Contribution to the theory of an electromagnetic signal from a hot quark-gluon plasma

V. G. Nosov and A. M. Kamchatnov

Kurchatov Atomic Energy Institute
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Various distribution functions that describe lepton pairs thermally emitted from an electromagnetically transparent plasma volume having a fixed temperature are obtained. It is shown that the energy distribution in the spectrum of the thermal-emission photons duplicates the Fermi distribution of the initial elementary emitters—the quarks. The hydrodynamic expansion, accompanied by cooling, is taken into account for a quark-gluon plasma realistically obtainable by collisions of ultrarelativistic nuclei. The resultant distributions of the electromagnetically interacting particles in energy and in angle are obtained in closed form. Recognizing the probable difficulties faced by experiment, the most suitable particles seem to be leptons, preferably muons.

I. INTRODUCTION

For all their attractive features, modern theoretical notions concerning strong interaction are characterized by serious difficulties in the case of problems outside the framework of perturbation theory. The most typical is here the seemingly fundamental property of confinement of colored particles, which prevent observation of quarks and gluons in a state of sufficiently prolonged free motion. We shall not discuss here the ensuing basic problems such as the transition to the classical limit, the meaning of a measurement, etc. We note only one circumstance that is without precedent in quantum electrodynamics and in other field theories of the traditional type. The specific features of confinement can enhance greatly the role of temperature in field-theoretical concepts.

This is precisely the reason for the active interest in the hypothesis concerning the phase transition whereby bound quarks ($T<T_c$) become freely moving ($T>T_c$). Besides the decrease of the effective strong-interaction constant α_s $(q)^2$, this circumstance simplifies substantially the physical picture. In fact, in the presence of thermal equilibrium, momenta $q\sim T(\hbar=c=1)$ predominate in the relativistic and ultrarelativistic cases. Figuratively speaking, the temperature T governs the characteristic value of the momentum transfer q, attributing it so to speak to most collisions. We are therefore justified in expecting at $T\gg T_c$ a gradual transition to the case of an ideal gas of quarks and gluons.

The situation, however, will be similar even in the absence of a phase transition, if

$$e^2\ll\alpha_s(T)\ll 1. \tag{1}$$

In other words, with rise of temperature the transition to an ideal (ultrarelativistic) gas plasma can have also a purely asymptotic character. If the criterion (1) is satisfied, colorless hadrons should in practice be annihilated by a disintegration similar to atom ionization in an ordinary plasma.

Quark-gluon plasma volumes that are not too large are transparent to electromagnetically interacting (colorless) particles, which one can attempt to use to obtain information on the true structure of matter of this type.[1] The photons and charged leptons created in it are referred to as the electromagnetic signal from the plasma. The possibility of using this signal to check on chromodynamic concepts has also been discussed in the literature,[2–6] but no complete theory of the phenomenon has been developed so far.

Consider the mechanism of thermal production of electromagnetic products. From the standpoint of gaskinetics, these products are created in reactions that take place when individual plasma particles collide. For example, quarks and antiquarks can be annihilated to form a lepton pair. Since the cross sections for such elementary processes are either known or easy to calculate, the problem reduces in essence to averaging the initial particles over the thermal distributions and to integrating by parts the phase space of the resultant particles needed to obtain the required distribution functions. In some specific cases this calculation is easy, and this is precisely how the results of the cited papers were obtained.

A different approach to the problem is, however, possible. The electric charges and currents in the plasma undergo thermal fluctuations that lead to the onset of real or virtual photons, and the latter decay into lepton-antilepton pairs.[1)] This treatment permits the use of the rather effective methods of the theory of electromagnetic fluctuations.[7,8] In the next section we obtain by this method a general equation for the pair-production rate in an element of the 6-dimensional space of the two leptons.

The question of the signal from a quark-gluon plasma is difficult to discuss without considering the feasibility of its production and subsequent evolution. The use of collisions between individual hadrons is somewhat problematic, and it is more likely that the plasma will be produced by collisions of ultrarelativistic nuclei. But their coalescence will inevitably be replaced by expansion of matter in the vacuum. The ensuing cooling will decrease the production of photons and leptons, in view of the temperature dependence. Nevertheless, this is not the only hydrodynamic-expansion manifestation capable in principle of decreasing the effect of interest to us. A moving plasma emits more slowly than an immobile one because of the universally known slowing-down of a moving clock.

It is known that it is precisely in its later three-dimensional stage that the expansion becomes most intense.[9–11] On the other hand, the preceding one-dimensional stage is characterized mainly by moderately relativistic velocities of hy-

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drodynamic flow, in which the slowing clock does not lower the order of magnitude of the effect. Thus, the three-dimensional stage, which is most substantial for the analysis of hadron emission, need not be taken into account in the present case. The signal of interest to us is formed mainly in a relatively quiescent and sluggish one-dimensional motion, still quite far from enabling individual hadrons to appear. On the other hand, even such moderate evolutions of the fluid alter radically the electromagnetic spectrum compared with the simplest variant of an immobile and uniform plasma (under thermal equilibrium).

This is precisely the subject of Sec. 2, the methods of which are needed later. In Sec. 3, one-dimensional plasma flow is taken into account with the aid of the exact solution obtained long ago by Khalatnikov.[12] The calculation of the angular and energy distributions of electromagnetic products of thermal origin is carried through here to conclusion. Section 4 is devoted to a discussion of the results.

2. STUDY OF AN IMMOBILE THERMODYNAMIC-EQUILIBRIUM QUARK-GLUON PLASMA

Assume that equilibrium obtains with respect to the color degrees of freedom, with a constant T specified. The currents fluctuating in time and in space generate lepton pairs in a specified momentum-space volume in accordance with the relation

$$dw=\overline{|M_{fi}|^2}\frac{d^3p_+d^3p_-}{4\varepsilon_+\varepsilon_-(2\pi)^6}, \tag{2}$$

where ε_- and ε_+ are the energies of the lepton and antilepton, while $\mathbf{p}-$ and $\mathbf{p}+$ are the corresponding momenta. The transition matrix element

$$M_{fi}=j^\mu D_{\mu\nu}(\omega,\mathbf{k})J^\nu(\omega,\mathbf{k}), \tag{3}$$
$$\omega=\varepsilon_++\varepsilon_-,\quad \mathbf{k}=\mathbf{p}_++\mathbf{p}_-.$$

Here $j^\mu = e\bar{u}(-p_+)\gamma^\mu u(p_-)$ is the lepton current of the transition and corresponds to the pair production; $D_{\mu\nu}(\omega,\mathbf{k})$ is the virtual-photon propagator; $J^\mu(\omega,\mathbf{k})$ is the Fourier component of the fluctuating quark current. The square of the matrix element (3) must be summed over the lepton polarizations and over the final states of the plasma, and averaged over its initial states. It is convenient to use in the calculations the gauge

$$D_{ij}(\omega,\mathbf{k})=\frac{4\pi}{\omega^2-k^2}\left(\delta_{ij}-\frac{k_ik_j}{\omega^2}\right), \tag{4}$$
$$\varphi=A_0=0,\quad D_{0i}=D_{i0}=D_{00}=0,$$

so that after squaring we need only the spatial components of the tensors $\overline{j_\mu j_\nu}$ and $\overline{J_\mu J_\nu}$. Summation over the lepton polarization yields

$$\overline{j_ij_j}=4e^2L_{ij},\quad i,j=1,2,3,$$
$$L_{ij}=p_{+i}p_{-j}+p_{-j}p_{+j}+(p_+p_-)\delta_{ij}, \tag{5}$$

where we have neglected the lepton mass: $m_l\ll T$. The thermodynamic averaging of J_iJ_j yields, after a simple calculation (see, e.g., Ref. 8), the Fourier component of the fluctuating-currents correlator:

$$\overline{J_i(\omega,\mathbf{k})J_j(\omega,\mathbf{k})}=Vt(J_iJ_j)_{\omega\mathbf{k}}. \tag{6}$$

We leave out hereafter the factor Vt and refer any type of emission from a uniform plasma at rest to unit 4-volume. According to the fluctuation-dissipation theorem, the spectral component of the current correlator is expressed in terms of the imaginary part of the dielectric constant of the plasma.[7,8] Taking the spatial dispersion into account, the corresponding expression takes the form

$$(J_iJ_j)_{\omega\mathbf{k}}=\frac{\omega^2}{2\pi}\frac{1}{e^{\omega/T}-1}\left[\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)\varepsilon_t''+\frac{k_ik_j}{k^2}\varepsilon_l''\right]. \tag{7}$$

The dielectric constant of an electron-positron plasma was calculated in Ref. 13. We can use this result, recognizing only that there exist several quark types with corresponding charges. For a lepton pair, the square of the invariant mass $M^2=\omega^2-k^2$ is positive. In this region of frequencies and wave vectors, the imaginary parts of the transverse and longitudinal dielectric constants are given in the ultrarelativistic limit $m_q\ll T$ by

$$\varepsilon_t''=\frac{e^2q^2}{8}\frac{\omega^2-k^2}{\omega^2}F_t,\quad \varepsilon_l''=\frac{e^2q^2}{8}F_l, \tag{8}$$

where q denotes the charge of the quark in units of $|e|$, and

$$F_t(\omega,k)=\int_{-1}^{1}(1+\beta^2)\,\mathrm{th}\frac{\omega+k\beta}{4T}\,d\beta,$$
$$F_l(\omega,k)=2\int_{-1}^{1}(1-\beta^2)\,\mathrm{th}\frac{\omega+k\beta}{4T}\,d\beta. \tag{9}$$

We introduce the number n_f of the excited quark flavors with $m_q<T$, and also the squared electric charge $\overline{q^2}$ averaged over them. Summing over the flavors and taking the color tripling into account, we obtain after making all the substitutions the general equation

$$dw=\frac{12\pi e^4n_f\overline{q^2}}{e^{\omega/T}-1}\frac{1}{k^2}\left[\omega^2F_t-2\varepsilon_+\varepsilon_-(F_t-F_l)-\frac{\omega^2-k^2}{2}(F_t+F_l)\right]$$
$$\times\frac{d^3p_+d^3p_-}{4\varepsilon_+\varepsilon_-(2\pi)^6} \tag{10}$$

for the rate of lepton-pair production.

It is of interest to find their joint distribution in energy and in mass M. Integration closed form over the remaining variables is not particularly difficult

$$dw(\omega,M^2)=\frac{e^4n_f\overline{q^2}}{\pi^3}\frac{T}{e^{\omega/T}-1}\ln\frac{\mathrm{ch}((\omega+k)/4T)}{\mathrm{ch}((\omega-k)/4T)}\,d\omega\,dM^2,$$
$$k=(\omega^2-M^2)^{1/2}. \tag{11}$$

The same result is obtained in the gaskinetic approach, to which we now turn in connection with the problem of the inclusive spectrum of individual leptons (antileptons).

In an ideal gas of colored particles (see also the Introduction) their distribution over the quantum states is given by

$$n_q=\frac{1}{e^{\varepsilon/T}+1},\quad n_g=\frac{1}{e^{\varepsilon/T}-1}, \tag{12}$$

where ε is the particle energy. Since only the binary collisions are of importance, their number per unit 4-volume can be conveniently obtained from the known Pauli equation[14]

$$dv=\sigma\frac{(p_1p_2)}{\varepsilon_1\varepsilon_2}\rho_1\rho_2. \tag{13}$$

Here $p_{1,2}$ are the 4-momenta of the reacting ultrarelativistic particles, and their spatial densities $\rho_{1,2}$, given by Eqs. (12), contain differentials of these variables, on which the cross sections σ of the considered reaction depend. Quantities referred to their c. m. system will be designated by a 0 subscript.

The cross section of the reaction of type

$$q+\bar{q}\to\mu+\bar{\mu} \tag{14}$$

is well known:

$$\sigma=\frac{\pi}{3}\frac{e^4q^2}{\varepsilon_0^2}=\frac{2\pi}{3}\frac{e^4q^2}{p_1p_2}. \tag{15}$$

The inclusive spectrum of interest to us, however, is influenced also by the form of the angular distribution

$$1+\cos^2\theta_0$$

of the products and the calculations become more complicated. This is due essentially to the mass-center motion following an arbitrary collision of the annihilating quarks in the gas.

The muon-momentum distribution function corresponding to an individual elementary act is taken to be normalized:

$$\int f(\mathbf{p})\,d^3p=1.$$

Recognizing that $\varepsilon f(\mathbf{p})=inv$, we verify that for the indicated angular indicatrix the relation

$$f(\mathbf{p})=\left\{\frac{3}{2}-12\frac{(p_1p)(p_2p)}{[(p_1+p_2)^2]^2}\right\}\frac{1}{\pi\varepsilon}\delta(2(p_1+p_2,p)-(p_1+p_2)^2) \tag{16}$$

is valid in any reference frame ($\varepsilon\equiv\varepsilon_\pm$). Integration over the directions of the vector $\mathbf{p}$ leads to a corresponding purely energy distribution, but the subsequent calculations become too complicated and we must confine ourselves to their description.

This distribution is obtained explicitly in the form

$$d\tau=\frac{d\tau}{d\varepsilon}(\varepsilon;\varepsilon_1,\varepsilon_2,\mu)\,d\varepsilon,\quad \mu=\cos\vartheta, \tag{17}$$

and depends parametrically also on the angle ϑ between the momenta of the initial fermions. Turning now to the fundamental equation (13), we carry out in it also a color tripling, followed by an additional doubling, so as to have the total number of muons of any charge instead of the number of events. Then

$$d\nu=d\varepsilon\frac{2}{\pi^3}e^4q^2\frac{\varepsilon_1\,d\varepsilon_1}{e^{\varepsilon_1/T}+1}\frac{\varepsilon_2\,d\varepsilon_2\,d\mu}{e^{\varepsilon_2/T}+1}\frac{d\tau}{d\varepsilon}. \tag{18}$$

The function $d\tau/d\varepsilon$, however, differs from zero only in the region

$$\varepsilon_{min}<\varepsilon<\varepsilon_{max},$$
$$\varepsilon_{min}={}^1/_2(\varepsilon_1+\varepsilon_2-(\varepsilon_1^2+\varepsilon_2^2+2\varepsilon_1\varepsilon_2\mu)^{1/2}), \tag{19}$$
$$\varepsilon_{max}={}^1/_2(\varepsilon_1+\varepsilon_2+(\varepsilon_1^2+\varepsilon_2^2+2\varepsilon_1\varepsilon_2\mu)^{1/2}),$$

and it is this which decides the limits of the integration with respect to μ.

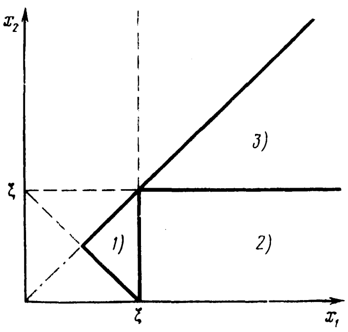

FIG. 1. Kinematic energy regions on the quark-antiquark plane.

Assume $\varepsilon_1>\varepsilon_2$ (this constraint is readily lifted by doubling the result). Subject in general to the inequality $\varepsilon_1+\varepsilon_2>\varepsilon$, which incidentally is dictated also directly by the energy conservation law, a distinction must be made between three kinematic regions: 1) $\varepsilon_1<\varepsilon$, 2) $\varepsilon_2<\varepsilon<\varepsilon_1$, 3) $\varepsilon_2>\varepsilon$. In terms of the dimensionless variables

$$x_{1,2}=\varepsilon_{1,2}/T,\quad \xi=\varepsilon/T \tag{20}$$

this subdivision is shown in Fig. 1, and the integration of (18) over the regions is carried out separately. For convenience, the integral that determines the form of the spectrum is normalized by the condition that $I(\xi)\approx\xi e^{-\xi}$ at $\xi\gg1$. It consists of three corresponding terms:

$$I(\xi)=I_1+I_2+I_3,$$
$$I_1=3\iint_{1)}\left\{\frac{2x_1x_2}{(x_1+x_2)^3}\xi^2-\frac{x_1^2+4x_1x_2+x_2^2}{(x_1+x_2)^2}\xi+x_1+x_2\right\}\frac{dx_1\,dx_2}{(e^{x_1}+1)(e^{x_2}+1)},$$
$$I_2=3\iint_{2)}\left\{\frac{2x_1x_2}{(x_1+x_2)^3}\xi^2-\frac{2x_1x_2}{(x_1+x_2)^2}\xi+x_2\right\}\frac{dx_1\,dx_2}{(e^{x_1}+1)(e^{x_2}+1)}, \tag{21}$$
$$I_3=3\iint_{3)}\left\{-\frac{2x_1x_2}{(x_1+x_2)^3}\xi^2+\frac{x_1^2+x_2^2}{(x_1+x_2)^2}\xi\right\}\frac{dx_1\,dx_2}{(e^{x_1}+1)(e^{x_2}+1)}.$$

A plot of $I(\xi)$ is shown in Fig. 2 (numerical calculation). The final result normalized to the number of produced particles takes the form

$$dw_l=\frac{4}{\pi^3}e^4n_j\overline{q^2}\,T^4I(\xi)\,d\xi. \tag{22}$$

In accordance with the form of Eq. (21), it is convenient to calculate the integral over the spectrum with the sequence of the operations reversed, beginning with elementary integration with respect to ξ. The remaining integrals with respect to $x_{1,2}$ are symmetrized and factorized, reducing thus to well known single integrals. In the upshot,

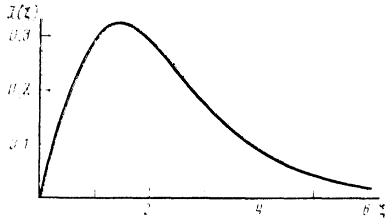

FIG. 2. Inclusive charged-lepton spectrum at fixed temperature.

$$\int_0^\infty I(\xi)\,d\xi=\frac{\pi^4}{144}. \tag{23}$$

Substitution in (22) leads to Shuryak's formula[2]

$$W_l=\pi e^4 n_f\overline{q^2}T^4/36 \tag{24}$$

for the total production of leptons and antileptons of the considered species, taken together.

The spectrum of the photons thermally emitted from a quark-gluon plasma is easier to determine. We consider it in the logarithmic approximation

$$\ln\frac{T}{m_q}\approx\ln\frac{T}{\Lambda}\gg 1 \tag{25}$$

(Λ is the dimensional renormalization parameter of the color charge[15]) an approximation that agrees in fact with the criterion (1). The latter, however, requires selection of the Feynman diagrams with the smallest number of vertices, primarily electrodynamics, but also the color ones. Attention is called therefore to the reaction

$$q+g\to q+\gamma \quad (\bar{q}+g\to\bar{q}+\gamma), \tag{26}$$

which is similar in many respects to the Compton effect. Trivial allowance for the chromodynamic character of the second vertex yields for the cross sections

$$d\sigma_{g\gamma}=\frac{e^2q^2\alpha_s}{12\varepsilon_0^2}\frac{d\theta_0}{(m_q/\varepsilon_0)^2+\theta_0^2} \tag{27}$$

$$\sigma_{g\gamma}\approx\frac{\pi e^2q^2\alpha_s}{12\varepsilon_0^2}\int_0^{\sim 1}\frac{2\theta_0\,d\theta_0}{(m_q/\varepsilon_0)^2+\theta_0^2}\approx\frac{\pi e^2q^2\alpha_s}{6\varepsilon_0^2}\ln\frac{\varepsilon_0}{m_q}, \tag{28}$$

with the γ-photon emission angle measured from the momentum $\mathbf{p}=\mathbf{p}_0$ of the initial fermion, and with the characteristics of the final quark marked by the superior bar. Here

$$\alpha_s(T)=2\pi/b\ln\frac{T}{\Lambda},\quad b=11-\frac{2}{3}n_f. \tag{29}$$

After substituting in (28), recognizing that $\varepsilon_0\sim T$, we cancel the logarithms, so that

$$\sigma_{g\gamma}=\frac{\pi^2}{3b}\frac{e^2q^2}{\varepsilon_0^2}=\frac{2\pi^2}{3b}\frac{e^2q^2}{(pp_g)}. \tag{30}$$

When counting the number of acts it is useful to bear the following circumstances in mind. Besides the variety of color states of the colliding quarks, antiquarks, and gluons, it is necessary to add to the right-hand side of (13) the projection factor $1-n_q(\varepsilon')$, due to the Pauli principle for the final fermion. At first glance its characteristics are not contained in the initial data, which refer only to the reacting quarks and gluons. It is seen from (27) and (28), however, that the angular distribution of the produced photons is peaked forward and

$$p_\gamma\approx p,\quad p'\approx p_g \tag{31}$$

with the logarithmic accuracy of (25). Since, however, they are written in invariant form, these relations between the 4-momenta remain valid also in the lab, where an individual collision certainly does not take place in its c. m. system. Therefore

$$1-n_q(\varepsilon')\approx 1-n_q(\varepsilon_g),$$

after which the integration with respect to the extra variable ε_g is easily carried out.

Similar properties are possessed also by the crossing-symmetry reaction

$$q+\bar{q}\to g+\gamma. \tag{32}$$

Without presenting here its cross sections, we note only the role of the factor $1+n_g(p_g)$ of the Bose enhancement of the annihilation on account of the gluons already present in the plasma. The argument can be identified with the characteristics of fermion p_1 after which the gluon was emitted, and the integration can then be carried out over the extra variable ε_1. Summing the contributions of reactions (26) and (32) (they are identical), we obtain ultimately

$$dw_\gamma=8e^2\frac{n_f\overline{q^2}}{b}T^2\frac{\varepsilon d\varepsilon}{e^{\varepsilon/T}+1}, \tag{33}$$

where ε is the photon energy. Integration with respect to this energy leads to

$$W_\gamma=\frac{2\pi^2}{3b}e^2n_f\overline{q^2}T^4 \tag{34}$$

for the total production of direct photons per unit 4-volume.

3. EXPANSION OF MATTER AFTER COALESCENCE OF NUCLEI, AND DISTRIBUTION OF ELECTROMAGNETIC PRODUCTS OF THE REACTION

After coalescence of nuclei of equal radius $R\sim A^{1/3}/\Lambda$ that have collided head-on, the quark-gluon plasma is contained in a volume V with a longitudinal dimension $2l$. Actually, $l\ll R$ in view of the Lorentz contraction. The expansion is next one-dimensional at $t\ll R$.

Matter of this type is subject to the Khalatnikov solution,[12] which is generally speaking quite complicated. However, during the greater part of the time of one-dimensional expansion in the region of concentration of the bulk of the emitting quarks (of the predominant part of the entire entropy of the liquid), this solution takes the simpler form:

$$v=\frac{x}{t},\quad\left(\frac{T}{T_0}\right)^3=\left(\frac{2}{\pi}\right)^{1/2}\frac{l/t}{\ln^{1/2}(t/l)}\gamma,$$
$$t\gg l,\quad y^2\ll\ln(T_0/T). \tag{35}$$

Here v is the hydrodynamic flow velocity, y its rapidity, $\gamma=\cosh y$ the Lorentz factor, and T_0 the temperature of the quark-gluon plasma immediately prior to its expansion. We measure the angle θ hereafter from the direction of the axis x of the nuclear reaction. Lorentz transformations make it possible to obtain the final distributions of the electromagnetic products in angle and in energy, by integrating over the space and over the expansion time.

Stipulating that the emitted muons are ultrarelativistic, we shall be guided by Eq. (22) and choose as the initial expression

$$dw_l=\frac{4}{\pi^3}e^4n_f\overline{q^2}T^3I(\xi)\,d\varepsilon_0\,d\tau_\varepsilon. \tag{36}$$

The 4-volume element by which it should be multiplied can now be referred to the laboratory frame.[2)] We also change here some of the notation: ε_0 is the energy of the particle in the rest frame of the plasma element in which they are emitted isotropically, and correspondingly

$$\xi=\varepsilon_0/T. \tag{37}$$

In addition, the right-hand side of (36) contains a factor of the suitably normalized distribution

$$d\tau_\varepsilon=\frac{(1-v^2)^{1/2}}{2v\varepsilon_0}d\varepsilon \tag{38}$$

with respect to the laboratory energies ε.

It is expedient to transform from the spatial differential to dv in accordance with the relation

$$\pi R^2dx=\pi R^2tdv, \tag{39}$$

and to replace the time by the time-dependent temperature. According to Eq. (35),

$$dt\approx\left(\frac{2}{\pi}\right)^{1/2}\frac{l}{\ln^{1/2}(R/l)}\frac{T_0^3}{(1-v^2)^{1/2}}d\left(\frac{1}{T^3}\right), \tag{40}$$

and analogously for t itself. The integration with respect to temperature (time) yields the factor

$$J=\int_0^\infty I(\xi)\xi^2d\xi=\frac{49\pi^6}{14\,400}+\frac{81}{80}[\zeta(3)]^2\approx4.73. \tag{41}$$

Its determination is similar to the calculation of the area under the spectrum (23); see also the text. The subsequent integration over the self-similar v-space reduces to

$$\int_{-1}^{1}\frac{1-v^2}{(1-v\cos\theta)^4}dv=\frac{4/3}{\sin^4\theta}.$$

As a result we obtain

$$dW_l=\frac{8J}{\pi^4}e^4n_f\overline{q^2}\frac{l^2R^2}{\ln(R/l)}T_0^6\frac{d\varepsilon\,d\theta}{\varepsilon^3\sin^4\theta} \tag{42}$$

for each such collision.

The inclusive distributions of the electromagnetic particles can be expressed in terms of the square of the maximum entropy

$$S=\pi^2\lambda T_0^3V,\quad \lambda=\frac{32+21n_f}{45}, \tag{43}$$

reached in the course of coalescence of the nuclei. Adding to the yield of the direct photons, calculated in the same manner, we represent the results in the form

$$dW_l=\frac{2J}{\pi^{10}}\frac{e^4}{c}\frac{n_f\overline{q^2}}{\lambda^2}\frac{S^2}{R^2\ln(R/l)}\frac{d^3p/\varepsilon}{p_\perp^4},$$
$$dW_\gamma=\frac{7e^2\hbar}{30\pi^3}\frac{n_f\overline{q^2}}{b\lambda^2}\frac{S^2}{R^2\ln(R/l)}\frac{d^3p/\varepsilon}{p_\perp^4}, \tag{44}$$

where $p_\perp=(\varepsilon/c)\sin\theta$ is the transverse momentum in the usual units. The question of the thickness $2l$ of the initial disk calls for a separate analysis. The usual Lorentz contraction along the x axis yields

$$l_L=\gamma_0^{-1}R=m_nc^2R/E_0, \tag{45}$$

where E_0 is the primary energy per nucleon. If its value is not too large, this is a perfectly acceptable estimate. However, $T_0\propto E_0^{1/2}$ and at ultrahigh energies the de Broglie length of the quark or gluon cannot be accommodated in the length (45). This means physically that in fact the further degradation of the color (the increase of the system entropy continues all the way to $l\sim\hbar c/T_0$. The acoustic perturbation becomes then capable of passing through the layer and the hydrodynamic expansion begins. Recognizing also that

$$E\sim AE_0\sim T_0^4R^2l/\hbar^3c^3, \tag{46}$$

we arrive at the following conclusion:

$$E_0\lessdot\frac{(m_nc^2)^{3/2}}{\Lambda^{1/2}}A^{2/3},\quad l\sim\frac{\hbar c}{\Lambda}\frac{m_nc^2}{E_0}A^{1/3},$$
$$E_0\gtrdot\frac{(m_nc^2)^{3/2}}{\Lambda^{1/2}}A^{2/3},\quad l\sim\frac{\hbar c}{\Lambda^{2/3}E_0^{1/3}}A^{-1/9}. \tag{47}$$

In the latter case $T_0\sim\Lambda^{\frac{2}{3}}E_0^{\frac{1}{3}}A^{\frac{1}{9}}$.

Obviously, $\varepsilon_0\sim T$. In the left-hand side of (41), the setting of the limits corresponds formally to integration over the entire range of the temperature as it cools down from its infinite value to zero. Estimates of the influence of aberration and of the Doppler effect on the radiation from a moving object make it possible to set more precisely the limits of such an analysis. It turns out that the equations for a real signal with allowance for expansion are valid in the region

$$\left(\frac{l/R}{\ln^{1/2}(R/l)\sin\theta}\right)^{1/3}T_0\ll cp_\perp<T_0. \tag{48}$$

It is easy to verify that under this validity condition the number of produced particles does not diverge at all in view of the smallness of the angles compared with unity. Angles $\sin\theta\ll1$ make a negligible contribution to the total electromagnetic production.

During the three-dimensional stage of expansion $ct\geqslant R$ the signal radiation is rapidly extinguished. Its total output has here a relative smallness $\sim(I/R)^{\frac{1}{3}}$ 1. The energy spectrum extends from zero to $\sim T_0$ because of the stronger Doppler effect. Rough estimates give grounds for assuming that thermal production of electromagnetic particles falls off outside the limits of the region (48).

4. DISCUSSION

An electromagnetic signal emitted by a quark-gluon plasma lends itself to a theoretical investigation. The corresponding experiment, however, meets with substantial difficulties. According to fundamental chromodynamic equation (29), the strong-interaction constant decreases extremely slowly. The problem is how to satisfy the conditions (1) and (25) for the transition to an ideal-gas state. Yet equations of the type (43) and (46) show that the temperature T_0 increases slowly enough with the primary energy. For example, at $E\sim1$ TeV $=10^3$ GeV per nucleon we have $T_0\sim4$ GeV.

Nonetheless, the required energies do not seem fantastic today. We note incidentally that requirements such as (1) or (25) reduce actually to $\ln(R/l)\gg1$, as was indeed implied in the derivation of the final equations.

The most difficult question is that of protection against the background of foreign particles of nonthermal origin. Their main source is the decay of the hadrons produced upon collision. The tremendous background of the practically instantaneous photons of the $\pi^0\rightarrow2\gamma$ decay can hinder the emission of direct photons. The relative weakness of the collateral branch $\pi^0\rightarrow e^+e^-\gamma$ might tempt to give preference to the electrons. But this argument meets with objections. A feature of the signal is that lepton generation is lower by three orders than the yield of the direct photons. Muons, on the other hand, are apparently quite free of the described shortcoming. Finally, if we single out (as is already the practice in an entirely different energy range) head-on collisions of nuclei in accordance with the multiplicity of the charged hadrons, it becomes necessary to detect also the hadrons.

On the whole, the inclusive signal (44) of the form $p_\perp^{-4}$ must be conceded to be not very informative from the structural-diagnostic standpoint. It is most probably some interesting independent phenomenon.

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[1)]The possibility of such an approach was indicated by Feĭnberg[1] even prior to the chromodynamic considerations. He was the first to suggest the use of an electromagnetic signal as a means of obtaining information on the structure of hot hadron matter.

[2)]To obtain initial temperatures T_0 needed for this purpose, a very high energy is needed to excite the composite system. Since the most suitable for this purpose is the experimental colliding-ion-beam technique, we assume the c. m. system of the colliding nuclei be that of the laboratory.

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