Influence of the sign of the charge of an ion on the friction force in electron cooling

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Experiments have been carried out on measurement of the friction force acting on ions in their motion in a cold magnetized electron beam. It is observed that there is an appreciable difference in the friction forces of positive and negative ions. A qualitative explanation of the observed difference in the friction forces is given.

INTRODUCTION

Electron cooling was proposed by Budker in 1966.[1] An experimental study of the method of electron cooling was carried out for the first time at Novosibirsk in the proton storage ring NAP-M. The minimum cooling time observed was 0.083 s, which is approximately an order of magnitude smaller than the theoretically expected cooling time. This difference was due to the magnetization (magnetic confinement) of the transverse motion of the electrons, which leads to a reduction of their effective temperature[2] and to a rise in the friction force at small particle velocities ("fast electron cooling"). In the experiments in the NAP-M a proton beam was cooled to very low temperatures, several orders of magnitude less than the transverse temperature of the electrons. In addition, it was noted that the transverse angular spread in the cooled beam was an order of magnitude greater than the longitudinal spread. This was explained by the fact that a proton on entry into an electron beam attracts the nearest electron, which during the interaction time oscillates in the longitudinal direction about the proton, leading to a strong increase of the transverse diffusion.[3] In the case of cooling of negative particles this electron is repelled and this effect should not be observed.

Fast electron cooling, which permits efficient cooling of a beam of heavy particles to unusually low temperatures, opened up fundamentally new possibilities in experiments on elementary-particle and nuclear physics.[2,4] Many accelerator laboratories in the world engaged in mastery and realization of the possibilities of electron cooling. Successful experiments on electron cooling were carried out at CERN and at Fermilab.[5] Further investigation of the physics of fast electron cooling was carried out at Novosibirsk in the "solenoid model" apparatus,[6] the parameters of which permitted study of cooling in a completely magnetized beam. In this apparatus friction forces close to the maximum theoretically possible value $F_{\max} \approx e^2 n^{2/3}$ were achieved. In the present work we described experiments on study of the magnetized friction force for both positive and negative cooled particles. The importance of cooling of negative particles is determined by the possibility of use of the method for cooling antiprotons.

1. PICTURE OF THE INTERACTION

The basic method of electron cooling is thermal exchange between a beam of heavy charged particles and an electron beam moving with the same average velocity.[1,2] On decrease of the relative velocities of the electrons and ions, the efficiency of the interaction rises, which leads to an increase of the cooling rate. In the electrostatic acceleration of an electron beam the longitudinal temperature of the electrons will be much lower than the transverse temperature.[7] A longitudinal magnetic field which confines the electron beam "magnetizes" the transverse motion of the electrons, which for a small longitudinal temperature results in a rise of the contribution to cooling of collisions with large impact parameters.[8,9]

Neglecting the thermal motion of the electrons along the lines of force of the magnetic field, the components of the friction force longitudinal and transverse relative to the magnetic field $F_\parallel$ and $F_\perp$ can be written in the following form[3,9]:

$$F_\parallel = -\frac{2\pi n e^4 L_c}{mv^2}\frac{2v_\perp^2 v_\parallel}{v^3},$$

$$F_\perp = -\frac{2\pi n e^4 L_c}{mv^2}\frac{v_\perp(v_\perp^2 - v_\parallel^2)}{v^3}. \qquad (1)$$

Here n is the density of electrons, e, and m are the charge and mass of the electrons, $v_\parallel$ and $v_\perp$ are the components of the ion velocity longitudinal and transverse to the magnetic field, $L_C = \ln(\rho_{\max}/\rho_{\min})$ is the Coulomb logarithm, and $\rho_{\max}$ and $\rho_{\min}$ are the impact parameters of the collisions:

$$\rho_{max} = v\left(\frac{m}{4\pi n e^2}\right)^{1/2}, \qquad \rho_{min} = \frac{2e^2}{mv^2}.$$

The condition of applicability of Eq. (1) is that L_C be sufficiently large, namely

$$\rho_{max} \gg \rho_{min}. \qquad (2)$$

However, in the cooling process the heavy-particle velocity v decreases and the condition (2) is violated already for

$$v \approx (e^2 n^{1/3}/m)^{1/2}. \qquad (3)$$

This velocity value corresponds to the characteristic velocity arising in an electron beam with mutual repulsion of initially randomly distributed electrons. Roughly such values of the longitudinal velocity spread also are obtained in an electron beam after fast acceleration in a strong magnetic field.[6,7] On the other hand, the established values of the spread of the velocities of heavy particles in the cooling process is $(M/m)^{1/2}$ times smaller, and therefore near equilibrium the condition (2) is violated. Under this condition a difference unavoidably arises in the friction forces of positively and negatively charged particles moving in an electron flux. In its reference frame, a cooled particle with negative charge repels electrons which approach it with impact parameters

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$\rho < \rho_{min}$. This changes the electron momentum by $2mv$. On the other hand, such electrons will jump past a positively charged particle without changing their momentum. This effect leads to appearance of an additional contribution to the friction force for negatively charged particles:

$$\Delta F_{\parallel} = -\pi \rho^2_{min} n v \cdot 2mv = -\frac{2\pi n e^4}{mv^2} \cdot 4, \tag{4}$$

which under the conditions $L_C \sim 1$ appreciably increases the friction force. Calculations of the friction force taking into account higher approximations of perturbation theory are quite awkward, and therefore we carried out a numerical modeling of the friction force for particle velocities $v \gtrsim (e^2 n^{1/3}/m)^{1/2}$. In Fig. 1 we have shown the results of numerical calculations of the longitudinal friction force for different angles of particle motion relative to the magnetic field θ and for different signs of the charge. The curves in the figure have been drawn in accordance with the expression

$$F_{\parallel} = -\frac{2\pi n e^2 Q^2}{mv^2}\left(4.5 \cos\theta Z(Q) + 2\cos\theta \sin^2\theta \ln \frac{v^3}{(e^2 n^{1/3}/m)^{3/2}}\right), \tag{5}$$

where

$$Z(Q) = \begin{cases} 1, & Q<0 \\ 0, & Q \geqslant 0 \end{cases},$$

and Q is the charge of the particle. This expression is simply a certain combination of the expressions (1) and (2) which describes the results of the numerical modeling. Since in the numerical modeling the interaction of the electrons with each other was neglected, its results are valid only for $v > (e^2 n^{1/3}/m)^{1/2}$.

The longitudinal temperature of the electron beam in the co-moving system has a decisive value in the kinetics of cooling with a strong transverse magnetization. Immediately after acceleration, the temperature is determined by the temperature of the cathode and the mutual repulsion of the electrons[6]:

$$T_{\parallel} = T_c^2/4W + e^2 n^{1/3} \tag{6}$$

(W is the electron energy after acceleration) and under conditions characteristic for electron cooling amounts to $T_{\parallel} \approx 10^{-4}$ eV = 1 K. The transverse temperature does not change in the acceleration and is $T_{\perp} = T_c \gg T_{\parallel}$. On motion of the beam in the region of cooling there is a transfer of energy of the transverse motion into longitudinal motion as the result of the mutual scattering of the electrons, and the longitudinal temperature $T_{\parallel}$ of the electrons increases. However, a strong magnetic field, which makes the radius of Larmor rotation $\rho_L = (2T_c mc^2)^2/eB_0$ less than the distance between electrons $n^{-1/3}$, appreciably suppresses this process.[7]

It is evident from Eq. (1) that the friction force $F_{\parallel}$ increases with increase of $v_{\parallel}$ up to some maximum which depends on $v_{\perp}$ and on the spread of the velocities of the electron beam, and then rapidly drops. In the case in which the characteristic spread of the longitudinal velocities of the electrons is $v_e \lesssim (e^2 n^{1/3}/m)^{1/2}$, the maximum friction force will be

$$F_{max} = ce^2 n^{2/3}, \tag{7}$$

where c is a constant of the order of unity.

2. APPARATUS

A diagram of the apparatus for study of electron cooling[6,10] is shown in Fig. 2. The use of an injector of negative hydrogen ions permits experiments to be performed both with negatively and positively charged ions. Change of sign of the charge of the ions is accomplished by turning on at the entrance to the solenoid a special magnesium vapor target in which double ionization of the negative hydrogen ions occurs. Then the ion beam is directed into the solenoid where it is brought together (merged) (in space, and also in the direction and velocity) with an electron beam. The electron beam is shaped by an electron gun[11] located in the magnetic field of the solenoid[12] and is transported along the magnetic field of the solenoid to an electron collector. For local change of the magnetic field, in the region of the gun inside the main solenoid there is a short additional solenoid which permits adjustment of the size of the electron beam in the cooling region. The interaction of the ions with the electron beam leads to a change of their energy and transverse angles. After leaving the solenoid the ions enter an electrostatic spectrometer intended for measurement of the longitudinal friction force. The main parameters of the experiment are as follows: hydrogen-ion energy 850 keV, energy stability of the H^--ion injector $5 \cdot 10^{-5}$, hydrogen-ion current ~ 1 nA, angular divergence and radius of the ion beam in the cooling region ~ 0.7 mrad $\times$ 0.5 mm, electron energy 470 eV, electron beam current — 15 mA, electron-beam radius 1 mm, solenoid magnetic field 1–4 kG, magnetic field of the additional solenoid (-2)–($+2$) kG, departure from parallel of the solenoid magnetic field $B_{\perp}/B_0 \approx 5 \cdot 10^{-5}$, solenoid length 2.88 m, and length of cooling portion 2.4 m.

Performance of the experiment with low-energy beams permitted achievement of very small transverse and longitu-

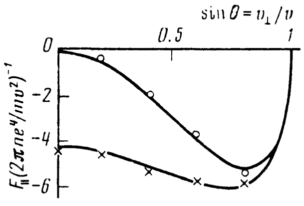

FIG. 1. Longitudinal friction force for positively charged particles (O) and negatively charged particles (×) for a particle velocity $v = 5(e^2 n^{1/3}/m)^{1/2}$. The curves have been drawn in accordance with Eq. (5).

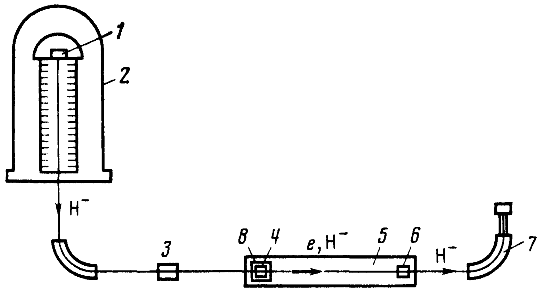

FIG. 2. Diagram of apparatus: 1—Source of H^- ions, 2—electrostatic accelerator, 3—magnesium vapor target, 4—electron gun, 5—solenoid, 6—electron collector, 7—spectrometer, 8—additional solenoid.

dinal relative velocities of the particles. For example, the transverse velocity due to distortions of the magnetic field $v_{\perp B} = v_0 B_\perp / B_0$ is of the order of $5 \cdot 10^4$ cm/sec, which is an order of magnitude smaller than the characteristic velocity $v_e = (e^2 n^{1/3}/m)^{1/2} \approx 5 \cdot 10^5$ cm/sec. The large value of the friction force with the relatively low ion energy permitted use of a single-pass measurement scheme.

Choice of the type of stripping target was determined by the following requirements: 1) the target thickness must be 10^{15}–10^{17} atoms/cm^2, so that practically complete stripping is achieved, and the change of velocity of the ion as the result of scattering in the target must still be sufficiently small: $\delta v / v_0 < 10^{-4}$; 2) the target must operate continuously and have a life of at least several hundred hours; 3) the target must be in the immediate vicinity of a high-vacuum bakeable apparatus with the electron beam ($p = 10^{-10}$ mm Hg), and therefore it must withstand heating and have the minimum possible gas flow into the high-vacuum region.

Experiments carried out with thin films of Formvar (0.07–0.1 μm) demonstrated the short life of targets of this type (several hours at a current density $\sim 5\ \mu$A/cm^2). The use of gases as a stripping target introduces complications in maintaining a sufficiently high vacuum in an apparatus with an electron beam. The requirements enumerated are satisfied by the magnesium vapor target developed by us. The target was made in the form of a long thin tube filled with magnesium vapor (see Fig. 3). With the chosen parameters (tube length 150 mm, diameter 2 mm, target thickness $5 \cdot 10^{15}$ atoms/cm^2) the expenditure of magnesium is 15 mg/hour, which is 10^3 times less than the expenditure for a standard target made in the form of a jet.[13] The target consists of a container with magnesium, on the axis of which is placed a central pipe for passage of the beam. Parallel to the axis are three more tubes in which tungsten heaters insulated by aluminum oxide are placed. The mass of magnesium in the container is 80 grams. In the middle of the central tube there are three openings for entry of magnesium vapor into the tube. To decrease the power required for heating the target, it is surrounded by a heat shield which decreases the power radiated from the surface of the target. The screen is made in the form of 12 layers of aluminum foil deformed in such a way that the thermal contact between neighboring layers is minimal. The power necessary to maintain a target working temperature of about 400 °C is 30 W.

A diagram of the beam-merging arrangement is shown in Fig. 4. The ion beam is introduced into the cooling region through a small opening in an electrode near the cathode of the electron gun. The gun is displaced from the solenoid axis by 3.5 mm. The electron beam is led to the solenoid axis where the ion beam is located by an inflection corrector. The inflection corrector creates a transverse magnetic field which distorts the lines of force of the guiding magnetic field of the solenoid and leads the electron beam to the axis. Measurement of the relative position of the beams was accomplished by a shadow method by means of a current-carrying frame fixed at one end and oscillating in the magnetic field of the solenoid. The frame was made of tantalum foil curved as shown in Fig. 4. Excitation of oscillations of the frame is accomplished by passing through it an alternating current with a frequency equal to the natural frequency of mechanical oscillations (about 12 Hz). In its motion the frame overlaps the beam, which leads to vanishing of the electron current at the collector and of the signal from the photomultiplier tube which serves to record the ion current. Merging of the beams is determined by the simultaneous vanishing of the electron and ion currents in the oscillations of the frame. To bring the beams together we used three pairs of frames measuring the vertical and horizontal merging of the beams at the beginning, center, and end of the cooling region. For visual recording of the merging, signals proportional to the currents of the electron and ion beams are fed to the inputs $Y1$ and $Y2$ of a two-beam oscilloscope, and a signal proportional to the deflection of the frame from its equilibrium position is fed to the X input. This signal is obtained by analog integration of the e.m.f. induced in the frame in its motion transverse to the magnetic field. For an amplitude of oscillations of the frame 1 cm ($B_0 = 3$ kG) the magnitude of this signal will be 6 mV. Excitation of oscillations of the frame by an external generator and integration of the e.m.f. are carried out "simultaneously." This is accomplished by means of a switch with a high repetition frequency (25 kHz). This switch alternately connects the frame to the excitation generator and the integrator. The accuracy of merging of the beams is 0.1–0.2 mm and is determined mainly by the nonuniformity of the current density over the cross section of the ion beam.

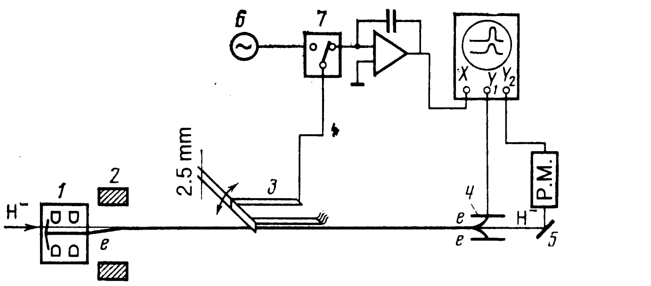

FIG. 4. Diagram of beam merging: 1—electron gun, 2—inflection corrector, 3—current-carrying frame, 4—electron collector, 5—luminescent screen, 6—alternating current generator, 7—switch with high repetition frequency.

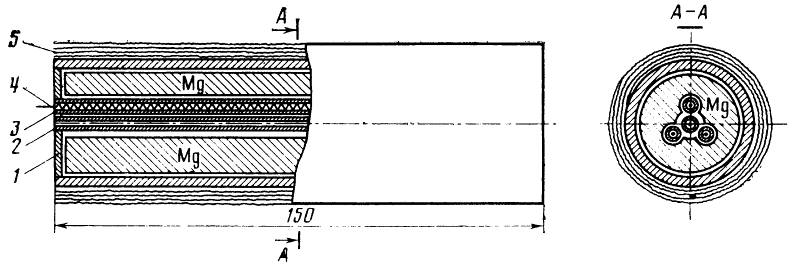

FIG. 3. Magnesium vapor target: 1—container with magnesium, 2—tube for passage of ion beam, 3—tubes for heaters, 4—heaters, 5—heat shield.

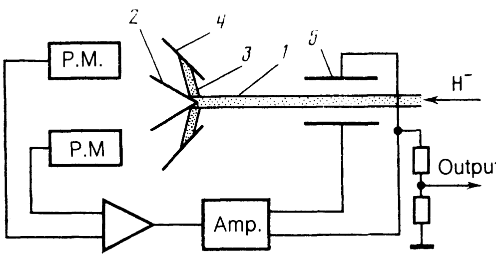

FIG. 5. Energy-deviation probe. 1—ion beam, 2—knife edge, 3—secondary electrons, 4—luminescent screens, 5—deflecting plates.

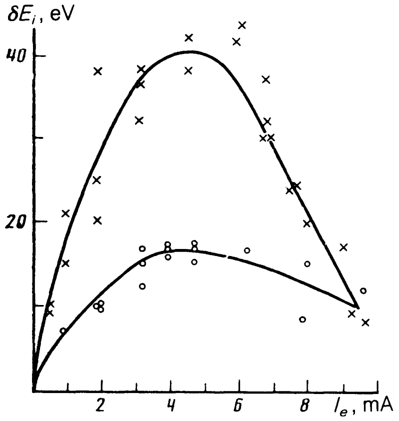

FIG. 7. Friction force as a function of electron current. $B_0 = 3$ kG, $\times - H^-$, $\bigcirc - H^+$.

Measurement of the energy deviation of the ions is carried out by means of a probe placed at the spectrometer exit. The design of the probe is shown in Fig. 5. The ion beam 1 strikes a metallic knife-edge 2. Secondary electrons 3 are accelerated to 4 kV and strike a luminescent screen 4. The light pulses are amplified by photomultipliers whose outputs are connected to a differential amplifier. To obtain a signal linear with variation of the energy and also to exclude a dependence of the output signal on the ion current, the system incorporates strong negative feedback. The signal from the output of the differential amplifier is fed to a high-voltage phase-splitting amplifier, the outputs of which are connected to the plates 5 which deflect the beam. The system described acts on the beam in such a way that the signals from the two photomultipliers are equal. In this case the signal from the divider of the phase-splitting amplifier will be proportional to the energy deviation of the ions. Conversion of the ions into secondary electrons greatly increases the useful life of the probe, since the destruction of the luminescent material under the action of the electron beam is significantly less than under the action of H^- ions or protons.

3. RESULTS OF MEASUREMENTS

The value of the longitudinal friction force was determined by means of an electrostatic spectrometer on the basis of the change in the energy of the ions after traversal of the cooling region. For this purpose at a fixed value of the ion-injector energy the dependence of the energy of the ions leaving the solenoid on the energy of the electrons was measured. When the velocities of the ion and electron beams are the same, the friction force is equal to zero and the ion energy is not changed. On deviation of the electron energy from the equilibrium value, a friction force arises, which leads to a change of the ion energy. The energy change δE_i will be proportional to the friction force $F_\parallel$ and to the length of the cooling region ($\delta E_i = F_\parallel l$). The relative magnitude of the change of ion energy is small ($\delta E_i / E_i \lesssim 5\cdot 10^{-5}$) and is comparable with the stability of the accelerating voltage of the ion injector. To separate the useful signal from the noise background of the accelerating voltage, repeated measurements were made, the results of which were summed. The duration of a single measurement was 0.2 sec, and the total number of measurements in a cycle was 1000. In Fig. 6 we have shown an example of the dependence of the change in energy of H^- and H^+ ions on the electron beam energy obtained in this way with a magnetic field of 4 kG and a current of 3 mA. It can be seen that the value of the friction force for negative ions is about 2.5 times greater than for positive ions. The dependence of the maximum longitudinal friction force on the current for a magnetic field of 3 kG for ions with different signs of the charge is shown in Fig. 7. At small currents, both for positive and for negative ions, the friction force rises in accordance with a law $c_i e^2 n^{2/3}$. On increase of the current the friction force becomes saturated and for currents above 6 mA begins to decrease. The decrease of the friction force with increase of the electron-beam current is

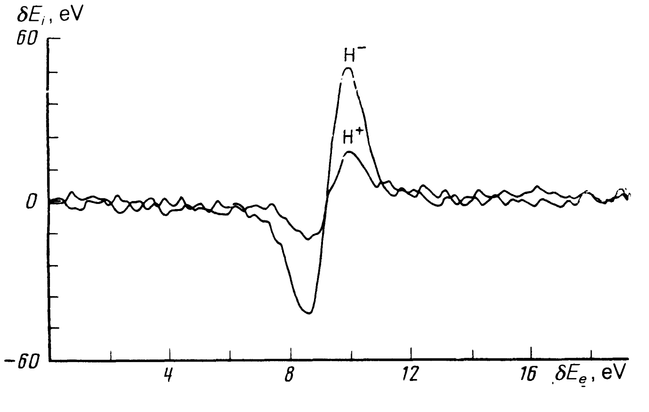

FIG. 6. Change of energy of ions of different signs as a function of electron energy for $B_0 = 4$ kG and $I_e = 3$ mA.

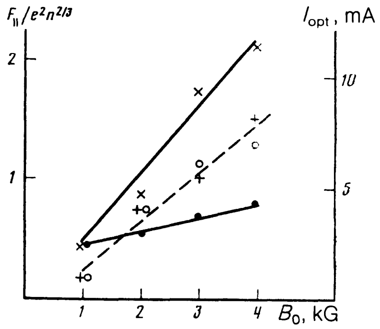

FIG. 8. Maximum friction force as a function of magnetic field strength (solid lines, $\times - H^-$, $\bullet - H^+$,), expressed in units of $e^2 n^{2/3}$, and optimal electron beam current (dashed line, $+ - H^-$, $\bullet - H^+$).

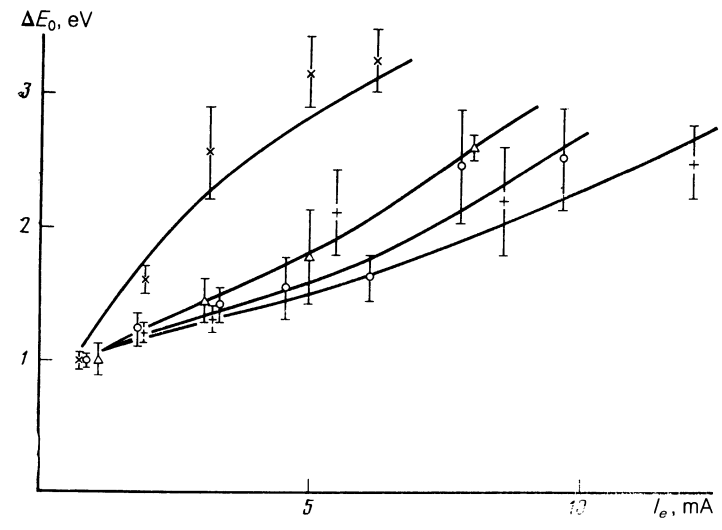

FIG. 9. Energy width ΔE_0 as a function of electron current in magnetic fields of strengths 4(+), 3(O), 2 (▲), and 1 (×) kG.

explained by the action of several factors, the relative contribution of which is hard to determine. The first factor is the increase of the longitudinal temperature of the electrons along the beam, which is due to internal collisions in the electron flux. The second factor is the absence of complete cancellation of space charge of the electron beam, which leads to a defocusing (focusing for H^+) of the beam of ions by the radial electric field and to an increase of the transverse angles in the cooling region, and also the excitation of transverse angles in the input region as the result of action on the ion beam of the field of the uncompensated electron beam in the electron gun. The third factor is the influence, on the friction force, of the nonmonochromaticity of the electron beam over its cross section, due to the action of the space charge of the beam.

Measurements of the friction force were carried out for magnetic fields from 1 to 4 kG. In Fig. 8 we have shown the maximum friction force for positive and negative ions, expressed in units of $e^2n^{2/3}$ as a function of the magnetic field Also in that figure we have shown the electron-beam current at which the friction force reaches its maximum. It can be seen that for a field of 1 kG the friction forces for positive and negative particles are equal and with increase of the magnetic field the friction force for H^- rises more rapidly, whereas for H^+ it remains almost constant.

The weak dependence of the friction force on the magnetic field for positive ions signifies an already rather strong magnetization of the collisions and is due only to some increase of the quality of the electron and ion beams. It is distinctly evident that the contribution of collisions of electrons with reflection of them from a moving negative ion increases greatly with the field, which also results in a difference in the friction forces for H^+ and H^- at large magnetic fields.

One of the important characteristics of the friction force is the characteristic energy width ΔE_0, which was determined by approximation of the dependence of the friction force on electron energy by the expression

$$F_{\parallel}(\delta E_e)=F_0\delta E_e/(\delta E_e^2+\Delta E_0^2)^{3/2}. \tag{8}$$

In the first approximation the value of ΔE_0 is determined by the longitudinal temperature of the electrons and by the angular divergence of the ion beam.[6] In the case of a strong magnetic field and a small electron current the longitudinal temperature is $T_{\parallel} \approx e^2n^{1/3}$ and changes only slightly with change of the current. In this case the energy width also depends weakly on the electron current. In Fig. 9 we have shown the dependence of the characteristic energy width on the electron current for various magnetic field values. With increase of the electron current the characteristic energy width rises more rapidly, the weaker is the magnetic field. This is due to enhancement of the transfer of energy of the transverse motion of the electrons into longitudinal motion, i.e., to rise of the longitudinal temperature, which determines the characteristic energy width.

CONCLUSIONS

Measurements of the friction force of ions in their motion in a cold magnetized electron beam have shown that there is an appreciable difference in the friction forces for positively and negatively charged particles. Theoretical calculations under these conditions (small relative velocities of the particles) are made difficult by the necessity of taking into account the higher approximations of perturbation theory. The observed effects are very important for understanding the values established for the spreads of the transverse and longitudinal velocities in the concluding stage of electron cooling.

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