Current and voltage fluctuations in microjunctions between normal metals and superconductors

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The nonequilibrium current-current correlation function is evaluated using the method of quasiclassical Green's functions for a microjunction between two normal metals, and between a normal metal and a superconductor, for an arbitrary transmission coefficient at the boundary between the metals. The noise current spectrum of a normal junction equals the sum of an equilibrium noise current through a resistance R_1 which is independent of the applied voltage V and nonequilibrium noise in the tunnel junction whose resistance is R_2. The "noise" resistances $R_{1,2}$ $(R_1^{-1} + R_2^{-1} = R_N^{-1})$ are determined by the transmission coefficient of the boundary. For a contact between a superconductor and a normal metal the low-frequency spectral density of the current and voltage fluctuations is calculated for arbitrary V and T. In this case an additional noise current appears, because there are now two possible processes by which electrons with energy greater than the gap Δ are transmitted through the NS boundary: Andreev reflection and electronic transitions from the normal metal to states above the gap in the superconductor. In the absence of electron reflection from the boundary and scattering by impurities, these processes completely determine the current fluctuations at $T = 0$.

1. INTRODUCTION

Current and voltage fluctuations in metallic microjunctions of various types—small-area tunnel junctions, microbridges, point contacts etc.—provide an interesting example of nonequilibrium fluctuations when the current and applied voltage through the junction are large. Calculation of the noise characteristics of these junctions is necessary in order to determine the limiting sensitivity of systems containing them as nonlinear elements.

In states of thermodynamic equilibrium the spectrum of fluctuations is connected with the linear response functions according to the fluctuation-dissipation theorem (FDT).[1] In the general case the noise spectrum of the current is determined from the current-current correlation function, whose average is taken using nonequilibrium density matrices.

In tunnel junctions the nonequilibrium current-current correlation function is calculated in Refs. 2–4 for normal and superconducting metals, i.e., for NIN-, SIN- and SIS-structures. The most general expression for the current-current correlation function in superconducting tunnel junctions, which is valid for arbitrary nonequilibrium states of the superconductor, was obtained in a paper by Larkin and Ovchinnikov.[5] For a fixed total tunneling transition current, including the displacement current, the spectrum of voltage fluctuations was calculated by Zorin.[6] In Refs. 2–6, the tunneling Hamiltonian was used to find the current-current correlation function.

In microjunctions with constrictions designed to vary the transmission coefficient of the boundary between metals, the direct conductivity is replaced by the tunneling conductivity. Thus, one should expect a change in the noise properties of the junction.

In this paper the current-current correlation function and the current and voltage fluctuation spectrum are calculated in microjunctions wth constrictions for arbitrary transmission coefficients through the boundary between the contacting metals. In this case the tunneling Hamiltonian method is inapplicable.

The current-current correlation function can be expressed in terms of quasiclassical Green's functions integrated over energy.[7–9] For a structure with sharp boundaries between the metals the quasiclassical equations were obtained by Zaitsev,[10] who also introduced in place of the well-known quasiclasssical functions $\check{g}$ (Ref. 9) a new function $\check{\mathscr{G}}$ connected with the reflected wave amplitude from the boundary. Macroscopic quantities such as the current density or potential are expressed in terms of $\check{g}$. The function $\check{\mathscr{G}}$ also appears in the current-current correlation function, as will be shown below. In the limit of small transmission, the result for the current-current correlation function obtained here coincides with well-known expressions found using the tunneling Hamiltonian method.[2,5]

In what follows, junctions will be investigated between two normal metals (N_1cN_2) and between a normal metal and a superconductor (ScN; the c denote a constriction). In the ScN case, the I-V characteristic is nonlinear[11–13] and has been calculated[10,14] for arbitrary values of applied voltage V in the limit of small junction dimensions.

In this paper, the spectral density of current fluctuations S_I is evaluated for arbitrary voltages and temperatures. Basically, we will limit ourselves to the low-frequency limit $\omega \ll \{T,\Delta,eV\}$, where Δ is the energy gap in the superconductor. For $V=0$, S_I reduces to the Nyquist thermal noise $4T/R_d(V)$ in agreement with the FDT, where $R_d(V)$ is the differential resistance. For large voltages $eV \gg \{T,\Delta\}$, S_I contains a contribution $2eV/R_2$ which increases linearly with voltage; however, in contrast to shot noise in the tunnel junction,[2–4] the resistance R_2 exceeds the normal resistance R_N, and coincides with it only in the limit of a transparent barrier. For voltages which are the same order of magnitude as the gap there arises an additional noise mechanism, connected with Andreev reflection of electrons at the boundary between the normal and superconducting metals. In pure

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form, this mechanism determines the current fluctuations in an ideal ScN short (the transmission coefficient $D=1$) in the absence of impurity scattering for $T=0$.

The low-frequency noise voltage spectrum S_v is calculated from $S_v = S_I R_d^2$ in a way analogous to the method used for point superconducting junctions[15,16]; here S_I is found from the Nyquist formula for thermal noise in a normal resistive contact or from the formula for shot noise. In this paper $S_I(V)$ is calculated from microscopic Green's function equations.

In describing as "low", the frequencies for which the results obtained here are applicable we are implying that these frequencies are nevertheless bounded from below by the region where $1/f$ noise appears (see, e.g., Ref. 17). The growth of the noise as V increases can also possibly be due to heating of the region around the junction.[18] Also of interest to us is a region of significant nonlinearity in the I-V characteristics, i.e., voltages which do not exceed the gap Δ by too much, for which thermal heating of sufficiently high ohmic junctions is usually small.

2. JUNCTION MODEL; QUASICLASSICAL EQUATION FOR THE CURRENT-CURRENT CORRELATION FUNCTION

As a model of the microjunction with constriction, we will investigate a hole of radius a in a thin screen separating two metals,[19,20] a model which has been discussed earlier in papers on the Josephson effect. In what follows, quantities related to the left- and right-side metals will be denoted by the subscripts 1 and 2. In order to include the reflection of electrons at the boundaries, we will assume, following Ref. 10, that within the inner constriction there is a thin transition layer of thickness 2δ near the plane $z=0$ $(\delta \ll a)$, within which the potential $U(z,\boldsymbol{\rho})$ at the boundary changes. This potential is considered to vary smoothly in the transverse coordinate $\boldsymbol{\rho}$. In addition, we assume the inequality $a \gg p_{F1,2}^{-1}$ holds.

We will investigate the limit of a pure microjunction, which implies that the inequality

$$a \ll l_j, \quad v_{Fj}/T_c, \tag{1}$$

holds; here, l_j are the mean free paths of electrons in the metals 1 and 2, V_{Fj} are the Fermi velocities, T_c is the critical temperature.

Let us write down an expression for the current-current correlation function, obtained by averaging the Heisenberg current operators at times $t_{1,2}$ using a nonequilibrium density matrix:

$$K(t_1, t_2) = {}^1/_2\langle [I(t_1), I(t_2)]_+\rangle - \langle I(t_1)\rangle\langle I(t_2)\rangle. \tag{2}$$

The paired operator averages in (2) can be cast in the form of averaged values of ordered products of electron creation and annihilation operators, where the time ordering corresponds to the position of the arguments $t_{1,2}$ on the Keldysh contour C (Ref. 21) which runs along the time axis from $t=-\infty$ to $t=+\infty$ and back again. According to the rules of the diagram technique for nonequilibrium systems,[21] the Keldysh average introduced into (2) can be reduced to a product of single-particle nonequilibrium Green's functions which make up the Keldysh matrix:

$$\begin{pmatrix} \hat{G}^c; & \hat{G}^< \\ \hat{G}^>; & \tilde{G}^c \end{pmatrix}. \tag{3}$$

Each of the components of this matrix in turn is a 2×2 matrix, whose elements will be normal and anomalous electron Green's functions. The definition of the Green's functions used here coincides with that used in Ref. 9. The current-current correlation function is written in terms of the nondiagonal matrix elements (3) in the following way:

$$K(t_1,t_2) = -\frac{e^2}{8m^2}\int d^2\rho_1 \int d^2\rho_2 \left[\frac{\partial}{\partial z_1} - \frac{\partial}{\partial z_1'}\right]\left[\frac{\partial}{\partial z_2} - \frac{\partial}{\partial z_2'}\right]$$
$$\cdot \mathrm{Sp}\{\hat{G}^>(1',2)\hat{\tau}_3\hat{G}^<(2',1)\hat{\tau}_3$$
$$+\hat{G}^<(1',2)\hat{\tau}_3\hat{G}^>(2',1)\hat{\tau}_3\}_{z_1=z_1'=z_2=z_2'=\delta}. \tag{4}$$

Here, $1=(t_1,\boldsymbol{\rho}_1,z_1)$, $1'=(t_1,\boldsymbol{\rho}_1,z_1')$, etc.; $\hat{\tau}_3$ is a Pauli matrix. Taking into account the smooth variation of the potential barrier with $\boldsymbol{\rho}_{1,2}$, we transform to the total transverse coordinate $(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)/2$, and form the Fourier transform in the difference coordinate $\boldsymbol{\rho}_1-\boldsymbol{\rho}_2$; the transverse electron momentum $\mathbf{p}_\parallel$ is conserved to quasiclassical accuracy.

Let us separate out the rapidly-oscillating factors $\exp[\pm i p_{zj}(z-z')]$ and $\exp[\pm i p_{zj}(z+z')]$ in $\widehat{G}^>(z,z')$ and $\widehat{G}^<(z,z')$ where $p_{zj}=(p_{Fj}^2-p_\parallel^2)$ is the component of the momentum normal to the boundary for electrons at the Fermi surface in metals 1 and 2. In the absence of reflection from the boundary, when all quantities depend smoothly on the total coordinate $(\mathbf{r}+\mathbf{r}')/2$, the slowly-varying coefficients in these exponentials can be reduced to Green's functions integrated with respect to $\xi_\mathbf{p}$ (Ref. 7). This limit has been investigated in detail by Shelankov.[22] In the case treated here, in addition to the usual functions $\check{g}(t,t';\mathbf{R},\mathbf{p}_{Fj})$, integrated over energy, where $\mathbf{R}=(z,\boldsymbol{\rho})$ and $\mathbf{p}_{Fj}=(p_{zj},\mathbf{p}_\parallel)$ lies on the Fermi surface, it is also necessary to investigate the function $\check{\mathscr{G}}(t,t',\mathbf{R},\mathbf{p}_{Fj})$ (Ref. 10) which arises from the coefficients of the factors $\exp[\pm i p_{zj}(z+z')]$ which describes the wave reflected from the barrier. The correlation function (4) contains only nondiagonal Keldysh functions; therefore the slowly-varying coefficients are continuous at $z=z'$ and to within a constant factor coincide with the quasiclassical Green's functions.[10]

In calculating (4), the differential operator with respect to z acts only on the rapidly-oscillating exponents, consistent with our approximation. The nonzero contribution arises only from those terms in which the coordinates z',z_1' and also z_2,z_2' appear in the exponents with opposite signs. In the end we obtain an expression for the correlation function of the form

$$K(t_1,t_2) = -\frac{e^2 p_{F2}^2}{16\pi}\int d^2\rho \int_{-1}^{1} |\alpha_2|\, d\alpha_2\, \mathrm{Sp}\{\hat{g}^>(t_1,t_2;\alpha_2)$$
$$\cdot\hat{\tau}_3\hat{g}^<(t_2,t_1;\alpha_2)\hat{\tau}_3$$
$$-\hat{\mathscr{G}}^>(t_1,t_2;\alpha_2)\hat{\tau}_3\hat{\mathscr{G}}^<(t_2,t_1;-\alpha_2)\hat{\tau}_3 + (t_1\to t_2)\}, \tag{5}$$

where the functions $\check{g}^{>,<}$ and $\check{\mathscr{G}}^{>,<}$ are calculated to the left of the barrier at $z=+\delta$. The parameter α_2 defines the direction of the vector $\mathbf{p}_{F2}$, $\alpha_2=\cos\theta_2=p_{z2}/p_{F2}$.

Thus, the calculation of the current-current correlation function reduces to finding the quasiclassical functions $\check{g}$ and $\check{\mathscr{G}}$ in the vicinity of the barrier. They satisfy equations which are most conveniently written if we pass to a triangular representation of the Green's functions:

$$\check{g} = \begin{pmatrix} \hat{g}^R; & \hat{g}^K \\ \hat{0}; & \hat{g}^A \end{pmatrix}, \quad \check{\mathscr{G}} = \begin{pmatrix} \hat{\mathscr{G}}^R; & \hat{\mathscr{G}}^K \\ \hat{0}; & \hat{\mathscr{G}}^A \end{pmatrix}. \tag{6}$$

The quasiclassical equations for the nonequilibrium Green's functions[8,9] can be cast in the following compact form:

$$\mathbf{v}_{Fj}\partial\check{g}/\partial\mathbf{R} + [\check{H}, \check{g}]_- = 0, \tag{7}$$

$$v_{zj}\partial\check{\mathscr{G}}/\partial z + [\check{H}, \check{\mathscr{G}}]_+ = 0. \tag{8}$$

The matrix operator H contains the electric potential $\varphi(\mathbf{R},t)$ which appears as the electric current flows through the contact, and the complex order parameter $\Delta(\mathbf{R},t)$ in the superconductor:

$$\check{H} = \left[\check{\tau}_3\frac{\partial}{\partial t} + ie\varphi(\mathbf{R},t) - i\check{\Delta}(\mathbf{R},t)\right]\delta(t-t') + i\check{\Sigma}(t,t';\mathbf{R}). \tag{9}$$

The matrices which enter into (9) are defined thus:

$$\check{\tau}_3 = \begin{pmatrix} \hat{\tau}_3; & \hat{0} \\ \hat{0}; & \hat{\tau}_3 \end{pmatrix}, \quad \check{\Delta} = \begin{pmatrix} \hat{\Delta}; & \hat{0} \\ \hat{0}; & \hat{\Delta} \end{pmatrix}, \quad \hat{\Delta} = \begin{pmatrix} 0; & \Delta \\ -\Delta^*; & 0 \end{pmatrix}.$$

The Green's function in Eq. (5) can be expressed in terms of the matrix elements

$$\hat{g}^> = (\hat{g}^K + \hat{g}^R - \hat{g}^A)/2, \quad \hat{g}^< = (\hat{g}^K - \hat{g}^R + \hat{g}^A)/2 \text{ etc.}$$

The self-energy operator Σ in (7)–(9) has a form analogous to (6), and describes scattering of electrons by impurities and phonons. The products in Eqs. (7), (8) include matrix multiplication and convolution with respect to the internal time variable.

In order to solve (7), (8), we will use the boundary conditions for the quasiclassical Green's function at a barrier,[10] in which the transmission coefficient of the barrier D appears; in general, D depends on $\boldsymbol{\rho}$ and on the direction of the electron momentum. Outside the constriction, i.e., for $|\boldsymbol{\rho}| > a$, we will set $D = 0$. Reflection of electrons from the barrier, which is connected with the presence of isolating or strongly scattering layers at the boundary, and also reflection arising from the differing electronic parameters of the metals, gives rise to a jump in the part of the Green's function which is symmetric in momentum space: $\check{g}_s(z = +\delta) \equiv \check{g}_s(+)$ does not coincide with $\check{g}_s(z = -\delta) \equiv \check{g}_s(-)$, which is also the case for $\check{\mathscr{G}}_s(+), \check{\mathscr{G}}_s(-)$. The components $\check{g}_a$ and $\check{\mathscr{G}}_a$ which are antisymmetric in momentum space are continuous as we pass through the barrier region.

For an ideal junction between identical metals $(p_{F1} = p_{F2})$, $D = 1$, the function $\check{\mathscr{G}} \equiv 0$ and (5) reduces to the form

$$K(t_1,t_2) = -\frac{\pi}{4R_N}\{\mathrm{Sp}[\hat{g}^>(t_1,t_2)\hat{\tau}_3\hat{g}^<(t_2,t_1)\hat{\tau}_3 + (t_1\to t_2)]\}_s, \tag{10}$$

where $(\ldots)_s$ denotes separating out the symmetric part (in the case discussed here $\check{g}$ depends only on the sign of p_z), $R_N = 4\pi^2/e^2p_F{}^2S$ is the normal resistance of a "pure" contact in the form of a hole in the surface $S = \pi a^2$ in a thin partition between the metals.[23]

In the limit of small barrier transmission in (5) we obtain the current-current function in the tunneling junction, whose expression we will write as follows, distinguishing clearly, between the components which are symmetric and antisymmetric in momentum space:

$$K(t_1,t_2) = -\frac{e^2p_{F2}{}^2}{8\pi}\int d^2\rho\int_0^1 \alpha_2\,d\alpha_2\,\mathrm{Sp}\{\hat{g}_s{}^>(t_1,t_2)\hat{\tau}_3\hat{g}_s{}^<(t_2,t_1)\hat{\tau}_3$$
$$+\hat{g}_a{}^>(t_1,t_2)\hat{\tau}_3\hat{g}_a{}^<(t_2,t_1)\hat{\tau}_3 - \hat{\mathscr{G}}_s{}^>(t_1,t_2)\hat{\tau}_3\hat{\mathscr{G}}_s{}^<(t_2,t_1)\hat{\tau}_3$$
$$+\hat{\mathscr{G}}_a{}^>(t_1,t_2)\hat{\tau}_3\hat{\mathscr{G}}_a{}^<(t_2,t_1)\hat{\tau}_3 + (t_1\to t_2)\}. \tag{11}$$

Here all quantities are evaluated to the right of the barrier, i.e., $\check{g}_s = \check{g}_s(+)$ and $\check{\mathscr{G}}_s = \check{\mathscr{G}}_s(+)$.

Let us investigate the contribution to (11) which is linear in the transmission coefficient. The second term can be discarded, because the odd part $\check{g}_a$ is proportional to D. Also, the next term in (11) is negligible in the approximation under discussion here. We use the relation between $\check{g}_s$ and $\check{\mathscr{G}}_s$.[10]:

$$\check{\mathscr{G}}_s(+) = -R^{1/2}[\check{g}_s(+) + \check{g}_s(-)]/2 - R^{-1/2}[\check{g}_s(+) - \check{g}_s(-)]/2,$$

where $R = 1 - D$ is the reflection coefficient of electrons from the boundary. Retaining the terms linear in D, we obtain

$$\check{\mathscr{G}}_s(+) = \check{g}_s(+) - D\check{g}_s(-).$$

Let us substitute this expression into (11) and replace the functions $\check{g}_s(\pm)$ by their values for the zero transmission approximation, i.e., by the quasiclassical functions for the left-hand and right-hand metals. By doing this we arrive at the well-known expression for the current-current correlation function[5] which is derived using the tunneling Hamiltonian:

$$K(t_1,t_2) = -\frac{\pi}{8R_N}\mathrm{Sp}[\hat{g}_R{}^>(t_1,t_2)\hat{\tau}_3\hat{g}_L{}^<(t_2,t_1)\hat{\tau}_3$$
$$+\hat{g}_L{}^>(t_1,t_2)\hat{\tau}_3\hat{g}_R{}^<(t_2,t_1)\hat{\tau}_3 + (t_1\to t_2)]. \tag{12}$$

The resistance of the tunnel junction between normal metals equals

$$R_N{}^{-1} = \frac{e^2p_{F2}{}^2}{2\pi^2}\int d^2\rho\int_0^1 \alpha_2 D(\alpha_2)\,d\alpha_2. \tag{13}$$

From here on, we will use Eq. (11) for calculating the spectrum of fluctuations in N_1cN_2 and ScN microcontacts. In this stationary case the fluctuation spectrum $S_I(\omega)$ is related to the Fourier transform of the correlation function by $S_I(\omega) = 2K(\omega)$.

3. SPECTRAL DENSITY OF CURRENT AND VOLTAGE FLUCTUATIONS FOR A MICROJUNCTION WITH CONSTRICTION

For microjunctions in which the size of the constriction satisfies the inequality (1), we can seek a solution to Eq. (7) by expanding in the small ratio a/ζ, where $\zeta = \min\{l_j, v_{Fj}/T_c\}$. To lowest order in this parameter we can replace the order parameter Δ and potential φ in the operator H(9) by the asymptotic values of these quantities far from the hole.[19,20] Calculating the Green's functions in first nonvanishing approximation in a/ζ and a/l_ε (l_ε is the mean free path for inelastic phonon scattering[24,25]) yields nonlinear contributions to the I-V characteristics.

Far from the junction, for $z\to\pm\infty$ the function $\check{g}$ takes on values $\check{g}_{2,1}$. Let us henceforth assume that the right-hand metal is the superconductor; we will set the scalar potential there equal to zero far from the constriction, and choose the order parameter to be real. Then $\check{g}_2$ coincides with the Green's function $\check{g}(t)$ integrated over energy for a homogeneous equilibrium superconductor,[7–9] which is the energy representation equals

$$\check{g}(\varepsilon)=\begin{pmatrix} \hat{g}^R(\varepsilon); & [\hat{g}^R(\varepsilon)-\hat{g}^A(\varepsilon)]\,\mathrm{th}\dfrac{\varepsilon}{2T} \\ \hat{0}; & \hat{g}^A(\varepsilon) \end{pmatrix},$$

$$\hat{g}^{R(A)}(\varepsilon)=g^{R(A)}(\varepsilon)\hat{\tau}_3+f^{R(A)}(\varepsilon)i\hat{\tau}_2, \qquad (14)$$

where

$$g^{R(A)}(\varepsilon)=(\varepsilon/\Delta)f^{R(A)}(\varepsilon)=\varepsilon[(\varepsilon\pm i0)^2-\Delta^2]^{-1/2}.$$

In the left-hand normal metal we set φ far from the junction equal to the potential difference V. For $\check{g}_1$, after performing a phase gradient transformation, we obtain

$$\hat{g}_1^{R,A}=\pm\hat{\tau}_3, \quad \hat{g}_1^K(\varepsilon)=\beta(\varepsilon)\hat{\tau}_3-\alpha(\varepsilon)\hat{\tau}_0, \qquad (15)$$

where

$$\beta(\varepsilon)=\mathrm{th}\frac{\varepsilon_+}{2T}+\mathrm{th}\frac{\varepsilon_-}{2T}, \quad \alpha(\varepsilon)=\mathrm{th}\frac{\varepsilon_+}{2T}-\mathrm{th}\frac{\varepsilon_-}{2T},$$

$$\varepsilon_\pm=\varepsilon\pm eV.$$

In addition to the boundary conditions at infinity, we must include the boundary condition for the quasiclassical Green's function at the surface separating the metals,[10] which connects the component $\check{g}_a$ of the Green's function which is odd in momentum space and continuous through the barrier with the discontinuous even part, i.e., $\check{g}_s(\pm)$:

$$\check{g}_a[R(\check{g}_s^+)^2+(\check{g}_s^-)^2]=D\check{g}_s^-\check{g}_s^+. \qquad (16)$$

Here $\check{g}_s^\pm=[\check{g}_s(+)\pm\check{g}_s(-)]/2$.

In calculating $\check{g}$ and $\check{\mathscr{G}}$ we also make use of the normalization condition[9]

$$\check{g}^2=\check{1} \qquad (17)$$

and the connection between these functions, from which there follows in particular a relation of the form[10]

$$\check{g}(+)\check{\mathscr{G}}(+)=\mathrm{sign}\,p_{z2}\check{\mathscr{G}}(+). \qquad (18)$$

As a result, we obtain the following expressions for the matrix Green's function:

$$\check{g}_a=D\check{g}_-\check{g}_+[\check{1}-D(\check{g}_-)^2]^{-1},$$

$$\check{g}_s(+)=(\check{g}_++R\check{g}_-)[\check{1}-D(\check{g}_-)^2]^{-1}, \qquad (19)$$

$$\check{\mathscr{G}}_a=-R^{1/2}[\check{1}-D(\check{g}_-)^2]^{-1},$$

$$\check{\mathscr{G}}_s(+)=-R^{1/2}\check{g}_2[\check{1}-D(\check{g}_-)^2]^{-1}, \qquad (20)$$

$$\check{g}_\pm=(\check{g}_2\pm\check{g}_1)/2.$$

In order to calculate the I-V characteristic of the junction, we must know the coefficient of $\hat{\tau}_3$ in the expansion of $\check{g}_a{}^k$, while in order to find the current-current correlation function it is necessary to determine all the coefficients of the Pauli matrices in the expression for the components of the matrix functions (19), (20). For $D=1$ the equations for the Green's functions were presented in Ref. 25. We will not write out here the rather involved expressions for the matrix elements (19), (20) in the general case, but will turn to an investigation of the final results for the fluctuation spectrum which follows from Eqs. (11), (19) and (20).

3.1. The N_1cN_2 Junction

The spectral density of current fluctuations for a normal heterocontact with a constriction has the form

$$S_I(\omega)=\frac{2\omega}{R_1}\mathrm{cth}\frac{\omega}{2T}+\frac{(\omega+eV)}{R_2}\mathrm{cth}\frac{(\omega+eV)}{2T}+\frac{(\omega-eV)}{R_2}\mathrm{cth}\frac{(\omega-eV)}{2T}. \qquad (21)$$

The resistance $R_{1,2}$ equal

$$R_1^{-1}=R_0^{-1}\langle D^2\rangle, \quad R_2^{-1}=R_0^{-1}\langle RD\rangle,$$

$$R_1^{-1}+R_2^{-1}=R_N^{-1}=R_0^{-1}\langle D\rangle, \qquad (22)$$

where the angle brackets denote an average over the momentum directions,

$$\langle\ldots\rangle=2\int_0^1 d\alpha_2\alpha_2\ldots,$$

while the resistance R_0 coincides in form with the resistance of a pure junction: $R_0=4\pi^2/e^2p_{F2}{}^2S$.

From (21) it follows that with regard to noise properties the N_1cN_2 microjunction with finite transmission at the boundary separating the metals can be discussed as a parallel combination of an ideal microjunction with resistance R_1 in which an equilibrium noise current spectrum continues to be observed at any voltage and a normal tunneling resistance R_2 for which the current fluctuations change over thermal noise in the limit $eV, \omega\ll T$ to shot noise when $eV, \omega\gg T$.[2–4] Previous authors[26], by solving the Boltzmann equation, obtained the result that for a homogeneous NcN junction the noise spectrum is independent of voltage if we do not take into account electron-impurity and electron-phonon collisions. In Eqs. (21), (22) this corresponds to a reflection coefficient R equal to zero. The nonlinear properties of point ScN and ScS junctions were previously explained based on the model of a parallel combination of a metal short and a tunnel junction.[12,27] In the present case it is clear that such a representation exactly describes the noise properties of a normal point junction, while it is possible to obtain the magnitudes of the metal-metal boundary reflection and transmission coefficients averaged over direction from the "noise" resistance $R_{1,2}$.

The I-V characteristics of normal microjunction are ohmic; therefore, from (21) we can immediately find the spectral density of voltage fluctuations, which for $\omega\ll T$, eV equals

$$S_V(V)=4TR_N^2/R_1+(2eVR_N^2/R_2)\mathrm{cth}(eV/2T). \qquad (23)$$

3.2 The ScN junction

Finding the spectrum of current fluctuations for an ScN junction requires rather tedious calculations; therefore, we will limit ourselves to giving only the expressions for the low-frequency limit of the fluctuation spectrum, i.e., $\omega\ll\{T,\Delta,eV\}$, which we can cast in the form of a sum of two terms:

$$S_I(V)=S_1(V)+S_2(V),$$

where

$$S_1(V)=\frac{2}{R_0}\int_0^\Delta d\varepsilon\left\langle D^2\left\{4R\left[1-\left(\frac{\varepsilon}{\Delta}\right)^2\right]+D^2\right\}^{-2}\right.$$

$$\cdot\left\{D^2\left[\mathrm{ch}^{-2}\left(\frac{\varepsilon_+}{2T}\right)+\mathrm{ch}^{-2}\left(\frac{\varepsilon_-}{2T}\right)\right]+8R\left[1-\left(\frac{\varepsilon}{\Delta}\right)^2\right]\right.$$

$$\left.\left.\cdot\left[1-\mathrm{th}\left(\frac{\varepsilon_+}{2T}\right)\mathrm{th}\left(\frac{\varepsilon_-}{2T}\right)\right]\right\}\right\rangle, \qquad (24a)$$

$$S_2(V)=\frac{2}{R_0}\int_\Delta^\infty d\varepsilon\left\langle\left[\varepsilon+\left(\frac{2}{D}-1\right)(\varepsilon^2-\Delta^2)^{1/2}\right]^{-1}\right.$$

$$\cdot\left\{2\varepsilon-\left\{\varepsilon^2\left[\mathrm{th}^2\frac{\varepsilon_+}{2T}+\mathrm{th}^2\frac{\varepsilon_-}{2T}\right]+2(\varepsilon^2-\Delta^2)\mathrm{th}^2\frac{\varepsilon}{2T}\right.\right.$$

$$+\left(\mathrm{th}\frac{\varepsilon_+}{2T}+\mathrm{th}\frac{\varepsilon_-}{2T}\right)(\varepsilon^2-\Delta^2)^{1/2}\left[\left(\frac{2}{D}-1\right)\varepsilon-(\varepsilon^2-\Delta^2)^{1/2}\right]$$

$$\left.\left.\left.\cdot\,\mathrm{th}\frac{\varepsilon}{2T}\right\}\left[\varepsilon+\left(\frac{2}{D}-1\right)(\varepsilon^2-\Delta^2)^{1/2}\right]^{-1}\right\}\right\rangle. \qquad (24b)$$

For $T=0$ an integration over energy in (24) gives

$$S_I(V)=\frac{\Delta}{R_0}\left(\frac{2}{D}-1\right)^2\left\{4(Q+Q^{-1})\,\mathrm{arth}\frac{eV}{Q\Delta}\right.$$

$$\left.-\frac{D^2}{R}\frac{eV}{\Delta}\left[Q^2-\left(\frac{eV}{\Delta}\right)^2\right]^{-1}\right\}\qquad \text{for}\quad eV\leqslant\Delta, \quad (25a)$$

where $Q=(1+R)/2R^{1/2}$, and

$$S_I(V)=S_I(\Delta)+\frac{\Delta}{R_0}[f(u)-f(1)]\qquad\text{for}\quad eV>\Delta, \qquad (25b)$$

where the function $f(u)$ is defined in the following way:

$$f(u)=D\left[Ru-R^{-2}u^{-1}-\frac{D^2u}{2(u^2-R)}(1+R^{-2})+C\,\mathrm{arth}\frac{R^{1/2}}{u}\right],$$

$$u=\frac{eV}{\Delta}+\left[\left(\frac{eV}{\Delta}\right)^2-1\right]^{1/2},\qquad C=\frac{(1-R^2)}{2R^{5/2}}(3R^2-2R+3).$$

In Fig. 1 a plot is shown of S_I for various values of the reflection coefficient R. For simplicity we investigate a model in which D, R do not depend on the direction of momentum. It is clear that the intensity of the noise current depends strongly on the barrier transmission at the boundary of the contacting metals; this dependence strongly influences the behavior of the nonequilibrium spectral density S_I as R increases. For larger voltages S_I goes over to a linear function whose slope is determined by the resistance $R_{1,2}$. In the case of an ideal microshort between metals with the same Fermi momentum ($D=1$), the expression for $S_I(V)$ reduces to the form

$$S_I(V)=\frac{\Delta}{R_0}\theta(eV-\Delta)\left[\frac{8}{15}-u^{-1}\left(1-\frac{2}{3}u^{-2}+\frac{1}{5}u^{-4}\right)\right]. \qquad (26)$$

In the zero-temperature limit under discussion here, there is no Nyquist noise, while the shot noise, which also increases linearly with voltage for large V, also disappears since the transmission of the boundary equals unity. However, in contrast to a normal junction the current noise for $eV>\Delta$ has a spectral density different from zero (at zero frequency), which goes to the limiting value $8\Delta/15R_0$ as V increases.

The reason that an excess noise appears when we pass from a normal to an ScN contact is Andreev reflection processes at the normal-superconductor boundary,[28] which also leads to the appearance of excess current in the I-V characteristics of ScN microjunctions.[14,10]

Before a dependence of S_I on voltage can appear in a normal junction, it is necessary that a mechanism for scattering or reflection from the potential barrier be present at the boundary, which hinders the transmission of electrons through the constriction (see Subsec. 3.1 and Ref. 26). If we do not include electron-phonon collisions, then for $D=1$ there is also no such mechanism for a pure microjunction. In this case the contribution of a given electron to the current depends only on the direction of its momentum, which does not change as the electron moves along its trajectory through the constriction. Therefore, at least for frequencies smaller than the inverse transit time of electrons through the junction region $\tau_f\sim a/v_F$ (in the absence of electron-impurity scattering) the noise current is determined by the equilibrium fluctuations of the electron distribution function, and does not depend on voltage.

When one of the metals enters the superconducting state, the electrons undergo Andreev reflection at the boundary, i.e., in the normal metal there appears a reflected hole, while in the depths of the superconductor the current carries a Cooper pair.[28] If we measure the energy of an electron incident from the normal metal from the Fermi level in the equilibrium superconductor, then for $\varepsilon<\Delta$ Andreev reflection takes place with a probability of unity. This leads to an increase in the conductivity of a pure junction for $eV<\Delta$ and $T=0$ by a factor of two compared to its value in the normal state.[20] Here, the fluctuation current, as in a normal junction, is connected with equilibrium fluctuations far from the junction, and for zero temperatures and at zero frequency it disappears. If the energy $\varepsilon>\Delta$, then an electron from the normal metal can pass into the superconductor and occupy a quasiparticle state above the gap; however, there is a finite probability of Andreev reflection ("above-barrier" reflection), which decreases with the growth of ε. The presence of two scattering channels for a normal electron at the NS-boundary gives rise to a noise current.

At zero temperatures, in the absence of reflection from the boundary ($D=1$) and scattering by point impurities at the edge of the microjunction, fluctuations due to Andreev reflection fully determine the noise current, whose spectral density at zero frequency (26) is proportional to the integral over energy of the product $A(\varepsilon)[1-A(\varepsilon)]$, where

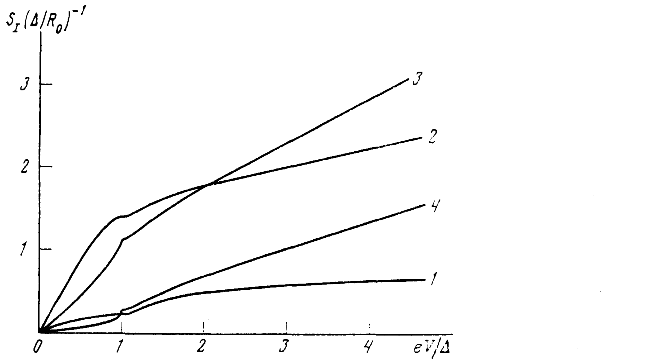

FIG. 1. Spectral density of current fluctuations at zero frequency as a function of voltage for an ScN junction at $T=0$ for various values of the electron reflection coefficient R at the boundary: 1—0.01, 2—0.1, 3—0.5, 4—0.8.

$$A(\varepsilon)=\left[\frac{\varepsilon}{\Delta}-\left[\left(\frac{\varepsilon}{\Delta}\right)^2-1\right]^{1/2}\right]^2$$

is the probability of above-barrier Andreev reflection for $\varepsilon>\Delta$. There is a clear analogy with the shot noise current in the normal case, whose spectrum is proportional to $R(1-R)$, where R gives the probability of the usual one-electron reflection from the boundary. An increase in R leads to growth of the noise current for $eV<\Delta$ (Fig. 1), while for $eV>\Delta$ the noise is related both with the one-electron and Andreev reflection. For small carrier transmission the amplitude of the fluctuation spectrum decreases because of the smallness of the conductivity.

At finite temperatures Eq. (24) for $V=0$ reduces in agreement with the FDT to the form

$$S_I(T,\ V=0)=4T/R_d(0).$$

Usually, in experiments the intensity of voltage fluctuations is measured at fixed displacement current. To first order in the fluctuating contribution at low frequencies for which the condition $\omega R_d C_{sh}\ll 1$ holds in addition to the inequalities indicated earlier (here C_{sh} is the capacity of the shunting junction), the noise voltage spectrum equals $S_v(V)=S_I(V)R_d^2$. Here, V is the average voltage at the junction. The differential resistance was calculated in Ref. 10. In Fig. 2 a plot of $S_v(V)$ obtained in this way is shown for finite temperatures, and also the corresponding curves $R_d(V)$. The family of curves $S_v(V)$ at fixed T and various values of the reflection coefficient R are illustrated in Fig. 3.

The function $S_v(V)$ changes from monotonic curves for small reflection coefficients to curves with a maximum lying within eV of $\Delta/2$. The minima on the curves of Figs. 2, 3 correspond to a minimum in the differential resistance. In contrast to the well-known result for point superconducting junctions,[15] in this case the functional dependence of S_v on V is related not only to the voltage dependence of R_d but also to the V dependence of the current fluctuation spectrum.

The nonmonotonic dependence of the spectrum of voltage fluctuations S_v in the voltage range $eV\sim\Delta$ is observed experimentally in ScN junctions.[29] The curves in Figs. 2, 3 with maxima are in qualitative agreement with the results of Ref. 29, which were obtained for a rather highly ohmic junction. At higher voltages the magnitude of the observed spectral noise power is smaller than predicted by the Schottky equation for shot noise. The theory developed here can explain this, in that the contribution to S_v which increases linearly with V (analogous to Eq. (23)) for $eV\gg\Delta$ is smaller than the shot noise for a tunnel junction with resistance R_N by a factor of R_N/R_2.

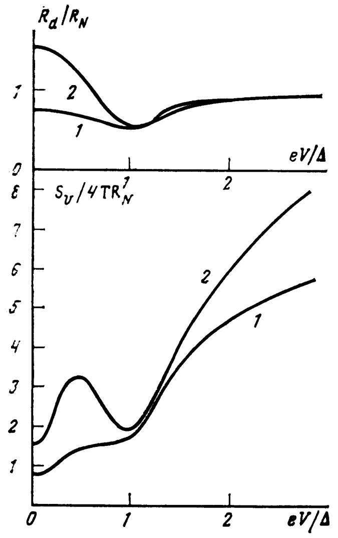

FIG. 2. Low-frequency spectral density of voltage fluctuations in an ScN junction at finite temperature ($\Delta/T=8$) and the corresponding dependence of the differential resistance on voltage for two values of the reflection coefficient R: 1—0.15, 2—0.4.

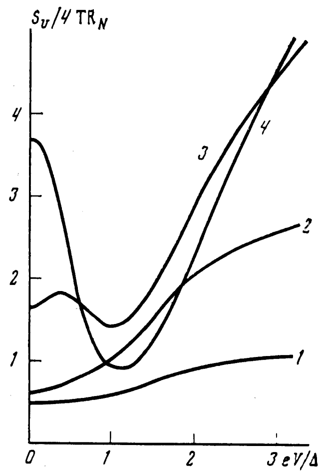

FIG. 3. Low-frequency spectral density of the noise voltage of an ScN junction at fixed temperature ($\Delta/T=4$) and for various values of R: 1—0.01, 2—0.1, 3—0.5, 4—0.7.

The authors of Ref. 29 also observed some functions S_v with two peaks for voltages smaller and larger than the gap; the present theory cannot account for such a shape of S_v. In connection with this, we note that the presence of strong impurity scattering, for which the electron motion through the constriction has a diffusive character, can significantly change the results related to the fluctuation properties of the junction. At the same time, the shape of the I-V characteristic, in particular the presence of a minimum in the differential resistance for voltages on the order of the gap, can just as easily be explained either by impurity scattering at the junction edge or by a finite-valued reflection coefficient R in the absence of impurities.[10,13]

If the impurity concentration is low, then the effect of electron scattering on the noise spectrum can be included by expanding all quantities in the small ratio a/l. Calculating the first-order correction to the quasiclassical Green's function at $z=0$ (for $D=1$) using the method described in Ref. 25 and substituting the result into (10) gives a shot noise contribution to the current fluctuation spectrum of an ScN junction which increases with voltage; for the resistance R_2 in (21) we must now substitute $R_0 l/a$. Thus, the presence of a small number of impurities in the junction region which can prevent electrons which collide with them from passing through the hole is equivalent to a small reflection coefficient from the potential barrier. This assertion does not apply if $l<a$, in which case additional investigation is required.

The possible influence of effects which are nonlinear in the fluctuation voltage on the magnitude of the low-frequency noise spectrum was also pointed out in Ref. 29; these effects are not included in the approximate treatment given here, which is linear in the fluctuations. In addition, it is possible for external noise to affect the system along with the intrinsic junction noise.

However, the dependence on external voltage of the nonequilibrium current fluctuation spectrum, which previously has been taken into account only by replacing the Nyquist thermal noise by shot noise for large applied voltages, must necessarily be accounted for in a correct description of the noise properties of junctions.

In conclusion, we note that the expression for the correlation function obtained in this paper is applicable also to ScS point junctions. In this case, the solutions to the equations for $\check{g}$ and $\check{\mathscr{G}}$ are significantly more complicated. A solution is easily found for a current through the junction which is smaller than the critical Josephson current, when there are no voltage oscillations and the noise current remains an equilibrium current. The current fluctuation spectrum calculated in this case from (11), according to the FDT, is then given in terms of the linear response function of the junction.

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