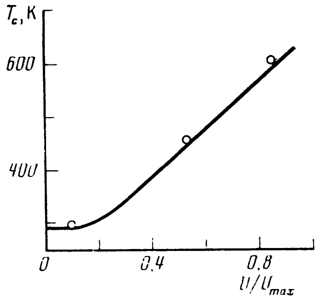

FIG. 6. Dependence of the plasma temperature on the energy of a pump pulse. The explanations are given in text.

(26)], exactly as at lower concentrations, but the rate is much less. We must bear in mind however that Eq. (26) itself is valid only at sufficiently high concentrations when $\tau_{qh\ \min} < \tau_a$ [see Eq. (10) for $\tau_{qh\,\min}$]. In the case of GaAs this means that the condition $n \gg 2\cdot 10^{18}(T_c/\hbar\omega)^{1/2}\tau_a^{-1}$ should be satisfied, where n is in reciprocal cubic centimeters and τ_a is in picoseconds.

5. COMPARISON WITH EXPERIMENTAL RESULTS

A detailed investigation of the bleaching spectra of GaAs was reported in Ref. 19. The experiments were carried out at room temperature. The duration of a pump pulse was 30 ps. which was considerably longer than the steady-state relaxation time under these conditions. An analysis of the bleaching spectrum demonstrated[18] that the temperature of a photoexcited plasma during a pulse reached 600 K and the concentration of electron-hole pairs was $\sim 5\times 10^{18}$ cm^{-3}. The change in the transparency of a sample determined using a probe light beam followed reversibly, beginning from the moment of saturation, the changes in the pump intensity in the course of a pulse and the delay was about 10 ps. This was in agreement with the above estimate, indicating that at high temperatures the dynamic relaxation time should exceed somewhat the anharmonic phonon decay time τ_a. It was suggested in Ref. 19 that the reversible change in the transparency was due to heating and subsequent cooling of the plasma and that the heating was due to intraband absorption of the pump radiation. The rate of pumping G could be estimated then from $G = U\alpha/\tau_0 S$, where U is the energy per pump pulse, τ_0 is the duration of a pump pulse, S is the area of the illuminated spot on the sample, and α is the intraband absorption coefficient of the pump radiation. Figure 6 shows the dependence of the plasma temperature T_c on the energy of a pump pulse, calculated from $nJ_{st} = G$, where J_{st} is described by Eq. (14). In these calculations we assume that $U_{\max} = 100\ \mu$J, $\tau_0 = 30$ ps, and $S = 0.5$ mm^2. It was postulated that α is proportional to the concentration and it amounts to $\alpha = 15$ cm^{-1} for $n = 10^{18}$ cm^{-3}. Figure 6 includes also the values of T_c obtained in Ref. 18 by analyzing the experimental data of Ref. 19.

The difference between the rates of losses of the energy by the plasma in the static and dynamic cases was demonstrated in Ref. 1.

The theoretically calculated rates of energy losses in the dynamic and static regimes are compared in Fig. 5. The nature of the dependences and also the order of magnitude of the loss rates are in agreement with those found experimentally in Ref. 11 (see Fig. 6 in Ref. 11).

We shall note one more qualitative result of the theory.

It is clear from Eq. (16) that at low temperatures the energy losses are proportional to $m^{3/2}$. This follows simply from the circumstance that J_{st} is proportional to the phase volume in which phonons are heated to the plasma temperature. Such a strong dependence on the carrier mass is in agreement with the experimentally established circumstance[20] of a much higher rate of cooling of holes compared with electrons in gallium arsenide.

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