Amplification of small signals near the threshold of a collective-acoustic instability in a system of parametric nuclear magnons

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(Submitted 15 May 1987)
Zh. Eksp. Teor. Fiz. **94**, 251–257 (January 1988)

Amplification of small signals was observed in a system of parametric nuclear magnons in antiferromagnetic $CsMnF_3$ near the threshold of excitation of elastic vibrations of the crystal. This effect confirmed the predictions of Wiesenfeld and McNamara [Phys. Rev. Lett. **55**, 13 (1985); Phys. Rev. A **33**, 629 (1986)] of the behavior of a dynamic system near a "cycle creation" bifurcation. A calculation of coupled oscillations of the magnon and phonon subsystems of a crystal, carried out using a model of a collective-acoustic instability proposed by Cherepanov [Sov. Phys. JETP **63**, 87 (1986)], provided a satisfactory description of the experimental results.

INTRODUCTION

One of the simplest and most convenient "objects" in the physics of nonlinear wave processes are spin waves excited in magnetic materials by the parametric resonance method in the microwave range. Above the threshold of a critical field h_c of a paramagnetic resonance a steady state is established in a system of spin waves and this state exhibits a long-range order.[3] An increase in the supercriticality h/h_c gives rise to various instabilities of the system in accordance with mechanisms which have been largely ignored. The fullest investigations have been made of an instability of parametric spin waves which is accompanied by generation of elastic vibrations of a sample (acoustic instability). This acoustic instability has been observed in antiferromagnets for systems of electrons[4,5] and nuclear[6] parametric spin waves. At present there are two fundamentally different models of the instability.

Model 1. According to this model the sound is excited as a result of decay of a parametric magnon into a magnon and a phonon (which is essentially a Cherenkov process); there is no phase correlation between the primary and secondary magnons. In accordance with the terminology of Ref. 7, this instability mechanism is called kinetic. It is important to note that in this model the source of primary magnons is not important but their total number is. The most detailed discussion of this mechanism of excitation of elastic vibrations was provided in a major experimental work of Smirnov.[5] Although all the results obtained in Ref. 5 can be explained on the basis of the kinetic model, Smirnov[5] reached the conclusion that the generation of sound was due to forced emission of a phonon by a spin wave accompanied by the formation of a secondary spin wave.

Model 2. This model[2] is largely based on the current concept of a system of parametrically excited spin waves put forward by Zakharov, L'vov, and Starobinets[3] (known as the S theory). According to the S theory the application of a microwave field to a magnetic medium creates, in a threshold manner, forced (at the frequency of the pump field ω_p) vibrations of intensity stabilized at some specific level by the phase mismatch. The forced vibrations are described at microscopic level by normal and anomalous correlation functions composed of pairs of operators describing creation and annihilation of magnons of half the frequency $\omega_p/2$ and with oppositely directed wave vectors **k**. The final dynamic equations contain just two variables: the intensity of the forced vibrations and the phase of their mismatch relative to the pump field. Deviations from the steady state of an excited system can be described in terms of collective vibrations similar to the second sound in superfluid helium. It was shown in Ref. 2 that the interaction between collective oscillations of a parametric system and elastic strains of a sample can sometimes result in excitation of magnetoelastic spontaneous oscillations. The mechanism of this instability is similar to a conventional oscillator with positive feedback: an accidental elastic deformation excites, because of the magnetoelastic interaction, collective oscillations of a parametric system of spin waves. During the subsequent evolution of these collective oscillations the energy is transferred back from the magnetic to the elastic system and the transfer coefficient can be so large that self-excitation of sound takes place (with the energy is "drawn" from the pump field).

An analysis of the experimental results[4-6] of observation of the acoustic instability in various magnetic systems supports the second model. In particular, an increase in the lifetime of acoustic vibrations on approach to the instability threshold[5] is evidence of coherence of acoustic vibrations before the threshold. This cannot be explained by the kinetic model which deals with noncoherent spin waves but is an obvious consequence of the second model. Moreover, an investigation of the acoustic instability in a system of nuclear parametric spin waves has revealed[6] that in addition to the instability excitation threshold, it has a threshold for vanishing when the supercriticality is high. This cannot be explained by the kinetic instability model where only the total number of parametric spin waves is important.

It is also pointed out in Ref. 6 that parametric spin waves carry information on the presence of vibrations of frequency ν_e well before the instability threshold h_e is reached. This information is provided by measurements of the modulation response α_m, i.e., of the depth of amplitude modulation of the microwave pump power absorbed by a sample, which is subjected not only to a static field H_0 and a pump field $h \cos 2\pi\nu_p t$, but also to a weak modulating field $H_m \cos 2\pi\nu_m t$ characterized by $\nu_m \ll \nu_p$ (Ref. 8). The dependence $\alpha_m(\nu_m)$ in the vicinity of the frequency ν_e exhibits a characteristic peak long before the h_e threshold ($h < h_e$). The existence of this peak demonstrates, in our opinion, that the observed instability is not kinetic, because in the case of the kinetic instability the elastic vibrations created by parametric spin waves should not be coherent below the threshold h_e and, consequently, they cannot create such a striking extremum in the modulation response.

The observed extremum of α_m can be explained in a natural manner using recent theoretical investigations[1] demonstrating that very different systems can amplify, near their instabilities (bifurcations), small signals of frequency corresponding to the period of passage of a path in the phase space. In the case of the acoustic instability we can adopt a model with "cycle creation" bifurcation (Hopf bifurcation).

We shall study the validity of this model in the case of an acoustic instability in a system of nuclear parametric spin waves and thus consider the mechanism of this instability. In addition, we shall check the predictions of the theory of Ref. 1 in the case of period-doubling bifurcation, which in the case of a system of parametric spin waves occurs in the case of a double parametric resonance,[9,10] i.e., in the case of parametric excitation of collective oscillations of parametric spin waves at a frequency Ω_0 by a modulating field at doubled frequency. Amplification of small signals nearest such bifurcations has been demonstrated experimentally for an NMR maser.[11] Another manifestation of such amplification was reported in Ref. 5 and it involved an increase in the lifetime of collective oscillations of parametric spin waves before the threshold of their parametric excitation by sound.

THEORY

For the reasons discussed in the Introduction we shall consider the second model and calculate the modulation response of a system of parametric spin waves near the threshold of a collective-acoustic instability. A rigorous description of the effect in Ref. 2 presupposes an analysis of correlation functions of the $\langle a_k^* a_{k+\varkappa}\rangle$ and $\langle a_k a_{-k-\varkappa}\rangle$ types, where a_k^* and a_k are the complex amplitudes of parametric spin waves, $\varkappa \approx \pi/L$, and L is the characteristic linear size of a sample. However, in view of the smallness of $\varkappa \ll k$, we can confine our treatment to a homogeneous approximation:

$$\langle a_k^* a_{k+\varkappa}\rangle \approx \langle a_k^* a_k\rangle, \quad \langle a_k a_{-k-\varkappa}\rangle \approx \langle a_k a_{-k}\rangle.$$

This was the approximation used to obtain the main result in Ref. 2. Information on the spatial inhomogeneity of oscillations in this approximation is retained in the equations of motion where it is represented by the amplitude of the magnetoelastic interaction, which is proportional to $\varkappa$. The dynamic equations describing the system can be reduced to

$$\frac{1}{2}\frac{d}{dt}\theta + hV\sin\theta = \frac{\omega_p}{2} - \tilde{\omega}_k - SN,$$
$$\frac{1}{2}\frac{d}{dt}N = N[hV\cos\theta - \gamma(N)],$$
$$\frac{d}{dt}b + (\gamma_e + i\omega_e)b = -2i\Psi N, \qquad (1)$$
$$\tilde{\omega}_k \equiv \omega_k + 2TN + \Psi(b+b^*) + \frac{\partial\omega_k}{\partial H}H_m\cos\omega_m t,$$

where N is the number of parametric spin waves per magnetic cell; θ is the angle of mismatch between the forced (of frequency ω_p) vibrations of the medium relative to the pump; V is the coefficient representing the coupling of magnons to an alternating magnetic field; T and S are the amplitudes of the four-magnon interaction[3]; Ψ is the amplitude of the magnetoelastic interaction [see Eq. (17) in Ref. 12]; b is the complex amplitude of the elastic deformation of a sample; $\gamma(N) = \gamma_0 + \eta N$ and γ_e are, respectively, the rates of relaxation of magnons and sound; $\tilde{\omega}_k = \omega_p/2$ under steady-state conditions ($d/dt = 0$) when $H_m = 0$; ω_e is the frequency of elastic vibrations.

We shall consider the modulation response α_m of the parametric system. The idea of the modulation response is quite simple. A system of spin waves excited parametrically by an external microwave field of frequency ω_p absorbs part of the energy of this field, which is demonstrated by measurements of the power transmitted through a resonator containing a sample. If an rf magnetic field is applied to such a system, oscillations of the microwave power at the frequency ω_m of the rf field will be observed at the exit from the resonator. Therefore, a magnetic sample supporting parametrically excited spin waves behaves as a nonlinear component or a modulator which mixes low- and high-frequency oscillations of the magnetic field. At low amplitudes H_m of the rf field, when this field has practically no effect on the properties of the parametric system, it is convenient to investigate the ratio $\alpha_m = \Delta P_m/H_m$, where ΔP_m is the depth of modulation of the transmitted microwave power at a frequency ω_m. The simplest variant when the depth of modulation ΔP_m is governed mainly by the dissipation of magnons, manifested by forced deviation of their total number from the steady-state value (modulation of the number of magnons) was discussed earlier in Ref. 8. Subsequently this treatment was modified[13] by including the considerable contribution made to ΔP_m by the terms associated with deviations of the phase mismatch between the forced vibrations of the medium and the pump field (phase modulation) and by the terms due to modulation of the magnon spectrum (frequency modulation). A calculation of α_m in Ref. 13 and in the present study was made allowing for all three contributions. Solving this system of Eq. (1), we obtained the following expression for the modulation response:

$$\alpha_m = 4\omega_p\left|\frac{\partial\omega_k}{\partial H}\right||S|N^2\left\{\left[(\gamma_0+2\eta N)A + \frac{\omega_m}{2}B\right]^2 + \left[(\gamma_0+2\eta N)B - \frac{\omega_m}{2}A - \frac{\omega_m}{4\omega_p SN}\right]^2\right\}^{1/2}, \qquad (2)$$

where

$$A = A_1/(A_1^2+B_1^2), \quad B = B_1/(A_1^2+B_1^2),$$
$$A_1 = \Omega_0^2 - \omega_m^2 + 4\gamma\eta N - \rho_s(\gamma_e^2 + \omega_e^2 - \omega_m^2),$$
$$B_1 = 2\omega_m(\gamma_0 + 2\eta N + \gamma_e\rho_s),$$
$$\rho_s = 16SN^2\Psi^2\omega_e/[(\gamma_e^2+\omega_e^2-\omega_m^2)^2 + (4\gamma_e\omega_m)^2],$$

and $\Omega_0 = 2N[S(2T+S)]^{1/2}$ is the frequency of collective oscillations when $\eta = 0$. The expression obtained above for α_m will be used to compare the experimental results with the theory ($\omega \equiv 2\pi\nu$).

EXPERIMENTAL RESULTS AND DISCUSSION

Nuclear parametric spin waves of frequency $\nu_k = 403$ MHz were excited in a single crystal of $CsMnF_3$ by the

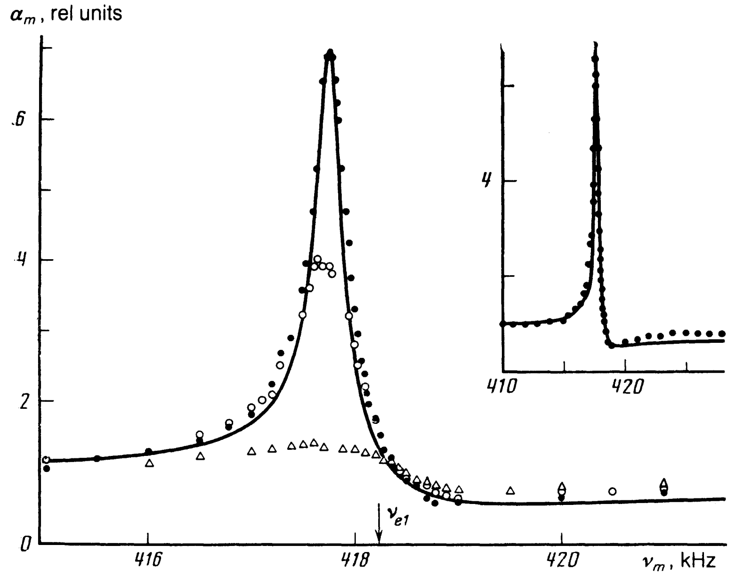

FIG. 1. Dependence of the modulation response α_m on the modulation frequency ν_m obtained for different degrees of approach of the pump amplitude h to the acoustic instability threshold h_e: ●) $h/h_e = 0.975 \pm 0.02$; ○) 0.95 ± 0.02; △) 0.79 ± 0.02. The continuous curve is calculated on the basis of Eq. (2) assuming the following values of the parameters: $\gamma_e = 418.2$ kHz; $\gamma_e/2\pi = 0.6$ kHz; $\gamma_0/2\pi = 40$ kHz; $h_e/h_c = 3.0$; $\eta = 0$; $\Psi/2\pi = 885$ kHz; $h/h_e = 0.99$; $\Omega_0/2\pi = 389$ kHz; $T = 4.2$ K; $H_0 = 0.77$ kOe. The inset shows the dependence $\alpha_m(\nu_m)$ obtained for $h/h_e = 0.975$ in a wide range of frequencies ν_m. All the values of α_m are normalized to α_m (410 kHz).

method of parallel microwave pumping at temperatures $T = 2.5$–4.2 K. The modulation response α_m was investigated by applying a field

$$H = H_0 + h\cos 2\pi\nu_p t + H_m \cos 2\pi\nu_m t$$

and measuring the amplitude of the absorbed microwave power at the modulation frequency ν_m (Ref. 8). In a study of signal amplification near the threshold of a double parametric resonance use was made of two modulating fields: an exciting field of frequency $\nu_{m1} = 80$ kHz and a weak probe field of frequency $\nu_{m2} \approx \nu_{m1}/2$.

The acoustic (elastic) instability threshold h_e exceeded approximately threefold the parametric resonance amplitude. The first to appear were elastic vibrations of frequency $\nu_{e1} = 418$ kHz and then other natural elastic modes of the sample were excited. We investigated the behavior of $\alpha_m(\nu_m)$ before the threshold of excitation of the mode ν_{e1}, i.e., in the region where the state of the system of parametric spin waves was still stable. We plotted in Fig. 1 the dependences $\alpha_m(\nu_m)$ near ν_{e1} for different degrees of approach of the amplitude of the microwave pump to the acoustic instability threshold. The dependences $\alpha_m(h/h_e)$ were obtained at frequencies corresponding to the maximum and minimum of the modulation response near the elastic vibration frequency ν_{e1} (Fig. 2). The following details of the effect were worth noting:

1) The value of α_m rose strongly at the maximum of the peak on approach of h to h_e and the maximum gain experienced by the signal in our experiments reached $\sim 10^2$,

2) The peak was clearly asymmetric and it indicated amplification of the modulation response at frequencies $\nu_m < \nu_{e1}$ and the attenuation at frequencies $\nu_m \gtrsim \nu_{e1}$.

The continuous curves in Figs. 1 and 2 represent calculations of α_m carried out using Eq. (2) and assuming the conditions in our experiments. The free parameters in these calculations were the rate of relaxation of elastic vibrations of a sample γ_e and the amplitude of the magnon-phonon interaction Ψ. The value of γ_e depended strongly on the method of clamping the sample and it governed the width of the peak, so that we substituted in Eq. (2) the observed width $\gamma_e/2\pi = 0.6$ kHz, corresponding to a fully reasonable quality factor $Q \sim 10^3$ of natural elastic vibrations of a clamped sample. The best description of the experimental curves was obtained for $\Psi/2\pi = 885$ kHz; its value was in good agreement with the estimate of Ψ obtained from Ref. 12 and amounting to $\Psi/2\pi \approx 10^3$ kHz.

Figures 1 and 2 demonstrate a good agreement between

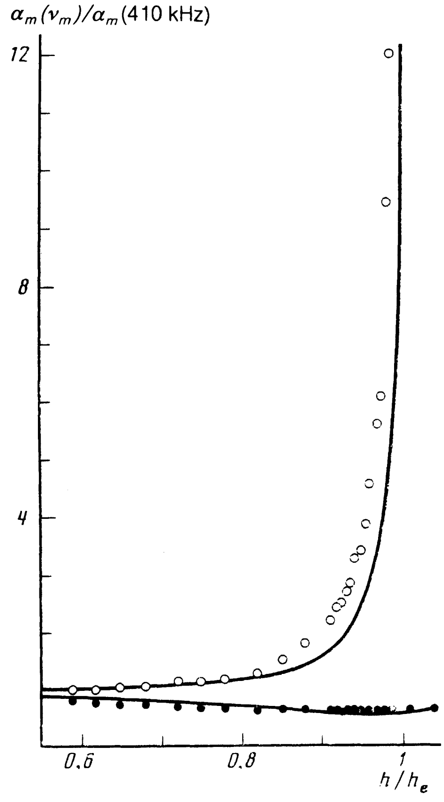

FIG. 2. Dependence of the reduced modulation response $\alpha_m(\nu_m)/\alpha_m$ (410 kHz) on the amplitude h of the microwave pump field for two typical frequencies: ○) max$\alpha_m(\nu_m)$ ($\nu_m = 417.8$ kHz); ●) min$\alpha_m(\nu_m)$ (ν_m 419.5 kHz). The continuous curves are calculated on the basis of Eq. (2) using the parameters given in the caption of Fig. 1.

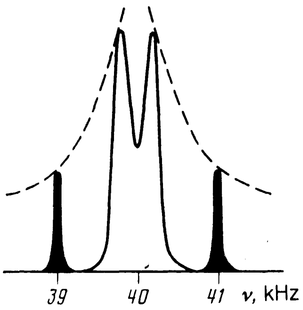

FIG. 3. Fourier spectra of collective oscillations of a system of parametric nuclear spin waves near the frequency $\nu_{m1}/2 = 40$ kHz when $\nu_{m2} = 39$ kHz (black peaks) and $\nu_{m2} = 39.8$ kHz (white peak).

the experimental and calculated results; the model describes not only the amplification of α_m near ν_{el}, but also details of the effect. The experimental results presented in Fig. 1 correspond to the case when the frequency of the elastic mode ν_{el} is higher than the frequency of collective oscillations of parametric spin waves $\Omega_0/2\pi$. In the case when $\Omega_0/2\pi > \nu_{el}$, the profile of the peak should be different, in accordance with Eq. (2): it should exhibit a slight attenuation of α_m in the range $\nu_m \lesssim \nu_{el}$ and a considerable amplification at $\nu_m \gtrsim \nu_{el}$. We were able to check this prediction by measuring α_m above the threshold of disappearance of the acoustic instability, i.e., at much higher values of h/h_c. It was found that in this case the peak profile did in fact change in full agreement with the calculation.

It is worth noting that the range of existence of the acoustic instability on the scale of the parameter h/h_c obeys quite well the condition $\Omega_0 \approx 2\pi\nu_{el}$. This was checked by us experimentally in a study of the temperature dependence of h_e/h_c. The natural frequency of collective oscillations depended on the supercriticality as follows:

$$\Omega_0 \propto \gamma[(h/h_c)^2-1]^{1/2},$$

and cooling, because of the decrease in γ, resulted in the condition $\Omega_0 \approx 2\pi\nu_{el}$ being satisfied at higher values of the supercriticality, i.e., one should expect an increase in h_e/h_c as a result of cooling. In our experiments when temperature was lowered from 4.2 to 2.5 K the rate of relaxation γ decreased by a factor of 3–4, whereas the value of h_e/h_c rose approximately threefold, which was again in agreement with the model adopted here.

It should be pointed out that a study of the acoustic instability of electron parametric spin waves in $MnCO_3$ (Ref. 5) failed to reveal disappearance of the instability right up to the maximum values h/h which were employed. Within the framework of the model of the collective-acoustic instability this result is to be expected because an investigation of the modulation response of a system of electrons parametric spin waves in $MnCO_3$ (Ref. 13) yielded a $\alpha_m(\nu_m)$ dependence in the form of a very wide plateau with the right-hand edge approaching the frequency $\nu_e \approx 2$ MHz for $h/h_c \approx 5$ (this supercriticality gave rise to the acoustic instability in Ref. 5), whereas the left-hand edge approached this frequency at $h/h_c > 100$ (this was an extrapolated value because the maximum h/h_c in our experiments and in Ref. 5 did not exceed 15). Naturally, it was not possible to reach the threshold of disappearance of the acoustic instability in a system of electron parametric spin waves, but one would rather expect suppression of this instability because of the appearance of some other instability.

Similar measurements of the signal amplification were made by us also near a period-doubling bifurcation which we selected to be the threshold of a double parametric resonance of nuclear magnons. In this case the interpretation of the results was more difficult because of the more complex nature of the effect. Close to the threshold of a double resonance we could expect not only the response of the system at frequencies ν_{m1} and ν_{m2}, but also the signal predicted in Ref. 1 and characterized by a frequency $\nu_{m1} - \nu_{m2}$, which was a "mirror image" of the frequency $\nu_{m1}/2$. The intensity of this signal was the same as that of the signal at ν_{m2} and on approach of ν_{m2} to $\nu_{m1}/2$ both signals increased simultaneously and merged at $\nu_{m2} = \nu_{m1}/2$ (see Fig. 2), which made it difficult to measure the gain (in our case the maximum gain was ≈ 4).

In these experiments we observed not only amplification of a signal, but also resonant suppression of a double parametric resonance by a field H_{m2} in a narrow frequency interval $\nu_{m2} = \nu_{m1}/2 \pm 0.5$ kHz; a strong influence was observed even in very weak fields $H_{m2} \lesssim 10^{-2}$ Oe. For comparison, one should mention that outside this frequency range the field H_{m2} had no influence on the double resonance threshold right up to $H_{m2} \approx 0.3$ Oe. This effect had not been observed before and it is not predicted by the theory of Ref. 1. It may be of interest on its own, but in our case it simply complicated even more the determination of the gain near the double resonance threshold.

We shall now formulate the main results of the present investigation.

1. We demonstrated that threshold excitation of natural elastic vibrations in a sample of $CsMnF_3$ by a system of nuclear parametric spin waves[6] was due to a collective-acoustic instability.

2. For the first time we observed clearly the predicted[1] amplification of a small signal near a cycle-creation bifurcation and provided a satisfactory theoretical explanation of the amplification on the basis of a model of a collective-acoustic instability.[2]

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Translated by A. Tybulewicz