Quantum effects in the absorption of a Rayleigh wave by two-dimensional electrons

L. I. Magarill and A. V. Chaplik

Institute of Semiconductor Physics, Siberian branch of the Academy of Sciences of the USSR
(Submitted 26 March 1987)
Zh. Eksp. Teor. Fiz. **93**, 2257–2263 (December 1987)

Quantum absorption of ultrasound by free carriers in low-dimensional systems in the presence of a magnetic field is analyzed theoretically. The main qualitative features distinguishing this absorption from its three-dimensional analog (the geometry of the absorption and the role of dynamic screening) are discussed. Estimates indicate that it should be possible in principle to observe giant quantum oscillations in existing gallium arsenide heterostructures.

Quantum effects are observed when sound propagates in solids in a strong magnetic field and at low temperatures. Specifically, the electron sound-absorption coefficient Γ oscillates with a large amplitude $\Gamma_{max}/\Gamma_{min} \gg 1$ ("giant" quantum oscillations).[1] The absorption is greatest when the magnetic field is such that the Fermi level crosses a Landau level, and the projection of the magnetic field along the direction of sound propagation must be nonzero. Recent experiments[2] on electron damping of Rayleigh waves in GaAlAs–GaAs heterostructures have stimulated interest in the study of giant quantum oscillations (GQO) in quasi-two-dimensional electron systems. The possibility that giant quantum oscillations might occur in a dimensionally quantized film with $\mathbf{q} \perp \mathbf{H}$ (where $\mathbf{q}$ is the wave vector of the acoustic wave, $\mathbf{H}$ the magnetic field) was discussed in Ref. 3; GQO's do not occur in three-dimensional systems under these conditions[3]. The sound absorption coefficient was calculated in Refs. 1, 3 by using the Fermi "golden rule" with an effective electron-phonon interaction constant, i.e., dynamic screening effects were neglected. Such a treatment is justified for typical metals. In the situation of interest to us however, dynamic screening substantially alters both the absolute absorption Γ and the ratio $\Gamma_{max}/\Gamma_{min}$. In addition, the temperature and magnetic field dependences of the GQO parameters change, while the phase velocity of the sound wave undergoes small but abrupt changes as the magnetic field varies.

In Sec. 1 we derive a general expression for the complex renormalization of the Rayleigh wave frequency caused by its interaction with a quasi-two-dimensional electron gas. In Sec. 2 we analyze a homogeneous two-dimensional model system in which the magnetic field makes an arbitrary angle with the vector $\mathbf{q}$. Section 3 is devoted to a study of a quasi-one-dimensional system (two-dimensional electrons in a strip of finite width), while in Sec. 4 we generalize the model to the case when an arbitrary potential confines the electrons along the normal to the surface.

1. GENERAL FORMULAS FOR THE ABSORPTION COEFFICIENT AND RENORMALIZATION OF THE SPEED OF SOUND

For definiteness we will consider a situation which has been realized experimentally (Refs. 2),[1)] in which two-dimensional electrons interact piezoelectrically with a Rayleigh wave. We assume that the Rayleigh wave propagates along the piezoactive direction [110] on the [001] surface of a cubic crystal. The x- and z-components u_x, u_z of the displacement vector and the electric potential φ are nonzero; the z axis is chosen normal to the surface, while the x axis is parallel to the direction [110]. In this geometry the equations governing the electric field and the motion of the crystal have the form

$$(\omega^2-c_l^2q^2)u_x+c_t^2u_x''+(c_l^2-c_t^2)iqu_z'-2i\beta q\varphi/\rho=0, \quad (1)$$

$$(\omega^2-c_t^2q^2)u_z+(c_l^2-c_t^2)iqu_x'+c_l^2u_z''+q^2\beta\varphi/\rho=0, \quad (2)$$

$$\chi(z)(\varphi''-q^2\varphi)+8\pi\beta(iqu_x'-u_zq^2/2)=-4\pi\rho_s\delta(z), \quad (3)$$

after elimination of a common factor $\exp(iqx-i\omega t)$. Here c_l and c_t are the longitudinal and transverse velocities of sound in the bulk of the crystal, β is the piezoelectric modulus, ρ is the density of the crystal, $\chi(z<0)$ is the dielectric constant of the crystal, $\chi(z>0)=\chi_1$ is the dielectric constant of the surrounding material, ρ_s is the variable component of the surface charge density associated with the wave, and primes denote derivatives with respect to z. The factor $\delta(z)$ appears on the right in (3) because we assume that the acoustic wavelength (and hence the characteristic scale of the perturbing potential) is much greater than the width of the conducting layer. The quantity ρ_s is expressed as usual in terms of the polarization operator:

$$\rho_s=-\int dz\,dz'\Pi(z,z';q,\omega)\varphi(0)\equiv-\Pi_s(\omega,q)\varphi(0).$$

In Eqs. (1) and (2) we have made the customary approximation that the crystal is acoustically isotropic in the absence of piezoelectric interaction. We have also neglected the mechanical loading of the surface, i.e., the Rayleigh wave is assumed to be the same as on a free surface; this is justified because for the wavelengths $\lambda \sim 1$ μm of interest, the mass of the adjoining dielectric is much less than the mass of the piezoelectric material involved in the wave motion. It follows that we have

$$u_x(z>0)=u_z(z>0)\equiv 0.$$

The system (1)–(3) must be supplemented with the boundary conditions

$$c_t^2[u_x(-0)+iqu_z(-0)]-(\beta/\rho)iq\varphi(0)=0,$$
$$i(c_l^2-2c_t^2)qu_x(-0)+c_l^2u_z'(-0)=0,\ \varphi(+0)=\varphi(-0), \quad (4)$$
$$\chi_1\varphi'(+0)-\chi\varphi(-0)-4\pi\beta iqu_x(-0)=4\pi\Pi_s(\omega,q)\varphi(0).$$

Mechanically, they describe a free surface not subject to any external force, while electrostatically they specify a jump in the potential corresponding to a given ρ_s. We proceed in the

 0038-5646/87/121289-04$04.00

usual way by seeking a solution as a superposition of exponentials $\exp(\varkappa_i z)$. The damping factors κ_i are the roots of the determinant of the system of algebraic equations obtained by substitution into (1)–(3). In our case there are three zeros (the wave has three components). In the absence of piezoelectric interaction ($\beta = 0$),

$$\varkappa_1 = \varkappa_l \equiv (q^2 - \omega^2/c_l^2)^{1/2}, \quad \varkappa_2 = \varkappa_t \equiv (q^2 - \omega^2/c_t^2)^{1/2}, \quad \varkappa_3 = |q|.$$

The equation $\mathscr{D} = 0$ follows from the conditions (4), where $\mathscr{D}$ is the determinant of the boundary conditions (see, e.g., Ref. 4). This equation determines the dispersion $\omega(q)$ of the surface wave. In general it is not possible to find a closed-form expression for $\omega(q)$. However, the piezoelectric modulus is generally small, and this can be exploited to find the complex correction $\Delta\omega(q)$ to the dispersion $\omega_0(q) = c_R|q|$ for an isotropic crystal with $\beta = 0$ and Rayleigh wave velocity c_R. A rather lengthy calculation leads to

$$\Delta\omega/\omega_0 = -\alpha(1+\Pi_s(\omega, q)/\Pi_0)^{-1}(\Pi_s(\omega, q)/\Pi_0), \qquad (5)$$

where $\Pi_0 = (\chi + \chi_1)|q|/4\pi$ and α, the "effective" electromechanical coupling coefficient for the surface wave, depends on the direction of $\mathbf{q}$, the orientation of the surface, and the ratios $c_R/c_l, c_R/c_t$. In our geometry

$$\alpha = \frac{4\pi\beta^2}{\beta c_R^2(\chi+\chi_1)}\frac{M^2}{N},$$

$$M = \frac{[1-(1-T)^{1/2}](3-2T)}{T} - \frac{3}{2L}(2-T)[1-(1-L)^{1/2}],$$

$$N = \frac{3-2T}{2(1-T)^{1/2}} + \frac{(1-T)^{1/2}}{2(1-L)} - \frac{2-T}{(1-L)^{1/2}},$$

$$T = (c_R/c_t)^2, \quad L = (c_R/c_l)^2.$$

We stress that $\Delta\omega(q)$ in Eq. (5) gives only the component of the frequency renormalization due to the presence of the electrons; this is the component which is affected by magnetic fields and measured experimentally.

The formulas for the absorption coefficient and the Rayleigh wave velocity normalization follow from (5); we have

$$\Gamma/2|q| = \alpha(\Pi_s''/\Pi_0)|1+\Pi_s/\Pi_0|^{-2}, \qquad (6)$$

$$\Delta c_R/c_R = -\alpha(\Pi_s'/\Pi_0 + |\Pi_s/\Pi_0|^2)|1+\Pi_s/\Pi_0|^{-2}, \qquad (7)$$

where $\Pi_s = \Pi_s' + i\Pi_s''$. The problem thus reduces to finding the polarization operator for a quasi-two-dimensional gas in a magnetic field.

The technique proposed by Ingebrigtsen[5] is widely used to calculate the electron damping of sound in multilayer systems. This method involves matching the impedances on either side of the interface; local electrodynamic equations are used to treat the conducting layer, i.e., it is assumed that $\Pi(z, z') = \Pi(z)\sigma(z-z')$. However, this is clearly invalid for systems such as quantum films. Although the resulting formulas are similar in structure to Eqs. (5)–(7) above, their derivation is thus open to question. In particular, it is not clear how to calculate the effective impedance for a quasi-two dimensional electron system with allowance for spatial dispersion. The derivation given here is more systematic and explains the significance of the "surface" quantities such as Π_s that appear in the result for the case of a thin electron layer (see below). In the next section we will calculate the absorption coefficient and phase velocity renormalization directly.

2. GQO IN A QUASI-TWO-DIMENSIONAL ELECTRON SYSTEM

We start with an exactly solvable model, in which the electron motion along the z axis is confined by a parabolic potential $U(z) = m\Omega^2 z^2/2$. We note that molecular beam epitaxy can be used to fabricate such parabolic potential wells in GaAs–GaAlAs structures, for example.[6] Assume first that the magnetic field lies in the plane of the structure and is perpendicular to $\mathbf{q}$ (i.e., $\mathbf{H}\|y$). The energy spectrum depends on three quantum numbers:

$$\varepsilon_\alpha = \varepsilon_n(k_x, k_y) = \tilde{\omega}(n+1/2) + k_x^2/2\tilde{m} + k_y^2/2m, \qquad (8)$$

where $\tilde{\omega} = (\omega_c^2 + \Omega^2)^{1/2}$, $\omega_c = |e|H/mc$ is the cyclotron frequency, $\tilde{m} = m(\omega/\Omega)^2$, m is the effective mass, and $\hbar = 1$.

We will use the relaxation time Ansatz[7,8] for the density matrix to calculate the polarization operator. In the calculations we again use the fact that the electron layer is thin compared to the wavelength; the potential $\varphi(z)$ and the correction to the chemical potential are taken outside the integrals (see, e.g., Eq. (20) in Ref. 8) by replacing them by their values at $z = 0$. The result for the polarization operator then becomes

$$\Pi_s(\omega, q) = \frac{e^2\bar{\omega}}{\omega}K_q(\omega)\left[1 + \frac{i}{\omega\tau}\frac{K_q(\omega)}{K_q}\right]^{-1}. \qquad (9)$$

Here $\bar{\omega} = \omega + i/\tau$,

$$K_q(\omega) = \frac{1}{S}\sum_{\alpha,\beta}\frac{f_\beta - f_\alpha}{\varepsilon_\alpha - \varepsilon_\beta - \bar{\omega}}|J_{\alpha\beta}(\mathbf{q})|^2, \qquad (10a)$$

$$\bar{K}_q = \frac{1}{S}\sum_{\alpha,\beta}\frac{f_\beta - f_\alpha}{\varepsilon_\alpha - \varepsilon_\beta}|J_{\alpha\beta}(\mathbf{q})|^2, \qquad (10b)$$

where S is the area of the system, α denotes the set of electron quantum numbers, the f_α are the Fermi occupation numbers, $J_{\alpha\beta}(q) = \langle\alpha|e^{iqx}|\beta\rangle$, and τ is a phenomenological relaxation time.

In the case of interest (large spatial dispersion), the damping of the acoustic wave can be described as a quantum-mechanical process in which phonons are absorbed by electrons (this is the essential physical feature of GQO). We consider the case $\omega\tau \ll 1, ql \gtrsim 1$ of greatest experimental interest (l is the Fermi mean free path). These two conditions are consistent, because the speed of sound is small compared to the typical electron velocity. In specializing the general expressions (9), (10) to our model system, we impose the conditions $T \ll \tilde{\omega}, q \ll (mT)^{1/2}$, which are generally necessary in order for GQO to be observed (T is the temperature). We can then replace $J_{\alpha\beta}(q)$ by $\delta_{n,n'}\delta_{ky',ky}\delta_{kx',kx+q}$, where $\beta = (n, k_x, k_y)$, $\alpha = (n', k_x', k_y')$. The expressions for Π_s', Π_s'' become

$$\Pi_s'' = e^2\omega\tau\frac{(m\tilde{m})^{1/2}}{2\pi}\sum_n F(A_n, \bar{B})\left\{1 - \sum_n F(A_n, \bar{B}) \times \left[\sum_n\left(1 + \operatorname{th}\left(\frac{A_n}{2}\right)\right)\right]^{-1}\right\}^{-2} \qquad (11)$$

$$\Pi_s' = \frac{e^2(m\tilde{m})^{1/2}}{2\pi}\sum_n\left(1 + \operatorname{th}\left(\frac{A_n}{2}\right)\right). \qquad (12)$$

Here

$$F(A_n,\tilde{B})=\int_0^\infty dx\,(1+2\tilde{B}^2x)^{-1/2}\,\mathrm{ch}^{-2}(A_n/2-x),$$
$$\tilde{B}=|q|\tau(2T/\tilde{m})^{1/2},\ A_n=[\varepsilon_F-\tilde{\omega}(n+1/2)]/T,$$

where ε_F is the Fermi energy.

It is clear from (11) and (12) that $\Pi_s''/\Pi_s'\sim\omega T\ll 1$, and furthermore $\Pi_s'/\Pi_0\sim qa$, where $a=(\chi_1+\chi_2)/2me^2$ is the effective Bohr radius. Since the parameter qa is small for all reasonable acoustic frequencies, we get the following final expressions for the absorption coefficient and phase velocity renormalization:

$$\frac{\Gamma}{2|q|}=\alpha qa\omega\tau\left(\frac{m}{\tilde{m}}\right)^{1/2}\sum_n F(A_n,\tilde{B})\times\left[\sum_n\left(1+\mathrm{th}\frac{A_n}{2}-F(A_n\tilde{B})\right)\right]^{-2},\qquad(13)$$

$$\frac{\Delta c_R}{c_R}=-\alpha\left\{1-qa\left(\frac{m}{\tilde{m}}\right)^{1/2}\left[\sum_n\left(1+\mathrm{th}\frac{A_n}{2}\right)\right]^{-1}\right\}.\qquad(14)$$

These results have the same form for an arbitrary angle θ between $\mathbf{q}$ and $\mathbf{H}$ ($\mathbf{H}$ lies in the x, y plane); one merely replaces $\tilde{B}$ by the quantity $(B^2\cos^2\theta+\tilde{B}^2\sin^2\theta)^{1/2}$, where $B=q\tau(2T/m)^{1/2}$.

As the magnetic field varies, for each $n=1,2,\ldots$, the quantity $F(A_n,\tilde{B})$ passes through a maximum lying in the region $|A_n|\ll 1$, so that the corresponding Landau level with hybrid frequency $\tilde{\omega}$ lies in the thermal layer with energy nearly equal to the Fermi energy. For A_n large and negative, $F(A_n,\tilde{B})$ decays exponentially, while in the limit $A_n\gg 1$ we have $F(A_n,\tilde{B})\sim(\tilde{B}^2A_n)^{-1/2}$. The quantity $F(A_0,\tilde{B})$ is of the order of $(\tilde{B}^2A_0)^{-1/2}\ll 1$ because the electron gas is highly degenerate, which implies $A_0\gg 1$. The denominator in expression (13) for Γ varies sharply near the nth maximum, from $2n$ for $A_n<0$ to $2(n+1)$ for $A_n>0$. The ratio $\Gamma_{\max}/\Gamma_{\min}$ is of the order of $\tilde{B}(\tilde{\omega}/T)^{1/2}\gg 1$, provided of course that the parameter $\tilde{B}$ characterizing the scattering is not too small. The magnetic field dependence of the absorption coefficient thus exhibits the characteristic features of giant quantum oscillations, and moreover, the maxima in Γ are highly asymmetric. We stress that this asymmetry is due to the effects of dynamic screening, which causes the denominator in (13) to have step-function dependence on the magnetic field. Formula (14) shows that a similar step dependence is also present for the renormalized phase velocity of the wave.

3. A QUASI-ONE DIMENSIONAL SYSTEM

Let us assume that a Rayleigh wave propagates along a narrow strip of quasi-two-dimensional electrons (along the x axis). The method discussed above clearly breaks down in this case, because for an acoustic wave unbounded along the y axis, the renormalization contributed by a single "strip" of electrons vanishes. Rigorously speaking, one must solve the problem for an acoustic beam of finite width, which is computationally very difficult. We will therefore find the absorption coefficient by a simpler method, which is based on directly calculating the energy losses of the wave within the conducting region.[9] Since the electromechanical coupling constant is small, to first order the displacement field within the Rayleigh wave may be assumed to be the same as in the absence of piezoelectric interaction. The problem then reduces to solving the equation

$$\chi(z)\left(\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}-q^2\varphi\right)+8\pi\beta\left(iq\frac{\partial u_z}{\partial z}-\frac{q^2}{2}u_z\right)=-4\pi\rho_L\delta(y)\delta(z).\qquad(15)$$

for the electrostatic potential, where the right-hand side is specified. The factor $\delta(y)$ appears in (15) because the width of the strip (as well as its thickness) is assumed much smaller than the wavelength. The quantity ρ_L is the varying part of the linear charge density; it is expressible in terms of the polarization operator for an "electron filament": $\rho_L=-\Pi_L(\omega,q)\varphi(0,0)$.

Since the components $u_{x,z}(z)$ of the displacement field are expressible as superpositions of exponentials $\exp(\kappa_{l,t}z)$ (see, e.g., Ref. 10), Eq. (15) is easily solved. The potential $\varphi(y,z)$ can be expressed in terms of the modified Bessel function $K_0(|q|(y^2+z^2)^{1/2})$, which diverges logarithmically as $y,z\to 0$. Clearly, this divergence must be cut off at distances comparable to the width of the strip. Denoting the cut-off logarithm in the expression for $\varphi(0,0)$ by Λ, we find that the absorption coefficient is

$$\Gamma=(2\pi\alpha/d)(\Pi_L''/\Pi_{L0})|1+\Lambda\Pi_L/\Pi_{L0}|^{-2},\qquad(16)$$

where α is as in (5), $\Pi_{L0}=(\chi+\chi_1)/4$, and d is the width of the acoustic beam.

Let us first consider the low-frequency limit $ql\ll l$ in the absence of a magnetic field; in this case $\Pi_L(\omega,q)$ is expressible in terms of the static conductivity $\sigma_L=n_Le^2\tau/m$, where n_L is the linear electron density. We then have

$$\Gamma\propto\sigma_L\omega\{1+[4\Lambda\omega\sigma_L/(\chi+\chi_1)c_R^2]^2\}^{-1},\qquad(17)$$

i.e., as a function of the acoustic frequency Γ has a maximum at $\omega=\omega_0=(\chi+\chi_1)c_R^2/4\Lambda\sigma_L$ (such is not the case for two- and three-dimensional systems). Assuming a strip of width 0.2 μm, we have $\omega_0\sim 10^5$–$10^6\ \mathrm{s}^{-1}$ for typical GaAs-based heterostructures.

We next consider giant quantum oscillations for a quasi-one-dimensional system; the potential confining the electron motion along the y axis is again assumed to be parabolic: $U(y)=m\Omega^2y^2/2$. We continue to assume that $\omega\tau\ll 1$, which implies that $\Pi_L'\gg\Pi_L''$. It should be noted that in this one-dimensional situation, dynamic screening effects should be less pronounced than in the three- and two-dimensional cases. Indeed, in the denominator in (16) we may neglect 1 compared with $\Lambda\Pi_L'/\Pi_{L0}$ only if the Coulomb parameter $e^2/\pi(\chi+\chi_1)v_T$ is sufficiently large. Estimates reveal that near the maxima $\Lambda\Pi_L'/\Pi_{L0}$ is greater than 1, although not by much, while between maxima it is of the order of unity. The final result is

$$\Gamma=\frac{2\pi\alpha}{d}\frac{\gamma\omega\tau Z^2\Phi}{(Z-\Phi)^2(1+\gamma\Lambda\Phi)^2},\qquad(18)$$

where

$$\gamma=e^2(2\tilde{m}/T)^{1/2}/\pi(\chi+\chi_1),$$
$$Z=\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}dx\,\mathrm{ch}^{-2}\left(\frac{A_n-x^2}{2}\right),$$
$$\Phi=\sum_{n=0}^{\infty}\int dx\,\mathrm{ch}^{-2}\left(\frac{A_n-x^2}{2}\right)(1+\tilde{B}^2x^2)^{-1}.$$

Equation (18) gives the estimate $\Gamma_{max}/\Gamma_{min} \sim \tilde{\omega}^{3/2} R_y^* T^{1/2} \Lambda^2$, where R_y^* is the effective Bohr energy. The magnetic field dependence of the giant quantum oscillations in this case thus proves to be the same as for a three-dimensional system ($\sim \tilde{\omega}^{3/2}$), while the temperature dependence ($\sim T^{-1/2}$) is the same as for a two-dimensional system.

4. ARBITRARY POTENTIALS $U(z)$

The treatment in Sec. 2 can be generalized to arbitrary potentials $U(z)$. Indeed, let the magnetic field be perpendicular to $\mathbf{q}$ (i.e., parallel to the y axis). Then we can replace (8) by

$$\varepsilon_n(k_x, k_y) = \varepsilon_n(k_x) + k_y^2/2m, \tag{19}$$

while the expression for the $J_{\alpha\beta}(q)$ becomes

$$J_{\alpha\beta}(q) = \delta_{k_y', k_y} \delta_{k_x', k_y+q} \int dz \varphi_{n', k_x+q}(z) \varphi_{n, k_x}(z), \tag{20}$$

where $\varphi_{n,kx}(z), \varepsilon_n(k_x)$ are the eigenfunctions and eigenvalues of the operator

$$\hat{p}_z^2/2m + U(z) + (k_x + eHz/c)^2/2m.$$

As before, the inequality $\varepsilon_{n+1} - \varepsilon_n \gg q^2/m$ holds by a wide margin (it replaces the condition $\tilde{\omega} \ll q^2/m$ for the case of a parabolic potential). The integral in (20) can then be replaced by $\delta_{nn'}$.

Calculations similar to the ones used in deriving (13) show that the oscillations in the absorption coefficient are due primarily to the integral

$$J_n = \int_{-\infty}^{\infty} dk_z\, dk_y \left[1 + q^2\tau^2 \left(\frac{\partial \varepsilon_n}{\partial k_x} \right)^2 \right]^{-1} \operatorname{ch}^{-2}\left(\frac{\varepsilon_n(k_x, k_y) - \varepsilon_F}{2T} \right).$$

The function $\varepsilon_n(k_x)$ also describes a "well" in k-space, regardless of the form of the potential confining the electrons along the z axis. If the Fermi level lies close to the minimum of the function $\varepsilon_n(k_x, k_y)$, in the sense that $|\varepsilon_F - \varepsilon_n^{min}| \lesssim T$ (and, of course, $\varepsilon_{n+1} - \varepsilon_n \gg T$), we can use the expansion $\varepsilon_n(k_x) = \varepsilon_n^{min} + (k_x - k_{xn}^{min})^2/2m_n$, where m_n is the effective mass in the nth subband. The integral J_n is then of order $J_n^{max} \equiv Y(mm_n)^{1/2}$ if the scattering is not too strong, i.e., $g\tau V_T \sim 1$. On the other hand, if ε_F is not close to the bottom of one of the subbands, i.e., $|\varepsilon_F - \varepsilon_n^{min}| \gg T$, the integral over k_y can be evaluated by approximating the hyperbolic cosine by a δ-function. The remaining integral over k_x is determined primarily by a small neighborhood of the point k_{xn}^{min}, at which the velocity $\partial\varepsilon_n/\partial k_x$ vanishes. One obtains the estimate

$$J_n \sim J_n^{max} (T/|\varepsilon_F - \varepsilon_n^{min}|)^{1/2}.$$

Giant quantum oscillations thus occur in this case also.

We close with an estimate for an electron gas in a GaAs–GaAlAs heterostructure. Assuming a mobility of $\sim 10^6\ \text{cm}^2/V \cdot S$ (this corresponds to $\tau \sim 4 \cdot 10^{-11}$ s), we have $B = \omega\tau V_T/c_R \sim 1$ at $T = 2$ K and frequency $\omega \sim 5 \cdot 10^8$ Hz. Taking 100–200 Å as the diameter of the potential well, we find that the size and magnetic quantizations are comparable for fields $H \sim 5$–10 T. We then obtain $\Omega \sim 100$ K, $\tilde{\omega} \sim 150$ K, and $\Gamma_{max}/\Gamma_{min} \sim 5$–$7$.

[1)] To prevent misunderstanding, we shall point out that in Ref. 2 the geometry of the experiment and the values of the special parameters correspond to the classical mechanism of the electrons and the sound wave interactions.

[1]V. L. Gurevich, V. G. Skobov, and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. **40**, 786 (1961) [Sov. Phys. JETP **13**, 552 (1961)].
[2]A. Wixforth, J. P. Kotthaus, and G. Weimann, Phys. Rev. Lett. **56**, 2104 (1986).
[3]V. A. Margulis, Pis'ma Zh. Eksp. Teor. Fiz. **12**, 269 (1970) [JETP Lett. **12**, 184 (1970)].
[4]M. K. Balakirev and I. A. Gilinskiĭ, Volny v P'ezokristallakh (*Waves in Piezoelectric Crystals*), Nauka, Novosibirsk (1982).
[5]K. A. Ingebrigtsen, J. Appl. Phys. **40**, 2681 (1969); J. Appl. Phys. **41**, 454 (1970).
[6]K. Arora and H. Spector, Surf. Sci. **176**, 669 (1986).
[7]M. P. Greene, H. J. Lee, J. J. Quinn, and S. Rodriguez, Phys. Rev. **177**, 1019 (1969).
[8]K. S. Yu and J. J. Quinn, Phys. Rev. B **27**, 1184 (1983).
[9]A. V. Chaplik, Pis'ma Zh. Tekh. Fiz. **10**, 1385 (1984) [Sov. Phys. Tech. Phys. Lett. **10**, 584 (1984)].
[10]L. D. Landau and E. M. Lifshitz, *Theory of Elasticity*, 2nd ed., Pergamon Press (1970).

Translated by A. Mason