Effect of temperature and density of an electron gas on the beta processes in a quantizing magnetic field

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Beta processes in an intensive magnetic field are investigated in a wide range of electron-gas temperatures and pressures, using the Furry variant of the interaction between a field and moving charged particles. A significant feature of the considered situation is that at certain ratios of the principal parameters of the problem the probabilities of the beta processes as well as the resultant neutrino luminosities are oscillatory. It is shown that in the degenerate case the interference effects, which reflect the non-analyticity of the considered expressions as the field goes to zero, exceed the perturbation-theory contributions. At high temperatures, the quantizing action of the magnetic field becomes greatly smoothed out. In ultrastrong magnetic fields, the leading factor is the linear growth of the probabilities of the beta-processes and of the neutrino luminosities with increase of the field. It is shown that if the investigated reactions are in kinetic equilibrium, the densities of the neutrons and protons as well as of the global neutrino luminosity can change substantially in the range of magnetic fields whose existence is predicted on the basis of an analysis of the data on the magnetic moments of x-ray pulsars.

1. INTRODUCTION

It is known that an important role is played in the collapse of massive nuclei in stars at high temperatures and densities by the formation of ν and $\bar{\nu}$ via beta reactions of the free nucleons.[1] Beta interaction of electrons and positrons both with free nucleons and with nuclei has been repeatedly studied in view of the importance of understanding the true mechanisms that determine the neutron losses from hot stellar matter. Account was also taken in these studies of the influence exerted on the beta processes by intense electromagnetic fields that can arise in the course of catastrophic compression of the stars at the instant of collapse.[2–4] It must be noted, however, that a consistent calculation of the field effects at relativistic parameter values involves multiparameter integral equations whose analysis is difficult, and sometimes even impossible, in a large number of regions. The results of the cited papers were therefore valid in rather narrow regions.

We have investigated in the present paper the beta processes $n \to pe^-\tilde{\nu}$, $e^-p \to n\nu$, and $e^+n \to p\tilde{\nu}$ in a quantizing magnetic field, with the temperature and density varied in a wide range. We take into account here also the possibility of generalizing the results for the interaction between electrons or positrons and free nucleons to include the case of their interaction with nuclei.

An essential feature of the situation considered is that at certain ratios of the principal parameters H/H_c, $\Phi = kT/mc^2$ and μ of the considered problem, where $H_c = m^2c^3/e\hbar$ and μ is the electron-gas chemical potential expressed in units of mc^2, the neutrino luminosities calculated by us are patently oscillatory. Note that such a possibility was pointed out earlier[5] in an investigation of beta decay in the field of an intense electromagnetic wave. It is known that the primary cause of the oscillations of a number of the characteristics of a charged Fermi gas in an intense magnetic field is the presence, in the density of the electron energy levels, of a strongly pronounced periodic structure stemming from the quantization conditions. In particular, the resonant character of the electron energy spectrum in beta decay of a neutron in a magnetic field was first pointed out in Ref. 7. It is important, however, to emphasize that the oscillations in a magnetic field, of such properties as the total probabilities[8] or neutrino luminosities cannot be adequately explained within the framework of perturbation theory with respect to the external field. The investigation reported here is based on an exact account of the influence of an intense magnetic field on the motion of charged particles, carried out in the Furry variant with allowance for the temperature and density of the fermion gas. It was this which made it possible to study the behavior of the expressions for the probabilities of the processes, and for neutrino luminosities in a wide range of variation of the parameters H/H_c, Φ and μ.

In the second section we consider the processes $n \to pe^-\tilde{\nu}$ and $e^-p \to n\nu$ under conditions of degeneracy of an electron gas acted upon by a quantizing magnetic field.

The third section is devoted to the study of the beta processes in a strong magnetic field in the high-temperature limit.

In the fourth section we investigate the conditions for kinetic equilibrium of the beta processes in an intense magnetic field at high temperatures, while in the fifth we discuss the summary neutrino luminosity due to the set of processes $n \to pe^-\tilde{\nu}$, $e^-p \to n\nu$, and $e^+n \to p\tilde{\nu}$ under the action of a strong magnetic field.

2. BETA PROCESSES UNDER CONDITIONS OF A DEGENERATE NONLINEAR GAS

The main result of the action of an external electromagnetic field of the half-life in the case of allowed beta transitions is known to be connected with increase of the phase volume on account of the interaction between the produced electron and the field. No account is usually taken here of the interaction of the parent and daughter nuclei with the external field, and the change of the matrix elements of

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the process in the field is neglected. In view of the smallness of the energy released in the decays and of the mass of the electron compared with the mass of the nuclei, this approximation can be regarded as fully justified. The same approach was used in Ref. 8 to calculate the probabilities and the cross-symmetric reactions of electron and positron capture by nuclei in an external intensive field.

Using the foregoing arguments and taking rigorous account, in the form of solutions of th relativistic quantum equations of motion,[9] of the action of an intense magnetic field on light charged particles, we can represent the neutrino luminosities due to the considered beta processes in the form

$$\frac{L_\nu^{(1)}}{L_0}=\frac{H}{H_c}\sum_{n=0}^{[N_{\varepsilon_0}]}\left(1-\frac{1}{2}\delta_{n,0}\right)\times\int_{b_n}^{\varepsilon_0}\frac{\varepsilon(\varepsilon_0-\varepsilon)^3\,d\varepsilon}{(\varepsilon^2-b_n^2)^{1/2}}[1-f(\varepsilon,\mu,\Phi)], \tag{1}$$

$$\frac{L_\nu^{(2)}}{L_0}=\frac{H}{H_c}\left\{\sum_{n=0}^{[N_{\varepsilon_0}]}\int_{\varepsilon_0}^{\infty}+\sum_{n=[N_{\varepsilon_0}]+1}^{\infty}\int_{b_n}^{\infty}\right\}\times\frac{\varepsilon(\varepsilon-\varepsilon_0)^3\,d\varepsilon}{(\varepsilon^2-b_n^2)^{1/2}}\left(1-\frac{\delta_{n,0}}{2}\right)f(\varepsilon,\mu,\Phi), \tag{2}$$

$$\frac{L_\nu^{(3)}}{L_0}=\frac{H}{H_c}\sum_{n=0}^{\infty}\int_{b_n}^{\infty}\frac{\varepsilon(\varepsilon+\varepsilon_0)^3 d\varepsilon}{(\varepsilon^2-b_n^2)^{1/2}}\left(1-\frac{1}{2}\delta_{n,0}\right)f(\varepsilon,-\mu,\Phi). \tag{3}$$

Here $[N_{\varepsilon_0}]$ denotes the integer part of $[N_{\varepsilon_0}]$, and $\delta_{n,0}$ is the Kronecker delta. The parameter ε_0 characterizes the mass difference between the parent and daughter nuclei, expressed in units of mc^2:

$$\varepsilon^2=1+2n(H/H_c)+x^2,$$

where εmc^2 and xmc are the electron energy and momentum component along the magnetic-field direction,

$$f(\varepsilon,\mu,\Phi)=\left[1+\exp\frac{\varepsilon-\mu}{\Phi}\right]^{-1}$$

is the distribution function of the electrons and positrons in a magnetic field. The function L_0 is of the form

$$L_0=\frac{\ln 2mc^2}{(ft)}(\rho/M_{\text{nuc}}),\quad (ft)=\frac{2\pi\hbar^7}{m^5c^4}\frac{\ln 2}{G_F^2}\frac{1}{|M|^2},$$
$$N_{\varepsilon_0}=(\varepsilon_0^2-1)/(2H/H_c),\ b_n^2=1+2n(H/H_c),$$

where M is the nuclear matrix element, $|M|^2=1+3\alpha^2$; $\alpha=|G_A/G_V|$ for the case of beta processes on nucleons, M_{nuc} is the nuclear mass, ρ the density, G_A and G_B the axial and vector constants of the V-A model of weak interaction, and G_F the Fermi constant. The leeway in the energy-release parameter ε_0 makes it possible to generalize the results for beta processes on nucleons to nuclear beta-decay reactions of the type of superallowed transitions, for example for $T\to H_e^3+e^-+\tilde{\nu}$, $\varepsilon_0=1{,}036$.

One of the interesting limiting cases that admit of an analysis of expressions (1)–(3) in analytic form is the case of a degenerate electron gas. In this limit, neglecting temperature corrections, it suffices to replace the Fermi distribution function by the theta function $\theta(\varepsilon-\mu)$. Then, by integrating expressions (1) and (2) they can be represented as sums of partial contributions from electronic states on fixed Landau levels:

$$\frac{L_\nu^{(1,2)}}{L_0}=\pm\frac{H}{H_c}\left\{\sum_{n=0}^{[N_1]}\left(1-\frac{\delta_{n,0}}{2}\right)I(\varepsilon_0,b_n)-\sum_{n=0}^{[N_2]}\left(1-\frac{\delta_{n,0}}{2}\right)I(\mu,b_n)\right\}, \tag{4}$$

$$I(q,b_n)=\varepsilon_0 s^3+\left[\varepsilon_0^3-\frac{3}{2}\varepsilon_0^2q+3\varepsilon_0b_n^2-\frac{q^3}{4}-\frac{3}{8}qb_n^2\right]s-\left(\frac{3\varepsilon_0^2b_n^2}{2}+\frac{3}{8}b_n^4\right)\ln\frac{q+s}{b_n},\quad s=(q^2-b_n^2)^{1/2},$$
$$N_q=\frac{q^2-1}{2(H/H_c)},\quad N_1=N_{\varepsilon_0},\quad N_2=N_\mu. \tag{5}$$

The contribution of the process with positron capture turns out to be exponentially small in the considered approximation. Equations (4) and (5) are convenient for the calculation of the neutrino luminosities in strong enough magnetic fields, when the main contribution is made by relatively few terms in the sums. If, however, $H/H_c\ll 1$ these expressions can be represented in a form more convenient for analysis by calculating the sums directly.

By using the Abel-Plana summation formula (see, e.g., Ref. 10) it is possible to write down, after a nmber of rather cumbersome transformations, the result in the form of a series in powers of the parameter (H/H_c):

$$L_\nu^{(1)}/L_0=G(\varepsilon_0)-G(\mu)\quad 1<\mu<\varepsilon_0, \tag{6}$$

$$L_\nu^{(2)}/L_0=G(\mu)-G(\varepsilon_0)\quad \mu>\varepsilon_0, \tag{7}$$

where

$$\begin{aligned}G(q)&=G_0(q)+(H/H_c)^2G_1(q)+(H/H_c)^4G_2(q)\\&+(2H/H_c)^{3/2}\zeta(-1/2,\{N_q\})(\varepsilon_0-q)^3/2\\&+(2H/H_c)^{5/2}\zeta(-3/2,\{N_q\})(\varepsilon_0-q)^2/2q\\&+(2H/H_c)^{7/2}\zeta(-5/2,\{N_q\})(\varepsilon_0^2-q^2)/10q^3\\&+(2H/H_c)^{9/2}\zeta(-7/2,\{N_q\})(3\varepsilon_0^2-q^2)/70q^5,\\G_0&=(3\varepsilon_0^2/8+1/16)\ln|q+s_1|-s_1(q^5/6-3\varepsilon_0q^4/5\\&+3\varepsilon_0^2q^3/4-q^3/24+\varepsilon_0q^2/5-\varepsilon_0^3q^2/3\\&-3\varepsilon_0^2q/8-q/16+\varepsilon_0^3/3+2\varepsilon_0/5),\\G_1&=(\varepsilon_0^2/4+1/8)\ln|q+s_1|+(q^3/4-3\varepsilon_0q^2/2-3\varepsilon_0^2q/2-3q/4\\&+\varepsilon_0^3/2+3\varepsilon_0)/6s_1,\\G_2&=(q^5-2\varepsilon_0^2q^5+5\varepsilon_0^2q^3-5\varepsilon_0q^2-\varepsilon_0^3+2\varepsilon_0)/240s_1^5,\\s_1&=(q^2-1)^{1/2},\end{aligned} \tag{8}$$

(a) denotes the fractional part of a.

It follows indeed from (6)–(8) that the field corrections in the expressions for the neutrino luminosities are essentially nonlinear. It must be noted that when determining the oscillating contributions whose order can be easily estimated by using the Hurwitz substitution for the generalized zeta function[10]

$$\zeta(s,v)=2(2\pi)^{s-1}\Gamma(1-s)\sum_{n=1}^{\infty}n^{s-1}\sin(2\pi nv+\pi s/2),$$

the expansion begins with a contribution proportional to $(H/H_c)^{3/2}$, so that the oscillating corrections turn out in the considered limit to be the leading ones compared with the so-called monotonic contribution $\sim (H/H_c)^{2p}$, where $p = 1, 2, \ldots$. The presence of such oscillations is a most distinctive feature of the expressions for processes in the presence of an intense electromagnetic field,[3,5,11] These interference effects reflect the nonanalyticity of the considered expansion at zero field. It can be stated by the same token that expressions (6) and (7) constitute an example of processes for which contributions that are non-analytic as $H\to 0$ exceed the perturbation-theory terms. We note also that a decisive role in the determination of the contribution of the oscillating corrections can be played by the relation between the value of the electron gas density and the energy release (absorption) in the reaction.

3. HIGH-TEMPERATURE LIMIT

It is known that with rise of temperature the beta-decay probability tends to saturation and depends quite weakly on the temperature.[1] The role of the magnetic field $H/H_c \ll 1$ can be tracked in this case by retaining the first terms of the expansion of the electron distribution function in reciprocal powers of he parameter Φ and by using the summation method used to obtain (6) and (7). From expression (1) for the neutrino luminosity due to this process we get

$$\begin{aligned}L_\nu^{(1)}/L_0 &= (3\varepsilon_0^2/16+{}^1/_{32})\ln|\varepsilon_0+s_1|+s_1(\varepsilon_0^5/30-7\varepsilon_0^3/30\\ &-27\varepsilon_0/40)/4+(H/H_c)^2[(2\varepsilon_0^2+1)\ln|\varepsilon_0+s_1|\\ &-3\varepsilon_0 s_1]/16-(H/H_c)^4\varepsilon_0 s_1/240+(2H/H_c)^{9/2}\zeta(-{}^7/_2,\{N_{\varepsilon_0}\})/70\varepsilon_0^3\\ &+\{s_1(\varepsilon_0^6/70-3\varepsilon_0^4/70+421\varepsilon_0^2/840+16/105\\ &-\varepsilon_0^5\mu/30+7\varepsilon_0^3\mu/30+27\varepsilon_0\mu/40)-(\varepsilon_0^3/4+3\varepsilon_0/8\\ &+3\varepsilon_0^3\mu/4+\mu/8)\ln|\varepsilon_0+s_1|+(H/H_c)^2[(17\varepsilon_0^4/3-\varepsilon_0^2/3\\ &-{}^{16}/_3+9\mu\varepsilon_0^3-9\mu\varepsilon_0)/s_1-(2\varepsilon_0^3+9\varepsilon_0+6\mu\varepsilon_0^2+3\mu)\ln|\varepsilon_0\\ &+s_1|]/12\}/8\Phi, \quad s_1=(\varepsilon_0^2-1)^{1/2}. \end{aligned} \tag{9}$$

This asymptotic representation, obtained for $H/H_c \ll 1$ and differs from expression (6), which is valid in the case of a degenerate electron gas, primarily in that the parameter (H/H_c) is of higher power in the rapidly oscillating part. The temperature rise leads to a smoothing of the interference effects, as is manifested by the absence of terms proportional to $(H/H_c)^{p/2}$, where $p<9$. This qualitative difference in the behavior of the interference corrections is due to the weaker influence of the terms that govern the density of the electron gas when the temperature is increased (see (6), (7), and the concluding remarks of Sec. 2).[1)] Note that in Eq. (9) the monotonic corrections begin as before with the term quadratic in the field.

At high temperatures and in the case of a strong magnetic field $H/H_c \gg (\varepsilon_0^2-1)/2$, the expression for the neutrino luminosity corresponding to beta decay takes the form

$$\begin{aligned}L_\nu^{(1)}/L_0 = HH_c^{-1}\Big\{&\frac{s_1}{4}(\varepsilon_0^3/4+13\varepsilon_0/8)-(3\varepsilon_0^2/8+{}^3/_{32})\ln|\varepsilon_0+s_1|\\ &+[s_1(\varepsilon_0^4/20-137\varepsilon_0^2/120-{}^8/_{15}-\mu\varepsilon_0^3/4-13\varepsilon_0\mu/8)\\ &+(\varepsilon_0^3/2+9\varepsilon_0/8+3\varepsilon_0^2\mu/2+3\mu/8)\ln|\varepsilon_0+s_1|]/8\Phi\Big\},\\ &s_1=s_1(\varepsilon_0).\end{aligned} \tag{10}$$

Equation (10) illustrates clearly the linear growth of $L_\nu^{(1)}$ with increase of field intensity. note that the beta decay behaves in exactly the same manner in this limit.[8]

In the case of electron or positron capture, the probabilities of the processes and the ensuing neutrino luminosities increase in the absence of a field in power-law fashion.[1] If account is taken of the influence of the field on these reactions in the considered region $\Phi \gg \mu$, compared with the corresponding calculations for beta decay, additional complications are encountered. They are due primarily to the fact that in this case the number of Landau levels over which the summation extends turns out to be infinite. It is therefore more convenient to carry out in expressions (2) and (3) first the summation and then the integration. The sum-calculation technique remains here exactly the same as before.

Recognizing that the considered processes $e^-p\to n\nu$ and $e^+n\to p\tilde{\nu}$ yield at high temperatures one of the main contributions to the neutrino radiation, we give the explicit form of the neutrino spectrum for these reactions, calculated by the method described above:

$$\begin{aligned}\frac{d(W^{(2,3)}/W_0)}{d\varkappa} &= \frac{\varkappa^2(\varkappa\pm\varepsilon_0)}{1+\exp\{(\varkappa\pm\varepsilon_0\mp\mu)/\Phi\}}\\ &\times\Big\{\frac{H}{2H_c}\frac{1}{((\varkappa\pm\varepsilon_0)^2-1)^{1/2}}\\ &-\left(\frac{H}{2H_c}\right)^{1/2}\zeta\left(\frac{1}{2},\frac{(\varkappa\pm\varepsilon_0)^2-1}{2H/H_c}\right)\\ &+\left(\frac{H}{2H_c}\right)^{1/2}\zeta\left(\frac{1}{2},\left\{\frac{(\varkappa\pm\varepsilon_0)^2-1}{2H/H_c}\right\}\right)\Big\},\\ W_0 &= \frac{m^5c^4}{2\pi^3\hbar^7}G_F^2|M|^2,\end{aligned} \tag{11}$$

where $\varkappa$ is the neutrino energy, and the curly brackets in the argument of the zeta function correspond to taking the fractional part of the expression contained in them. It is obvious from (11) that the neutrino spectrum is resonant in a magnetic field to the same degree as the spectrum of charged particles acted upon by a field.[7] Singularities arise whenever the second argument of the generalized zeta function turns out to be close to zero, as follows from the asymptotic series

$$\zeta({}^1/_2, v)\sim {}^1/_2 v^{-1/2}$$

under this condition.[10]

As to the neutrino luminosity due to e^+e^- capture processes, we can obtain for it in the high-temperature limit, using an analog of expression (11), the following expansion in the temperature and in the magnetic field:

$$\begin{aligned}\frac{L_\nu^{(2,3)}}{L_0} &= \Phi^6\frac{465}{4}\zeta(6)\pm\Phi^5(5\mu-3\varepsilon_0)\frac{45}{2}\zeta(5)\\ &+\Phi^4(3\varepsilon_0^2-12\mu\varepsilon_0+10\mu^2-{}^1/_2)\\ &\times\frac{21}{4}\zeta(4)\pm\Phi^3\Big(10\mu^3-18\mu^2\varepsilon_0+9\mu\varepsilon_0^2-\varepsilon_0^3\\ &+\frac{3}{2}\varepsilon_0-\frac{3}{2}\mu\Big)\frac{3}{2}\zeta(3)\\ &+\Phi^2\left(5\mu^4-12\varepsilon_0\mu^3+9\mu^2\varepsilon_0^2-2\mu\varepsilon_0^3-\frac{3}{2}(\varepsilon_0-\mu)^2-\frac{1}{8}\right)\frac{\zeta(2)}{2}\\ &\pm\Phi\left(\frac{3\varepsilon_0}{8}+\left(\mu^2-\frac{1}{2}\right)(\mu-\varepsilon_0)^3-\frac{\mu}{8}\right)\\ &\times\ln 2-\left(\frac{H}{H_c}\right)^2\Phi^2\frac{\zeta(2)}{24}\\ &\pm\frac{1}{12}\left(\frac{H}{H_c}\right)^2\Phi(3\varepsilon_0-\mu)\ln 2.\end{aligned} \tag{12}$$

As seen from (12), at high temperatures the neutrino-emission energy in e^+e^- capture reveals at H/H_0 a certain dependence, nonlinear as before, on the magnetic field. Attention must also be called to the fact that at the leading correction in the magnetic field is negative for the processes (2) and (3). Thus, in the considered region of temperatures the field weakens the neutrino luminosity, albeit to a sufficiently low degree. The field-correction oscillations due to the threshold singularities in the spectrum appear here only in the later terms of the expansion in Φ. Thus, in electron–positron capture, the quantizing action of the magnetic field becomes smoothed out with rising temperature even more strongly than in the case of beta decay.

Assume now that, besides the condition $\Phi \gg \mu$, the condition $(H/H_c) \gg \Phi^2$ is also met. In this limit, we can write for the probabilities of e^+e^- captures and for the energies of the neutrinos released in these processes per unit time

$$\frac{W^{(2,3)}}{W_0}=\frac{H}{24H_c}\Big[\frac{5}{4}\Phi^3\pm(\mu-4\varepsilon_0)\Phi^2+\Big(\frac{9}{4}+\frac{9\varepsilon_0^2}{2}-3\mu\varepsilon_0\Big)\Phi\Big], \qquad (13)$$

$$\frac{L_\nu^{(2,3)}}{L_0}=\frac{H}{32H_c}\Big[\frac{6}{5}\Phi^4\pm(\mu-5\varepsilon_0)\Phi^3+\Big(\frac{4}{3}-4\mu\varepsilon_0+8\varepsilon_0^2\Big)\Phi\Big]. \qquad (14)$$

This means that when the magnetic field is so strong that the condition $(H/H_c) \gg \Phi^2$ is met, the increases, linear in the field, of the probabilities and of the energies again become dominant. It follows from (13) that the leading temperature dependence of $W^{(2,3)}$ or $L^{(2,3)}$ is the same as of Φ^3 and Φ^4, respectively.

4. NEUTRONIZATION IN A QUANTIZING MAGNETIC FIELD

As shown in Ref. 1, nuclei dissociate into nucleons in matter whose density ranges from 10^3 to 10^{10} and the temperature from 10^9 to 10^{10}. We consider further the kinetics of beta processes on free nucleons, assuming that at the matter densities considered the star remains transparent to neutrinos and antineutrinos. In this case, the processes (1) and (3) balance each other in the equilibrium system. Solving the kinetics equations for the neutron and proton densities (n_n and n_p)

$$-(dn_p/dt)=(dn_n/dt)=-n_n(W^1+W^3)+n_pW^2,$$

we obtain for the ratio of the equilibrium densities

$$(n_n/n_p)=W^2[W^1+W^3]^{-1}.$$

In the high-temperature limit, which is of greatest interest from the viewpoint of estimating the situation in the collapse of massive stellar nuclei, the corresponding expansions for the probabilities of the processes (1)–(3) yield

$$\frac{n_n}{n_p}=1+\frac{2\mu-\varepsilon_0}{\Phi}\frac{14\zeta(4)}{15\zeta(5)}+\frac{[\varepsilon_0^2+6\mu(\mu-\varepsilon_0)-{}^1/_2]^2}{\Phi^2}\Big[\frac{\zeta(3)}{15\zeta(5)}\Big]^2-\frac{(H/H_c)^2\ln 2}{135\Phi^4\zeta(5)}. \qquad (15)$$

We have retained here only the first term of the expansion in the magnnetic field intensity H, and the zero-field temperature corrections up to the quadratic term inclusive. It is clear from (15) that in the interval $(H/H_c) \ll 1$ the magnetic field is incapable of influencing to any extent the equilibrium value of (n_n/n_p). We note, however, the field hinders the neutronization process.

We consider now the situation with $(H/H_c) \gg \Phi^4$ and, as before, $\phi \gg \mu$. In this limit the ratio of the neutron and proton densities can differ noticeably from (15) under equilibrium conditions:

$$(n_n/n_p)=1+8(\mu-4\varepsilon_0)/5\Phi. \qquad (16)$$

Although Eq. (16) does not contain formally the field-intensity parameter, the difference between the coefficient of Φ^{-1} and the corresponding coefficient in (15) points to a substantial influence of the field on the conditions for kinetic equilibrium of the beta processes in this case. It must be pointed out that a change takes place not only in the numerical value of the coefficient, but also in the conditions that determine the sign of the first temperature correction. Thus, without allowance for the field, neutronization prevails if $\mu > \varepsilon_0/2$ (Ref. 1), and in the case of a strong magnetic field the analogous condition becomes more stringent, viz., $\mu > 4\varepsilon_0$.

5. DISCUSSION OF RESULTS

As already noted, beta processes in a strong magnetic field are of interest in astrophysics in view of the discovery of ultrastrong fields that quantize the electronic component of the plasma in the surface and inner layers of neutron stars. Obviously, both in neutron stars and near their surfaces, processes in which electrons take part can play an extraordinarily important role. In view of the possible extremal values of the parameters, it is necessary to develop for the study of these processes mathematical methods that do not involve perturbation theory.

Our results for relatively weak field intensities, $H/H_c \ll 1$ [see Eqs. (6)–(9)], point to a strongly nonlinear dependence of the principal characteristics of processes (1)–(3) on the field intensity (see, however, Ref. 2). Separation of the interference contributions (non-analytic as $H \to 0$), which predominate over the perturbation-theory contributions, may permit first of all observation of just these corrections, $\sim (H/H_c)^{3/2}$, in sufficiently strong magnetic fields.

Taking into account the importance of the high-temperature asymptote for the estimate of the real situation that sets in at a definite stage of the collapse of massive stellar nuclei, we present here expressions for the total neutrino luminosity due to processes (1)–(3) under equilibrium conditions. For magnetic fields satisfying the condition $H/H_c \ll 1$ we have

$$\frac{L_\nu}{L_0}=\frac{465}{4}\zeta(6)\Phi^6+\Big(5\mu\varepsilon_0-5\mu^2-\frac{3\varepsilon_0^2}{2}-\frac{1}{4}\Big)\frac{21}{2}\zeta(4)\Phi^4+\Big[\frac{7\zeta(3)\zeta(4)}{5\zeta(5)}\Big(\varepsilon_0^4-9\varepsilon_0^3\mu+29\varepsilon_0^2\mu^2-10\varepsilon_0\mu^3+20\mu^4-\mu^2+\frac{\varepsilon_0\mu}{2}\Big)-\frac{\zeta(2)}{2}\Big(15\mu^4-30\mu^3\varepsilon_0+19\mu^2\varepsilon_0^2-4\mu\varepsilon_0^3+\frac{3}{2}\varepsilon_0^2-\frac{7\mu^2}{2}+\frac{1}{8}\Big)-\Big(\frac{H}{H_c}\Big)^2\frac{\zeta(2)}{24}\Big]\Phi^2. \qquad (17)$$

It follows hence that at high temperatures the influence of the magnetic field decreases the neutrino flux from the collapsing star.

The analogous expression in the limit of ultrastrong magnetic fields takes the form

$$\frac{L_\nu}{L_0}=\frac{H}{32H_c}\left\{\frac{6}{5}\Phi^4+\left(\frac{4}{3}-8\varepsilon_0^2+\frac{16}{5}\mu\varepsilon_0-\frac{4}{5}\mu^2\right)\Phi^2\right\}. \tag{18}$$

Direct comparison of expressions (17) and (18) shows that the role of the magnetic field in the determination of the neutrino luminosity under conditions of kinetic equilibrium with respect to the considered processes can become decisive if $H>301\xi(6)\Phi^2 H_c$. Starting with these field values, the field-induced growth of the radiation energy will exceed the purely temperature contribution. For example, at temperatures on the order of 10^9–10^{10} K, which are typical of the estimates for the collapse process, values $H\sim 10^{16}$–10^{17}G are necessary. The existence of magnetic fields of this strength is predicted on the basis of an analysis of the data on the magnetic moments of x-ray pulsars.[14]. It is therefore necessary to take also into account the action of intense magnetic fields in the calculation of the processes that lead to neutronization of stellar matter and cause the neutrino energy loss at the instant of collapse at relativistic temperatures and densities.

[1)]The presence of interference terms of sufficiently low degree, which are absent for electromagnetic fields having a different configuration, can apparently be attributed to a manifestation of the discrete character of the electronic states in a magnetic field.

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