Instability of the resistive state in two-dimensional superconductors

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The stability of the resistive state of two-dimensional superconductors against perturbations of the plane fluxoid density is studied by linear analysis of the stability of the differential equations for the vortex plasma density. The system stability loss current and equilibrium value of the vortex plasma density, about which the instability develops, are determined.

In two-dimensional superconductors a low-temperature phase transition is associated with the decay (dissociation) of molecules from vortex fluctuation structures with overall zero vorticity into individual fluxoids at the Kosterlitz-Thouless temperature[1] T_{2D}. The possibility of detecting both the state of the gas of vortex molecules ($T < T_{2D}$) and the plasma arising from the dissociation of the fluxoids ($T > T_{2D}$) implies (as this takes place in a type-II superconductor at a high magnetic field exceeding the lower critical field) to an Ohm's-law variation in the voltage across the sample.[2] In the low-temperature phase this effect can be neglected, since the density of thermally generated free fluxoids is exponentially small.[3] To establish the existence of molecules from the vortex structures below T_{2D} it is necessary to first disrupt the molecules, and then determine the contribution of the vortices released to the resistance to the superfluid electron current flow. For sufficiently large transport currents theory[4] predicts a nonohmic behavior of the current-voltage characteristics.

In the present work we investigate the stability of the resistive state of two-dimensional superconductors for temperatures below the phase transition at T_{2D}. To this end a system of self-consistent equations is written down which determines the value of the vortex plasma density in the resistive phase of the sample in terms of the value of the transport current. We estimate the magnitude of the threshold current (below which the system is stable) and the background value n_0 of the equilibrium density of the vortex plasma.

The conductive state of two-dimensional superconductors below T_{2D} is metastable; this is connected with the existence of a potential barrier equal in magnitude to the formation energy of two free plane fluxoids.[4] Proceeding from a kinetic treatment of the evolution of Gibbs nuclei (in this case a pair from the vortex structures with zero total vorticity) in a first-order phase transition (see Ref. 5), we have for the rate of production Γ of nuclei of the critical dimension (that is, in fact, of free fluxoids)

$$\Gamma = D\left[\int_0^\infty dR\,\sigma^{-1}(R)\right]^{-1}. \tag{1}$$

Here the quantity D plays the role of coefficient of size diffusion of the nuclei. Following Ref. 6, in which the process of fluctuation formation and subsequent growth of vortex rings in superfluids is investigated, it is easy to see that in our case, when the nuclei represent a pair from the plane vortex structures, the coefficient D in Eq. 1 is the vortex diffusion coefficient.

In relation (1) $\sigma(R)$ is the equilibrium distribution function of the pairs as a function of size R, normalized per unit distance between fluxoids forming a pair, and per unit area. In a current situation, according to Refs. 1 and 6, it can be written in the form (in the non-interacting pair model)

$$\sigma(R) = 2\pi\alpha\xi^{-4} I_0\left(\frac{pv_s}{kT}\right)\exp\left(-\frac{pv_s}{kT}\right) R \exp\left[-\frac{E(R)}{kT}\right], \tag{2}$$

where α is a coefficient of order unity, ξ is a correlation length, $I_0(x)$ is a Bessel function of imaginary argument, and $E(R)$ is the energy of a vortex pair oriented along the direction of the velocity v_s in which the condensate is moving,

$$E(R) = q^2 \ln(R/\xi) - pv_s + 2\mu. \tag{3}$$

Here $q = (\pi\hbar^2 n_s/2m)^{1/2}$ is the effective "charge" of a vortex (n_s is the two-dimensional condensate density, m is the electron mass), $p = 2mRq^2\hbar^{-1}$ is its momentum, and μ is the energy connected with formation of the vortex core.

Using Eq. (2) for the flux Γ, we get (the integral is done by the method of steepest descents)

$$\Gamma = a\exp\left(-\frac{q^2}{kT}\ln\frac{j_c}{ej}\right), \tag{4}$$

where

$$a = (2\pi)^{1/2} D\xi^{-4}\alpha (q^2/kT)^{1/2} I_0(q^2/kT)\exp(-q^2/kT),$$

$j_c = \hbar e n_s/2m\xi$ denotes a pair-breaking current density and j is the two-dimensional transport supercurrent density.

As long as $q^2/kT \geqslant 4$ holds, where the equal sign corresponds to the transition point, we can use the asymptotic expression for $I_0(x)$ at large x. We then get for the coefficient a a value $\approx \alpha D\xi^{-4}$.

The process of establishing equilibrium in a plasma of free vortices, generated by destruction of vortex pairs, is described by the following equations for the densities n^+, n^- of fluxoids of different polarity:

$$dn^\pm/dt = \Gamma - \gamma n^+ n^-. \tag{5}$$

Here the second term on the right-hand side represents the destruction of free vortices by recombination (γ is the recombination coefficient). This process is associated with the generation of a vortex pair with zero total vorticity. Insofar as the behavior of a vortex plasma is analogous to the behavior of a two-dimensional Coulomb gas,[1] one can calculate the recombination rate directly from the recombination diameter d for vortex collision (in a two-dimensional situation) so that

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$$\gamma = vd, \tag{6}$$

where v is the relative speed of the incident vortices of opposite polarities.

An important difference in relation (6) compared to the expression for γ for the Coulomb gas is the lack in (6) of an averaging process over a Maxwellian distribution of speeds v. This fact is intimately connected with the zero mass of the vortex structure. In ideal superconductors, where the fluxoids are not pinned by inhomogeneities in the sample, a vortex flow rate is established in the direction of the Lorentz force:

$$\mathbf{v}^{\pm} = \pm \frac{\Phi_0}{c\eta}[\mathbf{jn}].$$

Here Φ_0 is the magnetic flux quantum, n is the unit vector along the direction of a vortex with positive vorticity, η is the coefficient of viscosity of a vortex taking its depth into account, and c is the speed of light.

Taking $d = 2R_k$ (R_k is the critical pair radius, at which a pair dissociates[1)] which according to equation (3) can be evaluated as $R_k = \xi(j_c/j)$, we get

$$\gamma = 8Dq^2/kT. \tag{7}$$

The physical prerequisite for the onset of instability is hidden in the sensitivity of Γ to the density $n = n^+ + n^-$. With an increase in n, because of the presence of the normal core, the actual condensate density $n_s = n_{s0}f(n)$ [$n_{s0} = n_s(n=0)$; $f(n)$ is a function characterizing the variation of n_s with n][2)] decreases, as does D, ξ and the effective vortex charge ($q^2 \sim n_s^{-1}$, $D \sim n_s^{-1}$, $\xi^2 \sim n_s^{-1}$). Therefore the variation in the rate of generation of vortex fluctuations because of change in the free vortex density $\delta\Gamma/\delta n$ is opposite in sign to the variation in the rate of fluxoid annihilation due to recombination. Thus the transition of a film specimen from the resistive to the normal state will be determined by the level of the perturbation n in the film. Let us suppose that due to internal or external factors a density n arises. This leads to a reduction of n_s, that is, the initial perturbations of the vortex plasma density give rise to a free-vortex production rate different from the initial value. For certain currents this can avalanche and lead to discontinuities in the current-voltage curves for two-dimensional superconductors.

The stability of the resistive state of a thin film for an initial small perturbation of the density n has been investigated by linear analysis of the stability of Eqs. (5).[3)] The characteristic equation for our system can be written in the form

$$(i\omega + \gamma n_0 - 2\delta\Gamma/\delta n)\,\delta n = 0.$$

Here n_0 is the background value of the equilibrium vortex plasma density, satisfying the equation

$$n_0 = 2[\Gamma(n_0)/\gamma]^{1/2}. \tag{8}$$

The magnitude of the threshold current j_k corresponding to a change in sign of the imaginary part of the frequency obeys the condition $\gamma n_0 = 2\delta\Gamma/\delta n$, from which (with use of Eq. (4) to determine Γ) it follows that, on the other hand, n_0 is the solution of the equation

$$\frac{n_0}{2}\frac{\partial f}{\delta n}\left[\frac{1}{f} - 2\ln y - \frac{q_0^2}{2kT}(1+\ln f)\right] = 1, \qquad y = \left(\frac{j_{c0}}{ej}\right)^{q_0^2/2kT}, \tag{9}$$

The index on q^2 and on j_c signifies these are the magnitudes for $n^+ = n^- = 0$.

With an increase in the vortex plasma density of $n \sim \xi^{-2}$ the function $f(n)$, characterizing the variation in n_s due to the presence of the normal framework, must go to zero. In the absence of vortices $f(n)$ must equal unity. A good approximation of this behavior is given by the dependence $f(n) = 1 - (\pi n \xi^2)^\beta$. We note that for small fluxoid densities at large values of the Ginzburg-Landau parameter $\varkappa$, when the vortices are clearly defined, the exponent β can be taken as unity. However, as calculation has shown, the theoretical value of the density n_0 on which instability will arise is such as to show that the approximation of well-localized vortices is not justified. It is important that in strong magnetic fields the resistivity of a superconducting sample is not directly proportional to the density of the vortex lattice, but grows significantly faster as the field increases.[7] Taking this fact into account, we assume $\beta < 1$. Then from the definition of f it follows that

$$n_0 = \frac{f(1-f)^{1/\beta}}{\pi\xi_0^2}, \qquad \frac{\delta f}{\delta n} = -\beta\pi\xi_0^2\frac{(1-f)^{1-1/\beta}}{f}\left(1-\beta\frac{1-f}{f}\right)^{-1}.$$

Substituting these relations in equations (8) and (9), we find that an instability in the resistive state of two-dimensional superconductors arises for current densities

$$j_k = \frac{j_{c0}}{e}\exp\left\{\frac{kT}{q_0^2}\left[\frac{q_0^2}{2kT}(1+\ln f) + \frac{1}{f} - \frac{2}{\beta(1-f)}\right]\right\}, \tag{10}$$

where the magnitude of f is given by solution of the equation

$$\ln(1-f) + \frac{\beta}{2}\ln\frac{2q_0^2 f}{\alpha\pi^2 kT} - \frac{\beta}{2} + \frac{f}{1-f} - \beta\frac{q_0^2}{4kT}f = 0. \tag{11}$$

In the region of temperatures near absolute zero, we have $f \approx 1$, which we can write it in the form $1-\delta$, where δ is the small deviation of f from unity. Expanding the left-hand side of equation (11) in δ, we get $\delta \approx 4kT/\beta q_0^2$. Using this result, we find that for $T = 0$ the threshold current in a two-dimensional superconductor is e times less than j_{c0}. This value of j corresponds to the boundary beyond which for $T = 0$ we find an avalanche increase in free fluxoids. However, at low temperatures the basic process governing the production of free fluxoids must be not the fluctuation growth of nuclei (vortex-antivortex pairs) to the critical size, but sub-barrier quantum-mechanical tunneling to the state with two free fluxoids of opposite polarity. Taking account of this fact would lower the theoretical value of the threshold current j_k for $T \to 0$.

At temperatures near the phase transition temperature T_{2D} we can assume $q_0^2/kT = 4$. In this case equation (11) is exactly solvable. For $\beta = 1$, assuming $\alpha = 1$, we get $f = 0.72$, For $\beta = \frac{1}{2}$, $f = 0.64$. For these values of f, $j_k \sim 10^{-1} j_{c0}$. The background value of the vortex plasma density for which the development of small perturbations will take place is $n_0 \sim 10^{-1}\xi_0^{-2}$. Since the coefficient α in formula (11) is inside the logarithm, the resulting f will be relatively insensitive to it.

We note that in our calculations we take a uniform distribution of transport current across the film. This is possible only for a superconducting specimen in the form of a cylindrical film. The choice of a convenient sample geometry also excludes the influence of edge effects.

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[1)]The critical size R_k satisfies the condition $\delta E/\delta R|_{R=R_k}=0$

[2)]The resistance of two-dimensional superconductors is determined by the unpaired fluxoids; therefore the contribution to n_s of the vortices participating in a dissipative response to a constant electrical current is taken out.

[3)]Calculation has shown that instability arises first for a uniform perturbation of the vortex plasma density; therefore, in the initial equations (4) the gradient terms (the terms describing vortex diffusion) are dropped for simplicity. The Maxwell equations, which would supplement relations (4) if a local magnetic induction were present, are also omitted.

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